Near Soft Connected Spaces

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Abstract

Near soft sets are considered as mathematical tools for dealing with ambiguities. In this study, we describe the near soft connectedness in near soft topological spaces, and introduce its concerned properties.

Keywords: Near set, soft set, near soft set, near soft topological spaces, near soft connectedness

Yakın Esnek Bağlantılı Uzaylar

Öz

Yakın esnek kümeler, belirsizlikler ile başa çıkmak için matematiksel araçlar olarak kabul edilir. Bu çalışmada, yakın esnek topolojik uzaylarda yakın esnek bağlantılığı tanımlamakta ve ilgili özelliklerini sunmaktayız.

Anahtar Kelimeler: Yakın küme, esnek küme, yakın esnek küme, yakın esnek topolojik uzaylar, yakın esnek bağlantılılık.

1. Introduction

Classical mathematics requires certain ideas. However, there are imprecise concepts in other fields such as engineering, medicine and economics. For example, "high (or low) salary" in the economy is not certain. Such uncertain concepts are called ambiguous. The noted theories on mathematical tools to deal with dubiousness are: theory of fuzzy sets, theory of rough sets, theory of soft sets and theory of near sets. We need to have some knowledge about these tools to begin.

Being one of them, the theory of fuzzy sets was introduced by Zadeh in 1965. In 2001, Maji et al. combined fuzzy sets and soft sets and introduced the concept of fuzzy soft sets, and they presented an application of fuzzy soft sets in a decision making problem. Recently, many authors (Aygunoğlu, et al., 2019; Beaula and Gunaseeli, 2014; Beaula

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and Priyanga, 2015; Beaula and Raja, 2015), etc. studied on fuzzy soft sets.

Rough set theory proposed by Pawlak in 1982 is a mathematical method used in reasoning and knowledge extraction for expert systems (Grzymala-Busse, 1988; Orlowska, 1994). The rough set theory has emerged as a new mathematical approach to uncertainty. Theory essence of each object is location of thought to be paired with information. This thought leads us to information systems and is the first point of origin of theory. Areas of application for the rough set approach include data mining, economics, robotics, chemistry, biology, medicine and image analysis.

Molodtsov explained the notion of soft set as a set theory in 1999 (Molodtsov, 1999). This new theory quickly attracted attention, and many researchers focused on the application of soft set theory to the links between other branches of mathematics. In 2003, the basic set-theoretic process and properties of the soft set given by Maji et al. (2002). Ali et al. (2009), Maji et al. (2003) examined and expanded the process and features in a more detailed way. In the continuation of this study, Aygunoğlu and Aygun (2011), Cagman et al. (2011), Zorlutuna et al. (2012) and then Georgiou et al. (2013) made important contributions to the development of the notion of soft topological space. Pei and Miao (2005) stated that each information system is a soft set and defined some special soft sets and their relationship with information systems. First, Maji et al. (2002) showed that soft sets are a very useful tool in decision making problems. Aktas and Cagman (2007) and Acar et al. (2010) gave the concept of soft group and soft ring respectively and examined their related features.

Proximity lets us create correlation between two things in our daily life. The first entry in the concept of intimacy in mathematics was made in 1908 by Riesz with his article on the intimacy of two sets (Riesz, 1908). The "Near Set" theory was developed in 2002 as a generalization of approximated sets by Pawlak (Pawlak and Peters, 2002-2007). In near sets, data is obtained using real-valued functions. Studies on near sets gained momentum after Peters' article on near sets "General Theory about Nearness of Objects" (Peters, 2007). Peters and Wasilewski published the foundations of near sets (Peters, 2009) and Wolski published the new approaches for near sets and approximated sets (Wolski, 2010). In addition, Peters conducted researches on computerized applications (Peters, 2005; Peters and Henry, 2006; Peters and Ramanna, 2009; Peters, et al., 2007; Peters and Naimpally, 2012) and especially image analysis (Hassanien, et al., 2009; Henry, 2010; Peters, 2010; Henry and Peters, TR-2010-017; Henry and Smith, TR-2012-021). Then, many researchers published on approach spaces and near sets. Ozturk and Inan identified near groups and studied their key features (Inan and Ozturk, 2012).

Many new set of definitions have been made through these sets. One of these is the near soft set that Tasbozan et al. defined in 2017 using the feature of the concept of near set. Tasbozan et al. studied basic properties the concepts of near soft set and near soft topological spaces in 2017 (Tasbozan et al., 2017). In 2019, the continue of the theoretical studies of near soft sets and near soft topological spaces were introduced by Ozkan (Ozkan, 2019).

Intuitively when we say a space is connected to indicate that it has "one piece", whereas a

space is disconnected when it has several disjoint, "independent pieces" (Vu, 2019). Following authors, such as Cantor (1883) for cantor set; Zhao (1968) for fuzzy sets, Pawlak (1982) for rough sets, Peters (2007) for near sets, Hussain (2014) for soft sets, Mahanta et al. (2012) for fuzzy soft sets defined connectedness, which is one of the important topics in mathematics.

In this study, we aim to study the concepts of near soft connectedness in near soft topological spaces. Moreover, it is thought that these structures, whose basic properties are examined, will form the basis for future studies on near soft sets.

2. Preliminaries

In this section, we remember the definitions and conclusions.

Let $\mathcal{O}$ be a set of perceived objects and $X \subseteq \mathcal{O}$. An object description is defined via a tuple of function values $\phi(x)$ regarding object $x \in X$. The important point to be considered is to select of function $\phi_i \in B$ is used to identify the corresponding object. (Inan and Ozturk, 2012).

A nearness approximation space (NAS) is denoted by $NAS = (\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \mathcal{V}_{N_r})$ which is defined with a set of perceived objects $\mathcal{O}$, a set of probe functions $\mathcal{F}$ representing object features, an indiscernibility relation $\sim_{B_r}$ defined relative to $B_r \subseteq B \subseteq \mathcal{F}$, a collection of partitions (families of neighborhoods) $N_r(B)$, and a neighborhood overlap function N_r. The relation $\sim_{B_r}$ is the usual indiscernibility relation from rough set theory restricted to a subset $B_r \subseteq B$. The subscript r denotes the cardinality of the restricted subset B_r, where we consider $\binom{|B|}{r}$, i.e., $|B|$ functions $i \in \mathcal{F}$

taken r at a time to define the relation $\sim_{B_r}$. The overlap function $\mathcal{V}_{N_r}$ is defined by $\mathcal{V}_{N_r}: \mathcal{P}(\mathcal{O}) \times \mathcal{P}(\mathcal{O}) \to [0,1]$, where $\mathcal{P}(\mathcal{O})$ is the powerset of $\mathcal{O}$. The overlap function $\mathcal{V}_{N_r}$ maps a pair of sets to a number in $[0,1]$, representing the degree of overlap between sets of objects with features defined by he probe functions $B_r \subseteq B$. This relation defines a partition of $\mathcal{O}$ into non-empty, pairwise disjoint subsets that are equivalence classes denoted by $\bar{a}_{B_r}$, where $\overline{a}_{B_r} = \{a' \in \mathcal{O}: a \sim_{B_r} a'\}$. These classes form a new set called the quotient set $\mathcal{O}/\sim_{B_r}$, where $\mathcal{O}/\sim_{B_r} = \{\overline{a}_{B_r}: a \in \mathcal{O}\}$ (Peters, 2007).

Definition 2.1 A soft set F_A on the universe U, E is a set of parameters, $\mathcal{P}(U)$ is the power set of U, and $A \subset E$, is defined by the set of ordered pairs $F_A = \{(e, f_A(e)): e \in E, f_A(e) \in \mathcal{P}(U)\}$, where $f_A: E \to \mathcal{P}(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$.

Here, f_A is called an approximate function of the soft set F_A. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have non-empty intersection (Caǧman and Enginoǧlu, 2010).

Let $N_r(B)(X)$ be a family of neighborhoods of a set $X \subseteq \mathcal{O}$.

Proposition 2.2 Every family of neighborhoods may be considered a as soft set (Tasbozan et al., 2017).

Let $\mathcal{O}$ is an initial universe set in soft set and $\mathcal{F}$ is a collection of all possible parameters with respect to $\mathcal{O}$ and $A, B \subseteq \mathcal{F}$.

Definition 2.3 Let $\sigma = F_B$ be a soft set over $\mathcal{O}$ and $NAS = (\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \mathcal{V}_{N_r})$ be a nearness approximation space. The lower and upper near approximation of $\sigma = F_B$ with respect to NAS, respectively, are denoted by

$N_{r^*}(\sigma) = F_{B*}$ and $N_r^*(\sigma) = F_B^*$, which are soft sets over with the set-valued mappings given by

$$F_*(\phi) = N_{r*}\big(F(\phi)\big) = \cup \{a \in \mathcal{O} : \overline{a}_{B_r} \subseteq F(\phi)\},$$

$$F^*(\phi) = N_r^*\big(F(\phi)\big) = \cup \{a \in \mathcal{O} : \overline{a}_{B_r} \cap F(\phi) \neq \emptyset\}$$

where all $\phi \in B$. For two operators N_{r*} and N_r^* on soft set, we say that these are the lower and upper near approximation operators, respectively. If $Bnd_{N_{r}(B)}(\sigma) \geq 0$, then the soft set σ is said to be a near soft set (Tasbozan et al., 2017).

We consider only near soft sets F_B over a universe $\mathcal{O}$ in which all the parameter sets B are the same.

$NSS(\mathcal{O}_B)$ indicates the set of all near soft sets over $\mathcal{O}$.

Example 2.4 Let $\mathcal{O} = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{\phi_1, \phi_2, \phi_3\} \subset \mathcal{F} = \{\phi_1, \phi_2, \phi_3, \phi_4\}$ denote a set of perceptual objects and a set of functions, respectively. Sample values of the $\phi_i, i = 1,2,3,4$ functions are shown in Table 1.

	a_1	a_2	a_3	a_4	a_5
ϕ_1	2	0,09	2	2	0,09
ϕ_2	0,01	0,01	1	1	1
ϕ_3	2	2	3	3	0
ϕ_4	0,09	0,09	2	2	2

Table 1

Let $\sigma = F_B$, $B = \{\phi_1, \phi_2, \phi_3\}$ be a soft set defined by

$$F_B = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_2, a_4, a_5\}),$$

$$(\phi_3, \{a_2, a_3, a_4, a_5\})\}.$$

Then for $r = 1$;

$$\overline{a_1}_{\phi_1} = \{a_1, a_3, a_4\}, \ \overline{a_2}_{\phi_1} = \{a_2, a_5\},$$
$$\overline{a_1}_{\phi_2} = \{a_1, a_2\}, \overline{a_3}_{\phi_2} = \{a_3, a_4, a_5\},$$
$$\overline{a_1}_{\phi_3} = \{a_1, a_2\}, \quad \overline{a_3}_{\phi_3} = \{a_3, a_4\}, \ \overline{a_5}_{\phi_3} = \{a_5\}$$

$$N_*(\sigma) = \{(\phi_1, \emptyset), (\phi_2, \emptyset), (\phi_3, \{a_3, a_4, a_5\})\}$$

$$N^*(\sigma) = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_1, a_3\}),$$

$$(\phi_3, \{a_1, a_3, a_5\})\}.$$

$Bnd_{N_{r}(B)}(\sigma) \geq 0$, and then σ is a near soft set.

But $\lambda = G_B$, $\lambda = \{(\phi_1, \{a_1, a_4, a_5\}),$

$(\phi_2, \{a_1, a_3\}), (\phi_3, \{a_1\})\}$ is not a near soft set because $N_*(\lambda) = \emptyset$.

Definition 2.5 Let $F_B \in NSS(\mathcal{O}_B)$ and $a \in \mathcal{O}$. We say that $a \in F_B$ read as a belongs to the near soft set F_B, whenever $a \in F(\phi)$ for all $\phi \in B$.

Note that for $a \in \mathcal{O}$, $a \notin F_B$ if $a \notin F(\phi)$ for some $\phi \in B$ (Ozkan, 2019).

Definition 2.6 Let $F_B, G_B \in NSS(\mathcal{O}_B)$. Then F_B is a near soft subset of G_B, denoted by $F_B \subseteq G_B$ if $N_*(F_B) \subseteq N_*(G_B)$ for all $\phi \in B$, i.e., $N_*(F(\phi), B) \subseteq N_*(G(\phi), B)$ for all $\phi \in B$.

F_B is called a near soft superset of G_B; denoted by $F_B \supseteq G_B$; if G_B is a near soft subset of F_B (Tasbozan et al., 2017).

Definition 2.7 Let $F_B, G_B \in NSS(\mathcal{O}_B)$. If near soft sets F_B and G_B are subsets of each other, then they are called equal, denoted by $F_B = G_B$ (Ozkan, 2019).

Definition 2.8 Let $\mathcal{O}$ be an initial universe set and F be a universe set of parameters. Let $F_B, G_B \in NSS(\mathcal{O}_B)$ and $A, B \subset \mathcal{F}$.

  1) The extended intersection of F_A and G_B over $\mathcal{O}$ is the near soft set H_C, where $C = A \cup B$, and $\forall \phi \in C$,

$$H(\phi) = \begin{cases} F(\phi) & if \; \phi \in A - B \\ G(\phi) & if \; \phi \in B - A \\ F(\phi) \cap G(\phi) & if \phi \in A \cap B \end{cases}$$

We write $F_A \cap G_B = H_C$.

  2) The restricted intersection of F_A and G_B over $\mathcal{O}$ is the near soft set H_C, where $C = A \cap B$, and $H(\phi) = F(\phi) \cap G(\phi)$ for all $\phi \in C$. We write $F_A \cap G_B = H_C$.

  3) The extended union of F_A and G_B over $\mathcal{O}$ is the near soft set H_C, where $C = A \cup B$, and $\forall \phi \in C$,

$$H(\phi) = \begin{cases} F(\phi) & if \; \phi \in A - B \\ G(\phi) & if \; \phi \in B - A \\ F(\phi) \cup G(\phi) & if \phi \in A \cap B \end{cases}$$

We write $F_A \cup G_B = H_C$.

  4) The restricted union of F_A and G_B over $\mathcal{O}$ is the near soft set H_C, where $C = A \cap B$, and $H(\phi) = F(\phi) \cup G(\phi)$ for all $\phi \in C$.

We write $F_A \cup G_B = H_C$ (Ozkan, 2019).

Definition 2.9 Let $F_B \in NSS(\mathcal{O}_B)$. Then F_B is called:

  1) a null near soft set, denoted by $\emptyset_N$ if $F(\phi) = \emptyset$, for all $\phi \in B$.

  2) a whole near soft set, denoted by $\mathcal{O}_N$ if $F(\phi) = \mathcal{O}$, for all $\phi \in B$ (Tasbozan et al., 2017).

Definition 2.10 The relative complement of a near soft set F_B denoted by F_B^c, is defined where $F^c(\phi) = \mathcal{O} - F(\phi)$ for all $\phi \in B$ (Tasbozan et al., 2017).

Definition 2.11 Let $F_B \in NSS(\mathcal{O}_B)$. The $\mathcal{P}(F_B)$ set is defined as the near soft power set of F_B as follows:

$$\mathcal{P}(F_B) = \{F_B^i : F_B^i \subseteq F_B, i \in I \subseteq \mathbb{N}\}$$

We use the formula $|\mathcal{P}(F_B)| = 2^{\sum_{\phi \in B}|F(\phi)|}$ to find the number of elements of this near soft power set. Where $|F(\phi)|$ is the cardinality of $F(\phi)$.

Example 2.12 Let $= \{a_1, a_2, a_3, a_4, a_5\} \supset X = \{a_1, a_2, a_5\}$, $B = \{\phi_1, \phi_3\} \subset \mathcal{F} = \{\phi_1, \phi_2, \phi_3, \phi_4\}$. According to Table 1;

Let $\sigma = F_B$, $B = \{\phi_1, \phi_2\}$ be a soft set defined by

$F_B = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_2, a_5\})\}$. Since $Bnd_{N_r(B)}X(\sigma) \geq 0$, so σ is a near soft set. Then, since $|F(\phi_1)| = 2$ and $|F(\phi_2)| = 2$, $|\mathcal{P}(F_B)| = 2^{(2+2)} = 16$. All subsets of F_B are given below.

$F_B^1 = \{(\phi_1, \{a_1\})\}$,

$F_B^2 = \{(\phi_1, \{a_2\})\}$,

$F_B^3 = \{(\phi_1, \{a_1, a_2\})\}$,

$F_B^4 = \{(\phi_2, \{a_2\})\}$,

$F_B^5 = \{(\phi_2, \{a_5\})\}$,

$F_B^6 = \{(\phi_2, \{a_2, a_5\})\}$,

$F_B^7 = \{(\phi_1, \{a_1\}), (\phi_2, \{a_2\})\}$,

$F_B^8 = \{(\phi_1, \{a_1\}), (\phi_2, \{a_5\})\}$,

$F_B^9 = \{(\phi_1, \{a_1\}), (\phi_2, \{a_2, a_5\})\}$,

$F_B^{10} = \{(\phi_1, \{a_2\}), (\phi_2, \{a_2\})\}$,

$F_B^{11} = \{(\phi_1, \{a_2\}), (\phi_2, \{a_5\})\}$,

$F_B^{12} = \{(\phi_1, \{a_2\}), (\phi_2, \{a_2, a_5\})\}$,

$F_B^{13} = \{(\phi_1, \{a_1, a_2\}), (\phi_2, \{a_2\})\}$,

$F_B^{14} = \{(\phi_1, \{a_1, a_2\}), (\phi_1, \{a_5\})\}$,

$F_B^{15} = \emptyset_N, F_B^{16} = F_B$.

Definition 2.13 Let $F_B \in NSS(\mathcal{O}_B)$. A near soft topology on F_B, denoted by τ, is a collection of near soft subsets of F_B, B is the non-empty set of parameters, and then $\mathcal{O}_B$ is said to be a near soft topology on F_B if the following properties are satisfied:

1) $\emptyset_N, F_B \in \tau$ where $\emptyset(\phi) = \emptyset$ and $F(\phi) = F$, for all $\phi \in B$.

2) The intersection of any two near soft sets in τ belongs to τ.

3) The union of any number of near soft sets in τ belongs to τ.

The triplet $(\mathcal{O}, B, \tau)$ is said to be a near soft topological space (Tasbozan et al., 2017).

We type nsts in place of near soft topological space.

Definition 2.14 Let $(\mathcal{O}, B, \tau)$ be a nsts over $\mathcal{O}_B$, then the members of τ are said to be near soft open sets $\mathcal{O}$ (Tasbozan et al., 2017).

Definition 2.15 Let $(\mathcal{O}, B, \tau)$ be a nsts over $\mathcal{O}_B$. A near soft subset of $\mathcal{O}_B$ is called near soft closed if its complement is open and a member of τ (Tasbozan et al., 2017).

Example 2.16. If we consider the near soft sets given in the Example 2.12, then $\tau = \mathcal{P}(F_B)$ and $\tau = \{\emptyset_N, F_B\}$ sets are a near soft topology on F_B. These near soft topologies are called discrete near soft topology and near soft indiscrete topology, respectively.

Example 2.17. If we consider the near soft sets given in the Example 2.12, then $\tau = \{\emptyset_N, F_B, F_B^1, F_B^5, F_B^8, F_B^9, F_B^{11}, F_B^{14}\}$ is a near soft topology on F_B.

Definition 2.18 Let $(\mathcal{O}, B, \tau)$ be a nsts and $F_B \in NSS(\mathcal{O}_B)$. Then:

1) The set $\cap \{G_B \supseteq F_B : G_B$ is a near soft closed set of $\mathcal{O}_B\}$ is called the near soft closure of F_B in $\mathcal{O}_B$, denoted by $cl_n(F_B)$.

2) The set $\cup \{C_B \subseteq F_B : C_B$ is a near soft open set of $\mathcal{O}_B\}$ is called the near soft interior of F_B in $\mathcal{O}_B$, denoted by $int_n(F_B)$ (Ozkan, 2019).

Definition 2.19 Let $F_B \in NSS(\mathcal{O}_B)$. If for the element $\phi \in B, F(\phi) \neq \emptyset$ and $F(\phi') = \emptyset$ for all $\phi' \in B - \{\phi\}$, then F_B is called a near soft point in $\mathcal{O}_B$, denoted by ϕ_F^N (Ozkan, 2019).

Definition 2.20 Let $(\mathcal{O}, B, \tau)$ be a nsts and $G_B \in NSS(\mathcal{O}_B)$. If there exists a near soft open set H_B such that $\phi_F^N \in H_B \subseteq G_B$, then G_B is called a near soft neighbourhood (written near soft nbd) of the near soft point $\phi_F^N \in \mathcal{O}_B$ (Ozkan, 2019).

Definition 2.21 Let $(\mathcal{O}, B, \tau)$ be a nsts and $G_B \in NSS(\mathcal{O}_B)$. If there exists a near soft open set H_B such that $F_B \subseteq H_B \subseteq G_B$, then G_B is called a near soft nbd of the near soft set F_B (Ozkan, 2019).

Definition 2.22 Let $NSS(\mathcal{O}_A)$ and $NSS(V_B)$ be the families of all near soft sets over $\mathcal{O}$ and V, respectively. The mapping f is called a near soft mapping from $\mathcal{O}$ to V, denoted by $f: NSS(\mathcal{O}_A) \to NSS(V_B)$, where $u: \mathcal{O} \to V$ and $p: A \to B$ are two mappings.

1) Let F_A be a near soft set in $NSS(\mathcal{O}_A)$. Then for all $\omega \in B$ the image of F_A under f, written as $f(F_A) = (f(F), p(A))$, is a near soft set in $NSS(V_B)$ defined as follows:

$$f(F)(\omega)$$
$$= \begin{cases} \bigcup_{\phi \in p^{-1}(\omega) \cap A} u(F(\phi)), & p^{-1}(\omega) \cap A \neq \emptyset \\ \emptyset, & otherwise. \end{cases}$$

2) Let G_B be a near soft set in $NSS(V_B)$. Then for all $\phi \in A$ the near soft inverse image of G_B under f, written as $f^{-1}(G_B) = (f^{-1}(G), p^{-1}(B))$, is a near soft set in $NSS(\mathcal{O}_A)$ defined as follows:

$$f^{-1}(G)(\phi)$$
$$= \begin{cases} u^{-1}\left(G(p(\phi))\right), & p(\phi) \in B \\ \emptyset, & otherwise \end{cases}$$

(Ozkan, 2019).

Definition 2.23 A near soft map $f: NSS(\mathcal{O}_A) \to NSS(V_B)$ is said to be injective (resp. surjective, bijective) if f and p are injective (resp. surjective, bijective).

Throughout the paper, the space O and V stand for near soft topological spaces with the assumed $((\mathcal{O}, A, \tau)$ and $(V, B, \tau^*))$ unless otherwise stated and a near soft mapping $f: \mathcal{O} \to V$ stands for a mapping, where $f: (\mathcal{O}, A, \tau) \to (V, B, \tau^*)$, $u: \mathcal{O} \to V$, and $p: A \to B$ are assumed mappings unless otherwise stated and $A, B \subset \mathcal{F}$.

Definition 2.24 Let $(\mathcal{O}, A, \tau)$ and (V, B, τ^*) be two nstss. Let u be a mapping from $\mathcal{O}$ to V and p be a mapping from A to B. Let f be a mapping from $NSS(\mathcal{O}_A)$ to $NSS(V_B)$ and $\phi_F^N \in \mathcal{O}_A$. Then:

1) f is near soft continuous at $\phi_F^N \in \mathcal{O}_A$ if for each $G_B \in N_{\tau^*}(f(\phi_F^N))$, there exists a $H_A \in N_\tau(\phi_F^N)$ such that $f(H_A) \subseteq G_B$.

2) f is near soft continuous on $\mathcal{O}_A$ if f is near soft continuous at each near soft point in $\mathcal{O}_A$ (Ozkan, 2019).

Theorem 2.25 Let $(\mathcal{O}, A, \tau)$ and (V, B, τ^*) be two nstss. For a function $f: NSS(\mathcal{O}_A) \to NSS(V_B)$, the following are equivalent:

1) f is near soft continuous;

2) $\forall F_A \in NSS(\mathcal{O}_A)$, and the inverse image of every near soft nbd of $f(F_A)$ is a near soft nbd of F_A;

3) $\forall F_A \in NSS(\mathcal{O}_A)$ and for each near soft nbd H_B of $f(F_A)$, there is a near soft nbd G_A of F_A such that $f(G_A) \subseteq H_B$ (Ozkan, 2019).

Definition 2.26 Let $(\mathcal{O}, A, \tau)$ and (V, B, τ^*) be two nstss and $f: \mathcal{O} \to V$. f be a near soft mapping on $\mathcal{O}_A$. If the image of each near soft open (resp. near soft closed) set in $\mathcal{O}_A$ is a near soft open (resp. near soft closed) set in V_B, then f is called a near soft open (resp. near soft closed) function (Ozkan, 2019).

3. Near Soft Connected Spaces

We introduce the notions of near soft sets, their essential features, and actions like near soft disjoint sets, near soft separated sets and near soft connected spaces.

Thanks to the following definition, the theoretical studies of the near soft set will be accelerated and applications will be more comfortable.

Definition 3.1 Two near soft sets F_B and G_B over $\mathcal{O}_B$ are near soft disjoint sets, if $F_B \cap G_B = \emptyset_N$.

The near soft disjoint sets denoted by $F_B | G_B$.

Example 3.2 Let $(\mathcal{O}, B, \tau)$ be a nsts. Then for each $F_B \in NSS(\mathcal{O}_B)$, $F_B | F_B^c$.

Definition 3.3 Let $(\mathcal{O}, B, \tau)$ be a nsts. Two non-null subsets F_B, G_B over $\mathcal{O}_B$ are said to be near soft separated sets of $\mathcal{O}_B$ if $F_B \cap cl_n(G_B) = \emptyset_N$ and $cl_n(F_B) \cap G_B = \emptyset_N$.

Example 3.4 According to the τ topology on F_B given in Example 2.17, and hence $(\mathcal{O}, B, \tau)$ is a nsts. Clearly, the near soft closed sets $\emptyset_N, F_B, (F_B^1)^c, (F_B^5)^c, (F_B^8)^c, (F_B^9)^c, (F_B^{11})^c, (F_B^{14})^c$. Then, let us take $H_B = \{(\phi_1, \{a_1\})\}$; then $cl(H_B) = (F_B^1)^c$ and $G_B = \{(\phi_1, \{a_2\}), (\phi_2, \{a_5\})\}$; then $cl(G_B) = (F_B^{11})^c$ and $H_B \cap cl_n(G_B) = \emptyset_N$ and $cl_n(H_B) \cap G_B = \emptyset_N$, so H_B and G_B are near soft separated sets and also near soft disjoint sets of $\mathcal{O}_B$.

Remark 3.5 In a nsts any two near soft separated sets are near soft disjoint sets but two near soft disjoint sets are not necessary for soft separated sets.

Example 3.6 According to the τ topology on F_B given in Example 2.17,
$$H_B = \{(\phi_1, \{a_1\}), (\phi_2, \{a_2, a_5\})\},$$
$$G_B = \{(\phi_1, \{a_5\}), (\phi_2, \{a_1\})\}$$

so H_B and G_B are near soft disjoint sets and but not near soft separated of $\mathcal{O}_B$.

Example 3.7 Although each near soft set is disjoint with its own complement, it may not be a near soft separated set.

The following theorem gives us in which case the near soft disjoint set can be a near soft separated set.

Theorem 3.8 Two near soft closed (near soft open) subsets F_B, G_B of a nsts $(\mathcal{O}, B, \tau)$ are near soft separated sets if and only if they are near soft disjoint sets.

Proof. It's clear that any two near soft separated sets are near soft disjoint sets.

Conversely, let F_B, G_B are both near soft disjoint sets and near soft closed sets then so that $cl_n(F_B) \cap G_B = \emptyset_N$ and $F_B \cap cl_n(G_B) = \emptyset_N$. Showing that F_B, G_B are near soft separated. If F_B, G_B are both near soft disjoint sets and near soft open sets then F_B^c, G_B^c are both near soft closed. Then $F_B \subseteq G_B^c$, $G_B \subseteq F_B^c$, $cl_n(F_B) \subseteq cl_n(G_B)^c = G_B^c$ and $cl_n(G_B) \subseteq cl_n(F_B)^c = F_B^c$. Then $cl_n(F_B) \cap G_B = \emptyset_N$ and $cl_n(G_B) \cap F_B = \emptyset_N$, so F_B and G_B are near soft separated sets.

Definition 3.9 Let $(\mathcal{O}, B, \tau)$ be a nsts and $F_B \in NSS(\mathcal{O}_B)$. Then F_B is said to be near soft connected set, if there does not exist a pair G_B and H_C of non-null near soft disjoint subsets of $(\mathcal{O}, B, \tau)$ such that $F_B = G_B \cup H_C$ and $G_B \cap H_C = \emptyset_N$, otherwise F_B is said to be near soft disconnected set.

In above definition, if we take $\mathcal{O}_B$ instead of F_B, then the $(\mathcal{O}, B, \tau)$ is called near soft connected (disconnected) space.

We write non-empty near soft set instead of non-null near soft set.

Example 3.10 From Example 2.17 $F_B = \{(\phi_1, \{a_2, a_3\}), (\phi_2, \{a_2, a_4, a_5\}), (\phi_3, \{a_2, a_3, a_4, a_5\})$ is a near soft set, $G_A = \xi\{(\phi_1, \{a_2\}), (\phi_2, \{a_2, a_4\}), (\phi_3, \{a_3, a_4\})\}$ and $L_P = \{(\phi_1, \{a_3\}), (\phi_2, \{a_5\}), (\phi_3, \{a_2, a_5\})\}$ are near soft disjoint subsets according to Table 1. Then $F_B = G_A \cup L_P$ and $G_A \cap L_P = \emptyset_N$. Hence F_B is a near soft disconnected set.

Example 3.11 Each near soft indiscrete space is near soft connected, and that each near soft discrete non-trivial space is not near soft connected.

The next result gives an equivalent formulation of near soft connectedness.

Theorem 3.12 Let $(\mathcal{O}, B, \tau)$ be a nsts and $F_B \in NSS(\mathcal{O}_B)$. F_B is a near soft connected

set if and only if F_B cannot be written as a union of two near soft disjoint sets.

Proof. If $F_B = \emptyset_N$, it is clear. So let's be $F_B \neq \emptyset_N$. Let F_B be a near soft connected set and near soft disjoint sets of $G_B|H_C$ meet the $F_B \subseteq G_B \cup H_C$ condition.

$$G_B \cap H_C \subseteq G_B \cap cl_n(H_C) = \emptyset_N \text{ and } G_B \cap$$
$$H_C \subseteq cl_n(G_B) \cap H_C = \emptyset_N.$$

It is obtained from the coverage that the F_B is near soft disconnected. But this situation contradicts by hypothesis. So F_B cannot be written as a union of two near soft disjoint sets.

Conversely, suppose that F_B should not be written as a union of two near soft connected sets, but it is a near soft disconnected. Then, there are G_B and H_C in $\mathcal{O}_B$, such that $F_B = G_B \cup H_C$ and $G_B \cap H_C = \emptyset_N$. $G_B \cap cl_n(H_C) = cl_n(G_B) \cap H_C = \emptyset_N$ is obtained due to the closure of a near soft closed set is itself. This contradicts by hypothesis.

Lemma 3.13 Suppose then $F_B \subseteq G_B \subseteq (\mathcal{O}, B, \tau)$, then $cl_n^{G_B}(F_B) = G_B \cap cl_n^{\mathcal{O}_B}(F_B)$.

($cl_n^{G_B}(F_B)$: *near soft closure of F_B in G_B and $cl_n^{\mathcal{O}_B}(F_B)$: near soft closure of F_B in $\mathcal{O}_B$.*)

Proof. $G_B \cap cl_n^{\mathcal{O}_B}(F_B)$ is a near soft closed set in G_B that contains F_B, so

$$cl_n^{G_B}(F_B) \subseteq G_B \cap cl_n^{\mathcal{O}_B}(F_B) \ \dots\dots\dots(1).$$

On the other hand, suppose $a \in G_B \cap cl_n^{\mathcal{O}_B}(F_B)$. To show that $a \in cl_n^{G_B}(F_B)$, pick a near soft open set H_B in G_B in that contains a. We need to show $F_B \cap H_B \neq \emptyset_N$. There is a near soft open set L_B in $\mathcal{O}_B$ such that $L_B \cap G_B = H_B$. Since $a \in cl_n^{\mathcal{O}_B}(F_B)$, we have that $\emptyset_N \neq L_B \cap F_B = L_B \cap (G_B \cap F_B) = (L_B \cap G_B) \cap F_B = H_B \cap F_B$. Thus,

$$a \in cl_n^{G_B}(F_B) \ \dots\dots\dots\dots\dots(2).$$

As a result, $cl_n^{G_B}(F_B) = G_B \cap cl_n^{\mathcal{O}_B}(F_B)$ is obtained from (1) and (2).

The following technical theorem and its corollary are very useful in working with near soft connectedness in near soft subspaces.

Theorem 3.14 Let $(\mathcal{O}, B, \tau)$ be a nsts over $\mathcal{O}_B$ and V be a non-empty subset of $\mathcal{O}$. If F_B and G_B are near soft sets in V_B, then F_B and G_B are near soft separation sets of V_B if and only if F_B and G_B are near soft separation sets of $\mathcal{O}_B$.

Proof. From Lemma 3.13, we have $cl_n^{V_B}(G_B) = V_B \cap cl_n^{\mathcal{O}_B}(G_B) = \emptyset_N$, so $cl_n^{V_B}(G_B) \cap F_B = \emptyset_N$ iff $F_B \cap cl_n^{\mathcal{O}_B}(G_B \cap V_B) = \emptyset_N$ iff $(F_B \cap V_B) \cap cl_n^{\mathcal{O}_B}(G_B) = \emptyset_N$ iff $(cl_n^{\mathcal{O}_B}(G_B) \cap F_B) = \emptyset_N$.

Similar, we have

$$cl_n^{V_B}(F_B) \cap G_B = \left(cl_n^{\mathcal{O}_B}(F_B) \cap V_B\right) \cap G_B$$
$$= (cl_n^{\mathcal{O}_B}(F_B) \cap G_B)$$

Thus, the theorem holds.

Attention: According to Theorem 3.14, H_B is near soft disconnected iff $H_B = F_B \cup G_B$ where F_B and G_B are non-empty near soft separated sets in H_B iff $H_B = F_B \cup G_B$ where F_B and G_B are non-empty near soft separated sets in $\mathcal{O}_B$. Theorem 3.14 is very useful because it means that we don't have to distinguish here between "near soft separated in H_B" and "near soft separated in $\mathcal{O}_B$"- because these are equivalent.

In contrast, if we say that H_B is near soft disconnected when H_B is the union of two near soft disjoint sets, non-empty near soft open (or closed) sets F_B, G_B in H_B, then phrase "in H_B" cannot be omitted: the sets F_B, G_B, might not be near soft open (or closed) in $\mathcal{O}_B$.

The following lemma makes a simple but very useful observation.

Lemma 3.15 Let $(\mathcal{O}, B, \tau)$ be a nsts over O_B, and V is a non-empty subset of $\mathcal{O}$ such that (V, B, τ^*) is a near soft connected space. If F_B and G_B are a near soft separation of $\mathcal{O}_B$ such that $V_B \subseteq F_B \cup G_B$, then $V_B \subseteq F_B$ or $V_B \subseteq G_B$.

Proof. Since $V_B \subseteq F_B \cup G_B$, we have $V_B = (V_B \cap F_B) \cup (V_B \cap G_B)$. Theorem 3.14, $V_B \cap F_B$ and $V_B \cap G_B$ are a near soft separation of V_B. Since (V, B, τ^*) is a near soft connected space, we have $V_B \cap F_B = \emptyset_N$ or $V_B \cap G_B = \emptyset_N$. Thus, $V_B \subseteq F_B$ or $V_B \subseteq G_B$.

Near soft connectedness in the subspace can also be characterized by the following theorem.

Theorem 3.16 Let $(\mathcal{O}, B, \tau)$ be a nsts over O_B and (V, B, τ^*) be a near soft connected subspace of $(\mathcal{O}, B, \tau)$. If $V_B \subseteq A_B \subseteq cl_n(V_B)$, then (A, B, τ_A) is a near soft connected subspace of $(\mathcal{O}, B, \tau)$. In particular, $cl_n(V_B)$ is a near soft connected subspace of $(\mathcal{O}, B, \tau)$.

Proof. Let G_B and L_B be a near soft separation sets of (A, B, τ_A). By Lemma 3.15, we have $F_B \subseteq G_B$ or $F_B \subseteq L_B$. Without loss of generality, we may assume that $F_B \subseteq G_B$. By Theorem 3.14, we have $cl_n(G_B) \cap L_B = \emptyset_N$, which is a contradiction.

Theorem 3.17 Let $(\mathcal{O}, B, \tau)$ be a nsts. Then the following provisions are equivalent:

1) $(\mathcal{O}, B, \tau)$ has a near soft separation,

2) There exist two near soft closed disjoint sets F_B and G_B such that $F_B \cup G_B = \mathcal{O}_B$,

3) There exist two near soft open disjoint sets F_B and G_B such that $F_B \cup G_B = \mathcal{O}_B$,

4) $(\mathcal{O}, B, \tau)$ has a proper near soft open and near soft closed set in $\mathcal{O}_B$.

Proof. 1)$\Rightarrow$2). Let $(\mathcal{O}, B, \tau)$ have a near soft separation F_B and G_B. Then $F_B \cap G_B = \emptyset_N$ and

$$cl_n(F_B) = cl_n(F_B) \cap (F_B \cup G_B) =$$
$$(cl_n(F_B) \cap F_B) \cup (cl_n(F_B) \cap G_B) = F_B.$$

Hence, F_B is a near soft closed set in $\mathcal{O}_B$. Similar, we can see that G_B is also a near soft closed set in $\mathcal{O}_B$.

2)$\Rightarrow$3). Let $(\mathcal{O}, B, \tau)$ has a near soft separation F_B and G_B such that F_B and G_B are near soft closed in $\mathcal{O}_B$. Then F_B^c and G_B^c are near soft sets in $\mathcal{O}_B$. Then it is easy to see that $F_B^c \cap G_B^c = \emptyset_N$ and $F_B^c \cup G_B^c = \mathcal{O}_B$.

3)$\Rightarrow$4). Let $(\mathcal{O}, B, \tau)$ has a near soft separation F_B and G_B such that F_B and G_B are near soft open in $\mathcal{O}_B$. Then F_B^c and G_B^c are near soft sets in $\mathcal{O}_B$. Then F_B and G_B are also near soft closed in $\mathcal{O}_B$.

4)$\Rightarrow$1). Let $(\mathcal{O}, B, \tau)$ has a proper near soft open and near soft closed F_B in $\mathcal{O}_B$. Put $H_B = F_B^c$. Then H_B and F_B are non-empty near soft closed set in $\mathcal{O}_B$, $F_A \cap H_B = \emptyset_N$ and $F_A \cup H_B = \mathcal{O}_B$. Hence, H_B and F_B is a near soft separation of $\mathcal{O}_B$.

Theorem 3.18 Let $(\mathcal{O}, B, \tau)$ be a nsts over $\mathcal{O}_B$. $(\mathcal{O}, B, \tau)$ is a near soft connected (respt. near soft disconnected) space if and only if there does not exists (respt. exist) non-empty near soft set; which is both near soft open and near soft closed in $(\mathcal{O}, B, \tau)$.

Proof. Let $(\mathcal{O}, B, \tau)$ be a nsts over $\mathcal{O}_B$. Let us assume that there is both a near soft open and a near soft closed set of F_B in $(\mathcal{O}, B, \tau)$. Then $F_B \cup F_B^c = \mathcal{O}_B$ and $F_B \cap F_B^c = \emptyset_N$. Since F_B is a near soft closed set $F_B^c \in \mathcal{O}_B$. This contradicts $(\mathcal{O}, B, \tau)$ being near soft connected space.

Conversely, let there does not exist both near soft open and near soft closed set in

$(\mathcal{O}, B, \tau)$. Nevertheless, let $(\mathcal{O}, B, \tau)$ be near soft disconnected space.

In this case, there exist a pair G_B and H_C of non-empty near soft disjoint subsets of $(\mathcal{O}, B, \tau)$ such that $F_B = G_B \cup H_C$ and $G_B \cap H_C = \emptyset_N$.

If $F_B^c = G_B$ is taken from these two equations, a situation that contradicts the hypothesis occurs. So $(\mathcal{O}, B, \tau)$ is near soft connected space.

Note: By the Theorem 3.18, the near soft topological space in Example 2.14 is a near soft disconnected space since the near soft set F_B is near soft open and near soft closed in $\mathcal{O}_B$.

Theorem 3.19 Let $(\mathcal{O}, B, \tau)$ be a near soft connected space and $\sigma \subseteq \tau$. Then, $(\mathcal{O}, B, \sigma)$ is a near soft connected space.

Proof. $F_B, G_B \in \sigma$ cannot be found such that $F_B = G_B \cup H_C$ and $G_B \cap H_C = \emptyset_N$, since it is a near soft connected space. Since $\sigma \subseteq \tau$ is near soft open sets that satisfy these two conditions are not in the σ family. Hence $(\mathcal{O}, B, \sigma)$ is a near soft connected space.

Under a continuous function, the below theorem indicates that near soft connectedness in maintained.

Theorem 3.20 Let $(\mathcal{O}, A, \tau)$ and (V, B, τ^*) be two nstss, $F_B \in NSS(\mathcal{O}_A)$ and $f: \mathcal{O} \to V$ be a near soft continuous function. If F_B is a near soft connected set, then $f(F_B) \in NSS(V_B)$ is a near soft connected set.

Proof. Suppose that $F_B \in NSS(\mathcal{O}_A)$ is a near soft connected set and $f(F_B)$ is a near soft disconnected set. Then, there exist two near soft sets open sets G_B and H_C such that $F_B = G_B \cup H_C$ and $G_B \cap H_C = \emptyset_N$. Therefore, from Theorem 2.25

$$F_B \subseteq f^{-1}(f(F_B)) \subseteq f^{-1}(G_B \cup H_C)$$
$$= f^{-1}(G_B) \cup f^{-1}(H_C)$$

and

$$f^{-1}(G_B \cap H_C) = f^{-1}(G_B) \cap f^{-1}(H_C) = \emptyset_N.$$

Since f is a near soft continuous function, $f^{-1}(G_B), f^{-1}(H_C) \in \mathcal{O}_A$. Therefore, this situation contradicts with the near soft connected of the F_B.

Remark 3.21 In Theorem 3.20, if we take the $\mathcal{O}_A$ instead of F_B and if we take f onto mapping, whereas $(\mathcal{O}, A, \tau)$ nsts is near soft connected, (V, B, τ^*) nsts is also near soft connected space.

Remark 3.22 A inverse image of a near soft connected space under a near soft continuous function does not need to be near soft connected.

Example 3.23 Let $(\mathcal{O}, A, \tau)$ and (V, B, τ^*) be two nstss, $G_B \in NSS(\mathcal{O}_A)$ and $f: \mathcal{O} \to V$ is a near soft continuous function, where $\tau = \mathcal{P}(F_A)$ and τ^* is a single element near soft space. In this case, $f^{-1}(G_B)$ is not near soft connected.

Theorem 3.24 If F_B is a near soft subset in nsts $(\mathcal{O}, B, \tau)$ that provides $F_B \subseteq G_B \subseteq cl_n(F_B)$, G_B is near soft connected, specifically $cl_n(F_B)$ near soft connected.

Proof. Let H_B be both a near soft open and a near soft closed subset in G_B that is non-empty. Since $G_B \subseteq cl_n(F_B)$, H_B contains at least one point of F_B. Because every point of G_B is a point of $cl_n(F_B)$ and H_B is a near soft open set. Similarly, if H_B is non-empty, cuts F_B. $H_B \cap F_B$ is both near soft open and near soft closed, and non-empty. From Theorem 3.18, then $H_B \cap F_B = F_B$. For this reason $F_B \subseteq H_B$. Thus $H_B^c \cap F_B = \emptyset_N$, hence $H_B = G_B$. According to Theorem 3.18, G_B is near soft connected. This completes the proof.

Corollary 3.25 If F_B is a near soft connected subspace of a nsts $(\mathcal{O}, B, \tau)$, then $cl_n(F_B)$ is a near soft connected.

Now let's define the quotient space with near soft sets.

Definition 3.26 Let $(\mathcal{O}, A, \tau)$, (V, B, τ^*) be nstss. Then (V, B, τ^*) is said to be a near soft quotient space of $(\mathcal{O}, A, \tau)$, by denoted $(\mathcal{O}/\sim_{B_r}, B, \tau^*)$ if there exists a surjective mapping $f: \mathcal{O} \to V$ with the following property (1).

For each subset U_B of V_B, $U_B \in \tau^* \Leftrightarrow$
$$f^{-1}(U_B) \in \tau \ldots\ldots\ldots(1)$$

A surjective mapping f with above property is said to be a near soft quotient mapping.

Remark 3.27 From Definition 3.26 it is clear that every near soft quotient mapping is a near soft continuous map.

Remark 3.28 Property (1) is near soft equivalent to property (2).

For each subset U_B of V_B, U_B is near soft closed in $(\mathcal{O}/\sim_{B_r}, B, \tau^*) \Leftrightarrow f^{-1}(U_B)$ is near soft closed in $(\mathcal{O}, A, \tau)$. …..(2)

The f-quotient mapping defined from nsts $(\mathcal{O}, B, \tau)$ to near soft quotient space $(\mathcal{O}/\sim_{B_r}, B, \tau^*)$ does not have to be a near soft open (or near soft closed) mapping. But this mapping portrays some near soft open sets of space $\mathcal{O}$ over near soft open sets of quotient space $\mathcal{O}/\sim_{B_r}$. Let's examine this feature.

Definition 3.29 If an $V \subset \mathcal{O}$ subset has the following properties (according to the $\sim_{B_r}$), then it is called a saturated near soft set:

$$a \in \mathcal{O} \text{ and } b\sim_{B_r}a \Rightarrow b \in \mathcal{O}.$$

So, a saturated near soft set is a near soft set that consist of all the elements equal to each other.

Theorem 3.30 The inverse image of each subset of the quotient near soft set under the near soft quotient mapping is always a saturated near soft set.

Proof: Now $V \subset \mathcal{O}/\sim_{B_r} \Rightarrow a \in f^{-1}(V)$ and $a\sim_{B_r}b \Rightarrow f(a) \in V$

$$f(a) = f(b) \Rightarrow b \in f^{-1}(V)$$

and this proves what is desired.

Conversely, if $V \subset \mathcal{O}$ is a saturated near soft set, there is a $V \subset \mathcal{O}/\sim_{B_r}$ to be $\mathcal{O} = f^{-1}(V)$. Indeed, the set $V = f(\mathcal{O})$ will provide the desired. Because

$$\mathcal{O} \text{ saturated} \Rightarrow f^{-1}(\mathcal{O}) \circ f(\mathcal{O})$$

obtained. Also, since f is a near soft surjection, there will be

$$f(U) \circ f^{-1}(U) = U$$

for every $U \subset \mathcal{O}/\sim_{B_r}$.

Using these features, we can say the following proposition.

Proposition 3.31 Near soft open sets of near soft quotient topology consists of images under the near soft quotient mapping of saturated near soft open sets.

Of course, the above proposition does not mean that the near soft quotient mapping is a near soft open mapping. Because we know that this mapping will depict saturated near soft open sets on near soft open sets. It may not depict any near soft open set over a near soft open set. Similarly, we can say that the near soft quotient mapping does not have to be a near soft closed mapping.

Theorem 3.32 Each near soft quotient space of a near soft connected space is also a near soft connected space.

Proof: Let $(\mathcal{O}, B, \tau)$ be a near soft connected space, the correlation $\sim_{B_r}$ is an equivalence relation defined on $\mathcal{O}_B$. Since the near soft quotient mapping defined from $(\mathcal{O}, B, \tau)$ to $(\mathcal{O}/\sim_{B_r}, B, \tau^*)$ is a near soft continuous function (see Remark 3.27), the desired thing is obtained from Theorem 3.20.

There is no opposite to this theorem, but there is the following feature.

Proposition 3.33 If $(\mathcal{O}, B, \tau)$ is a near soft topological space and according to the $\sim_{Br}$ relation, each of the equivalence classes is a near soft connected set, then $(\mathcal{O}, B, \tau)$ space will also be near soft connected.

Proof: If $\mathcal{O}$ was not near soft connected, near soft sets F_B and G_B would exist such that $\mathcal{O}_B = F_B \cup G_B$, $F_B \cap G_B = \emptyset_N$. According to the $\sim_{B_r}$, F_B and G_B are two saturated near soft sets, because if there were $\overline{a}_{B_r} \cap G_B \neq \emptyset_N$ for one $\overline{a}_{B_r} \in F_B$ since $\overline{a}_{B_r}$ would be equal to the combination of sets $\overline{a}_{B_r} \cap F_B$ and $\overline{a}_{B_r} \cap G_B$, which are open and disjoint in subspace $\overline{a}_{B_r}$, the equivalence class $\overline{a}_{B_r}$ could not be connected. Since there can be no contradiction, F_B and G_B are two near soft saturated sets. Thus, in accordance with Proposition 3.31, the $f(F_B)$ and $f(G_B)$ images of these two near soft sets are near soft open under the quotient mapping f. Also, these two near soft sets do not intersect. They are not empty and their composition is equal to $\mathcal{O}/\sim_{B_r}$. This is contrary to the near soft connected of the quotient space $/\sim_{B_r}$. Since this contradiction does not exist, the $\mathcal{O}$ space is near soft connected.

4. Conclusion

This study introduces the notions of near soft connectedness, near soft connected topological spaces, near soft disjoint sets and some of their properties. In addition, it should be noted that the definitions and theorems presented in this paper are accepted as a general theory for the near soft set theory. With this paper, it is considered to pave the way for new studies (theoretical or applications) as a result of defining the connectedness of near soft set theory. Moreover, the contribution of this study is that it provides a near approach to proximity as an ambiguous concept that can be approximated to the state of objects and field knowledge. We foresee that problems of many fields that include uncertainties and will provide further study on near soft topology to fulfil general skeleton for the applications in practical life can be consulted with the findings in this study.

References

Acar, U., Koyuncu, F. and Tanay B. 2010. "Soft Sets and Soft Rings", *Comp. Math. Appl.*, 59, 3458-3463.

Aktas, H. and Cağman, N. 2007. "Soft Sets and Soft Groups", *Inform. Sciences*, 177, 2726-2735.

Ali, I. M., Feng, F., Liu, X. Y., Min, W. K. and Shabir, M., 2009. "On Some New Operations in Soft Set Theory", *Comp. Math. Appl.* 57, 1547-1553.

Aygunoğlu, A. and Aygun, H. 2011. "Some notes on soft topological spaces", *Neural Computing and Applications*, 21(1), 113-119.

Aygunoğlu, A., Aydoğdu, E and Aygun, H. 2019. "Fuzzy Soft Metric and Fuzzifying Soft Topology Induced by Fuzzy Soft Metric" *Filomat* 33 (2), 645–653.

Beaula, T., Gunaseeli, C. 2014. "On fuzzy soft metric spaces", *Malaya J. Math.* 2, 197–202.

Beaula, T., Priyanga, M. M. 2015. "A new notion for fuzzy soft normed space", *Internat. J. Fuzzy Math. Arch.* 9, 81–90.

Beaula, T., Raja, R. 2015. "Completeness in fuzzy soft metric spaces", *Malaya J. Math.* 2, 438–442.

Cağman, N., Karataş, S. and Enginoğlu, S. 2011. "Soft Topology", *Computers and Mathematics with Applications*, 62, 351-358.

Cağman, N., Enginoğlu, S. 2010. "Soft set theory and uni–intdecision Making". *European Journal of Operational Research,* 207 848–855.

Cantor, G., 1883. "Uber Unendliche, Lineare Punktmannigfaltigkeiten" 5, *Mathematische Annalen* 21: 545–586. In [Cantor, 1932].

Georgiou, D.N. Megaritis, A.C. and Petropoulos, V.I. 2013. "On Soft Topological Spaces", *Appl. Math. Inf. Sci.* 7(5), 1889-1901.

Grzymala-Busse, J. W. 1988. "Knowledge acquisition under uncertainty: A rough set approach", *Journal of Intelligent Robotic Systems*, 1.

Hassanien, A. E., Abraham, A. and Peters, J. F., Schaefer, G. and Henry, C. 2009. "Rough Sets and Near Sets in Medical Imaging: A Review, *IEEE Trans Info. Tech. in Biomedicine*, 13(6), 955-968.

Henry, C. 2010. "Near sets: Theory and applications", PhD Thesis, *Department of Electrical & Computer Engineering, University of Manitoba Canada.*

Henry, C. and Peters, J. F., "Near Set Evaluation and Recognition (NEAR) System V2.0", *University of Manitoba Computational Intelligence Laboratory Technical Report*, TR-2010-017.

Henry, C. and Smith, G. "Proximity System", *University of Manitoba Computational Intelligence Laboratory Technical Report*, TR-2012-021.

Hussain, S. 2015. "A Note On Soft Connectedness, *Journal of the Egyptian Matematical Society*, 23, 6-11.

Inan, E. and Ozturk, M. A. 2012 "Near Groups In Nearness Approximation Spaces", *Hacettepe Jr. of Math. and Stat*., 41, 544-558.

Mahanta J., Das P. K., 2012 "Results on fuzzy soft topological spaces", *arXiv:1203.0634v1.*

Maji, P. K., Biswas, R. and Roy, A. R. 200*3. "Soft Set Theory", Comput. Math. Appl.* 45, 555-562.

Maji, P. K., Roy, A. R. and Biswas, R. 2002. "An Application of Soft Sets In A Decision Making Problem", *Comput. Math. Appl.* 44, 1077-1083.

Molodtsov, D. 1999. "Soft Set Theory-First Results", *Computers & Mathematics with Applications*, 37(4-5), 19-31.

Orlowska, E. 1994. "Reasoning with incomplete information: rough set based information logics", *Incompleteness and Uncertainty in Information Systems,* New York, Springer-Verlag.

Ozkan, A. 2019. "On Near Soft Sets", *Turkish Journal of Mathematics*, 43(2), 1005-1017.

Pawlak, Z. 1982. "Rough Sets", *International Journal of Computer and Information Sciences*. 11, 341–356.

Pawlak Z. and Peters, J. F. 2002-2007. "Jack Blisko (How near) Systemy Wspomagania", 63 Decyzji I, 57, 109, ISBN:83-920730-4-5, 2002-2007.

Pei, D. and Miao, 2005. "From Soft Sets to Information Systems, In: Proceedings of Granular Computing (Eds: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R.R. Yager, B.Zhang)", *Institute of Electrical and Electronics Engineers*, 2, 617-621.

Peters, J. F. 2007. "Near Sets. Special Theory About Nearness of Objects", *Fund. Inform*, 57, 407-433.

Peters, J. F. 2007. "Near Sets. General Theory About Nearness of Objects", *Appl. Math.Sci.*, 2609-2629.

Peters, J. F. and Wasilewski, P. 2009. "Foundations of Near Sets", *Inform. Sci.*, 179, 3091-3109.

Peters, J. F. 2005. "Rough Ethology: Towards A Biologically-Inspired Study of Collective Behavior in Intelligent Systems with Approximation Spaces", *Transactions On Rough Sets*, III, *LNCS* 3400, 153-174.

Peters J. F. and Henry C. 2006. "Reinforcement Learning with Approximation Spaces", *Fund. Inform*, 71(2-3), 323-349.

Peters, J. F., Ramanna, S. 2009 "Affinities Between Perceptual Granules: Foundations and Perspectives", *In Human-centric information processing Through Granular

Zorlutuna, I., Akdağ, M., Min, W. K. and Atmaca, S. 2012. "Remark on soft topological spaces", *Annals of fuzzy mathematics and informatics*, 3(2), 171-185.

Modelling sci 182, Eds A, Bargiela and W. Pedrycz, Springer-Verlag, Berlin.

Peters, J. F, Henry, C. and Gunderson, D. S. 2007. "Biologically-Inspired Approximate Adaptive Learning Control Strategies: A Rough Set Approach", *International Journal of Hybrid Intelligent Systems*, 4(4), 203-216.

Peters, J. F. and Naimpally, S. 2012. "Applications of near sets", *Notices Amer. Math. Soc.*, 59(4), 536-542.

Peters, J. F. 2010. "Corrigenda and Addenda: Tolerance Near Sets and Image Correspondence, *Int. J. Bio-Inspired Comput.*, 2(5), 310-318.

Riesz, 1908. "Stetigkeitsbegriff und abstrakte mengenlehre", *Proceedings of the* International Mathematical Congress of Mathematicians, Series I, Fourth Congress, Rome, 2, 18-24; Kraus Reprint Limited, Nendeln/Liechtenstein, 1967.

Tasbozan, H., Icen, I., Bagirmaz N. and Ozcan, A. F. 2017. "Soft Sets and Soft Topology on Nearness Approximation Spaces, *Filomat*, 31(13), 4117-4125.

Vu, H. Q.2019. "Lecture notes on Topology" Version of February 10.

Wolski, M. 2010. "Perception and Classification. A Note On Near Sets and Rough Sets", *Fund. Inform.*, 101, 143-155.

Zadeh, L. A. 1965. "Fuzzy sets", *Inform. Control* 8, 338–353.

Zhao, X. D. 1986. "Connectedness On Fuzzy Topological Spaces", *Fuzzy Sets and Systems*, 20, 223-240.

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