



## Application of Discrete-Time Controllers to A Pilot Scale Packed Distillation Column

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**Abstract:** The process involved is a pilot scale packed distillation column that separates a mixture of methanol and water. The performances of two discrete-time controllers that different parameter tuning techniques were used for each of them were examined in the face of the set point tracking of the overhead product composition. The success of the discrete-time controllers was assessed by means of the rise time and integral of the square error (ISE) criteria. A discrete-time proportional-integral (PI) controller with two terms was obtained by using the velocity form of controller law. For this controller, firstly the proportional sensitivity term ( $K_c$ ) was determined by creating a root locus diagram. The other term that corresponded to the sampling time ( $T_s$ ) to integral time constant ( $\tau_I$ ) ratio multiplied by half of  $K_c$  value was then chosen by replacing the closed-loop poles to the appropriate location in unit circle of  $z$ -plane. The most successful proposed controller action was obtained with  $K_c$  of 0.95 and integral time constant of 2.375 min according to rise time of 3.7 min and ISE criteria of 1.08. The discrete-time proportional-integral-derivative (PID) controller with six parameters as  $P=0.009185$ ,  $I=0.01835$ ,  $D=0$ ,  $N=100$ ,  $b=1$ ,  $c=1$  that were tuned by using Simulink control design and 1 min sampling time was applied to the process. The same controller performance with six parameters of  $P=1.164$ ,  $I=0.5319$ ,  $D=-0.01481$ ,  $N=3.783$ ,  $b=0.1233$ ,  $c=0.01017$  was investigated by choosing 0.5 min sampling time. By comparing all the performances obtained, it can be said that the discrete-time controller with two terms tuned by the suggested approach can easily be applied to obtain the most desired performance.

**Keywords:** Discrete-time controller, packed distillation column, root locus, closed-loop pole placement.

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### INTRODUCTION

Discrete-time controller accuracy and speed have importance in industrial applications. A compromise is needed between cost and reliability (4). The corresponding controllers are called digital computers. System models are described in certain discrete-time domain, and their parameters are determined by using various methods based on experimental or well-simulated input and output data (2-3). The controlled systems dynamics are determined by the choice of controller parameters which are determined using various methods (1).

Discrete-time controllers are designed to

minimize a cost function or to locate closed-loop system poles in the  $z$ -plane unit circle. Obtaining a desired closed-loop system response of a controller also minimize indirectly a cost function. A cost effective design based on a closed-loop pole-placement approach was reported for discrete proportional-integral controller (7). For two-degrees-of-freedom (2DOF) PI current controller, an analytical discrete-time pole-placement technique was used (6). By separating a closed-loop system poles as a dominant pole pair and other poles and locating them in  $z$ -plane unit circle by targeting a certain distance between each set of poles, a discrete PI-proportional-derivate (PI-PD) controller was designed by Dincel and

Söylemez (2018). This technique is called a dominant pole-placement.

In the present work, two discrete-time controllers were used to track set point of the overhead product composition of a pilot-scale packed distillation column for performance comparison. The two tuning parameters of the proposed discrete PI controller were tuned by observing root locus diagram pole branches for proportional only control and by choosing the three poles location in z-plane according to a performance criteria of the system response with discrete PI controller. The two application performances of discrete-time PID controller (2DOF) that the related six parameters tuned by means of Simulink control design in MATLAB were also examined by choosing the sampling time ( $T_s$ ) as 1 min and 0.5 min respectively. As can be seen from the Simulink applications, the high value of the sampling time can destabilize the control or will at least result in the deviations of the controlled variable being much larger than necessary. If the chosen sampling time is larger than the real system delay, then a pole-placement based proposed discrete-time controller still produces acceptable set point tracking response.

## DISCRETE-TIME CONTROLLERS

Wardle and Hapoglu (1993) proposed a discrete-time transfer function of process without control. This transfer function response to a step increase in control signal was compared (10) with two-film approach model computer simulation that verified (9) by the experimental data (8) obtained from a pilot-scale packed distillation column where methanol water mixture was fed between 1 m and 0.9 m effective packing height in enriching and stripping sections.

The discrete system transfer function (10) without control which relates the deviation form of overhead product molar methanol composition ( $x_D'$ ) and the deviation form of reflux flow rate ( $L'$ ) was given in **Equation (1)**. Where, the  $T_s$  value was used as 1 min. The same system discrete transfer function can be written by using  $T_s$  of 0.5 min and zero order hold element in **Equation (2)**.

$$\frac{x'_D}{L'} = \frac{0.411z + 0.0119}{z^2 - 0.742z + 0.0183} \quad (1)$$

$$\frac{x'_D}{L'} = \frac{0.2122z - 0.01472}{z^2 - 1.006z + 0.1353} \quad (2)$$

The closed-loop system with proportional

control transfer function which relates overhead product composition in **Equation (1)** and set point was given in **Equation (3)**.

$$\frac{x'_D}{x'_{SET}} = a$$

$$a = \frac{K_c(0.411z + 0.0119)}{(z^2 - 0.742z + 0.0183) + K_c(0.411z + 0.0119)} \quad (3)$$

Where,  $x'_{SET}$  represents the set point value of the overhead product composition in deviation variable form.  $K_c$  is the proportional sensitivity (or the gain) of the proportional (P) controller. The characteristic equation of the transfer function given in **Equation (3)** was utilized to create root locus diagram data. The  $K_c$  value that locates two desired positive poles far from the edge of the unit circle was chosen to apply as a constant value in **Equation (9)**.

The proposed discrete-time PI controller transfer function was written as **Equation (4)**, because the changes of the manipulated variable are determined by varying two terms. **Equation (7)** is obtained by combining the discrete definition of the coefficients  $q_0$  and  $q_1$  which are given in **Equation (5)** and **Equation (6)** respectively. The velocity form control algorithm is utilized to create the terms in **Equation (5)** and **Equation (6)**. The tuning parameter of A was presented in **Equation (8)**.

$$G_c(z) = \frac{q_0z - q_1}{z - 1} \quad (4)$$

$$q_0 = K_c + \frac{K_c T_s}{2\tau_I} \quad (5)$$

$$q_1 = \frac{K_c T_s}{2\tau_I} - K_c \quad (6)$$

$$G_c(z) = \frac{(K_c + A)z - (A - K_c)}{z - 1} \quad (7)$$

$$A = \frac{K_c T_s}{2\tau_I} \quad (8)$$

$$G_c(z) = \frac{(0.95 + A)z - (A - 0.95)}{z - 1} \quad (9)$$

Where  $\tau_I$  is integral time constant which was varied by using different values of A (see **Equation (9)**).

$$ISE = 10^4 \sum_{n=0}^{n=N} T_s (x_{SET} - x_D)_n^2 \quad (10)$$

For controller performance comparison,

integral of the square error (ISE) criteria given in **Equation (10)** was used in discrete domain, where the error ( $x_{SET}-x_D$ ) is the deviation of the overhead product composition from the set point. The N value is the total number of sampling time step used to reach the final run time.

$$G_c(z) = P(b * x'_{set}) + I * T_s \frac{1}{z-1} (x'_{SET} - x'_D) + D \frac{N}{1 + N * T_s \frac{1}{z-1}} (c * x'_{SET} - x'_D) \quad (11)$$

The second discrete controller was obtained from Simulink Library in MATLAB. This discrete PID controller (2DOF) was presented in **Equation (11)**. Where P, I, D represent proportional, integral, derivative terms respectively. The parameters of b and c are the set point weights. The symbol of N is filter coefficient. The discrete PID controller block parameters was tuned by using Simulink Control Design in MATLAB. By substituting zero for D in **Equation (11)** the discrete PI controller Simulink block was obtained.

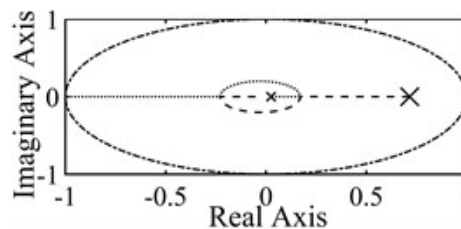
## RESULTS AND DISCUSSION

The discrete-time transfer function of the packed distillation column without control (10) which relates overhead product molar methanol composition as output and reflux flow rate as inputs was used to capture the

closed-loop behavior of the process with proportional only control. The proportional sensitivity ( $K_c$ ) of the closed-loop transfer function given in **Equation (3)** was varied to determine the poles location in z-plane by using MATLAB (see **Table 1**). Root locus diagram obtained was presented in **Figure 1**. The branches corresponding to the two poles of the proportional only control system begin at the two poles (0.7165 and 0.0255) of the system transfer function without control (These poles are represented by symbols of cross in **Figure 1**).

In **Table 1**, The  $K_c$  value was varied by considering the related root locus diagram to determine adequate location for the poles. For non-oscillatory stable case,  $K_c=0.95$  was chosen with two real positive closed-loop poles as 0.2119 and 0.1397.

At all set point tracking applications studied in the present work, an initial step change from 0.969 to 0.978 was introduced. For the proposed discrete-time PI controller parameters identification the  $K_c$  value selected from **Table 1** was used in **Equation (7)**. The coefficient of A in **Equation (9)** was selected based on closed-loop pole placement technique by considering the three poles location and the ISE criteria (**Table 2**). For this selection, the controlled variable behaviors versus time were also examined (**Figure 2-4**).

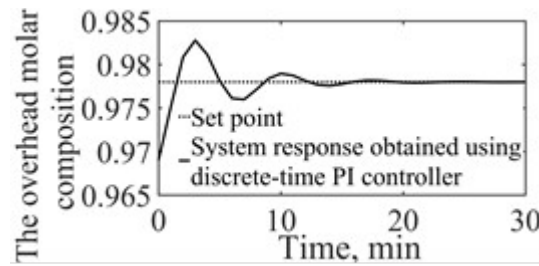


**Figure 1:** Root locus diagram generated using MATLAB for the closed-loop discrete system with P controller.

**Table 1:** Poles for the closed-loop discrete system with P controller.

$K_c$	poles	
1.1	0.1450-0.1019i	0.1450+0.1019i
1.0	0.1655-0.0530i	0.1655+0.0530i
0.98	0.1692-0.0346i	0.1692-0.0346i
(*) 0.95	0.2119	0.1397
0.90	0.2609	0.1112
0.80	0.3285	0.0847
0.009185	0.7124	0.0258

(\*) represents the selected poles and  $K_c$



**Figure 2:** The control of overhead product composition using the proposed discrete-time PI controller with  $T_s=1.0$  min,  $K_c=0.95$ ,  $\tau_I=0.28$  min,  $A=1.7$ .

**Table 2:** Poles for the discrete system with PI controller.

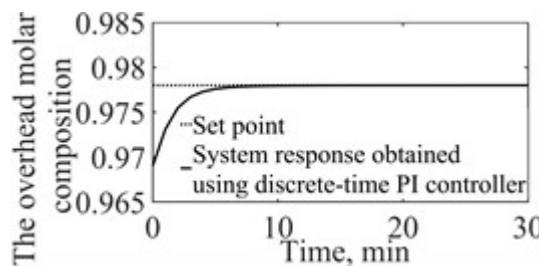
A	Three poles	ISE
1.70	$0.3222 \pm 0.9953i$ , $0.0086 + 0.0000i$	79.26
0.70	$0.5155 \pm 0.6143i$ , $0.0331 + 0.0000i$	1.41
0.40	$0.5683 \pm 0.4124i$ , $0.0504 + 0.0000i$	1.10
(*) 0.20	$0.5987 \pm 0.1466i$ , $0.0716 + 0.0000i$	1.08
0.18	$0.6016 \pm 0.0735i$ , $0.0748 + 0.0000i$	1.10
0.17	0.6527, 0.5529, 0.0764	1.11
0.16	0.7055, 0.5023, 0.0782	1.13
0.15	0.7400, 0.4700, 0.0800	1.14

The symbol (\*) represents the selected A value according to ISE criteria. The values of  $K_c$  and  $T_s$  are 0.95 and 1 min respectively.

In the cases studied (**Table 2**), with the proposed discrete-time PI controller, the offset problem did not exist. However, the addition of the integral (I) action with  $A=1.7$  ( $T_s=1.0$  min,  $\tau_I=0.28$  min) in the discrete controller that includes the previously chosen  $K_c$  value of 0.95 caused the oscillatory system response and increased the maximum deviation from the desired value (**Figure 2**). The system response to a change in set point may exhibit less oscillatory behavior with tolerable rise

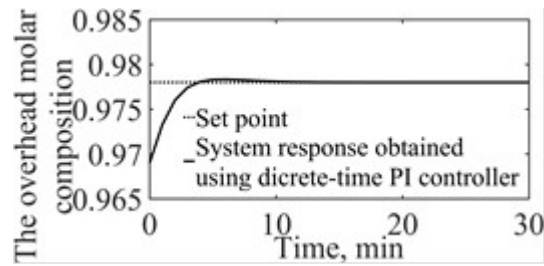
time or non-oscillatory behavior with high rise time as value of A decreases ( $\tau_I$  value increases).

**Figure 3** showed that the A value of 0.15 produced an overdamped response with high rise time of 8.8 min. The best response according to minimum ISE criteria and rise time of 3.7 min was shown in **Figure 4**.

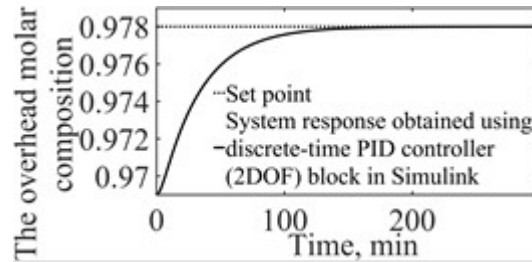


**Figure 3:** The control of overhead product composition using the proposed discrete-time PI controller with  $T_s=1.0$  min,  $K_c=0.95$ ,  $\tau_I=3.17$  min,  $A=0.15$ .

The discrete PID controller (2DOF) which is available in MATLAB/Simulink library was examined when applied to the overhead product composition control of the packed distillation column in the face of a set point change from 0.969 to 0.978. Its six tuning parameters were determined as  $P=0.009185$ ,  $I=0.01835$ ,  $D=0$ ,  $N=100$ ,  $b=1$ ,  $c=1$  using Simulink control design by using  $T_s$  of 1 min. It was noted that the PID controller given in **Equation (11)** converted to the PI control action by substituting zero ( $D=0$ ) for the last term. The overhead product composition set point tracking with this controller was shown in **Figure 5**. This control action performance according to ISE criteria was evaluated as 15.89. This response with 169 min rise time exhibited sluggishness.



**Figure 4:** The control of overhead product composition using the proposed discrete-time PI controller with  $T_s=1.0$  min,  $K_c=0.95$ ,  $\tau_i=2.375$ min,  $A=0.2$ .

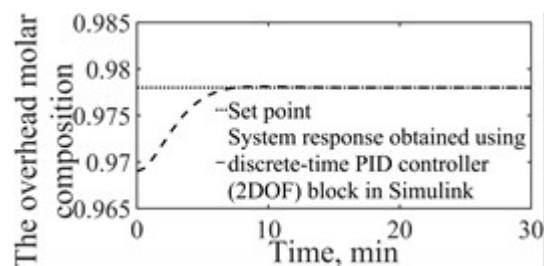


**Figure 5:** The closed-loop response of overhead product composition with the discrete-time PID controller (2DOF) that is available in MATLAB/ Simulink library.

By using  $T_s$  of 0.5 min and the system transfer function in **Equation (2)**, the discrete PID controller (2DOF) parameters were also determined as  $P=1.164$ ,  $I=0.5319$ ,  $D=-0.01481$ ,  $N=3.783$ ,  $b=0.1233$ ,  $c=0.01017$  using Simulink control design. The closed-loop system response obtained was illustrated in **Figure 6**. This set point tracking response obtained with the discrete PID controller (2DOF) performed with the ISE criteria of 3.68 and the rise time of 8 min. A decrease in  $T_s$  from 1 min to 0.5 min revealed 95.27 % and 76.84 % decreases in the ISE and in the rise

time values respectively.

In the present work, the theoretical cost-effective performance of the proposed controller was shown. This enhanced the economic performance of the modeled pilot-scale packed distillation column. Therefore, it is worth to use the discrete-time controller and evaluate the cost reduction of an industrial-scale packed column with the discrete-time controllers.



**Figure 6:** The control of overhead product composition using the discrete-time PID controller (2DOF) Simulink block that its parameters tuned as  $P=1.164$ ,  $I=0.5319$ ,  $D=-0.01481$ ,  $N=3.783$ ,  $b=0.1233$ ,  $c=0.01017$ .

## CONCLUSION

Clearly, the proposed discrete PI controller with the technique applied based on root locus search and pole-placement approach has a desired influence upon the set-point tracking performance of the overhead product composition of a packed distillation column. The discrete proposed PI controller exhibited much better performance than the one

obtained using the discrete PID (2DOF) controller in MATLAB/Simulink library. This controller can be recommended for relatively high sampling time applications.

The advantages of the proposed controller used in the present work were low rise time, low settling time and very smooth transient response. It was shown that the desired level of performance was guaranteed by using suitable tuning technique based on pole-

placement for the proposed controller parameters. There is currently a considerable need for the controller as described in the present work to be implemented on a full-scale industrial packed distillation column.

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