MUĞLA SITKI KOÇMAN ÜNİVERSİTESİ

Mugla Journal of Science and Technology

A FUZZY APPLICATION OF X-R CONTROL CHARTS OF AN ALUMINUM PRODUCTION PLANT USING FUZZY TRIANGULAR NUMBERS

Volkan ARSLAN*, General Directorate of Minerals Research and Exploration, Turkey, volkanarslan76@hotmail.com
(https://orcid.org/0000-0002-5594-1495)

Received: 22.04.2020, Accepted: 03.09.2020 Research Article
*Corresponding author DOI: 10.22531/muglajsci.725550

Abstract

In this study, fuzzy mean and range control charts were used to follow the feeding material and concentrate production line of Eti Aluminum Co. Fuzzy control charts were collected from the factory over a period of time and compared to Shewhart control charts used by the factory. The results showed that fuzzy control charts detected errors in the production process more accurately than Shewhart control charts. This method has increased quality and efficiency. Process capability indices (PCIs) provide numerical measures as to whether a process has confirmed the defined capability prerequisite. These were used to measure the process's ability to decide how well it meets specification limits (SLs). PCIs have been implemented by companies to evaluate quality and efficiency performance. Fuzzy process capability analysis using X-R control charts gave more accurate results.

Keywords: Statistical Process Control, Fuzzy Control Chart, Process Capability Indices, Aluminum

ALÜMİNYUM ÜRETİM TESİSİNİN ÜÇGEN BULANIK SAYILAR KULLANILARAK X-R KONTROL ŞEMALARININ BULANIK MANTIK UYGULAMASI

Özet

Bu çalışmada, Eti Alüminyum Şirketinin besleme malı ve konsantre üretim sürecini izlemek için bulanık ortalama ve aralık kontrol grafikleri kullanılmıştır. Bulanık kontrol çizelgeleri, belirli bir süre aralığında fabrikadan veri toplanmış ve fabrika tarafından kullanılan Shewhart kontrol çizelgeleri ile karşılaştırılmıştır. Sonuçlar, bulanık kontrol grafiklerinin Shewhart kontrol grafiklerine göre üretim sürecindeki hataları daha doğru bir şekilde tespit ettiği görülmüştür. Bu yöntem, kaliteyi ve verimliliği artırmıştır. İşlem yeteneği endeksleri (PCI'ler), bir işlemin tanımlanmış yetenek önkoşulunu onaylayıp onaylamadığına dair sayısal önlemler sağlar. Bunlar, sürecin şartname sınırlarını (SL'ler) ne kadar iyi karşıladığına karar verme yeteneğini ölçmek için kullanılmıştır. PCI'lar şirketler tarafından kalite ve verimlilik performansını değerlendirmek için uygulanmıştır. X-R kontrol grafikleri kullanılarak yapılan bulanık proses yeterlilik analizi daha doğru sonuçlar vermiştir.

Anahtar Kelimeler: İstatistiksel Proses Kontrolü, Bulanık Kontrol Kartı, Proses Yeterlilik İndeksleri, Alüminyum

Cite

Arslan, V., (2020). "A fuzzy application of mean and range control charts of an aluminum production plant using fuzzy triangular numbers", Mugla Journal of Science and Technology, 6(2),43-51.

1. Introduction

The control chart originated in the early 1920s, it has become a powerful tool in statistical process control (SPC) that is the most widely used in industrial processes. Control charts are designed to monitor the process of change in mean and variance; they also reflect the ability of the process. Control charts have two types: variable and attribute. Control chart technique is well-known as a key step in production process monitoring. The control chart has a major function in detecting the occurrence of assignable causes, so that the necessary correction can be made before non-conforming products are manufactured in a large amount. The control chart technique may be considered as both the graphical expression and operation of statistical hypothesis testing. It is recommended that if a control chart is employed to monitor process, some test parameters should be determined such as the sample size, the sampling interval between successive samples, and the control limits or critical regions of the chart. SPC is an efficient technique for improvement of a firm's quality and productivity. The main objective of SPC is similar to that of the control chart technique, that is, to rapidly examine the occurrence of assignable causes or process shifts [1].

The fuzzy set theory is a more suitable tool for handling attribute data since these data may be expressed in linguistic terms such as ''very good'', ''good'', ''medium'', "bad", and "very bad". The first application of fuzzy set theory in the area of SPC goes back to Bradshaw who used fuzzy sets as a basis for the explanation of the measurement of the conformity of each product units with the specifications [2]. Since then, several researchers [3-7] attempt to use fuzzy set theory in the area of SPC and control charts. These researchers have used the fuzzy set theory for the construction of fuzzy control charts. Since fuzzy data ubiquitously exist in the modern manufacturing process, for monitoring its sample average and variance, it offered the fuzzy X and R control charts, whose fuzzy control limits are obtained on the basis of the results of the resolution identity, a well-known theory in the fuzzy set field. By using the fuzzy dominance approach, which compares the fuzzy averages and variances to their respective fuzzy control limits, they are capable of determining whether the manufacturing process is needed to be adjusted or not [7-12]. Additionally, some researchers studied "fuzzy rule based method" to construct their fuzzy control charts. [13] proposed firstly the fuzzy rule method for evaluating the fuzzy control charts in their paper. Their method is based on some rules which define all possible patterns of a process. Also, they applied the suggested method for fuzzy X-R control charts by using a symmetric triangular fuzzy number. [14] applied fuzzy statistical quality control to a calcite grinding plant in Niğde. The data, used to in this study, were obtained from Eti Aluminum Company in Konya-Turkey (Fig. 1).

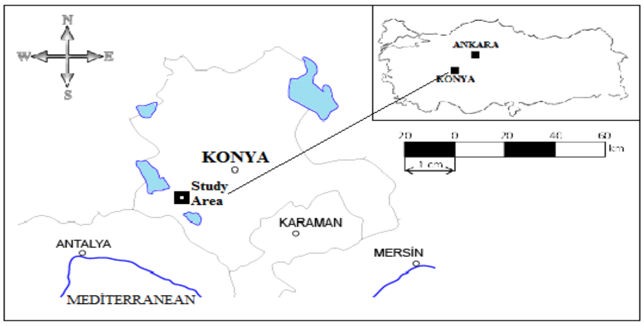

Figure 1. Location of studied area at Seydişehir/Turkey.

In this study, an application was presented for fuzzy method mean and range control charts. Fuzzy control charts were drawn using Eti Aluminum plant feeding material and concentrate data, and whether the process was in-control was analyzed. In addition, test samples were also made to chemically analysis with X-ray diffraction (XRD). This paper is organized as follows: In Section 2, general information about aluminum plant and chemical analysis of feeding material and concentrate were given. In addition, equations where the parameters used in drawing the X-R control charts were calculated. Finally, the main principles of fuzzy method control charts were explained. In Section 3, tables and graphics created with the data obtained from the plant were given. Finally, in Section 4, evaluation of the results and suggestions of solution presented to Eti Aluminum Company were given.

2. Materials and Methods

Eti Aluminum Company was established in 1973. It put into operation for production of alumina, raw aluminum and bulk products in 1973. Eti Aluminum was acquired by Cengiz Holding in 2005 in scope of privatization. Eti Aluminum, which is Turkey's biggest aluminum producer, is also one of the world's most important integrated plants capable of performing production from mine extraction until concentrate. The flowsheet of bayer process used in aluminum production is given in Fig. 2.

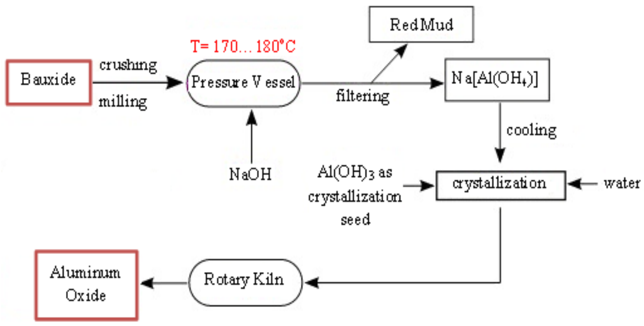

Figure 2. The flowsheet of bayer process used in Eti Aluminum Co.

The chemical analysis of the feeding material and concentrate were performed using lithium tetra borate fusion method (HCl digestion) and an atomic absorption spectrophotometer (AAS) analysis. Chemical composition of the samples was presented in Table 1.

Table 1. The results of chemical analysis of the samples.

Sample	Feeding Material (%)	Concentrate (Dry Hydrate) (%)
$Al(OH)_3$	-	99.84
Al_2O_3	59.09	65.28
Fe_2O_3	17.16	0.007
SiO_2	7.41	0.006
TiO_2	2.67	0.001
CaO	0.73	0.006
Na_2O	0.52	0.131
Moisture	3.57	2.28

2.1. Shewhart mean and range control charts

If process output can be measured with a numerical expression, control cards can be mentioned for the variables. The most commonly used control cards are the X-R control cards. These cards are used for variables whose process characteristics can be expressed numerically. In the mining industry, quality data are usually measurable and therefore X-R control cards are more suitable to use. While drawing the control cards, the data is marked according to the rational sampling. In this sampling, subgroups with a certain number (m) and size (n) are created. Subgroup samples can be sampled from the process at a specific time in a given time period or at different moments of the time period. The most important parameters that must be calculated in order to draw a control chart are the lower control limit (LCL) and the upper control limit (UCL). These limits refer to ± 3a distance from the center line showing the overall average of the subgroups, or 99.73% area of the standard normal distribution curve. As a result, these are calculated according to the following Equations (1-6).

$$\bar{\bar{X}} = \frac{\bar{X}_a + \bar{X}_b + \bar{X}_c + \cdots + \bar{X}_m}{m} \quad (1)$$

$$UCL_X = \bar{\bar{X}} + \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{X}} + A_2\bar{R} \quad (2)$$

$$LCL_X = \bar{\bar{X}} - \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{X}} - A_2\bar{R} \quad (3)$$

$$\bar{R} = \frac{R_a + R_b + R_c + \cdots + R_m}{m} \quad (4)$$

$$UCL_R = \bar{R} + 3d_3\frac{\bar{R}}{d_2} = D_4\bar{R} \quad (5)$$

$$LCL_R = \bar{R} - 3d_3\frac{\bar{R}}{d_2} = D_3\bar{R} \quad (6)$$

In the equations above; A_2, D_3 and D_4 are the constant values. In calculating the lower and upper control limits, suitable ones from these constant values in Table 2 are used [15-18].

Table 2. Constants for control charts [17].

Subgroup Size (n)	A_2	d_2	D_3	D_4
2	1.880	1.128	0.000	3.267
3	1.023	1.693	0.000	2.574
4	0.729	2.059	0.000	2.282
5	0.577	2.326	0.000	2.114
6	0.483	2.534	0.000	2.004
7	0.419	2.704	0.076	1.924

2.2. Fuzzy X and R control charts for triangular fuzzy numbers

Suppose a quality feature is defined as "approximately X". According to fuzzy sets, this value is converted to triangle fuzzy number (TFN) = (X_a, X_b, X_c). In this study, each observation is considered as a triangular fuzzy number $\tilde{X}_{ij} = (X_{a_{ij}}, X_{b_{ij}}, X_{c_{ij}})$; $i = 1,2,.,m$; $j = 1,2,.,n$ where *m* is the number of subgroup and *n* is the sample size in each subgroup. If $(X_{a_{i1}}, X_{b_{i1}}, X_{c_{i1}}), .., (X_{a_{in}}, X_{b_{in}}, X_{c_{in}})$ is a sample of *n* fuzzy observations in subgroup *i*, then $(\bar{X}_{a_i}, \bar{X}_{b_i}, \bar{X}_{c_i})$, the average of each sample and the range of the subgroup i is

$$\bar{X}_{a,b,c_i} = \frac{\sum_{j=1}^{n} X_{a,b,c_{ij}}}{n} \quad (7)$$

$$R_{a_i} = (\max X_{a_{ij}}) - (\min X_{c_{ij}}) \;;$$

$$R_{b_i} = (\max X_{b_{ij}}) - (\min X_{b_{ij}}) \;;$$

$$R_{c_i} = (\max X_{c_{ij}}) - (\min X_{a_{ij}}) \quad (8)$$

To draw the fuzzy X control chart, firstly; $\widetilde{CL}_{\bar{X}} = (\widetilde{CL}_a, \widetilde{CL}_b, \widetilde{CL}_c)$ must be calculated. $\widetilde{CL}_{\bar{X}}$, calculated according to Equation 9, is the fuzzy arithmetic mean of the observations. For calculating UCL_X, LCL_X, UCL_R and LCL_R, control limits of fuzzy mean and range graphics can be obtained using Equations (10-13) [17,19,20].

$$\widetilde{CL}_{\bar{X}} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) \quad ; \quad \bar{\bar{X}}_k = \frac{\sum_{i=1}^{m} \bar{X}_{k_i}}{m} \quad ;$$

$$\bar{R}_k = \frac{\sum_{i=1}^{m} R_{k_i}}{m} \quad (9)$$

$$\widetilde{UCL}_{\bar{X}} = \widetilde{CL}_{\bar{X}} + A_2\bar{R} = (\bar{\bar{X}}_{a,b,c} + A_2\bar{R}_{a,b,c}) \quad (10)$$

$$\widetilde{LCL}_{\bar{X}} = \widetilde{CL}_{\bar{X}} - A_2\bar{R} = (\bar{\bar{X}}_{a,b,c} - A_2\bar{R}_{a,b,c}) \quad (11)$$

$$\widetilde{UCL}_R = D_4\tilde{\bar{R}} = (D_4\bar{R}_a \; ; \; D_4\bar{R}_b \; ; \; D_4\bar{R}_c) \quad (12)$$

$$\widetilde{LCL}_R = D_3\tilde{\bar{R}} = (D_3\bar{R}_a \; ; \; D_3\bar{R}_b \; ; \; D_3\bar{R}_c) \quad (13)$$

2.3. Fuzzy process capability indices

Process capability indices (PCIs) are used to determine whether the products can be produced within the quality standards required by the customer. If the process meets the demands of the customer, this process is called “capable”. We determine how well a process fulfills the requests by process capability indices. The two most common PCI standards are C_p and C_{pk} [16]. The C_p index, which is the most used index in the literature, is the ratio of specification limits (USL-LSL) to process spread (6σ). The specification limits are the customer's demands. The larger the process variation, the smaller the C_p value. C_p is an indication of how much the process remains within the upper and lower specification limits. Process capability ratio C_p does not take into account some data in the process, so it usually does not reflect actual process performance. [21] developed the C_{pk} index to overcome problems caused by C_p. It shows how a process verifies its limits. The index is often used to associate “natural tolerances (3σ)” with the specification limits. C_{pk} shows how much the process is within specification limits according to the process average. C_{pk} shows how much the process is within the specification limits according to the process average. C_p, C_{pu}, C_{pl} and C_{pk} are calculated according to 14-15 Equations.

$$C_p = \frac{USL - LSL}{6\sigma} \quad ; \quad C_{pk} = \min\{C_{pu}, C_{pl}\} \quad (14)$$

$$C_{pu} = \frac{USL - \mu}{3\sigma} \quad ; \quad C_{pl} = \frac{\mu - LSL}{3\sigma} \quad (15)$$

where μ used in the calculation of Cpu and Cpl indicates the average process. The calculated Cp value gives us an idea of process efficiency. The relationship between quality conditions and Cp values is given in Table 3 [13,22,23]. When referring to specification limits (SLs), expressions such as "about" and "around" are preferred. Triangular fuzzy numbers (TFN) are used to convert this variable to fuzzy numbers. In addition, fuzzy process mean and standard deviation are calculated according to Equation 16. Fuzzy process capability indices are calculated as follows Equations 17-19.

$$\tilde{\mu} = \bar{\bar{X}} = \mathrm{TFN}(\mu_{1,2,3})\ ;\ \tilde{\sigma} = \frac{\bar{R}}{d_2} = \mathrm{TFN}(\sigma_{1,2,3}) \quad (16)$$

$$\tilde{C}_p = \frac{\widetilde{USL} - \widetilde{LSL}}{6\tilde{\sigma}} = \mathrm{TFN}(\frac{u_{1,2,3} - l_{1,2,3}}{6\sigma_{1,2,3}}) \quad (17)$$

$$\tilde{C}_{pu} = \frac{\widetilde{USL} - \tilde{\mu}}{3\tilde{\sigma}} = \mathrm{TFN}(\frac{u_{1,2,3} - \mu_{1,2,3}}{3\sigma_{1,2,3}}) \quad (18)$$

$$\tilde{C}_{pl} = \frac{\tilde{\mu} - \widetilde{LSL}}{3\tilde{\sigma}} = \mathrm{TFN}(\frac{\mu_{1,2,3} - l_{1,2,3}}{3\sigma_{1,2,3}}) \quad (19)$$

Table 3. Quality status and C_p values [22].

Quality Status	C_p Range
Super excellent	$C_p \geq 2.00$
Excellent	$1.67 \leq C_p < 2.00$
Satisfactory	$1.33 \leq C_p < 1.67$
Capable	$1.00 \leq C_p < 1.33$
Inadequate	$0.67 \leq C_p < 1.00$
Poor	$C_p < 0.67$

The percentage of fuzzy observation area (PA) is one of the important parameters to determine whether the process is whether the process is under control. According to this, if Xi is between the fuzzy upper and lower control limits, PA equals zero and the process is in-control (Fig. 3a). If Xi is outside the fuzzy upper or lower control limits, PA is equal to 1 and the process is out-of-control (Fig. 3b). Finally, if Xi intersects with fuzzy upper or lower control limits, PA is between 0 and 1. In this case, the process is very close to the limit values and must be monitored continuously (Fig. 3c) [20].

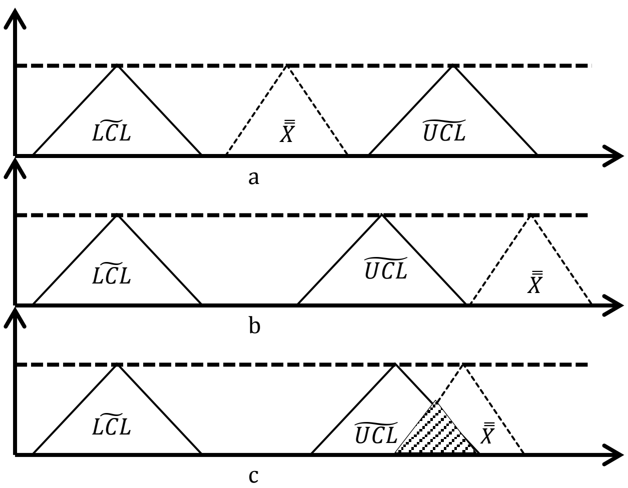

Figure 3. Process condition when the sample points and control limits are triangular fuzzy numbers **a**: in-control, **b**: out-of-control, **c**: rather-in-control or rather-out-of-control.

3. Results

Moisture, Fe_2O_3 and SiO_2 are among the most important parameters in aluminum production. For this reason, Eti Aluminum Company regularly monitors and controls the changes in these values. The company uses traditional Shewhart control charts at the stage of data control. In this study, fuzzy mean and range control charts were used to monitors the changes in the data of Eti Aluminum Co. Nowadays, this method has been widely used in many industrial areas (textile factories, machinery factories, ore dressing plants, etc.). 21-days data were collected to track the changes in the raw material delivered to the aluminum production facility in moisture, iron-oxide and quartz. The analyzed data were given in Figs. 4-6 (M = 21 (number of samples) and n = 4 (sample size)).

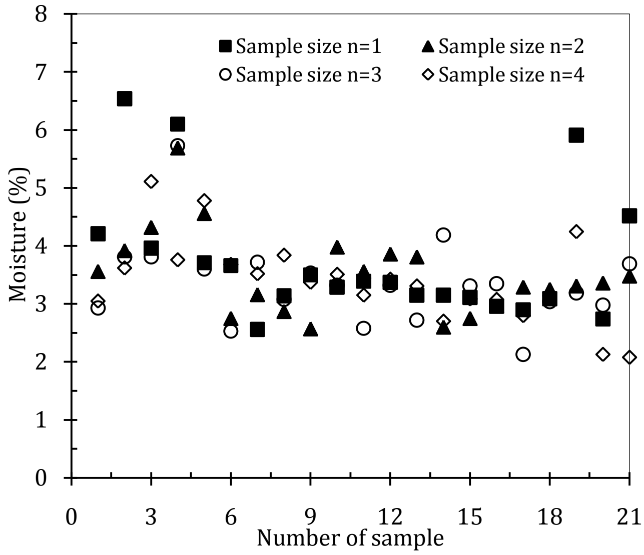

Figure 4. Case study data for moisture values.

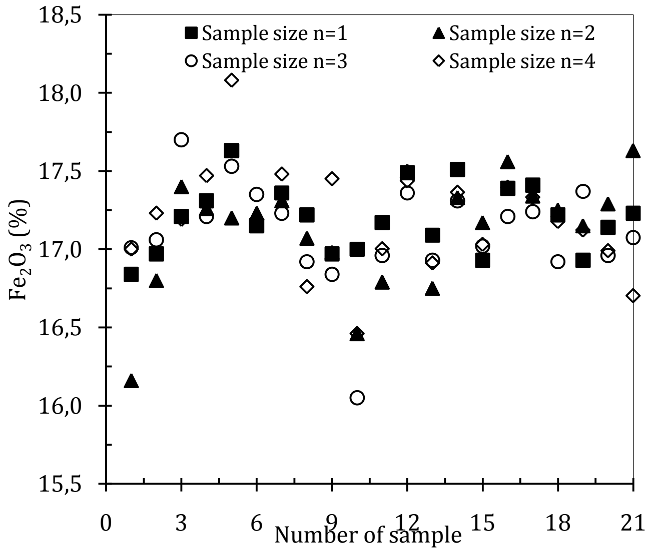

Figure 5. Case study data for Fe_2O_3 values.

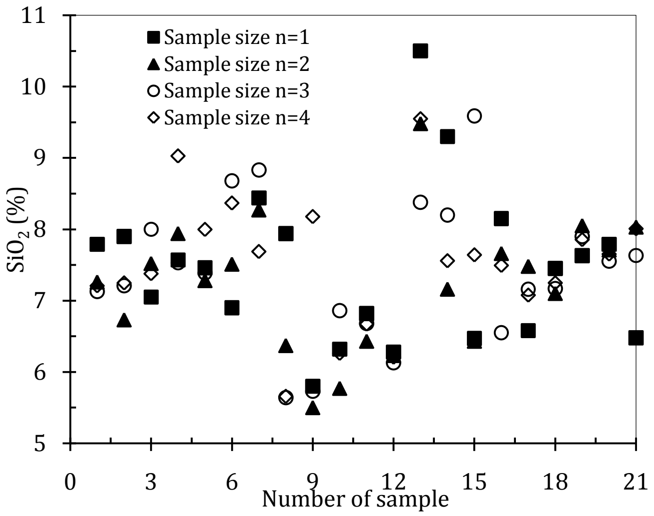

Figure 6. Case study data for SiO_2 values.

In this section, an example is given using the data from the aluminum production plant to prove that the fuzzy mean and range control charts give more accurate and illuminating results. The application was made on controlling feeding material and concentrate (dry hydrate) values in aluminum production plant. Thirty samples with sample sizes of four (total number of measurements 4x30 = 120) have been taken from the putative points that the process is in-control according to the quality control techniques applied by the factory.

Due to environmental conditions, each measurement item is expressed as a triangular fuzzy number (Xa, Xb, Xc). Tables 4 and 5 show the fuzzy triangular observations for feeding material and concentrate. Tables 6 and 7 show the result of each subgroup fuzzy mean and range calculation based on Equations 7-8 for feeding material and concentrate. Tables 6 and 7 also show the overall percentage of area (PA) of each observation remains outside the fuzzy control limits. When Tables 6 and 7 are examined carefully, it is seen that the vast majority of the analyzed data are in-control. However, some data is out-of-control (data which PA is greater than zero). These data should be regularly monitored and the necessary improvement works should be taken into effect immediately. Using equations 9-13, Fuzzy triangular control limits ($\widetilde{UCL}_{\bar{X}}, \widetilde{CL}_{\bar{X}}, \widetilde{LCL}_{\bar{X}}, \widetilde{UCL}_R, \widetilde{CL}_R$ and $\widetilde{LCL}_R$) are calculated as Table 8.

Table 4. Fuzzy triangular observations for feeding material.

	X_{a_1}	X_{b_1}	X_{c_1}	X_{a_2}	X_{b_2}	X_{c_2}	X_{a_3}	X_{b_3}	X_{c_3}	X_{a_4}	X_{b_4}	X_{c_4}
1	57.425	57.430	57.435	56.165	56.170	56.175	59.325	59.330	59.335	59.235	59.240	59.245
2	57.555	57.560	57.565	59.585	59.590	59.595	58.725	58.730	58.735	59.335	59.340	59.345
3	59.165	59.170	59.175	58.925	58.930	58.935	58.415	58.420	58.425	58.715	58.720	58.725
4	58.785	58.790	58.795	58.105	58.110	58.115	58.975	58.980	58.985	56.955	56.960	56.965
5	59.035	59.040	59.045	58.395	58.400	58.405	59.155	59.160	59.165	58.215	58.220	58.225
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
29	58.785	58.790	58.795	60.175	60.180	60.185	58.835	58.840	58.845	55.585	55.590	55.595
30	58.615	58.620	58.625	59.595	59.600	59.605	56.535	56.540	56.545	58.735	58.740	58.745

Table 5. Fuzzy triangular observations for concentrate (dry hydrate).

	X_{a_1}	X_{b_1}	X_{c_1}	X_{a_2}	X_{b_2}	X_{c_2}	X_{a_3}	X_{b_3}	X_{c_3}	X_{a_4}	X_{b_4}	X_{c_4}
1	99.849	99.854	99.859	99.847	99.852	99.857	99.842	99.847	99.852	99.855	99.860	99.865
2	99.849	99.854	99.859	99.858	99.863	99.868	99.836	99.841	99.846	99.835	99.840	99.845
3	99.840	99.845	99.850	99.860	99.865	99.870	99.851	99.856	99.861	99.847	99.852	99.857
4	99.840	99.845	99.850	99.858	99.863	99.868	99.835	99.840	99.845	99.835	99.840	99.845
5	99.826	99.831	99.836	99.847	99.852	99.857	99.852	99.857	99.862	99.835	99.840	99.845
.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.
29	99.794	99.799	99.804	99.820	99.825	99.830	99.802	99.807	99.812	99.813	99.818	99.823
30	99.821	99.826	99.831	99.816	99.821	99.826	99.826	99.831	99.836	99.748	99.753	99.758

Table 6. Fuzzy mean, range and PA for feeding material.

Subgroup	$\overline{X}_a$	$\overline{X}_b$	$\overline{X}_c$	$PA_{\overline{X}}$	R_a	R_b	R_c
1	58.038	58.043	58.048	0	3.15	3.16	3.17
2	58.800	58.805	58.810	0	2.02	2.03	2.04
3	58.805	58.810	58.815	0	0.74	0.75	0.76
4	58.205	58.210	58.215	0	2.01	2.02	2.03
5	58.700	58.705	58.710	0	0.93	0.94	0.95
6	59.193	59.198	59.203	0	2.15	2.16	2.17
7	58.480	58.485	58.490	0	1.20	1.21	1.22
8	60.678	60.683	60.688	0.158	2.67	2.68	2.69
9	60.863	60.868	60.873	0.389	3.21	3.22	3.23
10	59.436	59.441	59.446	0	3.43	3.44	3.45
11	59.705	59.710	59.715	0	1.23	1.24	1.25
12	60.284	60.289	60.294	0	0.44	0.45	0.46
13	57.259	57.264	57.269	0.225	2.06	2.07	2.08
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
30	58.370	58.375	58.380	0	3.05	3.06	3.07
Average	58.998	59.003	59.008		1.84	1.85	1.86

Table 7. Fuzzy mean, fuzzy range and PA for concentrate.

Subgroup	$\overline{X}_a$	$\overline{X}_b$	$\overline{X}_c$	$PA_{\overline{X}}$	R_a	R_b	R_c
1	99.848	99.853	99.858	0	0.01	0.01	0.02
2	99.845	99.850	99.855	0	0.01	0.02	0.03
3	99.850	99.855	99.860	0	0.01	0.02	0.03
4	99.842	99.847	99.852	0	0.01	0.02	0.03
5	99.840	99.845	99.850	0	0.02	0.03	0.04
6	99.848	99.853	99.858	0	0.03	0.04	0.05
7	99.850	99.855	99.860	0	0.01	0.02	0.03
8	99.850	99.855	99.860	0	0.02	0.03	0.04
9	99.848	99.853	99.858	0	0.03	0.04	0.05
10	99.833	99.838	99.843	0	0.03	0.04	0.05
11	99.819	99.824	99.829	0	0.03	0.04	0.05
12	99.799	99.804	99.809	0.92	0.07	0.08	0.09
13	99.829	99.834	99.839	0	0.04	0.05	0.06
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
30	99.802	99.807	99.812	0.47	0.07	0.08	0.09
Average	99.833	99.838	99.843		0.03	0.04	0.05

Table 8. Fuzzy triangular control limits.

	Feeding Material			Concentrate		
$\widetilde{UCL}_{\bar{X}}$	60.342	60.354	60.367	99.853	99.866	99.878
$\widetilde{CL}_{\bar{X}}$	59.081	59.086	59.092	99.833	99.838	99.843
$\widetilde{LCL}_{\bar{X}}$	57.653	57.651	57.649	99.813	99.811	99.809
$\widetilde{UCL}_{R}$	4.209	4.231	4.254	0.063	0.086	0.109
$\widetilde{CL}_{R}$	1.844	1.854	1.864	0.028	0.038	0.048
$\widetilde{LCL}_{R}$	0	0	0	0	0	0

When the PA values are examined in Tables 6 and 7, it is seen that the system is out-of-control for the data where the PA values are greater than zero (samples 8, 9 and 13 for feeding material, samples 18 and 30 for concentrate). This situation should be urgently analyzed and studies should be started to improve the process. To compare X-R control charts used to monitor quality parameters by the company and the fuzzy mean and range control charts proposed by me, the mean value of the fuzzy observations is calculated according to the Equation 20 as the crisp value to construct the X-R control charts [20].

$$E(\tilde{X}_{ij}) = \frac{X_{a_{ij}} + 2X_{b_{ij}} + X_{c_{ij}}}{4} \qquad (20)$$

Researchers who want to learn more about the mean and expected value of fuzzy numbers can refer to [24]. Figs. 7 and 8 show Shewhart mean and range control charts for feeding material and concentrate data. The use of the recommended fuzzy control charts instead of the statistical process control method currently used in Eti Aluminum Plant shows that it helps improve the quality of aluminum and dry hydrate as well as increase production efficiency.

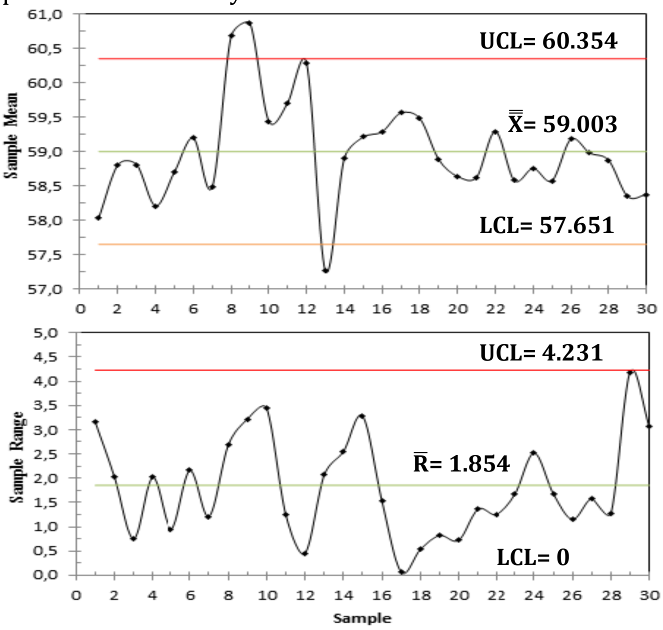

Figure 7. Mean and range charts for feeding material from aluminum production plant.

When Fig. 7, which is drawn using the feeding material data in the aluminum production plant, is examined, It is clearly seen that the samples 8, 9 and 13 are outside the control limits. In addition to, sample 12 is very close to the upper control limit. In the light of these data, the feeding material line should be carefully reviewed and the conditions causing the above errors should be corrected. If these errors are not corrected, this will cause larger problems in the future.

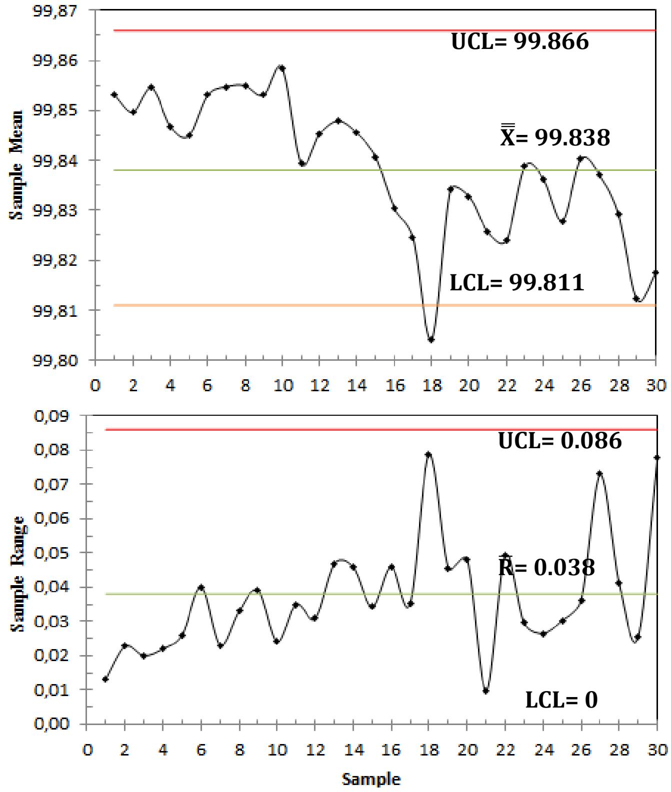

Figure 8. Mean and range charts for concentrate (dry hydrate) from aluminum production plant

When Fig. 8 is examined, analyses of the data from sample 18 for concentrate indicate that it is above the UCL and there are large fluctuations. The reason for this might be that the concentrate properties at the aluminum plant were not homogenous. Generally, it was not observed that there was a serious problem at this factory during the period of testing. However, the data in the concentrate line should be constantly monitored.

TFNs of (USL) and (LSL) were obtained from the company's quality control department of Eti Aluminum Co. The $\tilde{\mu}$, $\tilde{\sigma}$, $\tilde{C}_p$, $\tilde{C}_{pu}$ and $\tilde{C}_{pl}$ values were calculated using aluminum concentrate data. According to the data in Table 9, it is decided whether the process is in-control or out-of-control. The $\tilde{\mu}$, $\tilde{\sigma}$, $\tilde{C}_p$, $\tilde{C}_{pu}$ and $\tilde{C}_{pl}$ were calculated using equation 17-20. These indices are calculated as 2.917, 2.269, 1.775; 3.738, 2.907, 2.275 and 2.095 1.630, 1.375 respectively. While calculating these data, data collection was performed in three replications and calculated parameter values were greater than 1.33. As a result, the concentrated production efficiency and product quality of the aluminum production plant is adequate.

Table 9. Fuzzy capability indexes and values of $\tilde{\mu}$, $\tilde{\sigma}$, $\tilde{C}_p$, $\tilde{C}_{pu}$, $\tilde{C}_{pl}$ concentrate of aluminum plant.

	Concentrate (dry hydrate)		
$\widetilde{USL}$	99.990	99.995	100.00
$\widetilde{LSL}$	99.745	99.750	99.755
$\tilde{\mu}$	99.833	99.838	99.843
$\tilde{\sigma}$	0.014	0.018	0.023
$\tilde{C}_p$	2.917	2.269	1.775
$\tilde{C}_{pu}$	3.738	2.907	2.275
$\tilde{C}_{pl}$	2.095	1.630	1.375

4. Conclusion

A statistical process control study was carried out using the fuzzy process control technique of Eti Aluminum Company's feeding material and concentrate data and the quality grade of the plant was controlled. The results obtained from this study are summarized below;

With this study, it is shown that X-R control charts and fuzzy set theory can be used together. Primarily, fuzzy triangle numbers were calculated by using appropriate equations and then fuzzy mean and range control graphs were drawn with the help of calculated data. The number of errors data in the process is calculated using the amount of sample remaining above the UCL or below the LCL and the percentage of sample mean (PA).

Fuzzy mean and range control cards have been proposed to Eti Aluminum Company instead of the traditional control cards that it currently uses. With the use of the proposed fuzzy control charts, it is predicted that the quality of aluminum and dry hydrate will increase and also help increase production efficiency.

Statistical process control charts of the feeding material and concentrate data were drawn using the process capability index. Mean and range control charts created with feeding material and concentrate (dry hydrate) were observed to be in-control. In addition, the calculated $\tilde{C}_p$ values such as 2.917, 2.269 and 1.775 are higher than 1.33. Meanwhile, $\tilde{C}_{pu}$ and $\tilde{C}_{pl}$ values (3.738, 2.907, 2.275; 2.095, 1.630, 1.375 respectively) are higher than 1.33. According to these calculated data, it is clearly seen that the process is adequate. It has been determined that the mean and range control charts drawn using the feeding material and concentrate data are in-control.

When the results obtained from this study are investigated, it is clearly seen that the fuzzy statistical process control methods are very efficient in Eti Aluminum Company production line. Finally, company management was recommended to use the fuzzy statistical process control method instead of the Shewhart control charts currently use.

5. Acknowledgment

The author specially thanks Burak ÖZEN and the managements of Eti Aluminum Company in Seydişehir/Turkey for their help in providing the necessary data and author also gratitude General Directorate of Mineral Research and Exploration of Turkey for providing support.

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