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(3 + 1) boyutlu Kadomtsev-Petviashvili (KP) Denkleminin Yeni Tam Çözümleri

Yusuf PANDIR[1*], Tural AĞIR[2]

[1] Yozgat Bozok University, Faculty of Arts & Science, Department of Mathematics, 66100, Yozgat, Türkiye
[2] Yozgat Bozok University, The Graduate School of Natural and Applied Sciences, 66100 Yozgat, Türkiye
Yusuf Pandır ORCID No: 0000-0003-0274-7901
Tural Ağır ORCID No: 0000-0002-1521-7873

**Sorumlu yazar: yusuf.pandir@bozok.edu.tr*

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Anahtar Kelimeler
Genişletilmiş deneme denklemi yöntemi, (3+1)-boyutlu Kadomtsev-Petviashvili (KP) denklemi, Tam çözümler

Öz: Kısmi türevli diferansiyel denklemlerin soliton çözeltilerini ve Jacobi eliptik fonksiyon çözümlerini elde etmemizi sağlayan genişletilmiş deneme denklemi yöntemi araştırılmıştır. Bu yöntem (3+1)-boyutlu Kadomtsev-Petviashvili (KP) denklemine uygulanmış ve çeşitli yeni tam(kesin) çözümler elde edilmiştir. Bu yeni tam çözümler, literatürde yer almayan çözümlerdir. Ek olarak, elde edilen farklı tam çözümlerin fiziksel davranışlarını anlamak için iki ve üç boyutlu grafikler çizilmiştir.

New Exact Solutions of (3+1)-dimensional Kadomtsev-Petviashvili (KP) Equation

Keywords
Extended trial equation method, (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation, Exact solutions

Abstract: The extended trial equation method is investigated which allows us to achieve soliton solutions and Jacobi elliptic function solution of the partial differential equations. This method is implemented to the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation and various new exact solutions have been obtained. These new obtain exact solutions are solutions that are not known in the literature. Additionally, two and three-dimensional graphics were drawn to understand the physical behaviors of the distinct obtain exact solutions.

1. INTRODUCTION

The solutions of the nonlinear partial differential equations can guide the understanding of many problems. Therefore, recent studies on such equations have also increased. We can use the notion of the wave to make the solutions of such equations comprehensible. These nonlinear physical occurrences are commonly seen in the fields of optical fibers, plasma physics, chemical physics, fluid mechanics, solid-state physics, biology, geochemistry, chemical kinematics and engineering..

A solitary wave is a wave that spreads without changing over time. We know that the implementation areas of the waves are fairly high. Consequently, many distinct vigorous and influential methods have been developed by different scientists for the nonlinear partial differential equations. Herewith of these newly developed methods, many physical occurrences will be easier to understand with the determination of the new exact solution functions.

Therefore, many distinct approach methods have been proposed and developed. Sine-cosine method [1,2], Hirota's bilinear transformation method [3,4], (G'/G)-expansion method [5,6], trial equation method[7-10], extended trial equation method [11-13], modified Kudryashov method [14,15] can be given as an example for the developed exact solution methods.

A robust method to find the exact solutions of the nonlinear partial differential equations was recommended by Liu in 2005. Liu's main goal was to find the finite series solution functions using different integration methods for solutions of ordinary differential equations. Later, many scientists developed further this powerful method and brought different versions of this method into the literature. Recently, the proposed method by Liu has been further developed by Gürefe et al and introduced into the literature as an extended trial equation method. This developed method allowed us new and exact solutions of the nonlinear partial differential equations.

In this study the developed method is implemented to the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation. Codes are written according to the required algorithms of the extended trial equation method, and new and different exact solutions of the equations are obtained. In the next section, the outlines of the extended trial equation method are explained in detail.

2. MATERIAL AND METHOD

In this section, we aim to define the extended trial equation method to achieve the new exact solutions of the nonlinear partial differential equations. The extended trial equation method is clarified in detail. Let's suppose the general form of a nonlinear partial differential equation with independent variables $x, y, z, \ldots, t$ as

$$Q\left(u,u_x,u_y,u_z \ldots,u_t,\ldots,u_{xx},u_{xy},u_{xz},\ldots,u_{xt},\ldots\right)=0. \tag{1}$$

Consider travelling wave transformation for Eq. (1) as described below

$$u\left(x,y,z,\ldots,t\right)=u\left(\xi\right),\quad \xi=h_1x+h_2y+h_3z+\ldots+h_mt \tag{2}$$

where $h_j \neq 0\left(j=1,2,3,\ldots,m\right)$. Substituting Eq. (2) into Eq. (1) reduces a nonlinear ordinary differential equation

$$T\left(u,u',u'',u''',\ldots\right)=0. \tag{3}$$

Let's assume the solution of the Eq. (3) with the finite series approach as follows

$$u\left(\xi\right)=\sum_{i=0}^{\delta}\tau_i\Gamma^i\left(\xi\right) \tag{4}$$

where

$$\left(\Gamma'\right)^2=\Lambda\left(\Gamma\right)=\frac{\Phi\left(\Gamma\right)}{\Psi\left(\Gamma\right)}=\frac{\varepsilon_0+\varepsilon_1\Gamma+\varepsilon_2\Gamma^2+\cdots+\varepsilon_\theta\Gamma^\theta}{\zeta_0+\zeta_1\Gamma+\zeta_2\Gamma^2+\cdots\zeta_\epsilon\Gamma^\epsilon}. \tag{5}$$

Here the $\Gamma\left(\xi\right)$ functions are the solution functions of the nonlinear ordinary elliptical differential equation. Using Eq (4) and Eq (5), we get

$$\left(u'\right)^2\left(\xi\right)=\frac{\Phi\left(\Gamma\right)}{\Psi\left(\Gamma\right)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\left(\xi\right)\right)^2 \tag{6}$$

$$u''\left(\xi\right)=\frac{\Phi'\left(\Gamma\right)\Psi\left(\Gamma\right)-\Phi\left(\Gamma\right)\Psi'\left(\Gamma\right)}{2\Psi^2\left(\Gamma\right)}\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\left(\xi\right)+\frac{\Phi\left(\Gamma\right)}{\Psi\left(\Gamma\right)}\sum_{i=0}^{\delta}i\left(i-1\right)\tau_i\Gamma^{i-2}\left(\xi\right) \tag{7}$$

where $\Phi\left(\Gamma\right)$ and $\Psi\left(\Gamma\right)$ are polynomials. When the attained derivatives in Eq. (6) and Eq. (7) are analyzed, as stated in the solution function (4) turns into a polynomial expression dependent on a rational $\Gamma\left(\xi\right)$ function. The balance process according to Eq. (3) is implemented in terms of the polynomial equivalent of the highest-order term with the highest-order derivative term. With the balancing procedure, δ in solution function (4), θ and $\in$ values in Eq. (5) will be calculated. Some balancing terms are calculated as follows

$$uu''\rightarrow\Gamma^{\theta+2\delta-\epsilon-2},\ \left(u'\right)^2\rightarrow\Gamma^{\theta+2\delta-\epsilon-2},$$
$$u^2\rightarrow\Gamma^{2\delta},\ u^3\rightarrow\Gamma^{3\delta}. \tag{8}$$

Thus, when the calculated values are replaced in the expressions, zero polynomial related to the Γ function is attained. A system of algebraic equations is attained by synchronizing the related terms in this zero polynomial to zero. When the created algebraic equation system is solved with the help of the Mathematica 10 package program, $\varepsilon_0,\cdots,\varepsilon_\theta$, $\zeta_0,\cdots,\zeta_\epsilon$ and $\tau_0,\cdots,\tau_\delta$ coefficients are obtained. When the attained coefficients are rewritten in the Eq. (5), $\Gamma\left(\xi\right)$ functions are attained by calculating the integral

$$\pm\left(\xi-\xi_0\right)=\int\frac{d\Gamma}{\sqrt{\Lambda\left(\Gamma\right)}}=\int\sqrt{\frac{\Psi\left(\Gamma\right)}{\Phi\left(\Gamma\right)}}d\Gamma. \tag{9}$$

Then, $\Gamma\left(\xi\right)$ functions are substituted in solution function (4), respectively. Thus, by applying the transformation in the expression of (2) to the obtained $u(\xi)$ functions, new exact solutions of the Eq. (1) are attained.

3. RESULTS

In this section, the extended trial equation method is implemented to the (3+1)-dimensional Kadomtsev-Petviashvili equation. The (3+1)-dimensional Kadomtsev-Petviashvili equation was first introduced in 1970 by Boris B. Kadomtsev and Vladimir I. Petviashvili. This equation describes long wavelength water waves with weak nonlinear restore forces, waves in ferromagnetic media, and two-dimensional matter-wave pulses in Bose-Einstein condensates. Due to its importance, it has been extensively studied in the literature [16-18]. This equation has a structure that defines three-dimensional solitons in a weak fluid environment, especially fluid dynamics and plasma physics. The (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation is

$$\left(\frac{\partial u}{\partial t}+6u\frac{\partial u}{\partial x}+\frac{\partial^3 u}{\partial x^3}\right)_x-3\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 u}{\partial z^2}=0, \tag{10}$$

where u is a real valued function [18-23]. First let's suppose travelling wave transformation to implement the extended trial equation method to Eq. (10)

$$u(x,y,z,t)=u(\omega),\ \omega=kx+ly+mz+ct,$$

where k,l,m,c are arbitrary constants. When the integral is taken twice according to ω and the integration constants are selected as zero, Eq. (10) equation as follows

$$\left(kc-3\left(m^2+l^2\right)\right)u(\omega)+3k^2u^2(\omega)+k^4u''(\omega)=0. \tag{11}$$

Using the solution function (4) and differential equation (5), the related derivatives are calculated and replaced in Eq. (11). The balancing process is implemented to determine the δ, θ and $\in$ values in solution function (4) and differential equation (5). According to the extended trial equation method, the balancing process is determined between the term u'' containing the highest order derivative and the highest order nonlinear term u^2 as follows

$$u^2\to\Gamma^{2\delta},\ u''\to\Gamma^{\theta+\delta-\in-2}. \tag{12}$$

Accordingly, the balance term is attained as $\theta=\varepsilon+\delta+2$ from the equivalence of the obtained $u''\approx u^2$ terms. To determine the new solution of the Eq. (10), if the balance terms are selected as $\varepsilon=0$ and $\delta=1$, then $\theta=3$ is attained. When these balancing terms are written in solution function (4) and differential equation (5) respectively, we achieve

$$u(\omega)=\tau_0+\tau_1\Gamma(\omega) \tag{13}$$

$$(\Gamma')^2=\frac{\Phi(\Gamma)}{\Psi(\Gamma)}=\frac{\varepsilon_0+\varepsilon_1\Gamma+\varepsilon_2\Gamma^2+\varepsilon_3\Gamma^3}{\zeta_0} \tag{14}$$

The term u'' in the Eq. (11) is calculated as

$$u''=\frac{\tau_1\left(\varepsilon_1+2\varepsilon_2\Gamma+3\varepsilon_3\Gamma^2\right)}{2\zeta_0} \tag{15}$$

where $\varepsilon_3\neq 0$ and $\zeta_0\neq 0$.

When the calculated values are replaced in the Eq. (11), a polynomial expression based on the $\Gamma(\eta)$ function occurs. If we contemplate this polynomial as a zero polynomial, the coefficients of this polynomial are equalized to zero, resulting in a system of algebraic equations.

When this algebraic system is solved with the help of Mathematica 10 packet program, coefficients are found as follows

$$\varepsilon_0=\varepsilon_0,\ \ \varepsilon_1=\varepsilon_1,\ \ \varepsilon_2=\frac{3\zeta_0\left(l^2+m^2-2k^2\tau_0\right)}{k^4},$$
$$\varepsilon_3=\frac{-2\zeta_0\tau_1}{k^2},\ \ \zeta_0=\zeta_0,\ \ \tau_0=\tau_0,\ \ \tau_1=\tau_1,$$
$$c=\frac{6\zeta_0\tau_0\left(l^2+m^2-2k^2\tau_0\right)-k^4\varepsilon_1\tau_1}{2k\zeta_0}. \tag{16}$$

When these attained coefficients are replaced in Eq. (5) and Eq. (9), an integral is obtained

$$\pm(\omega-\omega_0)=H\int\frac{d\Gamma}{\sqrt{\frac{\varepsilon_0}{\varepsilon_3}+\frac{\varepsilon_1}{\varepsilon_3}\Gamma(\omega)+\frac{\varepsilon_2}{\varepsilon_3}\Gamma^2(\omega)+\Gamma^3(\omega)}}, \tag{17}$$

where $H=\sqrt{-\frac{k^2}{2\tau_1}}$, It is quite difficult to calculate the integral in Eq. (17). Integrating Eq. (17), we achieve the solutions of the Eq. (10) as follows:

$$\pm(\omega-\omega_0)=-\frac{2H}{\sqrt{\Gamma-\alpha_1}}, \tag{18}$$

$$\pm(\omega-\omega_0)=\frac{2H}{\sqrt{\alpha_2-\alpha_1}}\arctan\sqrt{\frac{\Gamma-\alpha_2}{\alpha_2-\alpha_1}},\ \ \alpha_2>\alpha_1, \tag{19}$$

$$\pm(\omega-\omega_0)=\frac{H}{\sqrt{\alpha_1-\alpha_2}}\ln\left|\frac{\sqrt{\Gamma-\alpha_2}-\sqrt{\alpha_1-\alpha_2}}{\sqrt{\Gamma-\alpha_2}+\sqrt{\alpha_1-\alpha_2}}\right|,\quad \alpha_1>\alpha_2, \tag{20}$$

$$\pm(\omega-\omega_0)=-\frac{2H}{\sqrt{\alpha_1-\alpha_3}}F(\varphi,l),\quad \alpha_1>\alpha_2>\alpha_3, \tag{21}$$

where $F(\varphi,l)=\int\limits_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}}$,

$$\varphi=\arcsin\sqrt{\frac{\Gamma-\alpha_3}{(\alpha_2-\alpha_3)}},\ l^2=\frac{(\alpha_2-\alpha_3)}{(\alpha_1-\alpha_3)}.$$

Also $\alpha_1,\alpha_2,\alpha_3$ and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\varepsilon_3}{\varepsilon_4}\Gamma^3+\frac{\varepsilon_2}{\varepsilon_4}\Gamma^2+\frac{\varepsilon_1}{\varepsilon_4}\Gamma+\frac{\varepsilon_0}{\varepsilon_4}=0.$$

Substituting the solutions (18-21) into (13) and using the wave transformation, we have

$$u_1(x,y,z,t)=\left[\tau_0+\tau_1\alpha_1+\frac{4H^2\tau_1}{\left(kx+ly+mz+\left(\frac{6\zeta_0\tau_0\left(l^2+m^2-2k^2\tau_0\right)-k^4\varepsilon_1\tau_1}{2k\zeta_0}\right)t-\omega_0\right)^2}\right] \tag{22}$$

$$u_2(x,y,z,t)=\left[\begin{array}{l}\tau_0+\tau_1\alpha_1\\ +\tau_1(\alpha_2-\alpha_1)\operatorname{sech}^2\left(\frac{\sqrt{\alpha_1-\alpha_2}}{2H}\left(kx+ly+mz+\left(\frac{6\zeta_0\tau_0\left(l^2+m^2-2k^2\tau_0\right)-k^4\varepsilon_1\tau_1}{2k\zeta_0}\right)t-\omega_0\right)\right)\end{array}\right] \tag{23}$$

$$u_3(x,y,z,t)=\left[\begin{array}{l}\tau_0+\tau_1\alpha_1\\ +\tau_1(\alpha_1-\alpha_2)\operatorname{cosech}^2\left(\frac{\sqrt{\alpha_1-\alpha_2}}{2H}\left(kx+ly+mz+\left(\frac{6\zeta_0\tau_0\left(l^2+m^2-2k^2\tau_0\right)-k^4\varepsilon_1\tau_1}{2k\zeta_0}\right)t-\omega_0\right)\right)\end{array}\right] \tag{24}$$

$$u_4(x,y,z,t)=\left[\tau_0+\tau_1\alpha_3+\left(\frac{\tau_1(\alpha_2-\alpha_3)}{sn^2\left[\pm\frac{\sqrt{\alpha_1-\alpha_3}}{2H}\left(kx+ly+mz+\left(\frac{6\zeta_0\tau_0\left(l^2+m^2-2k^2\tau_0\right)-k^4\varepsilon_1\tau_1}{2k\zeta_0}\right)t-\omega_0\right),\frac{(\alpha_2-\alpha_3)}{(\alpha_1-\alpha_3)}\right]}\right)\right] \tag{25}$$

If we take $\tau_0=-\tau_1\alpha_1$ and $\omega_0=0$ the equations (22)-(24) are obtained respectively rational function solutions:

$$u_1(x,y,z,t)=\left(\frac{\tilde{A}}{kx+ly+mz+ct}\right)^2 \tag{26}$$

where $\tilde{A}=2H\sqrt{\tau_1}$,

$$c=\frac{6\zeta_0\tau_0\left(l^2+m^2-2k^2\tau_0\right)-k^4\varepsilon_1\tau_1}{2k\zeta_0},$$ 1(bright)-soliton solution:

$$u_2(x,y,z,t)=\frac{\tilde{B}}{\cosh^2\left(B(kx+ly+mz+ct)\right)} \tag{27}$$

where $\tilde{B}=\tau_1(\alpha_2-\alpha_1)$, $B=\frac{\sqrt{\alpha_1-\alpha_2}}{2H}$, singular(dark) soliton solution:

$$u_3(x,y,z,t)=\frac{\tilde{C}}{\sinh^2\left(B(kx+ly+mz+ct)\right)} \tag{28}$$

where $\tilde{C}=\tau_1(\alpha_1-\alpha_2)$.

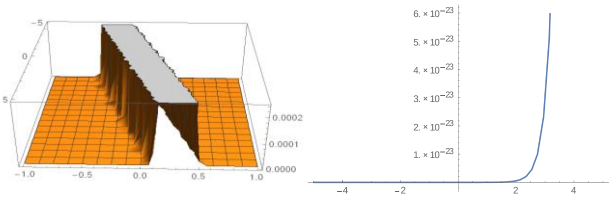

Figure 1. Three and two dimensional graphical for $\alpha_1=\xi_1=\tau_0=1,\ \alpha_2=k=\tau_1=2,\ \zeta_0=\frac{1}{3}$, of the solution (27).

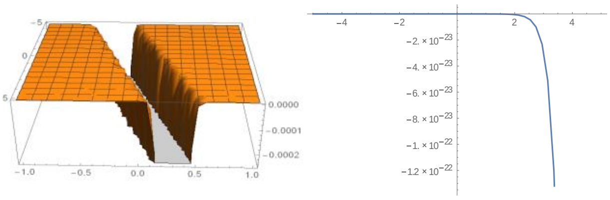

Figure 2. Three and two dimensional graphical for $\alpha_1=\xi_1=\tau_0=1,\ \alpha_2=k=\tau_1=2,\ \zeta_0=\frac{1}{3}$, of the solution (28).

If $\tau_0=-\tau_1\alpha_3$ is taken in Eq. (25), a Jacobi elliptic function solution is found as

$$u_4(x,y,z,t)=\frac{\tilde{D}}{sn^2(\varphi_2,l_2)} \tag{29}$$

where $\tilde{D}=\tau_1(\alpha_2-\alpha_3)$,

$$\varphi_2=\frac{\pm\sqrt{\alpha_1-\alpha_3}}{2H}(kx+ly+mz+ct),\ l_2^2=\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}.$$

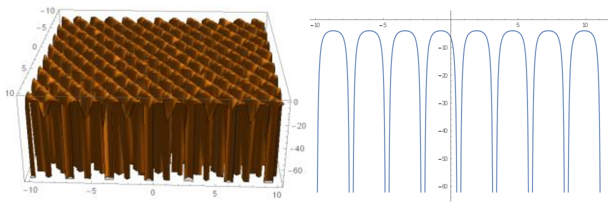

Figure 3. Three and two dimensional graphical for $\alpha_1=\xi_1=\tau_0=1,\ \alpha_2=k=\tau_1=2,\ \zeta_0=\frac{1}{3},\ \alpha_3=4$ of the solution (29).

Here, $\tilde{B}$ and $\tilde{C}$ demonstrate the amplitude of the soliton and B demonstrates the inverse width of the solitons and c demonstrates the velocity of the soliton.

Besides, if we get the module as $l \to 1$ in the elliptic solution Eq.(29), then the solution of the Eq. (10) turns into the following hyperbolic function solution

$$u_5(x,y,z,t) = \frac{\tilde{D}}{\tanh^2\left[\pm\frac{\sqrt{\alpha_1-\alpha_3}}{2H}(kx+ly+mz+ct)\right]} \tag{30}$$

where $\alpha_1 = \alpha_2$.

On the other hand; when the module is selected as $l \to 0$ in the Jacobi elliptic solution Eq. (29), then the solution of the Eq. (10) turns into the following periodic wave solution

$$u_6(x,y,z,t) = \frac{\tilde{D}}{\sin^2\left[\pm\frac{\sqrt{\alpha_1-\alpha_3}}{2H}(kx+ly+mz+ct)\right]} \tag{31}$$

where $\alpha_2 = \alpha_3$.

When all the solutions obtained from the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation are investigated; solutions (27), (28) and (30) are similar to the results obtained by Lu [19], respectively (4), (10) and (8). Other solutions are new and distinct exact solutions that are not included in the literature. With the proposed method, new exact solutions of this equation are attained. The graphs of the obtained solution functions are illustrated in Fig. 1-3.

4. CONCLUSION

The extended trial equation method has been used to get a new exact solutions of the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation. Through the extended trial equation method elliptic function solutions, periodic function solutions, singular soliton solutions and dark and bright soliton solutions were obtained. This method allows us to find different function solutions together. To understand which physical behaviors of these solutions show, two and three-dimensional graphics are drawn according to the different values of the coefficients in the soliton functions. We think that this method also can be applied to other nonlinear differential equations.

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