# Analysis of the Solutions of the Equation Modeled in the Field of Nonlinear Sciences

Tolga AKTÜRK[1*], Mahşure Kübra DİKİCİ[2]

**ABSTRACT:** In this article, the travelling wave solutions of the fourth-order nonlinear Ablowitz-Kaup-Newell-Segur (AKNS) equation are investigated using the modified exponential function method (MEFM). This method is used to find analytical travelling wave solutions of the AKNS equation. The different travelling wave solutions are obtained by determining the appropriate values for the parameters. Two and three dimensional graphics of the different wave solutions found in this way are plotted with the help of Mathematica package program by determining the appropriate parameters.

**Keywords:** Modified exponential function method, fourth-order nonlinear Ablowitz-Kaup-Newell-Segur equation, travelling wave solutions.

[1]Tolga AKTÜRK (**Orcid ID:** 0000-0002-8873-0424), Ordu University, Faculty of Education, Department of Mathematics and Science Education, Ordu, Turkey
[2]Mahşure Kübra DİKİCİ (**Orcid ID**: 0000-0003-0533-052X), Ordu University, Faculty of Science and Letters, Department of Mathematics, Ordu, Turkey

*Sorumlu Yazar/Corresponding Author: Tolga AKTÜRK, e-mail: tolgaakturk@odu.edu.tr

## INTRODUCTION

Solutions of the nonlinear partial differential are physically important. NPDEs, engineering, physics, medicine etc. each represents a scientific event in various fields of science. Therefore, various methods have been developed in the literature to obtain solutions of such equations. Some of them are as follows; $G'/G$ method (Wang, et al.,2008), the Hirota's bilinear operators (Hirota, R. 1973), the F-expansion method (Zhou, et al., 2004), the Jacobi elliptic function expansion (Liu, et al., 2001), the Bernoulli sub-ODE method (Zheng, et al., 2011, Baskonus, et al., 2016), the sine–cosine method (Yusufoğlu, et al.,2008), Improved Bernoulli Simple Equation Method (Bulut, et al., 2016), New Function Method (Shen, et al., 2013, Sun, 2014, Bulut, et al., 2014) and so on. In additon to these methods in the literature, the modified exponential function method (MEFM) (Bulut, et al., 2014), was used in our study. In this study, we consider the fourth-order nonlinear AKNS water wave equation as follow (Bruz´on, et al., 2003, Helal, et al., 2013),

$$4u_{xt} + u_{xxxt} + 8u_x u_{xy} + 4u_{xx} u_y - \gamma u_{xx} = 0. \tag{1}$$

Using the modified exponential function method (1), no study has been done on the travelling wave solutions of the equation. The solutions obtained help to understand physical events. In this study, MEFM is introduced in the second section, and in the third section, two and three-dimensional graphs are plotted by applying parameters of MEFM to the fourth-order AKNS equation and the solution functions obtained. Finally, in the fourth section, the comments of the results is given.

## MATERIALS AND METHODS

We consider the following nonlinear partial differential equation (NPDE):

$$P\left(u, u_x, u_y, u_{xx}, u_{xy}, u_{xt}, u_{xxxt}\right) = 0, \tag{2}$$

where $u(x, y, t)$ is unknown function, $P$ is the general form of the given nonlinear partial differential equation. If the travelling wave solution (3) is applied to equation (2), nonlinear ordinary differential equation (4) is obtained.

Step 1: Consider the following travelling wave transformation:

$$u(x, y, t) = u(\xi), \quad \xi = k\left(ax + by - ct\right). \tag{3}$$

Substituting Eq. (3) into Eq. (2), gives the following nonlinear ordinary differential equation (NODE);

$$N\left(u, u', \left(u'\right)^2, u''', \ldots\right) = 0. \tag{4}$$

Step 2: We get that the following the solution;

$$u(\xi) = \frac{\sum_{i=0}^{n} A_i [e^{-\vartheta(\xi)}]^i}{\sum_{j=0}^{m} B_j [e^{-\vartheta(\xi)}]^j} = \frac{A_0 + A_1 e^{-\vartheta} + \ldots + A_n e^{-n\vartheta}}{B_0 + B_1 e^{-\vartheta} + \ldots + B_m e^{-m\vartheta}}, \tag{5}$$

where $A_i, B_j, (0 \leq i \leq m, 0 \leq j \leq n)$. $m$, $n$ are positive integers that can be get by using the balancing procedure. The balancing procedure is the relationship between the highest order derivative term in the nonlinear ordinary differential equation and the highest grade nonlinear term.

$$\vartheta'(\eta) = e^{-\vartheta(\eta)} + k\, e^{\vartheta(\eta)} + \lambda. \tag{6}$$

Eq.(6) has the following families of solutions (Naher, et al., 2013):

**Family 1**: When, $\mu \neq 0$, $\lambda^2 - 4\mu > 0$,

$$\vartheta(\eta) = ln(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} tanh(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + EE)) - \frac{\lambda}{2\mu}). \tag{7}$$

**Family 2:** When, $\mu \neq 0$, $\lambda^2 - 4\mu < 0$,

$$\vartheta(\eta) = ln(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} tan(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\eta + EE)) - \frac{\lambda}{2\mu}). \tag{8}$$

**Family 3:** When, $\mu = 0$, $\lambda \neq 0$, $\lambda^2 - 4\mu > 0$,

$$\vartheta(\eta) = -ln(\frac{\lambda}{e^{\lambda(\eta + EE)} - 1}). \tag{9}$$

**Family 4:** When, $\mu \neq 0$, $\lambda \neq 0$, $\lambda^2 - 4\mu = 0$,

$$\vartheta(\eta) = ln(-\frac{2\lambda(\eta + EE) + 4}{\lambda^2(\eta + EE)}). \tag{10}$$

**Family 5:** When, $\mu = 0$, $\lambda = 0$, $\lambda^2 - 4\mu = 0$,

$$\vartheta(\eta) = ln(\eta + EE), \tag{11}$$

where, EE is a integral constant.

Step 3: If the derivatives required in the equation (4) are get in the equation (5) and replaced in the equation (4), an algebraic equation system is obtained. This system is solved using the Mathematica software program and solutions of the fourth-order nonlinear AKNS equation are obtained.

**RESULTS AND DISCUSSION**

In this section, we use the modified exponential function method (MEFM) to find the travelling wave solutions of the fourth-order nonlinear Ablowitz-Kaup-Newell-Segur (AKNS) equation,

$$4u_{xt} + u_{xxxt} + 8u_x u_{xy} + 4u_{xx} u_y - \gamma u_{xx} = 0. \tag{12}$$

When we apply the travelling wave transformation as $u(x, y, t) = u(\xi)$, $\xi = k(ax + by - ct)$, we get the nonlinear ordinary differantial equation (NODE) as following,

$$-4cau' - ca^3k^2u'' + 6a^2bk(u')^2 - \gamma a^2u' = 0. \tag{13}$$

In order to get the solution function, integrals must be taken in equation (13). However, if $u^{'} = v$ is taken to make these integration processes more simple, equation (13) follows,

$$\left(-4c - \gamma a\right)av - ca^3k^2v'' + 6a^2bkv^2 = 0. \tag{14}$$

If the balancing procedure is applied to the equation (14), if $m=1$ is selected, $n=3$ is obtained. When the equation (14) is solved, $v$ function is obtained. Using the $u^{'} = v$ relation, the two sides of equality are integrated. In this way, $u$ solution function is obtained for the equation (1),

$$u(\xi) = \frac{A_0 + A_1e^{-\theta} + A_2e^{-2\theta} + A_3e^{-3\theta}}{B_0 + B_1e^{-\theta}}. \tag{15}$$

Derivative terms required for equation (13) are obtained from equation (15). By substituting these terms (13) in the equation, an algebraic equation system consisting of coefficients is obtained. When this system of algebraic equations is solved, the following cases are obtained.

**Case 1:**

$$A_0 = -\frac{a^2k\gamma\mu B_0}{b\left(4 + a^2k^2\left(\lambda^2 - 4\mu\right)\right)};$$

$$A_1 = -\frac{a^2k\gamma\left(\lambda B_0 + \mu B_1\right)}{b\left(4 + a^2k^2\left(\lambda^2 - 4\mu\right)\right)};$$

$$A_2 = -\frac{a^2k\gamma\left(B_0 + \lambda B_1\right)}{b\left(4 + a^2k^2\left(\lambda^2 - 4\mu\right)\right)};$$

$$A_3 = -\frac{a^2k\gamma B_1}{b\left(4 + a^2k^2\left(\lambda^2 - 4\mu\right)\right)};$$

$$c = -\frac{a\gamma}{4 + a^2k^2\left(\lambda^2 - 4\mu\right)};$$

These coefficients are substituted in the equation (15) and the following conditions are get.

**Family 1:** When, $\mu \neq 0$, $\lambda^2 - 4\mu > 0$,

$$v_{1.1}(x,t) = \frac{a^2k\gamma\left(\lambda^2 - 4\mu\right)\mu Sech\left[\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left(EE + \xi\right)\right]^2}{b\left(4 + a^2k^2\left(\lambda^2 - 4\mu\right)\right)\left(\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left[\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left(EE + \xi\right)\right]\right)^2}, \tag{16}$$

is obtained. If the equation (16) is integrated, the solution function of equation (1) is as follows,

$$u_{1.1}(x,t) = \frac{a^2k\gamma\left(\lambda^3 - 4\lambda\mu + 2\sqrt{\lambda^2 - 4\mu}\mu\sinh\left[\sqrt{\lambda^2 - 4\mu}\left(EE + \xi\right)\right]\right)}{2b\left(4 + a^2k^2\left(\lambda^2 - 4\mu\right)\right)\left(\lambda^2 - 2\mu + 2\mu\cosh\left[\sqrt{\lambda^2 - 4\mu}\left(EE + \xi\right)\right]\right)}. \tag{17}$$
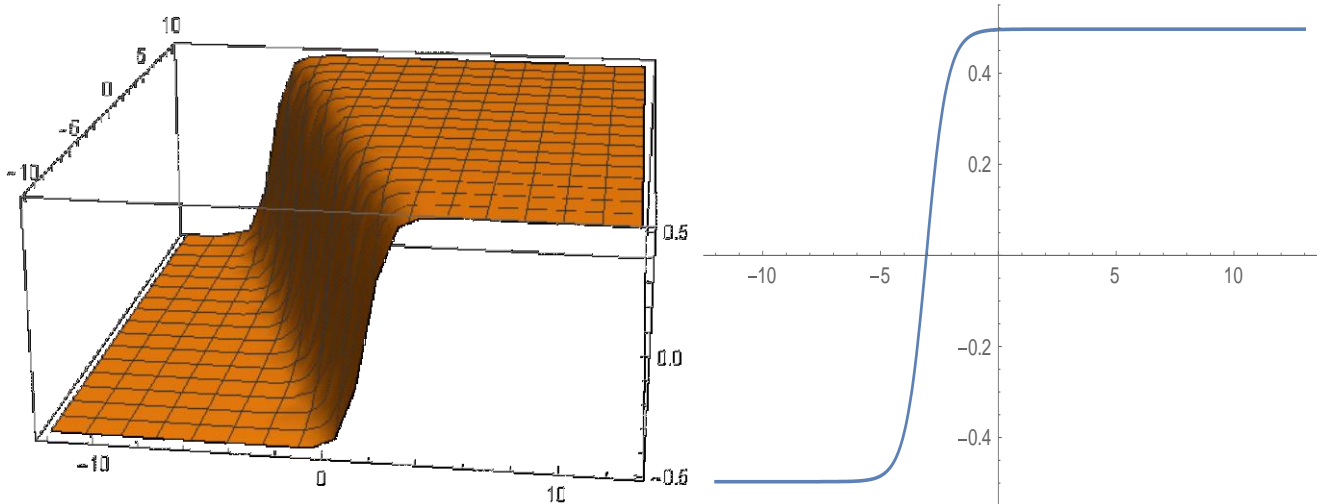
**Figure-1:** The 3D and 2D surfaces of Eq. (17) in $a=1,\ b=1,\ \gamma=4,\ k=1,\ \lambda=3,\ \mu=1,\ c=-\dfrac{4}{9},\ y=1,$

$EE=0.75$ and $t=1.$

**Family 2:** When, $\mu \neq 0,\ \lambda^2 - 4\mu < 0$,

$$v_{1.2}(x,t) = \frac{a^2 k \gamma \left(\lambda^2 - 4\mu\right)\mu \sec\left[\dfrac{1}{2}\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]^2}{b\left(4 + a^2 k^2 \left(\lambda^2 - 4\mu\right)\right)\left(\lambda - \sqrt{-\lambda^2 + 4\mu}\ \tan\left[\dfrac{1}{2}\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]\right)^2}, \tag{18}$$

is obtained. If equation (18) is integrated, the solution function of equation (1) is get,

$$u_{1.2}(x,t) = \frac{a^2 k \gamma \left(\lambda^2 - 4\mu\right)\left(\lambda + \dfrac{2\mu \sin\left[\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]}{\sqrt{-\lambda^2 + 4\mu}}\right)}{2b\left(4 + a^2 k^2 \left(\lambda^2 - 4\mu\right)\right)\left(\lambda^2 - 2\mu + 2\mu\cos\left[\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]\right)}. \tag{19}$$



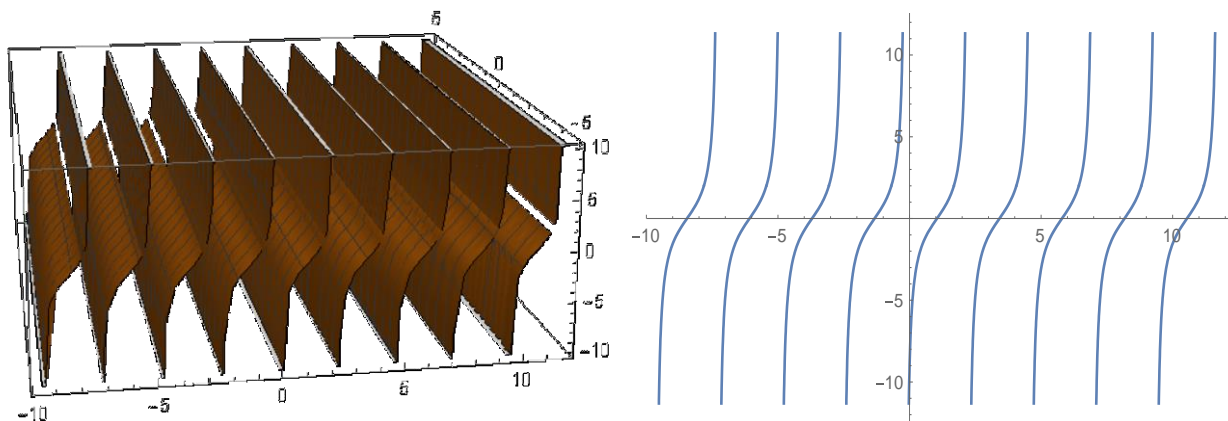**Figure-2:** The 3D and 2D surfaces of Eq. (19) in $a=1, b=1, \gamma=4, k=1, \lambda=1, \mu=2, c=-\dfrac{4}{3}, y=1,$

$EE=0.75$ and $t=1.$

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**Family 3:** When, $\mu = 0, \lambda \neq 0, \lambda - 4\mu > 0,$

$$v_{1.3}(x,t) = -\frac{a^2 k\gamma\left(4\mu + \lambda^2 Csch\left[\frac{1}{2}\lambda(EE+\xi)\right]^2\right)}{4b\left(4+a^2k^2\left(\lambda^2-4\mu\right)\right)}, \tag{20}$$

is get. If the equation (20) is integrated, the solution function of equation (1),

$$u_{1.3}(x,t) = \frac{a^2 k\gamma\left(-2\mu\xi + \lambda\coth\left[\frac{1}{2}\lambda(EE+\xi)\right]\right)}{2b\left(4+a^2k^2\left(\lambda^2-4\mu\right)\right)}. \tag{21}$$
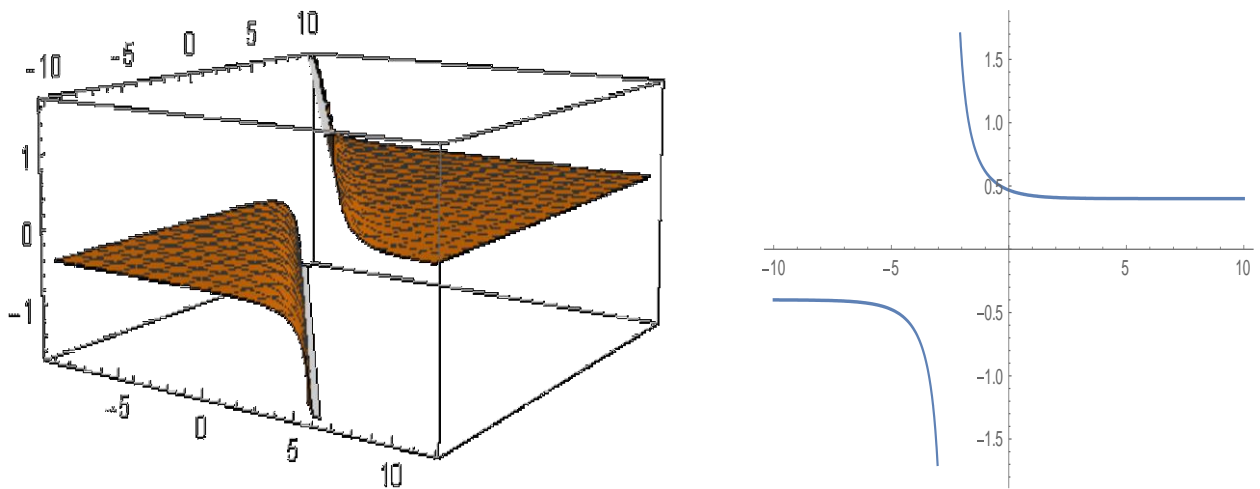


**Figure-3:** The 3D and 2D surfaces of Eq. (21) in $a=1, \gamma=4, k=1, \lambda=1, \mu=0, b=1, EE=0.75, y=1,$ $c=-\frac{4}{5}$ and $t=1$.

**Family 4:** $\lambda \neq 0$, $\mu \neq 0$, $\lambda^2 - 4\mu = 0$,

$$v_{1.4}(x,t) = \frac{a^2 k\gamma\left(-16\mu + \lambda\left(\lambda^2-4\mu\right)(EE+\xi)(4+\lambda(EE+\xi))\right)}{4b\left(4+a^2k^2\left(\lambda^2-4\mu\right)\right)(2+\lambda(EE+\xi))^2}, \tag{22}$$

is obtained . When the integrated form of equation (22) is written in equation (1), the solution function is get,

$$u_{1.4}(x,t) = \frac{a^2 k\gamma\left(\frac{4\lambda}{2+\lambda(EE+\xi)} + \frac{\left(\lambda^2-4\mu\right)(2+\lambda(EE+\xi))}{\lambda}\right)}{4b\left(4+a^2k^2\left(\lambda^2-4\mu\right)\right)}. \tag{23}$$

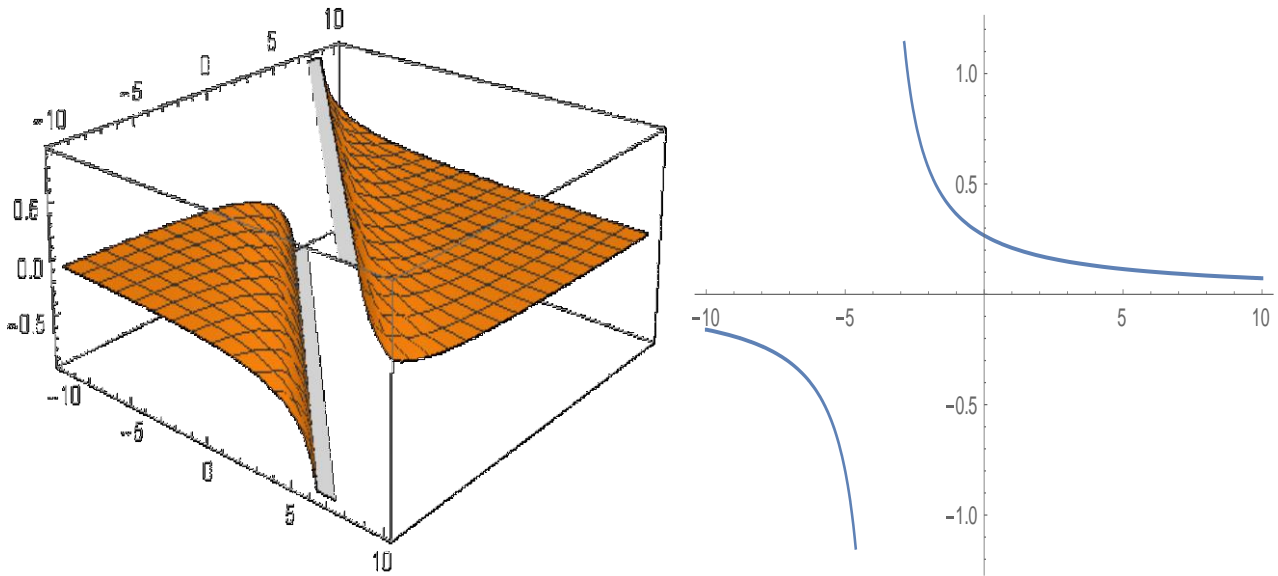**Figure-4:** The 3D and 2D surfaces of Eq. (23) in $a=1, \gamma=4, k=1, \lambda=2, \mu=1, c=-1, b=1, EE=0.75$ $y=1$ and $t=1$.

**Family 5:** $\lambda=0, \mu=0, \lambda^2-4\mu=0$,

$$v_{1.5}(x,t) = -\frac{a^2 k \gamma \left(1+\left(EE+\xi\right)\left(\lambda+\mu\left(EE+\xi\right)\right)\right)}{b\left(4+a^2 k^2 \left(\lambda^2-4\mu\right)\right)\left(EE+\xi\right)^2}. \tag{24}$$

If obtained equation (24) has been integrated, the soluttion function of equation is following,

$$u_{1.5}(x,t) = -\frac{a^2 k \gamma \left(-\dfrac{1}{EE+\xi}+\mu\left(EE+\xi\right)+\lambda \log\left[EE+\xi\right]\right)}{b\left(4+a^2 k^2 \left(\lambda^2-4\mu\right)\right)}. \tag{25}$$



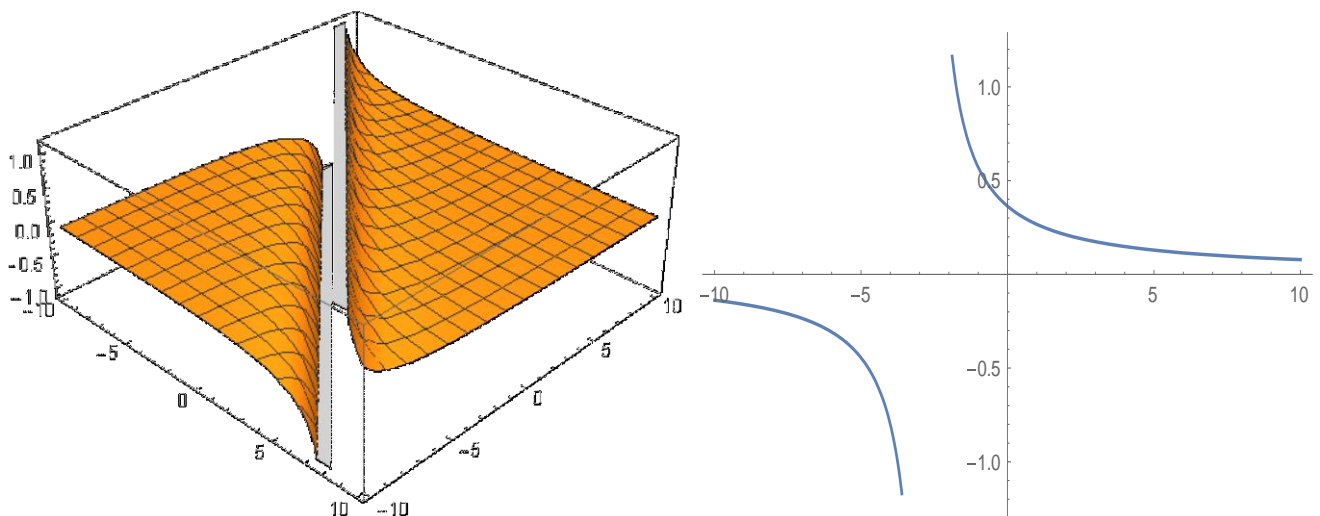**Figure-5:** The 3D and 2D surfaces of Eq. (25) in $a=1, \gamma=4, k=1, \lambda=0, \mu=0, b=1, c=-1, y=1,$ $EE=0.75$ and $t=1$.

**Case 2**

$$A_0 = \frac{ack\mu B_0}{b};$$

$$A_1 = \frac{ack(\lambda B_0 + \mu B_1)}{b};$$

$$A_2 = \frac{ack(B_0 + \lambda B_1)}{b};$$

$$A_3 = \frac{ackB_1}{b};$$

$$\gamma = -\frac{c(4 + a^2k^2(\lambda^2 - 4\mu))}{a};$$

Solutions written in the equation of these coefficients (15) are obtained.

**Family 1:** When, $\mu \neq 0$, $\lambda^2 - 4\mu > 0$,

$$v_{2.1}(x,t) = \frac{ack(-\lambda^2 + 4\mu)\left(\lambda + \frac{2\mu\sinh[\omega]}{\sqrt{\lambda^2 - 4\mu}}\right)}{2b(\lambda^2 - 2\mu + 2\mu\cosh[\omega])} \frac{ack(-\lambda^2 + 4\mu)\left(\lambda + \frac{2\mu\sinh[\omega]}{\sqrt{\lambda^2 - 4\mu}}\right)}{2b(\lambda^2 - 2\mu + 2\mu\cosh[\omega])}, \quad (26)$$

where $\left(\omega = \sqrt{\lambda^2 - 4\mu}(EE + \xi)\right)$.

If equation (26) is integrated,

$$u_{2.1}(x,t) = \left(-\frac{ack\left(\lambda^3 - 4\lambda\mu + 2\sqrt{\lambda^2 - 4\mu}\mu\sinh\left[\sqrt{\lambda^2 - 4\mu}(EE + \xi)\right]\right)}{2b\left(\lambda^2 - 2\mu + 2\mu\cosh\left[\sqrt{\lambda^2 - 4\mu}(EE + \xi)\right]\right)}\right). \quad (27)$$
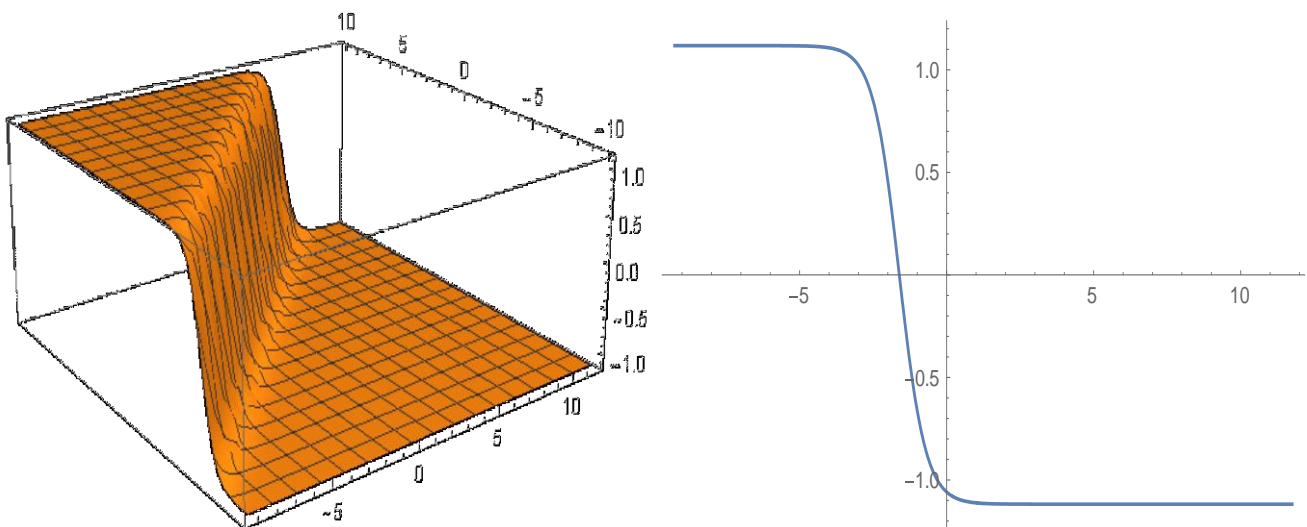


**Figure-1:** The 3D and 2D surfaces of Eq. (17) in $a = 1, b = 1, c = 1, k = 1, \lambda = 3, \mu = 1, \gamma = -9, EE = 0.75$, $y = 1$ and $t = 1$.

**Family 2:** When, $\mu \neq 0,\ \lambda^2 - 4\mu < 0$,

$$v_{2.2}(x,t) = \frac{ack\left(-\lambda^2 + 4\mu\right)\left(\lambda + \dfrac{2\mu \sin\left[\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]}{\sqrt{-\lambda^2 + 4\mu}}\right)}{2b\left(\lambda^2 - 2\mu + 2\mu\cos\left[\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]\right)}, \tag{28}$$

is obtained. If the equation (28) is integrated, the solution function of equation (1) is as follows,

$$u_{2.2}(x,t) = \left(-\frac{ack\left(\lambda^3 - 4\lambda\mu - 2\mu\sqrt{-\lambda^2 + 4\mu}\,\sin\left[\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]\right)}{2b\left(\lambda^2 - 2\mu + 2\mu\cos\left[\sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)\right]\right)}\right). \tag{29}$$
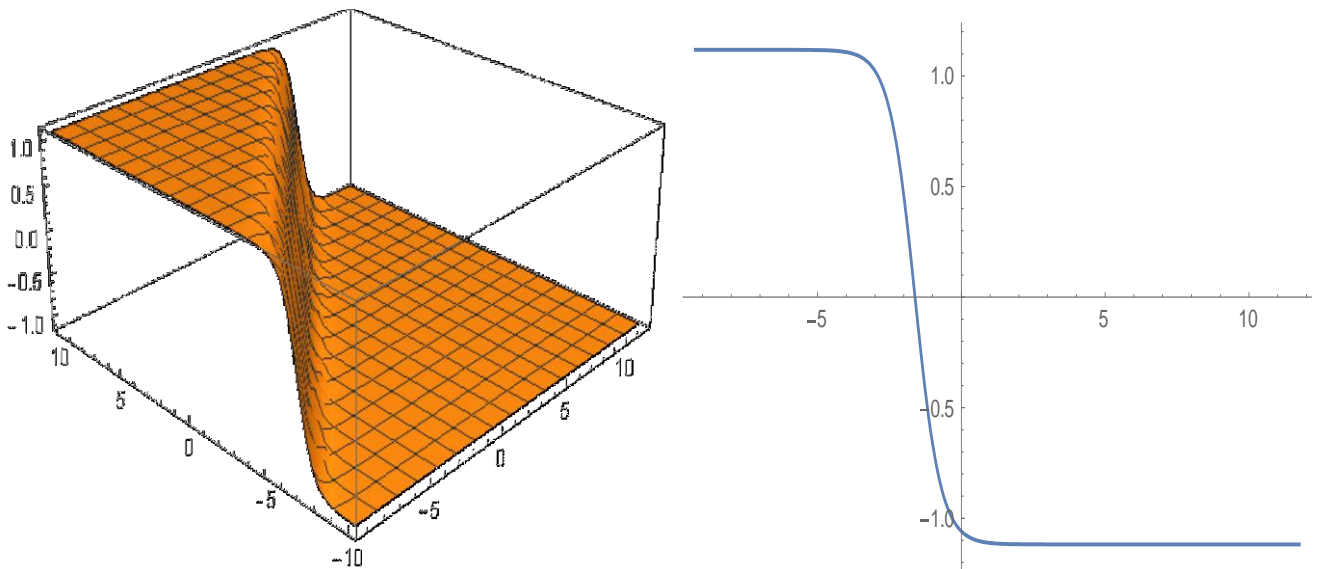


**Figure-2:** The 3D and 2D surfaces of Eq. (17) in $a = 1, b = 1, c = 1, k = 1, \lambda = 3, \mu = 1, \gamma = -1, EE = 0.75$, $y = 1$ and $t = 1$.

**Family 3:** When, $\mu = 0, \lambda \neq 0,\ \lambda - 4\mu > 0$,

$$v_{2.3}(x,t) = \left(\frac{ack\mu\xi}{b} - \frac{ack\lambda \coth\left[\dfrac{1}{2}\lambda\left(EE + \xi\right)\right]}{2b}\right), \tag{30}$$

is obtained. If the equation (30) is integrated, solution function $u$ of equation (1) is get as follows,

$$u_{2.3}(x,t) = \left(\frac{ack\left(2\mu\xi - \lambda\coth\left[\dfrac{1}{2}\lambda\left(EE + \xi\right)\right]\right)}{2b}\right). \tag{31}$$
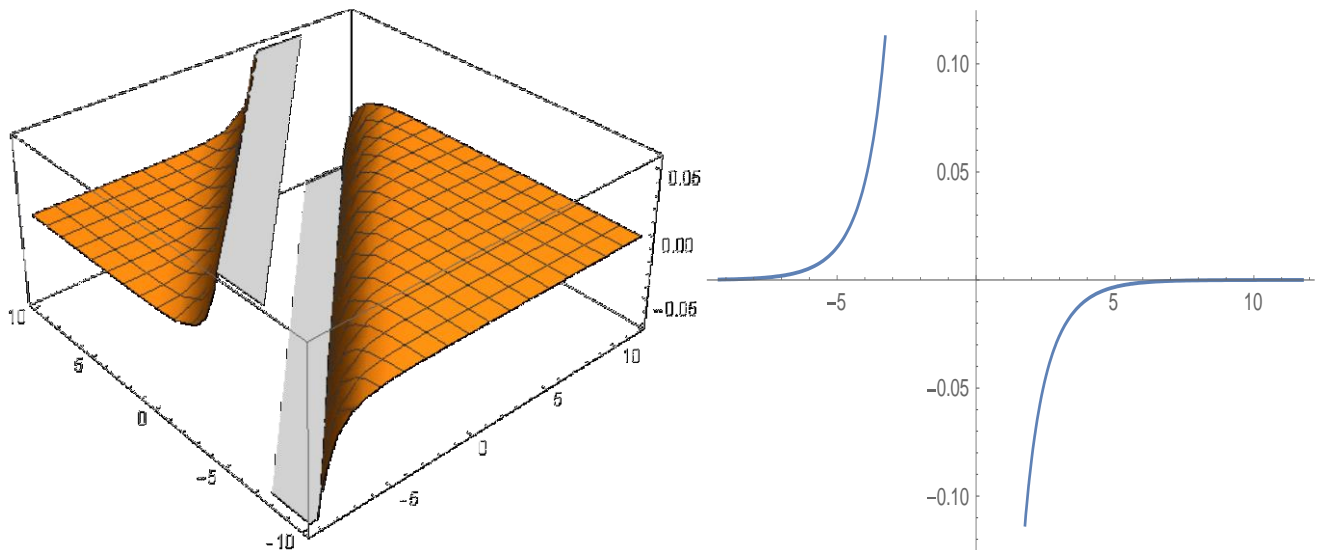
**Figure-3:** The 3D and 2D surfaces of Eq. (17) in $a=1, b=1, c=1, k=1, \lambda=1, \mu=0, \gamma=-5, EE=0.75$, $y=1$ and $t=1$.

**Family 4:** When, $\lambda \neq 0$, $\mu \neq 0$, $\lambda^2 - 4\mu = 0$,

$$v_{2.4}(x,t) = \left( \frac{ack\left( 4\mu\xi - \dfrac{4\lambda}{2+\lambda(EE+\xi)} - \lambda(2+\lambda(EE+\xi)) \right)}{4b} \right).$$  (32)

If the equation (32) is integrated, the solution function of equation (1) is obtained as follows,

$$u_{2.4}(x,t) = \left( \frac{ack\left( 4\mu\xi - \dfrac{4\lambda}{2+\lambda(EE+\xi)} - \lambda(2+\lambda(EE+\xi)) \right)}{4b} \right).$$  (33)
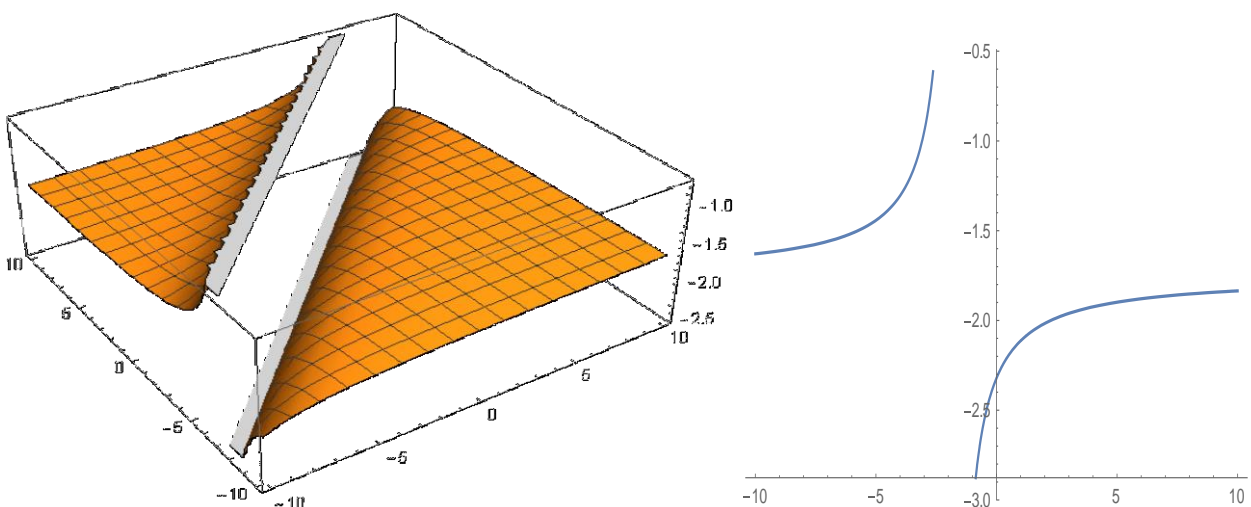


**Figure-4:** The 3D and 2D surfaces of Eq. (17) in $a=1, b=1, c=1, k=1, \lambda=2, \mu=1, \gamma=-4, EE=0.75$, $y=1$ and $t=1$.

**Family 5:** When, $\lambda = 0, \mu = 0, \lambda^2 - 4\mu = 0$ ,

$$v_{2.5}(x,t) = \left( \frac{ack\left(1 + (EE + \xi)(\lambda + \mu(EE + \xi))\right)}{b(EE + \xi)^2} \right),$$  (34)

If the equation (34) is integrated, the solution function of equation (1) is as follows,

$$u_{2.5}(x,t) = \left( \frac{ack\left(-\dfrac{1}{EE + \xi} + \mu(EE + \xi) + \lambda \log[EE + \xi]\right)}{b} \right).$$  (35)
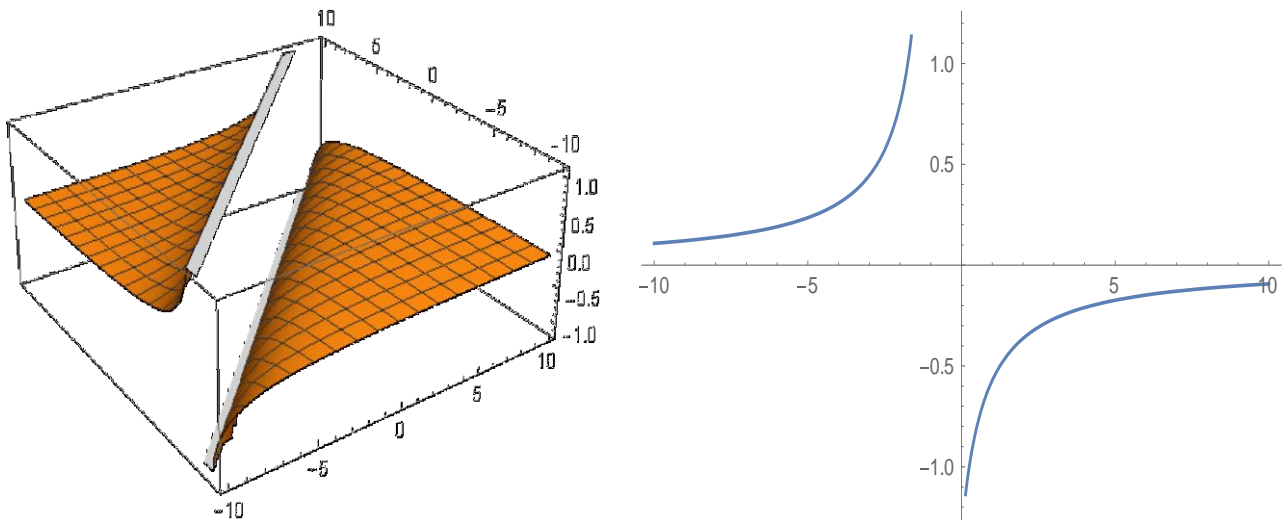


**Figure-5:** The 3D and 2D surfaces of Eq. (35) in $a = 1, b = 1, c = 1, k = 1, \lambda = 0, \mu = 0, \gamma = -4, EE = 0.75,$ $y = 1$ and $t = 1$.

**CONCLUSION**

In this article, some analytical solutions of the AKNS equation were obtained with the modified exponential function method. The modified exponential function method has been successfully used to reliable the significant wave solutions of nonlinear models; fourth-order nonlinear Ablowitz-Kaup-Newell-Segur equation. Whether the solutions obtained provide the AKNS equation was checked with the Mathematica program. Two and three dimensional graphs of the analytical solutions of the AKNS equation were plotted by determining the appropriate parameters with the same program. Various solutions of the AKNS equation are in the literature. Different wave solutions obtained using this effective method have been reported in this study. Otherwise, when we compare the results obtained by using this method in this article with the results obtained in [Helal, et al., 2013], some new travelling wave solutions have been introduced to the literature.

However, the solution functions in this study have not been obtained before.

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