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Flexural behavior of reinforced concrete beams with various layers of conventional and steel fiber reinforced concrete

Farklı geleneksel ve çelik lifli beton katmanlarına sahip betonarme kirişlerin eğilme davranışı

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Flexural Behavior of Reinforced Concrete Beams with Various Layers of Conventional and Steel Fiber Reinforced Concrete

Highlights

- *Flexural behavior of steel fiber reinforced concrete (SFRC) beams was studied.*
- *Various layers of conventional and SFRC were used.*
- *Adding SFRC at the tension of beam results in reasonable ductility.*

Graphical Abstract

Flexural behavior of reinforced concrete beams having various layers of conventional concrete and steel fiber reinforced concrete were investigated in this study.

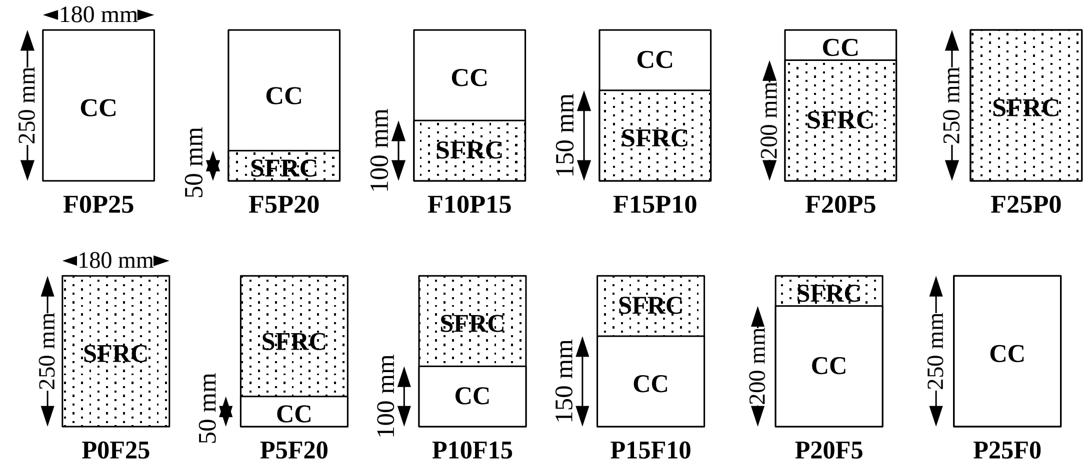

Figure. Beam Sections of F and P Groups

Aim

This research is performed to evaluate the behavior of beams having steel fibers at various locations throughout the cross-section.

Design & Methodology

The height of the cross-section of the beams was divided into 5 layers, each having 50 mm thicknesses. In one group of specimens, SFRC layers were added to the layers of a CC beam, starting from the bottom, as replacements of CC layers. In other group of specimens, CC layers were added to the layers of a SFRC beam, starting from the bottom, as replacements of SFRC layers.

Originality

This is the first study that used layered SFRC throughout the cross-section.

Findings

Addition of SFRC slightly increased the ultimate load capacity of the specimens, no matter where SFRC is added, from bottom or top. Addition of SFRC increased the toughness of the specimens, no matter where SFRC is added, from bottom or top.

Conclusion

Reasonable ductility may be achieved by adding SFRC at the tension side no matter how thick the layer is and where it is located.

Declaration of Ethical Standards

The author of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Flexural Behavior of Reinforced Concrete Beams with Various Layers of Conventional and Steel Fiber Reinforced Concrete

Araştırma Makalesi / Research Article

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ABSTRACT

Flexural behavior of reinforced concrete (RC) beams having various layers of conventional concrete (CC) and steel fiber reinforced concrete (SFRC) were investigated in this study. Two groups of five beams (180×250×3500 mm) were tested under four-point loading to evaluate the flexural behavior. Both of these groups of beams were reinforced with 4ϕ16 reinforcing bars. The main variable in this research was the concrete type of the layers throughout the height of the specimen. The height of the cross-section of the beams was divided into 5 layers, each having 50 mm thicknesses. In group "F" specimens, SFRC layers were added to the layers of a CC beam, starting from the bottom, as replacements of CC layers, i.e. F15P10 represented that the bottom 150 mm was cast using SFRC whereas the top 100 mm was cast using CC. In group "P" specimens, CC layers were added to the layers of a SFRC beam, starting from the bottom, as replacements of SFRC layers, i.e. P10F15 represented that the bottom 100 mm was cast using CC whereas the top 150 mm was cast using SFRC. Experimental load-deflection curves were evaluated based on ultimate load, service/post-peak stiffnesses, and flexural toughness. It can be concluded that reasonable ductility may be achieved by adding SFRC at the tension side no matter how thick the layer is and where it is located.

Keywords: Conventional concrete, steel fiber reinforced concrete, service/post-peak stiffness, flexural toughness.

Farklı Geleneksel ve Çelik Lifli Beton Katmanlarına Sahip Betonarme Kirişlerin Eğilme Davranışı

ÖZ

Bu çalışmada, farklı geleneksel ve çelik lifli beton katmanlarına sahip betonarme kirişlerin eğilme davranışı incelenmiştir. Boyutları 180×250×3500 mm olan toplamda 10 kiriş, iki grupa bölünerek dört nokta yüklemesi altında eğilme davranışı değerlendirmesi için test edilmiştir. Tüm kirişlerde çekme bölgesinde 4ϕ16 donatısı kullanılmıştır. Bu araştırmadaki ana değişken kiriş yüksekliğince oluşturulan katmanlardaki beton tipidir. Kirişin yüksekliği her biri 50 mm olan 5 katmana ayrılmıştır. "F" grubunda bulunan geleneksel beton kullanılan kirişlerde, çelik lifli beton katmanları aşağıdan başlayarak geleneksel beton katmanlarının yerlerine yerleştirilmiştir. Örnek olarak, F15P10 kirişinin yüksekliği boyunca aşağıdan 150 mm'si çelik lifli betondan, yukarıda kalan 100 mm'si ise geleneksel betondan imal edilmiştir. "P" grubunda bulunan çelik lifli beton kullanılan kirişlerde ise, geleneksel beton katmanları aşağıdan başlayarak çelik lifli beton katmanlarının yerlerine yerleştirilmiştir. Örnek olarak, P10F15 kirişinin yüksekliği boyunca aşağıdan 100 mm'si geleneksel betondan, yukarıda kalan 150 mm'si ise çelik lifli betondan imal edilmiştir. Kirişlerin yük-sehim eğrileri elde edilmiş ve bu eğriler azami yük, kullanım rijitliği, tepe sonrası rijitlik ve eğilme tokluğu açısından değerlendirilmiştir. Araştırma sonucunda göre, yeterli sünekliğin çekme bölgesinde bulunan çelik lifli beton katmanı ile sağlanabileceği belirlenmiştir. Bu katmanın, çekme bölgesinde olduğu sürece yüksekliğinin ve yerinin davranışı önemli bir şekilde etkilemediği görülmüştür.

Anahtar Kelimeler: Geleneksel beton, çelik lifli beton, kullanım/tepe-sonrası rijitliği, eğilme tokluğu.

1. INTRODUCTION

Fibers such as straws were used in mud bricks in Egypt and Middle East in the ancient times. The use of commercial steel fiber reinforced concrete (SFRC) and other types of synthetic fibers in various structural applications dates back to 1960's. The historical background of the use of FRC in details is presented in [1].

The behavior of SFRC was modeled by various researchers in tension ([2], [3], [4], and [5]) and compression ([2], [6], [7], and [8]). In many of the researches, these proposed models were used to estimate load deflection relationship of flexural members. It was observed that the analytical solutions using the models in the literature predicted the experimental flexural responses quite favorably. [9] used [5]'s model to evaluate moment–curvature, load–deflection relationships, and minimum flexural reinforcement ratio of hybrid SFRC beams. [10] modified the models proposed by [3] and [11] to predict the flexural strength steel fiber reinforced high-strength concrete fully/partially prestressed beams. It was stated that the estimated load deflection responses provided good

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comparison with the experimental results for high-strength prestressed concrete beams containing trough shape fibers. [12] used the tensile model proposed by [13] to develop new models to evaluate the flexural behavior of SFRC beams with and without steel bars. It was concluded that developed models provided appropriate safety margin for design. All these researches show that the models of SFRC in tension and compression in the literature can appropriately estimate the flexural behavior of SFRC beams.

Using fibers in concrete matrix increases the cost of concrete mixture. The high cost of steel fibers can restrict the extensive use of SFRC ([14]). [15] stated that using steel fibers increases construction cost in a considerable level despite the advantages. To reduce the amount of use of SFRC in flexural members consequently decrease the cost, fibers can be used in locations where necessary. This research is performed to evaluate the behavior of beams having steel fibers at various locations throughout the cross-section.

This study investigates the behavior of reinforced concrete (RC) beams in flexure having various layers of conventional concrete (CC) and SFRC. Beams reinforced with 4ϕ16 steel bars were tested under four-point bending. The main variable in this study was the concrete type of the layers throughout the height of the specimen. The height of the cross-section of the beams was divided into 5 layers, each having 50 mm thicknesses. In one group of specimens, SFRC layers were added to the layers of a CC beam, starting from the bottom, as replacements of CC layers. In other group of specimens, CC layers were added to the layers of a SFRC beam, starting from the bottom, as replacements of SFRC layers. Results were assessed according to load carrying capacity, service/post-peak stiffnesses, and flexural toughness.

2. EXPERIMENTAL PROGRAM

Two groups of beams (Group F and Group P) were tested in the scope of this research. The cross-sections of these two groups of beams are shown in Figure 1 and 2. The main variable in this research was the concrete type of the layers throughout the height of the specimen cross-section. The height of the cross-section of the beams was divided into 5 layers, each having 50 mm thicknesses. In group “F” specimens, SFRC layers were added to the layers of a CC beam, starting from the bottom, as replacements of CC layers, i.e. F15P10 represented that the bottom 150 mm was cast using SFRC whereas the top 100 mm was cast using CC. In group “P” specimens, CC layers were added to the layers of a SFRC beam, starting from the bottom, as replacements of SFRC layers, i.e. P10F15 represented that the bottom 100 mm was cast using CC whereas the top 150 mm was cast using SFRC. Descriptions of the test specimens are tabulated in Table 1.

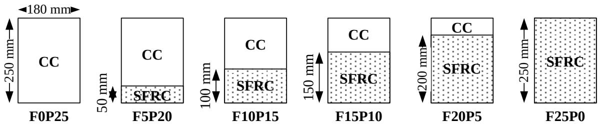

Figure 1. Cross-sections of Group F beam specimens

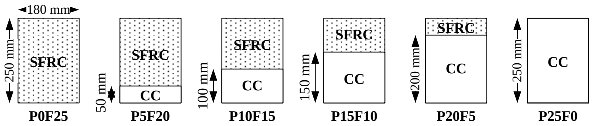

Figure 2. Cross-sections of Group P beam specimens

Table 1. Details of the test specimens

Group	Specimen	Depth of Concrete Layer from Bottom (mm)	Depth of Concrete Layer from Top (mm)
F Series	F25P0*	250 (SFRC)	0 (CC)
	F20P5	200 (SFRC)	50 (CC)
	F15P10	150 (SFRC)	100 (CC)
	F10P15	100 (SFRC)	150 (CC)
	F5P20	50 (SFRC)	200 (CC)
	F0P25**	0 (SFRC)	250 (CC)
P Series	P25F0**	250 (CC)	0 (SFRC)
	P20F5	200 (CC)	50 (SFRC)
	P15F10	150 (CC)	100 (SFRC)
	P10F15	100 (CC)	150 (SFRC)
	P5F20	50 (CC)	200 (SFRC)
	P0F25*	0 (CC)	250 (SFRC)

* Control specimens for full SFRC throughout the cross-section
** Control specimens for full CC throughout the cross-section

It should be noted that specimens F25P0 and P0F25 in Table 1 refer to the same specimen that has SFRC through the full depth. Similarly, specimens F0P25 and P25F0 refer to the same beam that has CC through the full depth.

Since the specimens were subjected to four point bending tests, the constant moment region is produced at the mid-span. All the beam specimens had the same dimensions, 180×250×3500 mm, and the same longitudinal steel reinforcement, 4ϕ16, at the tension side of the 500 mm mid-span region. The reinforcement ratio of all the beams was 2.13% which results in over-reinforced section behavior of full CC specimen. Two ϕ12 hanger bars were used to support the ϕ8/100 mm transverse reinforcement outside the mid-span region throughout the length of the beams. No compression and transverse reinforcement was used at the constant moment region of the beams to eliminate the confinement effects. The yield strength of steel used in this research was 420 MPa both for transverse and longitudinal reinforcement. Reinforcement configurations of beam specimens are shown in Figure 3.

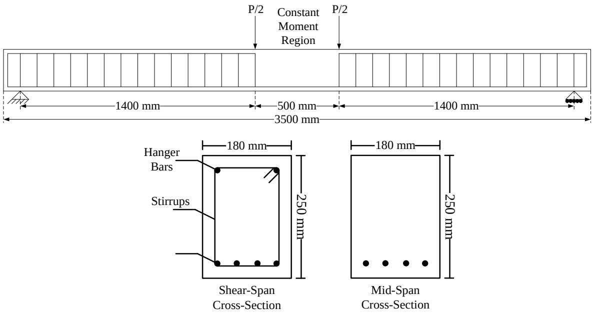

Figure 3. Reinforcement configurations of beam specimens

Crushed sand (4.4 mm), fine aggregate (4-16 mm), and coarse aggregate (15-25 mm) were mixed with PC 42.5 Portland Cement, Glenium ACE 30 superplasticizer, and tap water to obtain the mixtures used in this research. The proportions of these materials for the two mixtures used in this research are tabulated in Table 2.

Table 2. Proportion of materials for two mixtures

Materials	Quantity (kg/m^3)	
	CC	SFRC
Portland Cement	400	400
Sand (0-4.4 mm)	900	900
Fine aggregate (4-16 mm)	440	440
Coarse aggregate (15-25 mm)	580	580
Steel fiber	-	77
Water	200	220
Superplasticizer	-	1

Steel fibers used in SFRC mixture were Dramix (ZP-305). Manufacturer supplied specifications of these fibers are shown in Table 3. Also a photo of these fibers is shown in Figure 4.

Table 3. Specifications of Dramix (ZP-305) fibers

Effective Length (mm)	Equivalent Diameter (mm)	Aspect Ratio	Young's Modulus (GPa)	Tensile Strength (MPa)	Density (kg/m^3)
30	0.55	60	210	1345	7850

Figure 4. Dramix (ZP-305) fibers

2.1. Test Method and Test Set-Up

Beam specimens were tested under four-point bending. The beams were simply supported at the ends. A hydraulic jack (300 kN capacity) was used to apply the load and a load cell (200 kN capacity) was used to measure it. Constant moment region (500 mm) was obtained using a spreader beam. Test set-up is shown in Figure 5.

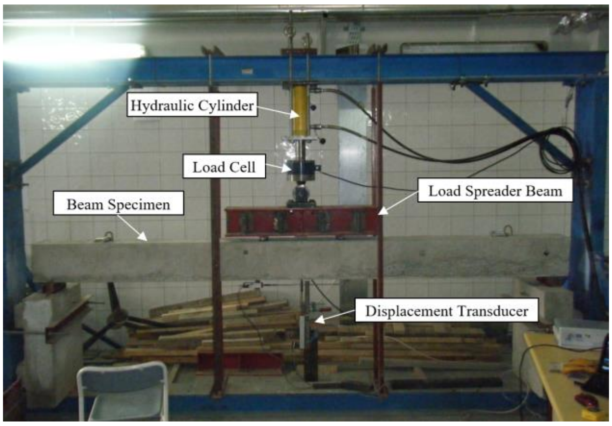

Figure 5. Test set-up for beams

A displacement transducer (150 mm stroke) was used to measure the vertical mid-span deflection at the bottom side of the beam.

The compressive strength of concrete mixtures at the beam testing day was measured according to [16] using three cylindrical concrete specimens (150×300 mm) collected at the casting day. The compression machine used in testing cylinders had a capacity of 1500 kN. It was observed that the cylinders of CC mixtures failed suddenly in a brittle failure mode whereas, cylinders of SFRC mixtures failed in a ductile manner. Average concrete strength values obtain from cylinders for each beam are shown in Table 4.

Table 4. Average concrete compressive strengths for beam specimens

Group	Specimen	f_{cSFRC} (MPa)	f_{cCC} (MPa)
F Series	F25P0	31.8	-
	F20P5	27.4	28.4
	F15P10	27.5	29.2
	F10P15	29.1	26.2
	F5P20	31.0	25.7
	F0P25	-	27.7
P Series	P25F0	-	27.7
	P20F5	20.8	25.7
	P15F10	24.4	24.0
	P10F15	29.5	30.9
	P5F20	27.7	27.9
	P0F25	31.8	-

3. TEST RESULTS AND DISCUSSIONS

Typical crack distributions at the constant moment region of the tested specimens for F and P Series specimens are shown in Figure 6. All beams failed due to crushing of concrete at the upper side of the constant moment region after initiation of multiple vertical flexural cracks in the same region. No significant difference was observed for the crack initiation load of all the beams.

Figure 6. Typical crack distributions of F and P Series specimens

The experimental load-deflection relationships of Group F and P Series specimens are shown in Figures 7 and 8, respectively. The comparisons related to Group F Series specimens showed that only the F0P25 specimen having CC in all the layers behaved in a less ductile manner compared to the other Group F Series specimens. When the Group P Series specimens are compared, specimens P25F0 and P20F5 showed less ductile behavior compared to the rest of the group specimens.

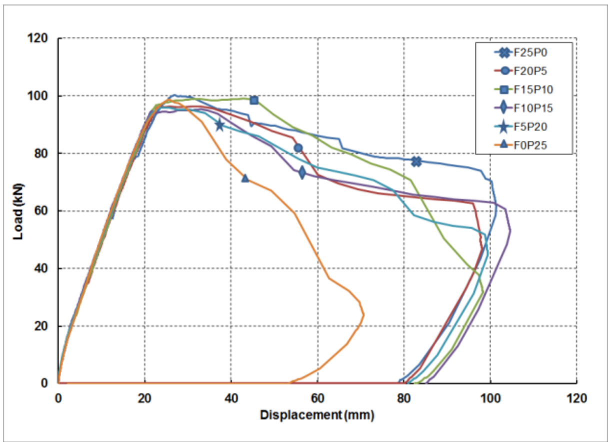

Figure 7. Experimental load-deflection graphs for Group F Series specimens

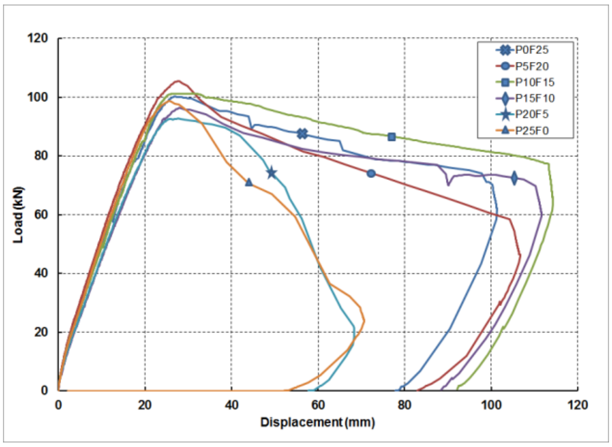

Figure 8. Experimental load-deflection graphs for Group P Series specimens

3.1. Ultimate Load

The achieved maximum load value is defined as the ultimate load capacity of the specimen. The ultimate load values of Group F and P Series specimens were compared in Table 5 and Figure 9. For Group F Series specimens, addition of SFRC from bottom increased the ultimate load capacity of the specimens. However, for Group P Series specimens, addition of CC from bottom decreased the ultimate load capacity of the specimens. This figure stated that the addition of SFRC slightly increased the ultimate load capacity of the specimens, no matter where SFRC is added, from bottom or top.

Table 5. Comparison of ultimate loads and service stiffnesses

Group	Specimen	Ultimate Load (kN)	Service Stiffness (kN/mm)
F Series	F25P0	100.2	3.5
	F20P5	96.2	4.0
	F15P10	99.0	4.1
	F10P15	94.8	4.1
	F5P20	96.0	4.1
	F0P25	93.7	3.8
P Series	P25F0	93.7	3.8
	P20F5	92.5	3.6
	P15F10	96.3	3.6
	P10F15	101.0	4.0
	P5F20	105.4	4.1
	P0F25	100.2	3.5

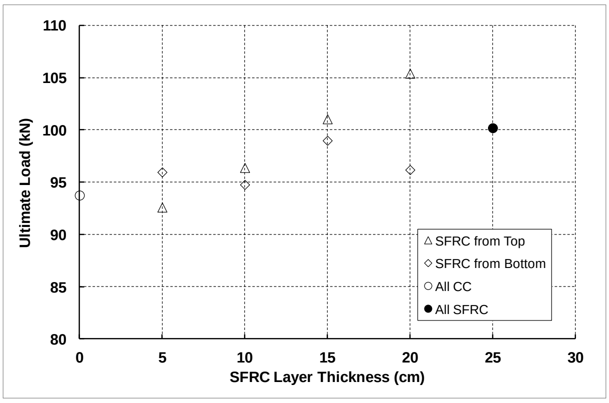

Figure 9. Ultimate loads for Group F and P Series specimens

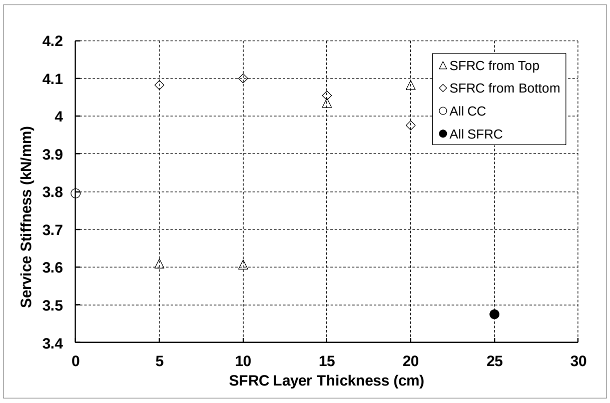

Figure 11. Service stiffness for Group F and P Series specimens

3.2. Service Stiffness

Since all economically-designed RC members would be cracked under realistic conditions, the stiffness of the ascending part of the load-deflection relationships at this state was used to evaluate the service stiffness of the specimens. When the design factors for materials and loads in the current design specifications are considered, the service load level is approximately equal to 60-70% of the ultimate load ([17]). Therefore, the slope of the line passing from points at the 50 and 80% of the ultimate load was selected to simulate this cracked behavior as shown in Figure 10.

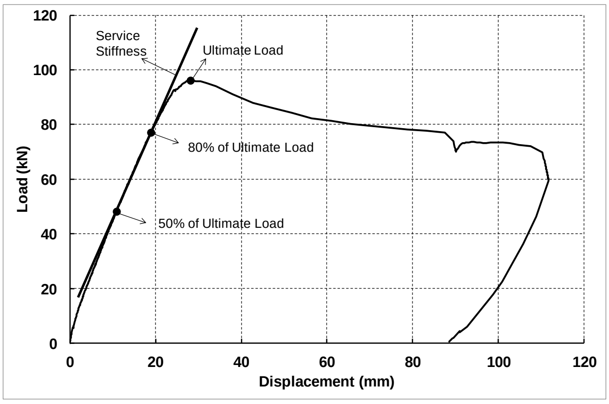

Figure 10. Definition of service stiffness between 50% and 80% of ultimate load

The service stiffness values of Group F and P Series specimens were compared in Table 5 and Figure 11. For Group F Series specimens, addition of SFRC from bottom slightly decreased the service stiffness of the specimens. There was no clear trend for Group P Series specimens due to the addition of CC from bottom. The figure indicates that the addition of SFRC did not have any clear effects on the service stiffness of the specimens, no matter where SFRC is added, from bottom or top.

3.3. Flexural Toughness

The area under the load-deflection relationship for a selected load value on the relationship was used to calculate the flexural toughness of each specimen. The selected load values used in this research were the ultimate load, 90 and 80% of the ultimate load on the descending part of the load-deflection relationship. An example of the determination of flexural toughness of a specimen for 80% of the ultimate load on the descending part of the load-deflection relationship is shown in Figure 12.

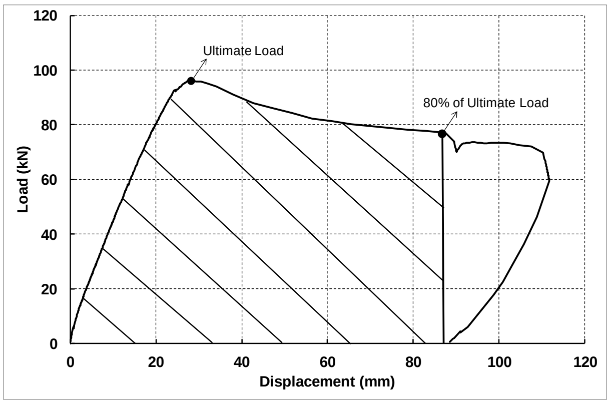

Figure 12. Determination of flexural toughness of a specimen for 80% of ultimate load on descending part of load-deflection relationship

The comparison of flexural toughness values of Group F and Group P Series specimens are given in Table 6, Figures 13, 14, and 15. Since toughness is an indicator of ductility of the members, the toughness values at ultimate load, 90%, and 80% of the ultimate load showed that addition of SFRC from bottom increased the toughness of the Group F Series specimens and addition of CC from bottom decreased the toughness of the Group P Series specimens. This behavior was less pronounced for toughness at ultimate load, more pronounced for toughness at 80% of the ultimate load. It can be concluded that addition of SFRC increased the toughness of the specimens, no matter where SFRC is added, from bottom or top.

Table 6. Comparison of flexural toughnesses

Group	Specimen	Flexural Toughness (kN.mm)		
		@ Ultimate Load	@ 90% of Ultimate Load	@ 80% of Ultimate Load
F Series	F25P0	1604	3319	5520
	F20P5	2193	3953	4451
	F15P10	1107	4305	5481
	F10P15	2224	3397	4022
	F5P20	1414	3344	4324
	F0P25	1461	2213	2552
P Series	P25F0	1461	2213	2552
	P20F5	1579	3067	3473
	P15F10	1601	3220	6527
	P10F15	1554	5033	8483
	P5F20	1414	3344	4324
	P0F25	1604	3319	5520

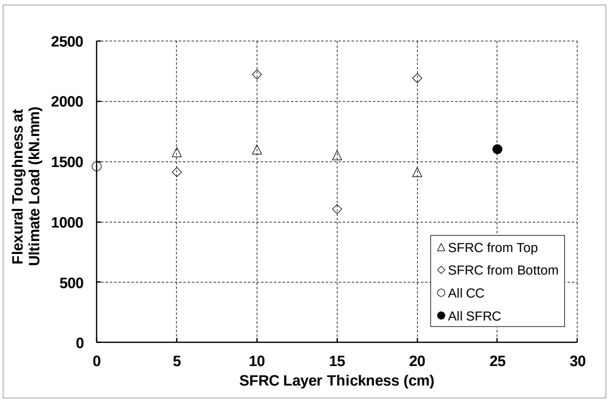

Figure 13. Flexural toughness at ultimate load for Group F and P Series specimens

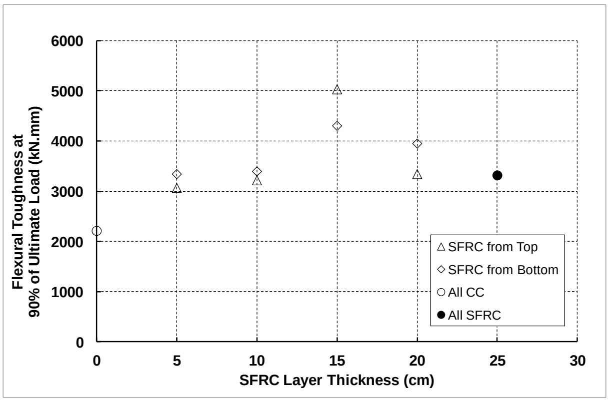

Figure 14. Flexural toughness at 90% of ultimate load on descending part of load-deflection relationship for Group F and P Series specimens

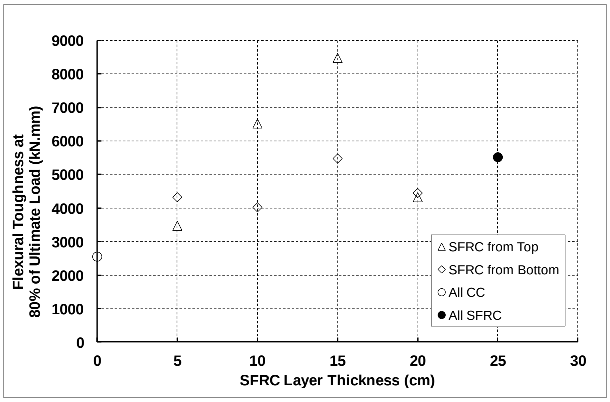

Figure 15. Flexural toughness at 80% of ultimate load on descending part of load-deflection relationship for Group F and P Series specimens

3.4. Post-Peak Stiffness

Two post-peak stiffnesses were calculated for each specimen using the slope between the two points (100%-90% and 100-80% of the ultimate load) on the descending part of the load-deflection relationship as shown in Figure 16.

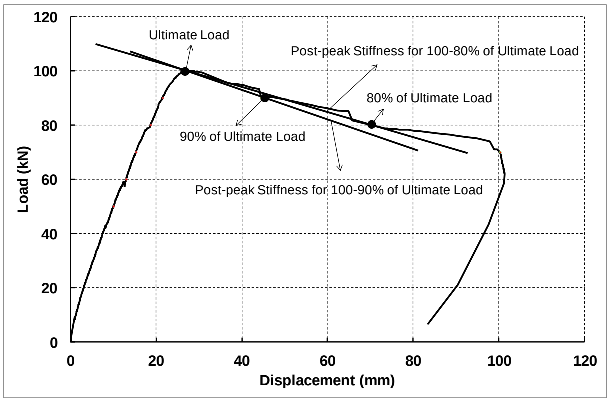

Figure 16. Definition of post-peak stiffness between 100-90% and 100-80% of ultimate load

The comparisons of post-peak stiffness values for 100%-90% and 100%-80% of the ultimate load for Group F and P Series specimens are shown in Table 7, Figures 17, and 18. The comparisons related to Group F and P Series specimens on post-peak stiffness values for 100%-90% of ultimate load showed that, only the F0P25 (P25F0) specimen having CC in all the layers had lower post-peak stiffness compared to the other Group F Series specimens. When the post-peak stiffness values for 100%-80% are compared, value for P20F5 was also lower than that of the other specimens in Group P Series. It can be concluded from the post-peak stiffness values that, the lower the post-peak stiffness, the brittle the behavior was. Therefore, F0P25 specimen behaved in a brittle manner compared to the other Group F Series specimens and P25F0 and P20F5 showed brittle behavior compared to the other Group P Series.

Table 7. Comparison of post-peak stiffnesses

Group	Specimen	Post Peak Stiffness (kN/mm)	
		@ 100-90% of Ultimate Load	@ 100-80% of Ultimate Load
F Series	F25P0	-0.56	-0.46
	F20P5	-0.34	-0.56
	F15P10	-0.43	-0.54
	F10P15	-0.39	-0.59
	F5P20	-0.46	-0.58
	F0P25	-1.12	-1.49
P Series	P25F0	-1.12	-1.49
	P20F5	-0.50	-0.78
	P15F10	-0.55	-0.33
	P10F15	-0.28	-0.33
	P5F20	-0.46	-0.58
	P0F25	-0.56	-0.46

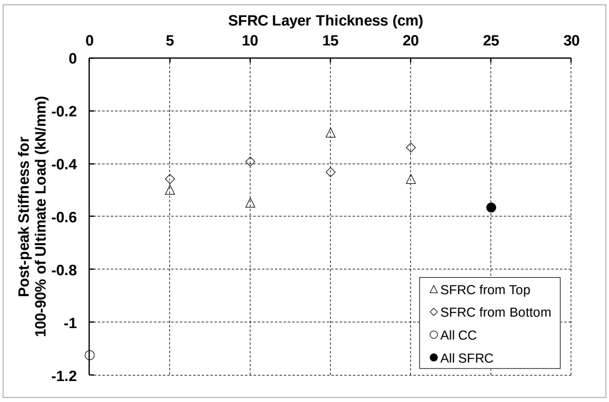

Figure 17. Post-peak stiffness for 100 and 90% of ultimate load of Group F and P Series specimens

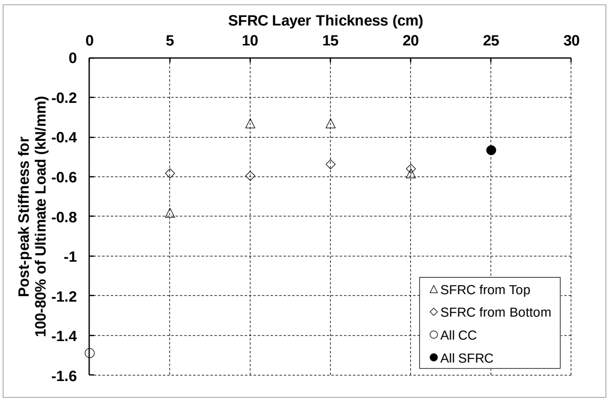

Figure 18. Post-peak stiffness for 100 and 80% of ultimate load of Group F and P Series specimens

4. CONCLUSIONS

Note that this study is limited to flexure critical beams and all discussions are for flexural behavior. The observations and conclusions are as follows:

- The comparisons related to Group F Series specimens showed that only the F0P25 specimen having CC in all the layers behaved in a less ductile manner compared to the other Group F Series specimens. When the Group P Series specimens are compared, specimens P25F0 and P20F5 showed less ductile behavior compared to the rest of the group specimens. It can be concluded that reasonable ductility may be achieved by adding SFRC at the tension side no matter how thick the layer is and where it is located.
- For Group F Series specimens, addition of SFRC from bottom increased the ultimate load capacity of the specimens. However, for Group P Series specimens, addition of CC from bottom decreased the ultimate load capacity of the specimens. It can be concluded that the addition of SFRC slightly increased the ultimate load capacity of the specimens, no matter where SFRC is added, from bottom or top.
- The addition of SFRC from bottom increased the toughness of the Group F Series specimens and addition of CC from bottom decreased the toughness of the Group P Series specimens. This behavior was less pronounced for toughness at ultimate load, more pronounced for toughness at 80% of the ultimate load. It can be concluded that addition of SFRC increased the toughness of the specimens, no matter where SFRC is added, from bottom or top.
- Only the F0P25 (P25F0) specimen having CC in all the layers had lower post-peak stiffness compared to the other Group F Series specimens. When the post-peak stiffness values for 100%-80% are compared, value for P20F5 was also lower than that of the other specimens in Group P Series. It can be concluded from the post-peak stiffness values that, the lower the post-peak stiffness, the brittle the behavior was. Therefore, F0P25 specimen behaved in a brittle manner compared to the other Group F Series specimens and P25F0 and P20F5 showed brittle behavior compared to the other Group P Series.

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DECLARATION OF ETHICAL STANDARDS

The author of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

AUTHORS' CONTRIBUTIONS

Halit Cenan MERTOL: Performed the experiments, analysed the the results, and wrote the manuscript.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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