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Dynamical Analysis, Electronic Circuit Design and Control Application of a Different Chaotic System

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ABSTRACT In this study, the dynamic behavior of a chaotic system is explored and its dynamical analysis is performed by Lyapunov exponents, fractional dimension, dependence to initial conditions and bifurcation diagram. In addition, the bifurcation analysis of the chaotic system is studied with respect to a certain parameter. The electronic circuit implementation of a chaotic system is realized and compared with the phase portraits obtained from Matlab and circuit realization. Also, passive control technique is applied to stabilize and suppress the chaos in the chaotic system. Numerical simulations are presented to verify the theoretical analysis and the effectiveness of the proposed control method.

KEYWORDS
Chaos
Chaotic systems
Circuit design
Control

INTRODUCTION

After the first chaotic system is discovered and introduced by Lorenz Lorenz (1963), many researches have been performed to explore new chaotic systems with different features Koyuncu *et al.* (2020); Akgul *et al.* (2016b,a). Chaos-based applications are an important scientific as well as engineering issue. A lot of novel chaotic systems have been discovered and presented from different disciplines especially during the last decades Lai *et al.* (2018); Wang *et al.* (2017); Ren *et al.* (2018); Pham *et al.* (2017); Varan and Akgul (2018). Comprehensive studies and implementations of various chaotic processes in different fields such as physics, control Wei *et al.* (2018), computer science, cryptology Çavuşoğlu *et al.* (2017), steganography Akgul *et al.* (2017), electronic circut implementation Kai *et al.* (2017); Li *et al.* (2018); Tuna and Fidan (2016), artificial neural network Vaidyanathan *et al.* (2020); Tuna *et al.* (2019) and synchronization Rajagopal *et al.* (2019) are introduced in the literature.

The chaotic oscillations and trajectories should be eliminated and diminished if they are undesirable. So, the control of the chaos or chaotic system has become important and paid attention by many researchers. After Ott et al. proposed the method for the chaos control named as OGY method Ott *et al.* (1990), many control methods are proposed to control the chaos or reduce-eliminate the chaotic oscillations occured in any applications such as sliding mode control Uyaroğlu *et al.* (2012); Kocamaz *et al.* (2017), adaptive control Asadollahi *et al.* (2020), linear feedback control Fu *et al.* (2020); Kocamaz *et al.* (2017), passive control Yu (1999); Qi *et al.* (2004); Uyaroglu and Emiroglu (2015); Emiroglu and Uyaroglu (2010). In this paper, control application and electronic circuit realizations are executed for chaos-based applications with dynamical analysis.

Inspring from previous studies, in the dynamical analysis and some applications of chaos is given in this article. After introduction section, the paper is organized as follows: The dynamic analysis of chaotic system is realized in Section 2. In Section 3, electronic circuit work is implemented on oscilloscope as real-time application. Control application of the chaotic system is presented in Section 4. Conclusion is given in last section.

THE USED CHAOTIC SYSTEM AND ITS DYNAMICAL ANALYSIS

The chaotic system is defined by the differential equation 1 as below Azarang *et al.* (2016):

$$\begin{aligned} \dot{x} &= -a(x+y) + bz + cyz \\ \dot{y} &= dx - y \\ \dot{z} &= -e(x^2+y^2-xy) - fz \end{aligned} \tag{1}$$

Initial values of system are taken as $x(0) = 0.1$, $y(0) = -0.1$ and $z(0) = 0.2$.This chaotic system has ten terms and six parameters (a,b,c,d,e and f). The values of these parameters are $a = 10$, $b = 1.5$,

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$c = 3$, $d = 70$, $e = 0.5$ and $f = 5$. The chaotic system has given by the following equation 2:

$$\begin{aligned} \dot{x} &= -10(x+y) + 1.5z + 3yz \\ \dot{y} &= 70x - y \\ \dot{z} &= -0.5(x^2 + y^2 - xy) - 5z \end{aligned} \quad (2)$$

Sensitivity to initial conditions

It is expected to be precisely independent from the initial conditions in the time series for a chaotic system. The potential state of the chaotic system will lead to substantially distant behaviour. Long-term prediction of the time series of the system is difficult due to the sensitive dependence on initial condition of the system. Time series of the chaotic system are given for initial conditions $x_1(0) = 0.1$ and $x_2(0) = 0.0001$ in Figure 1.

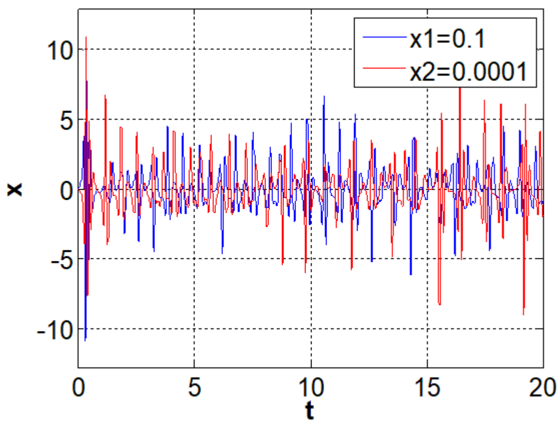


Figure 1 Time series for $x1(0) = 0.1$ and $x2(0) = 0.0001$

Phase portraits

The phase portraits of the chaotic system for the state variables x, y and z with the initial conditions (0.1, -0.1, 0.2). It can be seen from the phase portrait figures that this chaotic system shows rich dynamical behaviors. The phase plains of the chaotic system are illustrated in Figure 2.

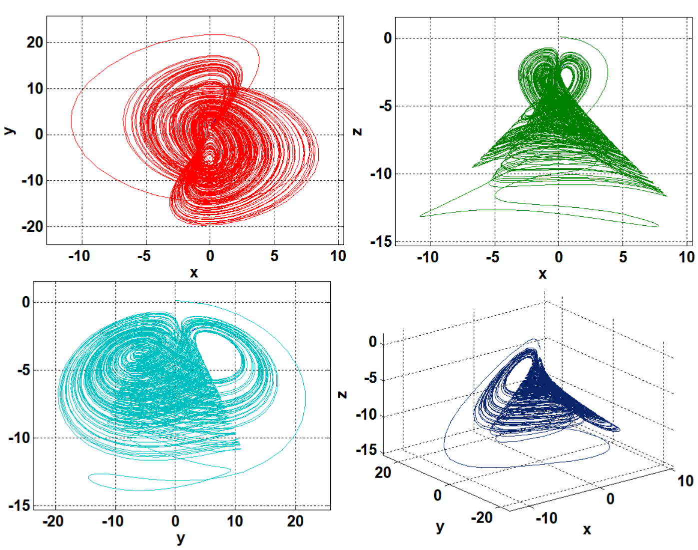


Figure 2 Phase portraits of the chaotic system

Lyapunov exponents spectrum and fractional dimension

The Lyapunov Exponent (LE) spectrum of the chaotic system with a varying parameter $d \in [0-1]$ is depicted in Figures 3 and 4. The 3D system is a chaotic system since the first LE (blue line), the second LE (green line) and the third LE (red line) are positive, zero and negative values respectively. The system shows chaotic behaviour because of at least one LE is positive Akgul *et al.* (2016b).

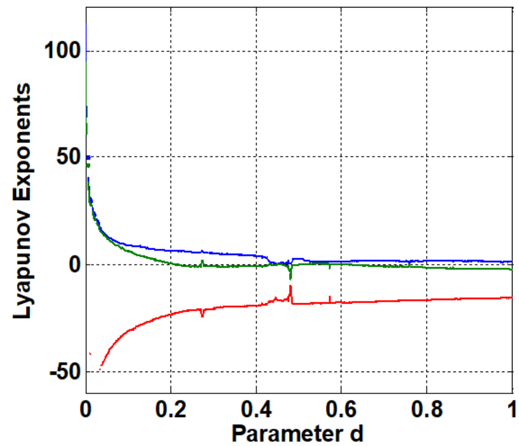


Figure 3 The LE spectrum of the system (for d parameter)

Figure 4 gives detailed LE spectrum with respect to parameter d changing in the range 0.37-0.5.

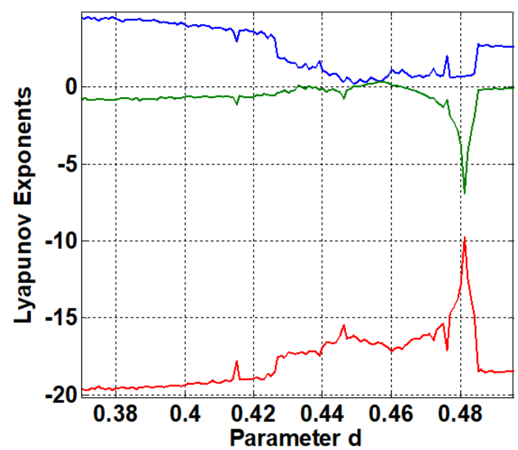


Figure 4 The LE spectrum of the system with respect to d varying between $0.37 - 0.5$

The LE of the system are calculated numerically with the parameter values $a = 10$, $b = 1.5$, $c = 3$, $d = 70$, $e = 0.5$ and $f = 5$ and the initial conditions as $x(0) = (0.1, -0.1, 0.2)$. The determined values of the LEs are

$$L1 = 2.4954, L2 = 0, L3 = -18.4945 \quad (3)$$

Obtaining the Lyapunov dimension of the system as

$$D_L = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 2 + \frac{L_1 + L_2}{|L_3|} = 2.134999. \quad (4)$$

Bifurcation Analysis

Figure 5 and Figure 6 show the bifurcation diagrams which corresponds to the maximum LE spectrum, shown in Figure 3 and 4.

As can be seen in Figure 5 the system behaves chaotic between the varying parameter d about $0 - 0.45, 0.46 - 1$ for parameters $a = 10$, $b = 1.5$, $c = 3$, $d = 70$, $e = 0.5$ and $f = 5$.

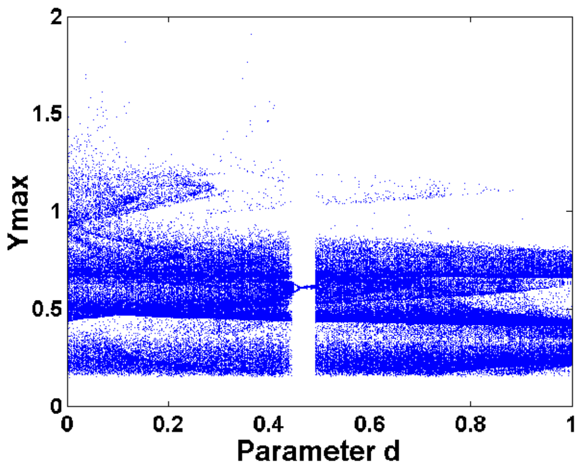


Figure 5 Bifurcation Diagram (between 0-1)

The detailed chaotic behaviour in bifurcation diagram with parameter d for varying in the range of 0-1 are given in Figure 6. System has not chaotic behaviour between about 0.45–0.46.

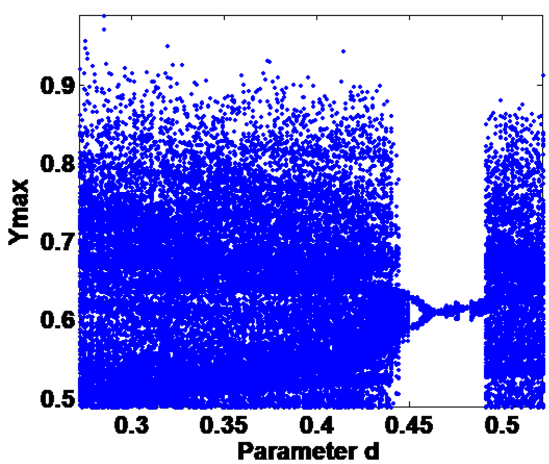


Figure 6 Bifurcation Diagram for varying paramater d (between 0.25-0.55)

ELECTRONIC CIRCUIT DESIGN

The circuit design of the system is given in Figure 7. Electronic circuit implementation of the chaotic system is designed for the parameters $a = 10$, $b = 1.5$, $c = 3$, $d = 70$, $e = 0.5$, $f = 5$ with initial conditions $x(0) = 0.1, y(0) = -0.1, z(0) = 0.2$.

TL081 opamp and the AD633 multipliers were used in electronic circuit and $R1 = 40Kohm$, $R2 = 40Kohm$, $R3 = 266Kohm$, $R4 = 13.3Kohm$, $R5 = 100Kohm$, $R6 = 100Kohm$, $R7 = 5.7Kohm$, $R8 = 399Kohm$, $R9 = 100Kohm$, $R10 = 100Kohm$, $R11 = 80Kohm$, $R12 = 80Kohm$, $R13 = 80Kohm$, $R14 = 80Kohm$, $R15 = 100Kohm$, $R16 = 100Kohm$ $C1 = C2 = C3 = 1nF$, $Vn = -15V$, $Vp = 15V$ are chosen.

The oscilloscope outputs of the system has been seen in Figure 8. The circuit implementation outputs verify the results of the chaotic system modelled in MATLAB.

PASSIVE CONTROL OF THE CHAOTIC SYSTEM

In this section, the passive control method is applied to the system (1) for controlling the chaos Yu (1999); Qi *et al.* (2004); Uyaroglu and Emiroglu (2015); Emiroglu and Uyaroglu (2010). The controlled system (5) is obtained by adding the designed controllers u_1 and u_2 as below.

$$\begin{aligned} \dot{x} &= -10(x+y) + 1.5z + 3yz + u_1 \\ \dot{y} &= 70x - y \\ \dot{z} &= -0.5(x^2 + y^2 - xy) - 5z + u_2 \end{aligned} \tag{5}$$

Suppose that state variables,

$$\begin{aligned} z &= y \\ y &= \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x \\ z \end{pmatrix} \end{aligned} \tag{6}$$

After that, the system can be described by generalized form defined in passive theory:

$$\begin{aligned} \dot{z} &= 70y_1 - z \\ \dot{y}_1 &= -10y_1 + 10z + 1.5y_2 + 3zy_2 + u_1 \\ \dot{y}_2 &= 0.5{y_1}^2 - 0.5z^2 + 0.5y_1z - 5y_2 + u_2 \end{aligned} \tag{7}$$

So, writing the generalized form of the system by using passive control theory given below Yu (1999); Qi *et al.* (2004),

$$\begin{aligned} \dot{z} &= f_0(z) + p(z,y)y, \\ \dot{y} &= b(z,y) + a(z,y)u \end{aligned} \tag{8}$$

where

$$f_0(z) = [-z]$$

$$p(z,y) = [70 \quad 0]$$

$$b(z,y) = \begin{bmatrix} -10y_1 + 10z + 1.5y_2 + 3zy_2 \\ 0.5{y_1}^2 - 0.5z^2 + 0.5y_1z - 5y_2 \end{bmatrix}$$

$$a(z,y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The storage function can be selected as,

$$W(z) = \frac{1}{2}{z_1}^2 \tag{9}$$

is the Lyapunov function of $f_0(z)$ by condition $W(0) = 0$.

Considering (9), the derivative of $W(z)$ can be obtained as follows.

$$\dot{W} = \frac{d}{dt}W(z) = \frac{\partial W(z)}{\partial z} f_0(z) = [z][-z] = -z^2 \leq 0 \tag{10}$$

It can be obtained that the chaotic system is going to be minimum phase with expression (11).

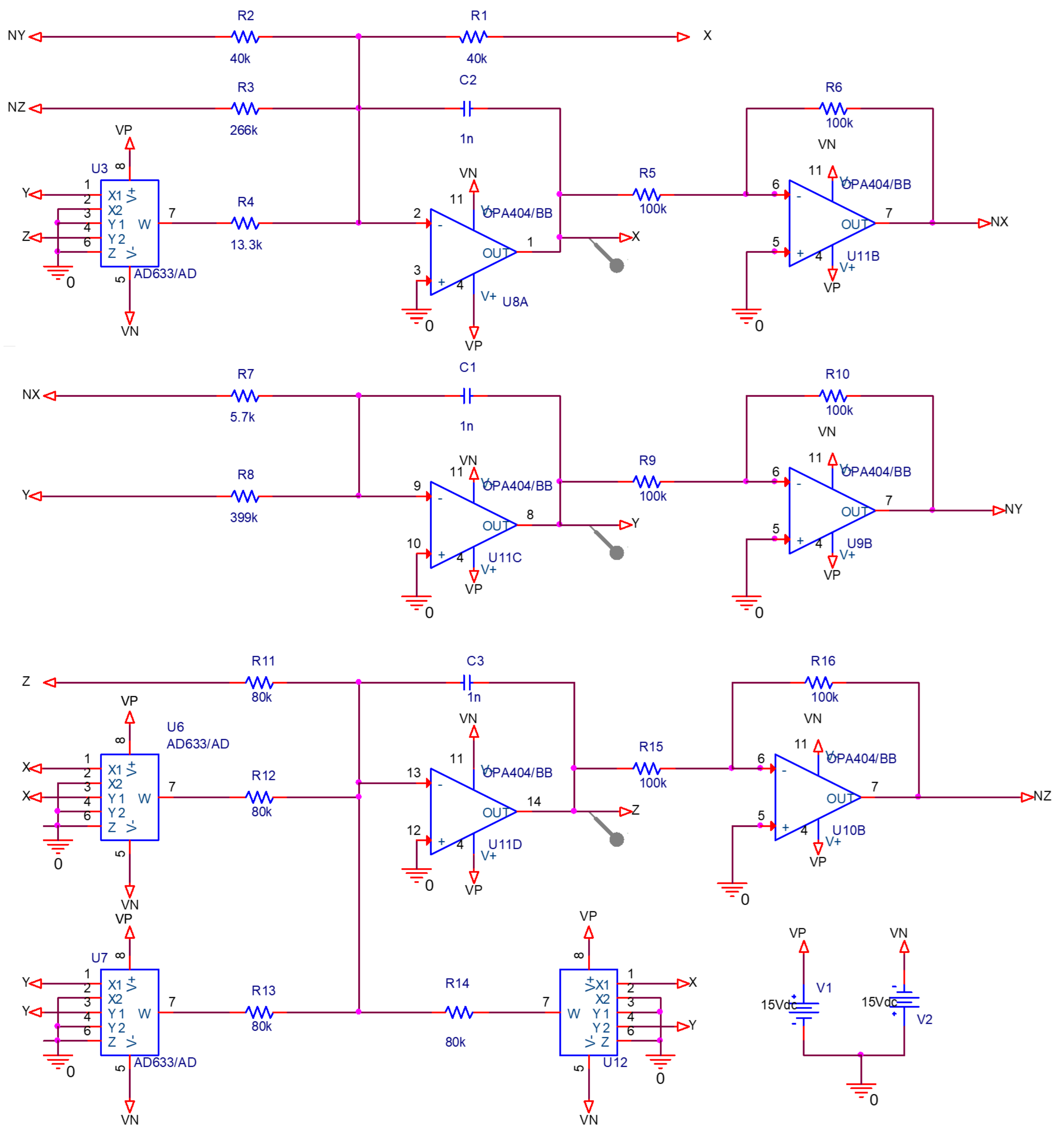
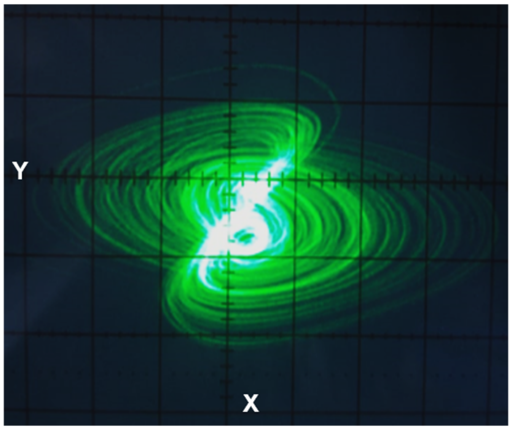
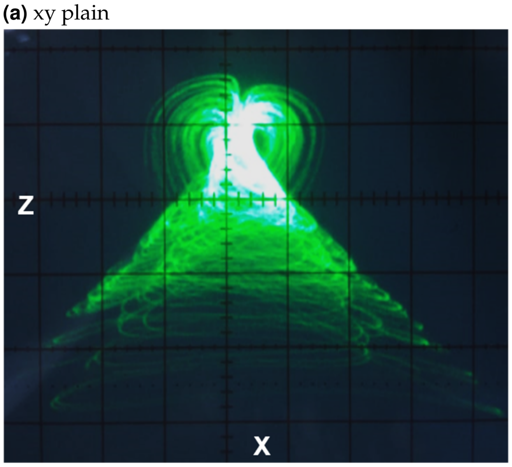


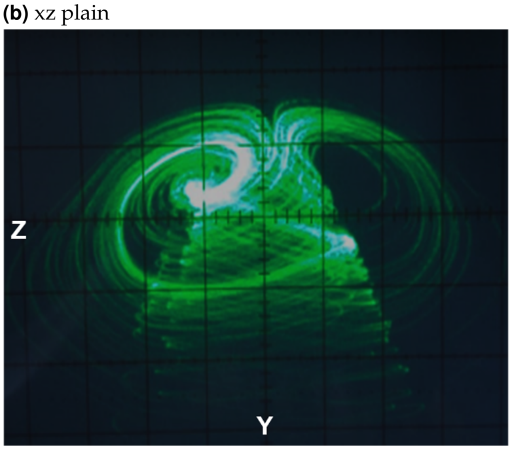
Figure 7 The experimental circuit of system



(a) xy plain



(b) xz plain



(c) yz plain

Figure 8 The phase portraits of chaotic system with a=10, b=1.5, c=3, d=70, e=0.5 and f=5 on the oscilloscope

$$\frac{\partial W(z)}{\partial z} f_0(z) \leq 0. \tag{11}$$

As $W(z) \geq 0$ and $\dot{W}(z) \leq 0$, the zero dynamics of the controlled system (5) is Lyapunov stable, so $f_0(z)$ is globally asymptotically stable. So, the system is said to be as a passive system with the use of following state feedback Yu (1999); Qi *et al.* (2004).

According to passive control theory Uyaroglu and Emiroglu (2015); Yu (1999); Qi *et al.* (2004), the system (5) can be called as passive system. From the property of passive control theory, the control signal u is determined as follows.

$$u = a(z,y)^{-1}\left[-b^T(z,y) - \frac{\partial}{\partial z}W(z)p(z,y) - \alpha y + v\right] \tag{12}$$

where $W(z)$, α and v are the Lyapunov function of $f_0(z)$, the positive real values and an external reference input respectivelyYu (1999); Qi *et al.* (2004); Emiroglu and Uyaroglu (2010).

The system with control signal u (5) can be considered as a passive system. It is asymptotically controlled around equilibrium point at origin globally by the state feedback controller as follows.

$$\begin{aligned} u_1 &= -(10+\alpha)x - 60y + 1.5z + 3yz + v \\ u_2 &= 0.5x^2 - 0.5y^2 + 0.5xy - (5+\alpha)z + v \end{aligned} \tag{13}$$

When passive controller is applied at $t = 25$ s to the system, it is showed the convergence of the system to its zero equilibrium point as shown in Figs. 9 and 10. Also the controlled chaotic system (controller is active from the beginning of the simulations-activated at $t = 0s$) converging to zero equilibrium points is shown in phase trajectory figure (Fig. 10).

The designed feedback controllers for control of the chaotic system are constructed based on passive theory. After the passivity based controllers are designed, the control signals with external signal $v = 0$ are applied to the chaotic system.

The time series of the controlled chaotic system with passivity based controller applied at $t = 25$ s are shown in Fig. 9. Also, phase space of the controlled system is shown in Fig. 10.

As it can be seen from the Figs. 9 and 10, the system can be controlled at its zero equilibrium point asymptotically and converges to the equilibrium point at origin, after the controllers are applied to the system. Passive controller provides the global asymptotical stability of the controlled system due to designing the controller by using Lyapunov stability theory.

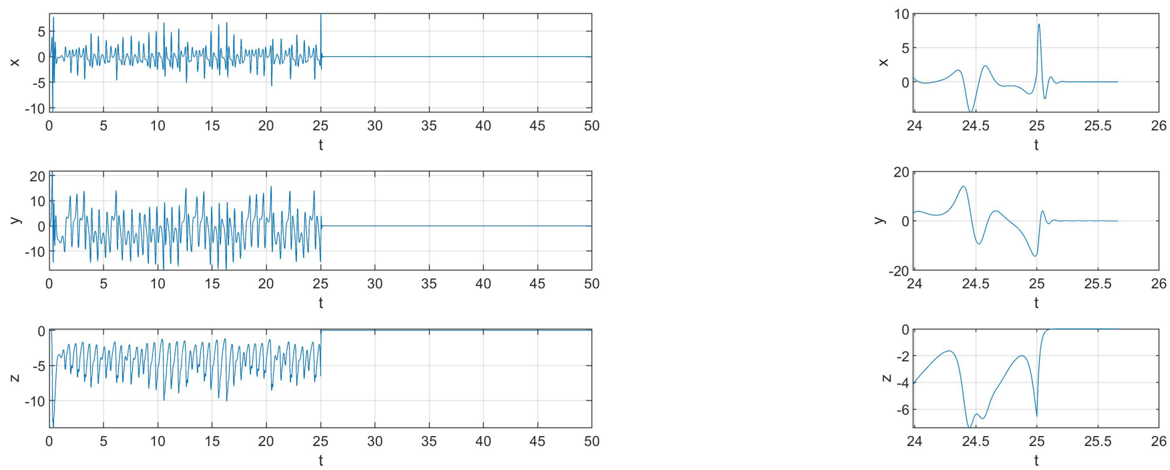


Figure 9 Time series of the controlled system with the controllers activated at 25 s.

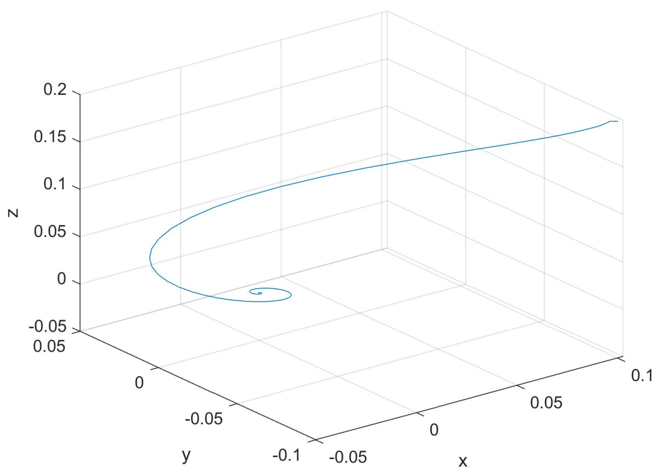


Figure 10 Phase space of the controlled system

RESULTS AND DISCUSSION

In this study, the dynamics of a chaotic system is explored such as Lyapunov exponents, Lyapunov dimension, phase portraits and bifurcation diagrams. The numerical simulations as well as theoretical analysis are also presented to indicate the chaotic dynamic behavior of a system. We determined the sensitivity to initial conditions on the chaotic system. Electronic circuit implementation of the chaotic system has been presented and compared to Matlab simulations. it is confirmed that comparisons between Matlab simulation and electronic circuit experimental results are consistent with each other and demonstrated from the phase portraits. Next, based on passive theory, the controllers are designed to stabilize the chaotic system to its zero equilibrium point. It is showed that chaos control can be achieved using passive control method and the stability of the controlled system is ensured by the controller designed based on Lyapunov stability theory.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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