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Some Notes on the New Sequence Space $b_p^{r,s}(D)$

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Highlights

- The $b_p^{r,s}(D)$ is defined by means of the composition of the Binomial matrix and triple band matrix.
- The space $b_p^{r,s}(D)$ is linearly isomorphic to the space l_p. where $1 \le p < \infty$.
- Some inclusion relations and Schauder basis of the space $b_p^{r,s}(D)$ are obtained.
- α-, β- and γ-duals of the space $b_p^{r,s}(D)$ are calculated .

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Abstract

In this paper, we describe the sequence space $b_p^{r,s}(D)$ originated by the composition of the Binomial matrix and generalized second order difference (triple band) matrix and indicate that the space $b_p^{r,s}(D)$ is linearly isomorphic to the space l_p, where $1 \le p < \infty$. Moreover, we obtain some inclusion relations and Schauder basis of the space $b_p^{r,s}(D)$. We also pinpoint α-, β- and γ-duals of the space $b_p^{r,s}(D)$. Finally, we classify some matrix classes related to the space $b_p^{r,s}(D)$.

1. INTRODUCTION

A sequence space is described as a vector subspace of w which is a vector space under poit-wise addition and scalar multiplication, where w is a set of all real (or complex) valued sequences. By l_∞, c, c_0 and l_p symbols, we mean correspondingly the classical sequence spaces of all bounded, convergent, null and absolutely p-summable sequences, where $1 \le p < \infty$.

A Banach sequence space is identified as a BK-space should each of the maps $p_n: X \longrightarrow \mathbb{C}$ be defined by $p_n(x) = x_n$ is continuous for all $n \in \mathbb{N}$ [1]. Taking this notion into account, it can be said that l_∞, c and c_0 are BK-spaces along with their usual sup-norm named by $\|x\|_\infty = \sup_{k\in\mathbb{N}}|x_k|$ and l_p is a BK-space with its p-norm defined by

$$\|x\|_p = \left(\sum_{k=0}^{\infty}|x_k|^p\right)^{\frac{1}{p}}$$

where $1 \le p < \infty$. To maintain straightforwardness, the summation without limits runs from 0 to ∞ in the remaining of the paper.

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Let $A=(a_{nk})$ be an infinite matrix of complex entries, X and Y be two sequence spaces and $x=(x_k)\in w$. The A-transform of x is, then, defined by

$$(Ax)_n=\sum_k a_{nk}x_k$$

and is supposed to be convergent for all $n\in\mathbb{N}$. By $(X:Y)$, we mean the class of all infinite matrices from X into Y represented as

$$(X:Y)=\{A=(a_{nk}): Ax\in Y\ for\ all\ x\in X\},$$

The matrix domain of $A=(a_{nk})$ in X is defined by

$$X_A=\{x=(x_k)\in w: Ax\in X\}$$

which is also a sequence space [2].

For bs and cs, in the given order, we write the sets of all bounded and convergent series which are defined by using the matrix domain of the summation matrix $S=(s_{nk})$ such that $bs=(l_\infty)_S$ and $cs=c_S$ where $S=(s_{nk})$ is defined by

$$s_{nk}=\begin{cases}1 & , \quad 0\le k\le n\\ 0 & , \quad k>n\end{cases}$$

for all $n,k\in\mathbb{N}$.

An infinite matrix $A=(a_{nk})$ is called a triangle provided the entries $a_{nk}=0$ for $k>n$ and $a_{nn}\neq 0$ for all $n,k\in\mathbb{N}$. A triangle matrix has an inverse which is unique and a triangle. If the otherwise is not stated, any term with negative subscript is assumed to be zero.

Many authors construct a new sequence space by using the matrix domain of an infinite matrix such as: $(l_p)_{N_q}$ and c_{N_q} in [3], X_p and X_∞ in [4], $l_\infty(\Delta), c_0(\Delta)$ and $c(\Delta)$ in [5], $l_\infty(\Delta^2), c_0(\Delta^2)$ and $c(\Delta^2)$ in [6], e_0^r and e_c^r in [7], e_p^r and e_∞^r in [8] and [9], $e_0^r(\Delta)$, $e_c^r(\Delta)$ and $e_\infty^r(\Delta)$ in [10], $e_0^r(\Delta^m)$, $e_c^r(\Delta^m)$ and $e_\infty^r(\Delta^m)$ in [11], $e_0^r(B^{(m)})$, $e_c^r(B^{(m)})$ and $e_\infty^r(B^{(m)})$ in [12], $\hat{l}_\infty, \hat{c}, \hat{c}_0$ and $\hat{l}_p$ in [13].

In the present paper, we describe the sequence space $b_p^{r,s}(D)$ originated by the composition of the Binomial matrix and generalized second order difference (triple band) matrix and indicate that the space $b_p^{r,s}(D)$ is linearly isomorphic to the space l_p, where $1\le p<\infty$. Additionally, we obtain some inclusion relations and Schauder basis of the space $b_p^{r,s}(D)$. We also pinpoint α-, β- and γ-duals of the space $b_p^{r,s}(D)$. Finally, we classify some matrix classes related to the space $b_p^{r,s}(D)$.

2. THE SEQUENCE SPACE $b_p^{r,s}(D)$

In this part, we briefly state the previous studies of Binomial matrix and Euler matrix, and define the sequence space $b_p^{r,s}(D)$. Moreover, we show that the sequence space $b_p^{r,s}(D)$ is linearly isomorphic to the sequence space l_p and is not a Hilbert space excluding the case $p=2$, where $1\le p<\infty$. Besides, we investigate some inclusion relations.

The usage of matrix domain of the Euler matrix was first motivated by the authors in [7-9]. They constructed the Euler sequence spaces e_0^r, e_c^r, e_∞^r and e_p^r as:

$$e_0^r = \left\{ x = (x_k) \in w: \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k x_k = 0 \right\},$$

$$e_c^r = \left\{ x = (x_k) \in w: \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k x_k \ \ exists \right\},$$

$$e_\infty^r = \left\{ x = (x_k) \in w: \sup_{n\in\mathbb{N}} \left| \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k x_k \right| < \infty \right\}$$

and

$$e_p^r = \left\{ x = (x_k) \in w: \sum_{n} \left| \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k x_k \right|^p < \infty \right\}$$

where $1 \le p < \infty$, $0 < r < 1$ and the Euler matrix of order r is defined by

$$e_{nk}^r = \begin{cases} \binom{n}{k} (1-r)^{n-k} r^k & , \quad 0 \le k \le n \\ 0 & , \quad k > n \end{cases}$$

for all $n, k \in \mathbb{N}$.

Afterwards, Altay and Polat [10] improved works in [7-9] by defining the sequence spaces $e_0^r(\Delta)$, $e_c^r(\Delta)$ and $e_\infty^r(\Delta)$ in [10] as:

$$e_0^r(\Delta) = \left\{ x = (x_k) \in w: \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k (x_k - x_{k-1}) = 0 \right\},$$

$$e_c^r(\Delta) = \left\{ x = (x_k) \in w: \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k (x_k - x_{k-1}) \ \ exists \right\}$$

and

$$e_\infty^r(\Delta) = \left\{ x = (x_k) \in w: \sup_{n\in\mathbb{N}} \left| \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k (x_k - x_{k-1}) \right| < \infty \right\}$$

Recently, Bişgin [14,15] has further generalized works in [7-9] by describing the Binomial sequence spaces $b_0^{r,s}$, $b_c^{r,s}$, $b_\infty^{r,s}$ and $b_p^{r,s}$ in [14,15] as :

$$b_0^{r,s} = \left\{ x = (x_k) \in w: \lim_{n\to\infty} \frac{1}{(s+r)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k x_k = 0 \right\},$$

$$b_c^{r,s} = \left\{ x = (x_k) \in w: \lim_{n\to\infty} \frac{1}{(s+r)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k x_k \ \ exists \right\},$$

$$b_{\infty}^{r,s}=\left\{x=(x_k)\in w:\sup_{n\in\mathbb{N}}\left|\frac{1}{(s+r)^n}\sum_{k=0}^{n}\binom{n}{k}s^{n-k}r^kx_k\right|<\infty\right\}$$

and

$$b_{p}^{r,s}=\left\{x=(x_k)\in w:\sum_{n}\left|\frac{1}{(s+r)^n}\sum_{k=0}^{n}\binom{n}{k}s^{n-k}r^kx_k\right|^p<\infty\right\}$$

where $1\le p<\infty$ and the Binomial matrix $B^{r,s}=\left(b_{nk}^{r,s}\right)$ is defined by

$$b_{nk}^{r,s}=\begin{cases}\frac{1}{(s+r)^n}\binom{n}{k}s^{n-k}r^k & , \quad 0\le k\le n\\ 0 & , \quad k>n\end{cases}$$

for all $n,k\in\mathbb{N}$, $r,s\in\mathbb{R}$ and $s.r>0$. In the matrix above, we obtain the Euler matrix of order r provided we take $r+s=1$.

Subsequently, when the Binomial matrix and generalized difference matrix $G=(g_{nk})$ is considered, the sequence space $b_p^{r,s}(G)$ has been defined by Bişgin in [16] as follows:

$$b_{p}^{r,s}(G)=\left\{x=(x_k)\in w:\sum_{n}\left|\frac{1}{(s+r)^n}\sum_{k=0}^{n}\binom{n}{k}s^{n-k}r^k(tx_k+ux_{k-1})\right|^p<\infty\right\}$$

where $1\le p<\infty$ and generalized difference matrix $G=(g_{nk})$ is defined by

$$g_{nk}=\begin{cases}t & , \quad k=n\\ u & , \quad k=n-1\\ 0 & , \quad \text{otherwise}\end{cases}$$

for all $n,k\in\mathbb{N}$ and $t,u\in\mathbb{R}\backslash\{0\}$. Note that, if we take $t=1$ and $u=-1$, we obtain the difference matrix Δ. So, generalized difference matrix generalizes the difference matrix [13].

Now, we define the sequence space $b_p^{r,s}(D)$ by using triple band matrix and Binomial matrix such that

$$b_{p}^{r,s}(D)=\left\{x=(x_k)\in w:\sum_{n}\left|\frac{1}{(s+r)^n}\sum_{k=0}^{n}\binom{n}{k}s^{n-k}r^k(tx_k+ux_{k-1}+vx_{k-2})\right|^p<\infty\right\}$$

where $1\le p<\infty$ and triple band matrix $D=(d_{nk})$ is defined by

$$d_{nk}=\begin{cases}t & , \quad k=n\\ u & , \quad k=n-1\\ v & , \quad k=n-2\\ 0 & , \quad \text{otherwise}\end{cases}$$

for all $n,k\in\mathbb{N}$ and $t,u,v\in\mathbb{R}\backslash\{0\}$. If $t=1,u=-2$ and $v=1$, we obtain the difference matrix Δ^2. Moreover, if we take $v=0$, we obtain the generalized difference matrix $G=(g_{nk})$. So, we generalize the sequence space $b_p^{r,s}(G)$.

Using the domain of the triple band matrix, we define the sequence space $b_p^{r,s}(D)$ by

$$b_p^{r,s}(D) = \left(b_p^{r,s}\right)_D \ . \tag{1}$$

Also, by constructing a matrix $H^{r,s} = \left(h_{nk}^{r,s}\right)$ so that

$$h_{nk}^{r,s} = \begin{cases} \dfrac{s^{n-k-2}r^k}{(s+r)^n}\left[ts^2\binom{n}{k} + usr\binom{n}{k+1} + vr^2\binom{n}{k+2}\right] & , \quad 0 \le k \le n \\ 0 & , \quad k > n \end{cases}$$

for all $n, k \in \mathbb{N}$, we redefine the sequence space $b_p^{r,s}(D)$ by aid of the $H^{r,s} = \left(h_{nk}^{r,s}\right)$ matrix as follows:

$$b_p^{r,s}(D) = \left(l_p\right)_{H^{r,s}} \ . \tag{2}$$

So, for given $x = (x_k) \in w$, the $H^{r,s}$-transform of x is defined by

$$y_k = (H^{r,s}x)_k = \frac{1}{(s+r)^k}\sum_{i=0}^{k}\binom{k}{i} s^{k-i}r^i(tx_i + ux_{i-1} + vx_{i-2}) \tag{3}$$

or

$$y_k = (H^{r,s}x)_k = \frac{1}{(s+r)^k}\sum_{i=0}^{k}\left[ts^2\binom{k}{i} + usr\binom{k}{i+1} + vr^2\binom{k}{i+2}\right]s^{k-i-2}r^i x_i \tag{4}$$

for all $k \in \mathbb{N}$.

Theorem 2.1. The sequence space $b_p^{r,s}(D)$ is a BK-space with its norm defined by

$$\|x\|_{b_p^{r,s}(D)} = \|H^{r,s}x\|_p = \left(\sum_{k=0}^{\infty}|(H^{r,s}x)_k|^p\right)^{\frac{1}{p}}$$

where $1 \le p < \infty$.

Proof. It is known that l_p is a BK-space according to its p-norm and (2) holds. Also, the matrix $H^{r,s} = \left(h_{nk}^{r,s}\right)$ is a triangle. By combining these results and Theorem 4.3.12 of Wilansky [2], we deduce that the sequence space $b_p^{r,s}(D)$ is a BK-space, where $1 \le p < \infty$.

Theorem 2.2. The sequence space $b_p^{r,s}(D)$ is linearly isomorphic to the sequence space l_p, where $1 \le p < \infty$.

Proof. Let L be a transformation such that $L: b_p^{r,s}(D) \longrightarrow l_p$, $L(x) = H^{r,s}x$. Then, we should show that L is a linear bijection. The linearity of L is obvious. On the other hand, It can be easily shown that $x = \theta$ whenever $Tx = \theta$. So, L is injective.

Now, let us define a sequence $x = (x_n)$ such that

$$x_n = \frac{1}{t}\sum_{k=0}^{n}\sum_{i=k}^{n}\sum_{\vartheta=0}^{n-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{n-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\, y_k$$

for all $n \in \mathbb{N}$, where $y = (y_k) \in l_p$ and $1 \le p < \infty$. Then, we have

$$\|x\|_{b_p^{r,s}(D)} = \|H^{r,s}x\|_p$$

$$= \left(\sum_{n=0}^{\infty} |(H^{r,s}x)_n|^p \right)^{\frac{1}{p}}$$

$$= \left(\sum_{n=0}^{\infty} \left| \frac{1}{(s+r)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (tx_k + ux_{k-1} + vx_{k-2}) \right|^p \right)^{\frac{1}{p}}$$

$$= \left(\sum_{n=0}^{\infty} \left| \frac{1}{(s+r)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k \sum_{j=0}^{k} \binom{k}{j} (-s)^{k-j} (r+s)^j r^{-k} y_j \right|^p \right)^{\frac{1}{p}}$$

$$= \left(\sum_{n=0}^{\infty} |y_n|^p \right)^{\frac{1}{p}}$$

$$= \|y\|_p < \infty.$$

Therefore, L is norm preserving and $x = (x_n) \in b_p^{r,s}(D)$ for all $y = (y_k) \in l_p$, namely L is surjective.

As a consequence, L is a linear bijection as desired.

Theorem 2.3. The sequence space $b_p^{r,s}(D)$ is not a Hilbert space for $1 \leq p < \infty$ with $p \neq 2$.

Proof. Suppose $p = 2$. By Theorem 2.1, the sequence space $b_2^{r,s}(D)$ is a BK-space with its norm defined by

$$\|x\|_{b_2^{r,s}(D)} = \|H^{r,s}x\|_2 = \left(\sum_{k=0}^{\infty} |(H^{r,s}x)_k|^2 \right)^{\frac{1}{2}}$$

which is also generated by an inner product such that

$$\|x\|_{b_2^{r,s}(D)} = \langle H^{r,s}x, H^{r,s}x \rangle^{\frac{1}{2}}.$$

So, $b_2^{r,s}(D)$ is a Hilbert space.

On the other hand, assuming that $p \in [1,\infty)\backslash\{2\}$, we define two sequences $y = (y_k)$ and $z = (z_k)$ as follows:

$$y_k = \frac{1}{t} \sum_{i=0}^{k} \sum_{\vartheta=0}^{k-i} \left(\frac{-u + \sqrt{u^2 - 4vt}}{2t} \right)^{k-i-\vartheta} \left(\frac{-u - \sqrt{u^2 - 4vt}}{2t} \right)^{\vartheta} \left(-\frac{s}{r} \right)^{i-1} \frac{-s + i(r+s)}{r}$$

and

$$z_k = \frac{1}{t} \sum_{i=0}^{k} \sum_{\vartheta=0}^{k-i} \left(\frac{-u + \sqrt{u^2 - 4vt}}{2t} \right)^{k-i-\vartheta} \left(\frac{-u - \sqrt{u^2 - 4vt}}{2t} \right)^{\vartheta} \left(-\frac{s}{r} \right)^{i-1} \frac{-s - i(r+s)}{r}$$

for all $k \in \mathbb{N}$. Then we get

$$\|y+z\|^2_{b_p^{r,s}(D)} + \|y-z\|^2_{b_p^{r,s}(D)} = 8 \neq 2^{\frac{2}{p}+2} = 2\left[\|y\|^2_{b_p^{r,s}(D)} + \|z\|^2_{b_p^{r,s}(D)}\right].$$

Therefore, the norm of the sequence space $b_p^{r,s}(D)$ does not satisfy the parallelogram equality, namely the norm can not be generated by an inner product. As a consequence, the sequence space $b_p^{r,s}(D)$ is not a Hilbert space for $p \in [1,\infty)\backslash\{2\}$.

Theorem 2.4. The inclusion $l_p(D) \subset b_p^{r,s}(D)$ strictly holds, where $1 \leq p < \infty$.

Proof. We give the proof of theorem for $1 < p < \infty$. In case of $p = 1$, the proof can be done similarly.

For a given arbitrary sequence $x = (x_k) \in l_p(D)$, from the definition of the sequence space $l_p(D)$,

we have

$$\sum_k |tx_k + ux_{k-1} + vx_{k-2}|^p < \infty$$

where $1 < p < \infty$. By the Hölder's inequality, we write

$$|(H^{r,s}x)_k|^p = \left|\frac{1}{(s+r)^k}\sum_{j=0}^{k}\binom{k}{j} s^{k-j}r^j\left(tx_j + ux_{j-1} + vx_{j-2}\right)\right|^p$$

$$\leq \left(\frac{1}{|s+r|^k}\right)^p \left[\left(\sum_{j=0}^{k}\binom{k}{j}|s|^{k-j}|r|^j\right)^{p-1} \times \left(\sum_{j=0}^{k}\binom{k}{j}|s|^{k-j}|r|^j\left|tx_j + ux_{j-1} + vx_{j-2}\right|^p\right)\right]$$

$$= \frac{1}{|s+r|^k}\sum_{j=0}^{k}\binom{k}{j}|s|^{k-j}|r|^j\left|tx_j + ux_{j-1} + vx_{j-2}\right|^p$$

$$= \sum_{j=0}^{k}\binom{k}{j}\left|\frac{s}{s+r}\right|^k\left|\frac{r}{s}\right|^j\left|tx_j + ux_{j-1} + vx_{j-2}\right|^p$$

where $1 < p < \infty$. Then we obtain

$$\sum_k |(H^{r,s}x)_k|^p \leq \sum_k\sum_{j=0}^{k}\binom{k}{j}\left|\frac{s}{s+r}\right|^k\left|\frac{r}{s}\right|^j\left|tx_j + ux_{j-1} + vx_{j-2}\right|^p$$

$$= \sum_j\left|tx_j + ux_{j-1} + vx_{j-2}\right|^p\sum_{k=j}^{\infty}\binom{k}{j}\left|\frac{s}{s+r}\right|^k\left|\frac{r}{s}\right|^j$$

$$= \left|\frac{s+r}{s}\right|\sum_j\left|tx_j + ux_{j-1} + vx_{j-2}\right|^p$$

where $1 < p < \infty$. If we connect this result and comparison test, $H^{r,s}x \in l_p$, namely $x = (x_k) \in b_p^{r,s}(D)$. Hence, $l_p(D) \subset b_p^{r,s}(D)$.

Define a sequence $z = (z_k)$ such that

$$z_k=\frac{1}{t}\sum_{i=0}^{k}\sum_{\vartheta=0}^{k-i}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{k-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-1)^i$$

for all $k\in\mathbb{N}$. Then, one can see that $Dz=\left((-1)^k\right)\notin l_p$ and $H^{r,s}z=\left(\left(\frac{s-r}{s+r}\right)^k\right)\in l_p$, namely $z=(z_k)\notin l_p(D)$ and $z=(z_k)\in b_p^{r,s}(D)$ which shows $l_p(D)\subset b_p^{r,s}(D)$.

Theorem 2.5. The inclusion $b_p^{r,s}(D)\subset b_q^{r,s}(D)$ strictly holds in case of $1\le p<q<\infty$.

Proof. It is known that the inclusion $l_p\subset l_q$ holds in case of $1\le p<q<\infty$. Suppose $x=(x_k)\in b_p^{r,s}(D)$. Then, we have $H^{r,s}x\in l_p$. By combining these two facts, we write $H^{r,s}x\in l_q$, namely $x=(x_k)\in b_q^{r,s}(D)$. This shows us that the inclusion $b_p^{r,s}(D)\subset b_q^{r,s}(D)$ holds.

Define $g=(g_k)$ by

$$g_k=\frac{1}{t}\sum_{j=0}^{k}\left[\sum_{i=j}^{k}\sum_{\vartheta=0}^{k-i}\binom{i}{j}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{k-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-j}(r+s)^j r^{-i}\right](j+1)^{-\frac{1}{p}}$$

for all $k\in\mathbb{N}$. Then, it is clear that $H^{r,s}g=\left(\frac{1}{(k+1)^{\frac{1}{p}}}\right)\in l_q\backslash l_p$, namely $g=(g_k)\in b_q^{r,s}(D)\backslash b_p^{r,s}(D)$ in case of $1\le p<q<\infty$. Therefore the inclusion $b_p^{r,s}(D)\subset b_q^{r,s}(D)$ strictly holds.

Theorem 2.6. The sequence spaces $b_p^{r,s}(D)$ and $l_\infty(D)$ overlap but do not include each other, where $p\in[1,\infty)$.

Proof. Define three sequences $x=(x_k)$, $y=(y_k)$ and $z=(z_k)$ as follows

$$x_k=\frac{(-1)^k}{t}\sum_{i=0}^{k}\sum_{\vartheta=0}^{k-i}\left(\frac{u-\sqrt{u^2-4vt}}{2t}\right)^{k-i-\vartheta}\left(\frac{u+\sqrt{u^2-4vt}}{2t}\right)^{\vartheta},$$

$$y_k=\frac{1}{t}\sum_{i=0}^{k}\sum_{\vartheta=0}^{k-i}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{k-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}$$

and

$$z_k=\frac{1}{t}\sum_{i=0}^{k}\sum_{\vartheta=0}^{k-i}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{k-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}\left(-\frac{s}{r}\right)^i$$

for all $k\in\mathbb{N}$, where $\left|\frac{s}{r}\right|>1$.Then $Dx=\left((-1)^k\right)\in l_\infty$, $H^{r,s}x=\left(\left(\frac{s-r}{s+r}\right)^k\right)\in l_p$, $Dy=e\in l_\infty$, $H^{r,s}y=e\notin l_p$, $Dz=\left(\left(-\frac{s}{r}\right)^k\right)\notin l_\infty$ and $H^{r,s}z=(1,0,0,\dots)\in l_p$, namely $x\in l_\infty(D)\cap b_p^{r,s}(D)$, $y\in l_\infty(D)\backslash b_p^{r,s}(D)$ and $z\in b_p^{r,s}(D)\backslash l_\infty(D)$. As a consequence of these the spaces, $b_p^{r,s}(D)$ and $l_\infty(D)$ overlap but do not include each other, where $p\in[1,\infty)$.

3. THE SCHAUDER BASIS AND $\alpha-,\beta-,\gamma$ −DUALS OF THE SPACE $b_p^{r,s}(D)$

In this section, we determine the Schauder basis and α-, β-, γ-duals of the sequence space $b_p^{r,s}(D)$.

A sequence $y=(y_k)$ is called a Schauder basis of a normed space $(X,\|.\|_X)$, if for each $x=(x_k)\in X$, there exists a unique sequence $\lambda=(\lambda_k)$ of scalars such that

$$\lim_{m\to\infty}\left\|x-\sum_{k=0}^{m}\lambda_k y_k\right\|_X=0.$$

Then the expansion of $x=(x_k)$ with respect to $y=(y_k)$ is written by

$$x=\sum_{k=0}^{\infty}\lambda_k y_k$$

By [17] X_A has an Schauder basis if and only if X has a Schauder basis whenever $A=(a_{nk})$ is a triangle. Also, the sequence $\left(e^{(k)}\right)$ is a Schauder basis for l_p and the matrix $H^{r,s}=\left(h_{nk}^{r,s}\right)$ is a triangle, where $e^{(k)}$ is a sequence with 1 in the k-th place and zeros elsewhere.

By combining these results, we can give next corollary.

Corollary 3.1. Let $\mu^{(k)}(r,s)=\left\{\mu_n^{(k)}(r,s)\right\}_{n\in\mathbb{N}}$ be a sequence defined by

$$\mu_n^{(k)}(r,s)=\begin{cases}\dfrac{1}{t}\displaystyle\sum_{i=k}^{n}\sum_{\vartheta=0}^{n-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{n-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i} & , \quad n\geq k\\ 0 & , \quad 0\leq n<k\end{cases}$$

for all fixed $k\in\mathbb{N}$. Then, the Schauder basis of the sequence space $b_p^{r,s}(D)$ is the sequence $\left\{\mu^{(k)}(r,s)\right\}_{k\in\mathbb{N}}$ and every $x=(x_k)\in b_p^{r,s}(D)$ can be uniquely written of the form

$$x=\sum_k\sigma_k\mu^{(k)}(r,s)$$

where $\sigma_k=(H^{r,s}x)_k$ for all $k\in\mathbb{N}$.

By connecting the results of Theorem 2.1 and Corollary 3.1 , one more result can be given.

Corollary 3.2. The sequence space $b_p^{r,s}(D)$ is separable.

A set defined by

$$M(X,Y)=\{y=(y_k)\in w:\ xy=(x_k y_k)\in Y\ \text{ for all } x=(x_k)\in X\}$$

is called the multiplier space of the sequence spaces X and Y. Then, the α-, β- and γ-duals of the sequence space X are defined by means of the multiplier space, l_1, cs and bs such that

$$X^\alpha=M(X,l_1)\ ,\ X^\beta=M(X,cs)\ \text{ and }\ X^\gamma=M(X,bs)$$

respectively.

Lemma 3.3. (see [18]) Let $A=(a_{nk})$ be an infinite matrix. Then, the following statements hold

i-) $A=(a_{nk})\in(l_1:l_1)$ iff

$$\sup_{k\in\mathbb{N}}\sum_n|a_{nk}|<\infty\ , \tag{5}$$

ii-) $A=(a_{nk})\in(l_1:l_\infty)$ iff

$$\sup_{n,k\in\mathbb{N}}|a_{nk}|\ , \tag{6}$$

iii-) $A=(a_{nk})\in(l_1:c)$ iff (6) holds and

$$\lim_{n\to\infty}a_{nk}=a_k \text{ for all } k\in\mathbb{N}\ , \tag{7}$$

Lemma 3.4. (see [18])

Let $A=(a_{nk})$ be an infinite matrix. Then, the following statements hold

i-) $A=(a_{nk})\in\left(l_p:l_1\right)$ iff

$$\sup_{K\in\mathcal{F}}\sum_k\left|\sum_{n\in K}a_{nk}\right|^q<\infty\ , \tag{8}$$

ii-) $A=(a_{nk})\in\left(l_p:l_\infty\right)$ iff

$$\sup_{n\in\mathbb{N}}\sum_k|a_{nk}|^q<\infty\ , \tag{9}$$

iii-) $A=(a_{nk})\in\left(l_p:c\right)$ iff (7) and (9) hold

where $\frac{1}{p}+\frac{1}{q}=1\ ,1<p<\infty$ and $\mathcal{F}$ is the collection of all finite subset of $\mathbb{N}$.

Theorem 3.5. Let $\xi_1^{r,s}(\mathrm{D})$ and $\xi_2^{r,s}(\mathrm{D})$ be two sets defined by

$$\xi_1^{r,s}(\mathrm{D})=\left\{a=(a_k)\in w:\ \sup_{K\in\mathcal{F}}\sum_k\left|\frac{1}{t}\sum_{n\in K}\sum_{i=k}^{n}\sum_{\vartheta=0}^{n-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{n-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\,a_n\right|^q<\infty\right\}$$

and

$$\xi_2^{r,s}(\mathrm{D})=\left\{a=(a_k)\in w:\ \sup_{k\in\mathbb{N}}\sum_n\left|\frac{1}{t}\sum_{i=k}^{n}\sum_{\vartheta=0}^{n-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{n-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\,a_n\right|<\infty\right\}.$$

Then $\{b_1^{r,s}(D)\}^\alpha=\xi_2^{r,s}(\mathrm{D})$ and $\left\{b_p^{r,s}(D)\right\}^\alpha=\xi_1^{r,s}(\mathrm{D})$, where $1<p<\infty$.

Proof. Consider the sequence $x=(x_n)$, which is defined by

$$x_n=\frac{1}{t}\sum_{k=0}^{n}\left[\sum_{i=k}^{n}\sum_{\vartheta=0}^{n-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{n-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\right]y_k \tag{10}$$

for all $n\in\mathbb{N}$. Then, for given $a=(a_n)\in w$, we write

$$a_nx_n=\sum_{k=0}^{n}\left[\frac{1}{t}\sum_{i=k}^{n}\sum_{\vartheta=0}^{n-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{n-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\,a_n\right]y_k=\sum_{k=0}^{n}p_{nk}^{r,s}y_k=(P^{r,s}y)_n$$

for all $n\in\mathbb{N}$. By taking into account the equality above, we observe that $ax=(a_nx_n)\in l_1$ whenever $x=(x_k)\in b_1^{r,s}(D)$ or $x=(x_k)\in b_p^{r,s}(D)$ iff $P^{r,s}y\in l_1$ whenever $y=(y_k)\in l_1$ or $y=(y_k)\in l_p$, respectively where $1 < p < \infty$. So, we obtain that $a = (a_n) \in \{b_1^{r,s}(D)\}^\alpha$ or $a = (a_n) \in \{b_p^{r,s}(D)\}^\alpha$ iff $P^{r,s} \in (l_1 : l_1)$ or $P^{r,s} \in (l_p : l_1)$, respectively, where $1 < p < \infty$. By connecting these results, (5) and (8), we deduce that

$$a = (a_n) \in \{b_1^{r,s}(G)\}^\alpha \Leftrightarrow \sup_{k\in\mathbb{N}} \sum_n \left| \frac{1}{t} \sum_{i=k}^{n} \sum_{\vartheta=0}^{n-i} \binom{i}{k} \left(\frac{-u+\sqrt{u^2-4vt}}{2t} \right)^{n-i-\vartheta} \left(\frac{-u-\sqrt{u^2-4vt}}{2t} \right)^{\vartheta} (-s)^{i-k} (r+s)^k r^{-i} a_n \right| < \infty$$

and

$$a = (a_n) \in \{b_p^{r,s}(G)\}^\alpha \Leftrightarrow \sup_{K\in\mathcal{F}} \sum_k \left| \frac{1}{t} \sum_{n\in K} \sum_{i=k}^{n} \sum_{\vartheta=0}^{n-i} \binom{i}{k} \left(\frac{-u+\sqrt{u^2-4vt}}{2t} \right)^{n-i-\vartheta} \left(\frac{-u-\sqrt{u^2-4vt}}{2t} \right)^{\vartheta} (-s)^{i-k} (r+s)^k r^{-i} a_n \right|^q < \infty$$

where $1 < p < \infty$. These yield us that $\{b_1^{r,s}(D)\}^\alpha = \xi_2^{r,s}(\mathrm{D})$ and $\{b_p^{r,s}(D)\}^\alpha = \xi_1^{r,s}(\mathrm{D})$,where $1 < p < \infty$.

Theorem 3.6. Define the sets $\xi_3^{r,s}(\mathrm{D})$, $\xi_4^{r,s}(\mathrm{D})$ and $\xi_5^{r,s}(\mathrm{D})$ by

$$\xi_3^{r,s}(\mathrm{D}) = \left\{ a = (a_k) \in w : \frac{1}{t} \sum_{j=k}^{\infty} \sum_{i=k}^{j} \sum_{\vartheta=0}^{j-i} \binom{i}{k} \left(\frac{-u+\sqrt{u^2-4vt}}{2t} \right)^{j-i-\vartheta} \left(\frac{-u-\sqrt{u^2-4vt}}{2t} \right)^{\vartheta} (-s)^{i-k} (r+s)^k r^{-i} a_j \text{ exists for all } k \in \mathbb{N} \right\}$$

$$\xi_4^{r,s}(\mathrm{D}) = \left\{ a = (a_k) \in w : \sup_{k,n\in\mathbb{N}} \left| \frac{1}{t} \sum_{j=k}^{n} \sum_{i=k}^{j} \sum_{\vartheta=0}^{j-i} \binom{i}{k} \left(\frac{-u+\sqrt{u^2-4vt}}{2t} \right)^{j-i-\vartheta} \left(\frac{-u-\sqrt{u^2-4vt}}{2t} \right)^{\vartheta} (-s)^{i-k} (r+s)^k r^{-i} a_j \right| < \infty \right\}$$

and

$$\xi_5^{r,s}(\mathrm{D}) = \left\{ a = (a_k) \in w : \sup_{n\in\mathbb{N}} \sum_{k=0}^{n} \left| \frac{1}{t} \sum_{j=k}^{n} \sum_{i=k}^{j} \sum_{\vartheta=0}^{j-i} \binom{i}{k} \left(\frac{-u+\sqrt{u^2-4vt}}{2t} \right)^{j-i-\vartheta} \left(\frac{-u-\sqrt{u^2-4vt}}{2t} \right)^{\vartheta} (-s)^{i-k} (r+s)^k r^{-i} a_j \right|^q < \infty \right\}$$

where $1 < q < \infty$.

Then the following statements hold:

(I) $\{b_1^{r,s}(D)\}^\beta = \xi_3^{r,s}(\mathrm{D}) \cap \xi_4^{r,s}(\mathrm{D})$,

(II) $\{b_p^{r,s}(D)\}^\beta = \xi_3^{r,s}(\mathrm{D}) \cap \xi_5^{r,s}(\mathrm{D})$, where $1 < p < \infty$,

(III) $\{b_1^{r,s}(D)\}^\gamma = \xi_4^{r,s}(\mathrm{D})$,

(IV) $\{b_p^{r,s}(D)\}^\gamma = \xi_5^{r,s}(\mathrm{D})$, where $1 < p < \infty$.

Proof. Since the proofs of the parts (II), (III) and (IV) may be obtained by using a same way, we prove the theorem for only the part (I). Let $a = (a_n) \in w$ be arbitrarily given. Consider the sequence $x = (x_n)$ defined by the relation (10). Then, we write

$$\sum_{k=0}^{n} a_k x_k = \sum_{k=0}^{n} \left[\frac{1}{t} \sum_{j=0}^{k} \sum_{i=j}^{k} \sum_{\vartheta=0}^{k-i} \binom{i}{j} \left(\frac{-u+\sqrt{u^2-4vt}}{2t} \right)^{k-i-\vartheta} \left(\frac{-u-\sqrt{u^2-4vt}}{2t} \right)^{\vartheta} (-s)^{i-j} (r+s)^j r^{-i} y_j \right] a_k$$

$$= \sum_{k=0}^{n} \left[\frac{1}{t} \sum_{j=k}^{n} \sum_{i=k}^{j} \sum_{\vartheta=0}^{j-i} \binom{i}{k} \left(\frac{-u+\sqrt{u^2-4vt}}{2t} \right)^{j-i-\vartheta} \left(\frac{-u-\sqrt{u^2-4vt}}{2t} \right)^{\vartheta} (-s)^{i-k} (r+s)^k r^{-i} a_j \right] y_k$$

$$= (Q^{r,s} y)_n$$

for all $n \in \mathbb{N}$, where the matrix $Q^{r,s} = (q_{nk}^{r,s})$ is defined by

$$q_{nk}^{r,s} = \begin{cases} \frac{1}{t}\sum_{j=k}^{n}\sum_{i=k}^{j}\sum_{\vartheta=0}^{j-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{j-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\,a_j & , \quad 0 \le k \le n \\ 0 & , \quad k > n \end{cases}$$

for all $n, k \in \mathbb{N}$. So, $ax = (a_n x_n) \in cs$ whenever $x = (x_k) \in b_1^{r,s}(D)$ iff $Q^{r,s}y \in c$ whenever $y = (y_k) \in l_1$. This yields us that $a = (a_n) \in \{b_1^{r,s}(D)\}^{\beta}$ iff $Q^{r,s} \in (l_1 : c)$. By connecting this result and (7), we obtain that $a = (a_n) \in \{b_1^{r,s}(D)\}^{\beta}$ iff

$$\sup_{k,n\in\mathbb{N}}\left|\frac{1}{t}\sum_{j=k}^{n}\sum_{i=k}^{j}\sum_{\vartheta=0}^{j-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{j-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\,a_j\right| < \infty$$

and

$$\frac{1}{t}\sum_{j=k}^{\infty}\sum_{i=k}^{j}\sum_{\vartheta=0}^{j-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{j-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\,a_j \quad \text{exists for all } k \in \mathbb{N}$$

This result shows that $\{b_1^{r,s}(D)\}^{\beta} = \xi_3^{r,s}(\mathrm{D}) \cap \xi_4^{r,s}(\mathrm{D})$.

4. SOME MATRIX CLASSES

In this section, we characterize some matrix classes related to the sequence space $b_p^{r,s}(G)$, where $1 \le p < \infty$.

For notation simplicity, we prefer to use following equality throughout the section 4.

$$h_{nk}^{r,s,D} = \frac{1}{t}\sum_{j=k}^{\infty}\sum_{i=k}^{j}\sum_{\vartheta=0}^{j-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{j-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^k r^{-i}\,a_{nj}$$

for all $n, k \in \mathbb{N}$.

Theorem 4.1. Given an infinite matrix $A = (a_{nk})$, the following statements hold

(i) $A = (a_{nk}) \in (b_1^{r,s}(D) : l_\infty)$ iff

$$\sup_{k,n\in\mathbb{N}}\left|h_{nk}^{r,s,D}\right| < \infty, \tag{11}$$

(ii) $A = (a_{nk}) \in \left(b_p^{r,s}(D) : l_\infty\right)$ iff

$$\sup_{n\in\mathbb{N}}\sum_k\left|h_{nk}^{r,s,D}\right|^q < \infty \tag{12}$$

$$\{a_{nk}\}_{k\in\mathbb{N}} \in \xi_5^{r,s}(\mathrm{D}) \tag{13}$$

where $1 < p < \infty$.

Proof. Let $p \in (1, \infty)$. We take any $x = (x_k) \in b_p^{r,s}(D)$ by assuming that the (12) and (13) hold. Then, it is obtained that $\{a_{nk}\}_{k\in\mathbb{N}} \in \left\{b_p^{r,s}(D)\right\}^{\beta}$. This result implies the existence of the A transform of x. From the (10), we have

$$\sum_{k=0}^{m} a_{nk}x_k = \sum_{k=0}^{m}\left[\frac{1}{t}\sum_{j=0}^{k}\sum_{i=j}^{k}\sum_{\vartheta=0}^{k-i}\binom{i}{j}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{k-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-j}(r+s)^{j}r^{-i}y_j\right]a_{nk}$$

$$= \sum_{k=0}^{m}\sum_{j=k}^{m}\left[\frac{1}{t}\sum_{i=k}^{j}\sum_{\vartheta=0}^{j-i}\binom{i}{k}\left(\frac{-u+\sqrt{u^2-4vt}}{2t}\right)^{j-i-\vartheta}\left(\frac{-u-\sqrt{u^2-4vt}}{2t}\right)^{\vartheta}(-s)^{i-k}(r+s)^{k}r^{-i}\right]a_{nj}y_k\,. \quad (14)$$

By taking limit of the (14) side by side as $m \to \infty$, we obtain that

$$\sum_k a_{nk}x_k = \sum_k h_{nk}^{r,s,D}y_k \quad (n \in \mathbb{N}). \quad (15)$$

Then, we derive by taking l_∞-norm of the (15) side by side and by applying Hölder's inequality that

$$\|Ax\|_\infty = \sup_{n\in\mathbb{N}}\left|\sum_k h_{nk}^{r,s,D}y_k\right|$$

$$\leq \sup_{n\in\mathbb{N}}\left(\sum_k\left|h_{nk}^{r,s,D}\right|^q\right)^{\frac{1}{q}}\left(\sum_k|y_k|^p\right)^{\frac{1}{p}} < \infty.$$

As a result of this, we obtain that $Ax \in l_\infty$, namely $A = (a_{nk}) \in \left(b_p^{r,s}(D): l_\infty\right)$.

Conversely, assume that $A = (a_{nk}) \in \left(b_p^{r,s}(D): l_\infty\right)$. This gives us to $\{a_{nk}\}_{k\in\mathbb{N}} \in \left\{b_p^{r,s}(D)\right\}^\beta$ for all $n \in \mathbb{N}$. Then, the necessity of (13) is immediate and $\left\{h_{nk}^{r,s,D}\right\}_{k,n\in\mathbb{N}}$ exists. On account of $\{a_{nk}\}_{k\in\mathbb{N}} \in \left\{b_p^{r,s}(D)\right\}^\beta$, we can see that the (15) holds and the sequences $a_n = (a_{nk})_{k\in\mathbb{N}}$ define the continuous linear functionals f_n on $b_p^{r,s}(D)$ by

$$f_n(x) = \sum_k a_{nk}x_k$$

for all $n \in \mathbb{N}$. Also, by Theorem 2.2, $b_p^{r,s}(D)$ and l_p are norm isomorphic. By connecting this result and the equation (15), we obtain that

$$\|f_n\| = \left\|\left(h_{nk}^{r,s,D}\right)_{k\in\mathbb{N}}\right\|_q$$

which yields that the functionals f_n are pointwise bounded. Moreover, we derive from the Banach-Steinhaus theorem that the functionals f_n are uniformly bounded, namely there exists a constant $M > 0$ such that

$$\left(\sum_k\left|h_{nk}^{r,s,D}\right|^q\right)^{\frac{1}{q}} = \|f_n\| \leq M$$

for all $n \in \mathbb{N}$, which shows us that the condition (12) holds. The part (i) can be done similarly.

Lemma 4.2 (see [18]) Let $A = (a_{nk})$ be an infinite matrix. Then, $A = (a_{nk}) \in \left(l_1: l_p\right)$ iff

$$\sup_{k\in\mathbb{N}}\sum_n|a_{nk}|^p < \infty$$

where $1 < p < \infty$.

Theorem 4. 3. Let an infinite matrix $A=(a_{nk})$ be given. Then, $A=(a_{nk})\in\left(b_1^{r,s}(D):l_p\right)$ iff

$$\sup_{k\in\mathbb{N}}\sum_n\left|h_{nk}^{r,s,D}\right|^p<\infty \tag{16}$$

where $1\leq p<\infty$.

Proof. Let a sequence $x=(x_k)\in b_1^{r,s}(D)$ be given. Assume that the condition (16) holds. Then, it is clear that $y=(y_k)\in l_1$ and $\{a_{nk}\}_{k\in\mathbb{N}}\in\{b_1^{r,s}(D)\}^\beta$ for all $n\in\mathbb{N}$, namely A-transform of x exists. As a result of this, the series $\sum_k h_{nk}^{r,s,D}y_k$ are absolutely convergent for all $n\in\mathbb{N}$ and $y=(y_k)\in l_1$. By applying the Minkowsky inequality to (15), we can write

$$\left(\sum_n|(Ax)_n|^p\right)^{\frac{1}{p}}\leq\sum_k|y_k|\left(\sum_n\left|h_{nk}^{r,s,D}\right|^p\right)^{\frac{1}{p}}$$

which yields that $Ax\in l_p$, namely $A=(a_{nk})\in\left(b_1^{r,s}(D):l_p\right)$.

Conversely, we suppose that $A=(a_{nk})\in\left(b_1^{r,s}(D):l_p\right)$, where $1\leq p<\infty$, namely $Ax\in l_p$ for all $x=(x_k)\in b_1^{r,s}(D)$. So, $\{a_{nk}\}_{k\in\mathbb{N}}\in\{b_1^{r,s}(D)\}^\beta$ for all $n\in\mathbb{N}$, which shows us that the (15) holds. These results give us that $H^{r,s,D}=\left(h_{nk}^{r,s,D}\right)\in(l_1:l_p)$. By combining last result and Lemma 4.2, we obtain that the condition (16) holds. This completes the proof.

5. CONCLUSION

The domain of Binomial matrix $B^{r,s}=\left(b_{nk}^{r,s}\right)$ in the sequence space l_p has been introduced by Bişgin in [15]. Also, the domain of triple band matrix $D=(d_{nk})$ in some sequence spaces was used and studied by many authors. Since $H^{r,s}=\left(h_{nk}^{r,s}\right)$ is composition of $B^{r,s}=\left(b_{nk}^{r,s}\right)$ and $D=(d_{nk})$, and $H^{r,s}=\left(h_{nk}^{r,s}\right)$ is stronger than $D=(d_{nk})$, our results are more general.

CONFLICTS OF INTEREST

No conflict of interest was declared by the author.

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