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Coefficient Estimates for Certain General Subclasses of Meromorphic Bi-Univalent Functions

Serap Bulut[1*], N. Magesh[2] and J. Sivapalan[3]

[1]*Kocaeli University, Faculty of Aviation and Space Sciences, Arslanbey Campus, 41285 Kartepe-Kocaeli, Turkey*
[2]*Post-Graduate and Research Department of Mathematics, Government Arts College for Men, Krishnagiri 635001, Tamilnadu, India*
[3]*Department of Mathematics, Govt Arts and Science College, Hosur 635109, Tamilnadu, India*
[*]*Corresponding author*

Abstract

In the present investigation, we introduce two interesting general subclasses of meromorphic and bi-univalent functions. Further, we find estimates on the initial coefficient $|b_0|$ and $|b_1|$ for functions belonging to these subclasses. Some other closely related results are also represented.

Keywords: *Analytic functions, bi-univalent functions, meromorphic bi-univalent functions, strongly bi-pseudo-starlike functions.*
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1. Introduction

Let $\mathscr{A}$ be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} := \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. We also denote by $\mathscr{S}$ the subclass of the normalized analytic function class $\mathscr{A}$ consisting of all functions which are also univalent in $\mathbb{U}$.
Since univalent function are one-to-one, they are invertible and the inverse functions need not be defined on the entire unit disk $\mathbb{U}$. In fact, the Koebe one-quarter theorem ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathscr{S}$ contains a disc of radius $1/4$. Thus every function $f \in \mathscr{S}$ has an inverse f^{-1}, which is defined by

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f^{-1}(f(w)) = w \qquad \left(|w| < r_o(f);\, r_o(f) \geq \frac{1}{4}\right).$$

A function $f \in \mathscr{A}$ is said to be bi-univalent in the open unit disk $\mathbb{U}$ if both the function f and its inverse f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of analytic and bi-univalent functions in $\mathbb{U}$ given by the Taylor-Maclaurin series expansion as in (1.1). For a brief history and interesting examples of functions in the class Σ, see [13]. In fact, the aforecited work of Srivastava et al. [13] essentially revived the investigation of various subclasses of the bi-univalent function class Σ in very recent years
In this paper, the concept of bi-univalency is extended to the class of meromorphic functions defined on $\Delta := \{z : z \in \mathbb{C} \text{ and } 1 < |z| < \infty\}$.
The class of functions

$$g(z) = z + \sum_{n=0}^{\infty} \frac{b_n}{z^n} \tag{1.2}$$

that are meromorphic and univalent in Δ is denoted by σ, and every univalent function g has an inverse g^{-1} satisfy the series expansion

$$g^{-1}(w) = w + \sum_{n=0}^{\infty} \frac{B_n}{w^n}, \tag{1.3}$$

Email addresses: serap.bulut@kocaeli.edu.tr (Serap Bulut), nmagi_2000@yahoo.co.in (N. Magesh), jsskavya2007@gmail.com (J. Sivapalan)

where $0<M<|w|<\infty$. Analogous to the bi-univalent analytic functions, a function $g\in\sigma$ given by (1.2) is said to be meromorphic and bi-univalent if both g and g^{-1} are meromorphic and univalent in Δ. The class of all meromorphic and bi-univalent functions denoted by $\sigma_{\mathscr{M}}$. A simple calculation shows that

$$h(w)=g^{-1}(w)=w-b_0-\frac{b_1}{w}-\frac{b_2+b_0b_1}{w^2}-\frac{b_3+2b_0b_2+b_0^2b_1+b_1^2}{w^3}+\cdots. \tag{1.4}$$

The history and examples of the various subclasses of meromorphic bi-univalent functions one could refer the recent works [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 15] as well as references therein.
Recently, Sakar [9] introduced and investigated the following two subclasses with initial coefficient estimates:

Definition 1.1. *[9] A function $g\in\sigma_{\mathscr{M}}$ given by (1.2) is said to be in the class $\mathscr{T}^{\alpha}_{\sigma_{\mathscr{M}}}$ if the following conditions are satisfied:*

$$\left|\arg\left(\frac{z^2g'(z)}{[g(z)]^2}\right)\right|<\frac{\alpha\pi}{2}\ \text{and}\ \left|\arg\left(\frac{w^2h'(w)}{[h(w)]^2}\right)\right|<\frac{\alpha\pi}{2}\qquad(z,w\in\Delta,\ 0<\alpha\le 1),$$

where the function h is given by (1.4).

Theorem 1.2. *[9] Let the function $g\in\sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\mathscr{T}^{\alpha}_{\sigma_{\mathscr{M}}}$, $0<\alpha\le1$. Then*

$$|b_0|\le\sqrt{\frac{2}{3}\alpha}\qquad\text{and}\qquad|b_1|\le\begin{cases}\frac{2}{3}\alpha & ,\quad 0<\alpha\le\frac{\sqrt{2}}{2}\\[2mm] \frac{2\sqrt{2}}{3}\alpha^2 & ,\quad \frac{\sqrt{2}}{2}\le\alpha\le1\end{cases}.$$

Definition 1.3. *[9] A function $g\in\sigma_{\mathscr{M}}$ given by (1.2) is said to be in the class $\mathscr{T}_{\sigma_{\mathscr{M}}}(\mu)$ if the following conditions are satisfied:*

$$\Re\left(\frac{z^2g'(z)}{[g(z)]^2}\right)>1-\mu\ \text{and}\ \Re\left(\frac{w^2h'(w)}{[h(w)]^2}\right)>1-\mu\qquad(z,w\in\Delta,\ 0<\mu\le1),$$

where the function h is given by (1.4).

Theorem 1.4. *[9] Let the function $g\in\sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\mathscr{T}_{\sigma_{\mathscr{M}}}(\mu)$, $0<\mu\le1$. Then*

$$|b_0|\le\sqrt{\frac{2\mu}{3}}\qquad\text{and}\qquad|b_1|\le\frac{2\sqrt{2}}{3}\mu.$$

Very recently, Srivastava et al. [12] introduced and studied meromorphically strongly λ-bi-pseudo-starlike functions and meromorphically λ-bi-pseudo-starlike functions in Definitions 1.5 and 1.7, respectively, which are analogous to analytically case introduced and studied by Joshi et al. [7].

Definition 1.5. *[12] A function $g\in\sigma_{\mathscr{M}}$ given by (1.2) is said to be in the class $\sigma_{\mathscr{B},\lambda^*}(\alpha)$ if the following conditions are satisfied:*

$$\left|\arg\left(\frac{z[g'(z)]^{\lambda}}{g(z)}\right)\right|<\frac{\alpha\pi}{2}\qquad\text{and}\qquad\left|\arg\left(\frac{w[h'(w)]^{\lambda}}{h(w)}\right)\right|<\frac{\alpha\pi}{2}\qquad(z,w\in\Delta,\ 0<\alpha\le1,\ \lambda\ge1),$$

where the function h is given by (1.4).

Theorem 1.6. *[12] Let the function $g\in\sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\sigma_{\mathscr{B},\lambda^*}(\alpha)$, $0<\alpha\le1$ and $\lambda\ge1$. Then*

$$|b_0|\le2\alpha\qquad\text{and}\qquad|b_1|\le\frac{2\sqrt{5}}{1+\lambda}\alpha^2.$$

Definition 1.7. *[12] A function $g\in\sigma_{\mathscr{M}}$ given by (1.2) is said to be in the class $\sigma_{\mathscr{B}^*}(\lambda,\beta)$ if the following conditions are satisfied:*

$$\Re\left(\frac{z[g'(z)]^{\lambda}}{g(z)}\right)>\beta\qquad\text{and}\qquad\Re\left(\frac{w[h'(w)]^{\lambda}}{h(w)}\right)>\beta\qquad(z,w\in\Delta,\ 0\le\beta<1,\ \lambda\ge1),$$

where the function h is given by (1.4).

Theorem 1.8. *[12] Let the function $g\in\sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\sigma_{\mathscr{B}^*}(\lambda,\beta)$, $0\le\beta<1$ and $\lambda\ge1$. Then*

$$|b_0|\le2(1-\beta)\qquad\text{and}\qquad|b_1|\le\frac{2(1-\beta)\sqrt{4\beta^2-8\beta+5}}{1+\lambda}.$$

Remark 1.9. *For $\lambda=1$, we get the classes $\sigma_{\mathscr{B},1^*}(\alpha)=\tilde{\sigma}^*_{\mathscr{B}}(\alpha)$ and $\sigma_{\mathscr{B}^*}(1,\beta)=\sigma^*_{\mathscr{B}}(\beta)$ introduced and studied by Halim et al. [3].*

In the present investigation, two general subclasses of meromorphic bi-univalent functions are defined and general estimates for the coefficients $|b_0|$ and $|b_1|$ of functions in the newly introduced two subclasses are obtained.

Definition 1.10. *Througout this paper, we assume that the functions $\phi,\psi:\Delta\to\mathbb{C}$ be analytic functions and*

$$\phi(z)=1+\frac{\phi_1}{z}+\frac{\phi_2}{z^2}+\frac{\phi_3}{z^3}+\cdots;\ \ \psi(z)=1+\frac{\psi_1}{z}+\frac{\psi_2}{z^2}+\frac{\psi_3}{z^3}+\cdots$$

such that

$$\min\{\Re(\phi(z)),\ \Re(\psi(z))\}>0\qquad(z\in\Delta).$$

Definition 1.11. *A function $g \in \sigma_{\mathscr{M}}$ given by (1.2) is said to be in the class $\mathscr{T}_{\sigma_{\mathscr{M}}}(\phi,\psi)$ if the following conditions are satisfied:*

$$\frac{z^2 g'(z)}{[g(z)]^2} \in \phi(\Delta) \qquad \text{and} \qquad \frac{w^2 h'(w)}{[h(w)]^2} \in \psi(\Delta) \qquad (z,w \in \Delta),$$

where the function h is given by (1.4).

Definition 1.12. *A function $g \in \sigma_{\mathscr{M}}$ given by (1.2) is said to be in the class $\sigma_{\mathscr{B},\lambda}(\phi,\psi)$ if the following conditions are satisfied:*

$$\frac{z[g'(z)]^{\lambda}}{g(z)} \in \phi(\Delta) \qquad \text{and} \qquad \frac{w[h'(w)]^{\lambda}}{h(w)} \in \psi(\Delta) \qquad (z,w \in \Delta,\ \lambda \geq 1),$$

where the function h is given by (1.4).

There are many choices of ϕ and ψ which would provide interesting subclasses of classes $\mathscr{T}_{\sigma_{\mathscr{M}}}(\phi,\psi)$ and $\sigma_{\mathscr{B},\lambda}(\phi,\psi)$, we illustrate as examples:

Example 1.13. *If we take*

$$\phi(z) = \psi(z) = \left(\frac{1+\frac{1}{z}}{1-\frac{1}{z}}\right)^{\alpha} = 1 + \frac{2\alpha}{z} + \frac{2\alpha^2}{z^2} + \frac{2\alpha^3}{z^3} + \cdots \qquad (0 < \alpha \leq 1,\ z \in \Delta) \tag{1.5}$$

in Definition 1.11 and Definition 1.12, then we get the classes

$$\mathscr{T}_{\sigma_{\mathscr{M}}}(\phi,\psi) = \mathscr{T}^{\alpha}_{\sigma_{\mathscr{M}}} \qquad \text{and} \qquad \sigma_{\mathscr{B},\lambda}(\phi,\psi) = \sigma_{\mathscr{B},\lambda^*}(\alpha)$$

defined in Definition 1.1 and Definition 1.5, respectively. It is clear that the functions ϕ and ψ satisfy the condition of Definition 1.10.

Example 1.14. *If we take*

$$\phi(z) = \psi(z) = \frac{1+\frac{1-2\beta}{z}}{1-\frac{1}{z}} = 1 + \frac{2(1-\beta)}{z} + \frac{2(1-\beta)}{z^2} + \frac{2(1-\beta)}{z^3} + \cdots \qquad (0 \leq \beta < 1,\ z \in \Delta) \tag{1.6}$$

in Definition 1.11 and Definition 1.12, then we get the classes

$$\mathscr{T}_{\sigma_{\mathscr{M}}}(\phi,\psi) = \mathscr{T}_{\sigma_{\mathscr{M}}}(1-\beta) \qquad \text{and} \qquad \sigma_{\mathscr{B},\lambda}(\phi,\psi) = \sigma_{\mathscr{B}^*}(\lambda,\beta)$$

defined in Definition 1.3 and Definition 1.7, respectively. It is clear that the functions ϕ and ψ satisfy the condition of Definition 1.10.

2. Coefficient bounds for the function class $\mathscr{T}_{\sigma_{\mathscr{M}}}(\phi,\psi)$

In the following theorem, we obtain the initial coefficient estimates for functions belonging to the meromorphically bi-univalent function class $\mathscr{T}_{\sigma_{\mathscr{M}}}(\phi,\psi)$.

Theorem 2.1. *Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\mathscr{T}_{\sigma_{\mathscr{M}}}(\phi,\psi)$. Then*

$$|b_0| \leq \min\left\{\frac{|\phi_1|}{2};\ \sqrt{\frac{|\phi_2|+|\psi_2|}{6}}\right\} \tag{2.1}$$

and

$$|b_1| \leq \min\left\{\sqrt{\frac{|\phi_2|^2+|\psi_2|^2}{18} + \frac{|\phi_1|^4}{16}};\ \frac{|\phi_2|+|\psi_2|}{6}\right\}.$$

Proof. From Definition 1.11, we have

$$\frac{z^2 g'(z)}{[g(z)]^2} \in \phi(\Delta) \qquad \text{and} \qquad \frac{w^2 h'(w)}{[h(w)]^2} \in \psi(\Delta).$$

Let us set

$$\phi(z) = \frac{z^2 g'(z)}{[g(z)]^2} \qquad (z \in \Delta)$$

and

$$\psi(w) = \frac{w^2 h'(w)}{[h(w)]^2} \qquad (w \in \Delta).$$

Now expressing in terms of power series, we have

$$\frac{z^2 g'(z)}{[g(z)]^2} = 1 - \frac{2b_0}{z} + \frac{3b_0^2 - 3b_1}{z^2} + \cdots$$

and

$$\frac{w^2h'(w)}{[h(w)]^2} = 1 + \frac{2b_0}{z} + \frac{3b_0^2+3b_1}{z^2} + \cdots,$$

respectively. Upon equating the coefficients of $\frac{z^2g'(z)}{[g(z)]^2}$ with those of $\phi(z)$ and coefficients of $\frac{w^2h'(w)}{[h(w)]^2}$ with those of $\psi(w)$, we get

$$-2b_0 = \phi_1, \tag{2.2}$$

$$-3b_1 + 3b_0^2 = \phi_2, \tag{2.3}$$

$$2b_0 = \psi_1, \tag{2.4}$$

$$3b_1 + 3b_0^2 = \psi_2. \tag{2.5}$$

From (2.2) and (2.4), we obtain

$$\phi_1 = -\psi_1, \tag{2.6}$$

$$b_0 = -\frac{\phi_1}{2} = \frac{\psi_1}{2} \tag{2.7}$$

and

$$4b_0^2 = \phi_1^2 + \psi_1^2. \tag{2.8}$$

Adding (2.3) and (2.5), we have

$$6b_0^2 = \phi_2 + \psi_2. \tag{2.9}$$

It follows from the equations (2.6) and (2.7)-(2.9) that

$$|b_0| = \frac{|\phi_1|}{2} = \frac{|\psi_1|}{2},$$

$$|b_0|^2 = \frac{|\phi_1|^2}{2}$$

and

$$|b_0|^2 \leq \frac{|\phi_2| + |\psi_2|}{6},$$

respectively. So, we get the desired estimate on the coefficient $|b_0|$ as asserted in (2.1). Next, in order to find the bound on the coefficient $|b_1|$, we subtract (2.3) from (2.5), thus we have

$$6b_1 = \psi_2 - \phi_2. \tag{2.10}$$

By squaring and adding (2.3) and (2.5), using (2.8) in the computation leads to

$$b_1^2 = \frac{\phi_2^2 + \psi_2^2}{18} - b_0^4. \tag{2.11}$$

If we set the values of b_0 from the equalities (2.7)-(2.9) in (2.11), we find that

$$b_1^2 = \frac{\phi_2^2 + \psi_2^2}{18} - \frac{\phi_1^4}{16},$$

$$b_1^2 = \frac{\phi_2^2 + \psi_2^2}{18} - \frac{\phi_1^4}{4}$$

and

$$b_1^2 = \frac{\phi_2^2 + \psi_2^2}{18} - \frac{(\phi_2 + \psi_2)^2}{36}.$$

Therefore, we obtain from the above equations that

$$|b_1| \leq \sqrt{\frac{|\phi_2|^2 + |\psi_2|^2}{18} + \frac{|\phi_1|^4}{16}}$$

and

$$|b_1| \leq \frac{|\phi_2| + |\psi_2|}{6}.$$

This evidently completes the proof of Theorem 2.1. □

By setting ϕ and ψ as given in (1.5) in Theorem 2.1, we conclude the following result:
Corollary 1. Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\mathscr{T}^{\alpha}_{\sigma_{\mathscr{M}}}$ $(0 < \alpha \leq 1)$. Then

$$|b_0| \leq \sqrt{\frac{2}{3}}\alpha \qquad \text{and} \qquad |b_1| \leq \frac{2}{3}\alpha^2.$$

Remark 2.2. *Note that Corollary 2 is an improvement of the estimates obtained in Theorem 1.2.*

By setting ϕ and ψ as given in (1.6) in Theorem 2.1, we conclude the following result:
Corollary 2. Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\mathscr{T}_{\sigma_{\mathscr{M}}}(1-\beta)$ $(0 \leq \beta < 1)$. Then

$$|b_0| \leq \begin{cases} \sqrt{\dfrac{2(1-\beta)}{3}} & , \quad 0 \leq \beta \leq \frac{1}{3} \\ 1-\beta & , \quad \frac{1}{3} \leq \beta < 1 \end{cases} \qquad \text{and} \qquad |b_1| \leq \frac{2(1-\beta)}{3}.$$

Remark 2.3. *Note that Corollary 2 is an improvement of the estimates obtained in Theorem 1.4.*

3. Coefficient bounds for the function class $\sigma_{\mathscr{B},\lambda}(\phi,\psi)$

In the following theorem, we obtain the initial coefficient estimates for functions belonging to the meromorphically bi-univalent function class $\sigma_{\mathscr{B},\lambda}(\phi,\psi)$.

Theorem 3.1. *Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\sigma_{\mathscr{B},\lambda}(\phi,\psi)$. Then*

$$|b_0| \leq \min\left\{|\phi_1|;\ \sqrt{\frac{|\phi_2|+|\psi_2|}{2}}\right\} \tag{3.1}$$

and

$$|b_1| \leq \min\left\{\sqrt{\frac{|\phi_2|^2+|\psi_2|^2}{2(1+\lambda)^2}+\frac{|\phi_1|^4}{(1+\lambda)^2}};\ \frac{|\phi_2|+|\psi_2|}{2(1+\lambda)}\right\}. \tag{3.2}$$

Proof. From Definition 1.12, we have

$$\frac{z[g'(z)]^{\lambda}}{g(z)} \in \phi(\Delta) \qquad \text{and} \qquad \frac{w[h'(w)]^{\lambda}}{h(w)} \in \psi(\Delta).$$

Let us set

$$\phi(z) = \frac{z[g'(z)]^{\lambda}}{g(z)} \qquad (z \in \Delta)$$

and

$$\psi(w) = \frac{w[h'(w)]^{\lambda}}{h(w)} \qquad (w \in \Delta).$$

Now expressing in terms of power series, we have

$$\frac{z[g'(z)]^{\lambda}}{g(z)} = 1 - \frac{b_0}{z} + \frac{b_0^2-(1+\lambda)b_1}{z^2} + \cdots$$

and

$$\frac{w[h'(w)]^{\lambda}}{h(w)} = 1 + \frac{b_0}{w} + \frac{b_0^2+(1+\lambda)b_1}{w^2} + \cdots,$$

respectively. Upon equating the coefficients of $\frac{z[g'(z)]^{\lambda}}{g(z)}$ with those of $\phi(z)$ and coefficients of $\frac{w[h'(w)]^{\lambda}}{h(w)}$ with those of $\psi(w)$, we get

$$-b_0 = \phi_1, \tag{3.3}$$

$$-(1+\lambda)b_1 + b_0^2 = \phi_2, \tag{3.4}$$

$$b_0 = \psi_1, \tag{3.5}$$

$$(1+\lambda)b_1 + b_0^2 = \psi_2. \tag{3.6}$$

From (3.3) and (3.5), we get

$$\phi_1 = -\psi_1, \tag{3.7}$$

$$b_0 = -\phi_1 = \psi_1 \tag{3.8}$$

and

$$2b_0^2 = \phi_1^2 + \psi_1^2. \tag{3.9}$$

Adding (3.4) and (3.6), we get

$$2b_0^2 = \phi_2 + \psi_2. \tag{3.10}$$

It follows from the equations (3.7) and (3.8)-(3.10) that

$$|b_0| = |\phi_1| = |\psi_1| \tag{3.11}$$

and

$$|b_0|^2 \leq \frac{|\phi_2| + |\psi_2|}{2} \tag{3.12}$$

respectively. So, we get the desired estimate on the coefficient $|b_0|$ as asserted in (3.1). Next, in order to find the bound on the coefficient $|b_1|$, we subtract (3.4) from (3.6), we thus get

$$2(1+\lambda)b_1 = \psi_2 - \phi_2. \tag{3.13}$$

By squaring and adding (3.4) and (3.6), using (3.9) in the computation leads to

$$b_1^2 = \frac{\phi_2^2 + \psi_2^2}{2(1+\lambda)^2} - \frac{b_0^4}{(1+\lambda)^2}. \tag{3.14}$$

If we set the values of b_0 from the equalities (3.8)-(3.10), we find that

$$b_1^2 = \frac{\phi_2^2 + \psi_2^2}{2(1+\lambda)^2} - \frac{\phi_1^4}{(1+\lambda)^2}$$

and

$$b_1^2 = \frac{(\phi_2 - \psi_2)^2}{4(1+\lambda)^2}.$$

Therefore, we obtain from the above equations that

$$|b_1| \leq \sqrt{\frac{|\phi_2|^2 + |\psi_2|^2}{2(1+\lambda)^2} + \frac{|\phi_1|^4}{(1+\lambda)^2}}$$

and

$$|b_1| \leq \frac{|\phi_2| + |\psi_2|}{2(1+\lambda)}.$$

This evidently completes the proof of Theorem 3.1. □

By setting ϕ and ψ as given in (1.5) in Theorem 3.1, we conclude the following result:

Corollary 3. Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\sigma_{\mathscr{B},\lambda^*}(\alpha)$ $(0 < \alpha \leq 1,\ \lambda \geq 1)$. Then

$$|b_0| \leq \sqrt{2}\alpha \qquad \text{and} \qquad |b_1| \leq \frac{2}{1+\lambda}\alpha^2.$$

Remark 3.2. *Note that Corollary 3 is an improvement of the estimates obtained in Theorem 1.6.*

By setting ϕ and ψ as given in (1.6) in Theorem 3.1, we conclude the following result:

Corollary 4. Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\sigma_{\mathscr{B}^*}(\lambda, \beta)$ $(0 \leq \beta < 1,\ \lambda \geq 1)$. Then

$$|b_0| \leq \begin{cases} \sqrt{2(1-\beta)} & , \quad 0 \leq \beta \leq \frac{1}{2} \\ 2(1-\beta) & , \quad \frac{1}{2} \leq \beta < 1 \end{cases} \quad \text{and} \quad |b_1| \leq \frac{2(1-\beta)}{1+\lambda}.$$

Remark 3.3. *Note that Corollary 3 is an improvement of the estimates obtained in Theorem 1.8.*

Letting $\lambda = 1$ in Corollary 3 and Corollary 3, we obtain following two consequences, respectively.

Corollary 5. Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\tilde{\sigma}^*_{\mathscr{B}}(\alpha)$ $(0 < \alpha \leq 1)$. Then

$$|b_0| \leq \sqrt{2}\alpha \qquad \text{and} \qquad |b_1| \leq \alpha^2.$$

Corollary 6. Let $g \in \sigma_{\mathscr{M}}$ given by (1.2) be in the function class $\sigma^*_{\mathscr{B}}(\beta)$ $(0 \leq \beta < 1)$. Then

$$|b_0| \leq \begin{cases} \sqrt{2(1-\beta)} & , \quad 0 \leq \beta \leq \frac{1}{2} \\ 2(1-\beta) & , \quad \frac{1}{2} \leq \beta < 1 \end{cases} \quad \text{and} \quad |b_1| \leq 1-\beta.$$

Remark 3.4. *Corollary 3 and Corollary 3 are improvements of the estimates obtained by Halim et al. [3, Theorem 2 and Theorem 1].*

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