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Directly modulated semiconductor ring lasers: Chaos synchronization and applications to cryptography communications

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ABSTRACT This paper numerically investigates the synchronization and the chaos-based encryption using unidirectional coupled chaotic semiconductor ring lasers (SRLs) with direct modulation. A good quality of synchronization and transmission of messages are realized for specific choice of system parameters. The mismatches of typical internal SRLs parameters have an influence on the quality of synchronization. While the communication performance of the system has a good robustness to the mismatches of typical internal SRLs parameters. Finally, the encrypted message can be recorvered without using any filter.

KEYWORDS
Chaos,
Chaos synchronization,
Chaos-based communications,
Chaos modulation technique.

INTRODUCTION

Communication with synchronized nonlinear lasers was demonstrated experimentally and numerically for Nd: Yttrium Aluminum Garnet and CO2 lasers Colet and Roy (1994); Posadas-Castillo *et al.* (2008), erbium-doped fiber ring lasers Vanwiggeren and Roy (1998) and semiconductor lasers Goedgebuer *et al.* (1998); Rogister *et al.* (2001); Argyris *et al.* (2005); Scire *et al.* (2003); Lee *et al.* (2004); Kouomou and Woafo (2003).

Most of investigations on chaos synchronization characteristics and data encryption in semiconductor laser has been performed on unidirectional coupled conventional edge-emitting semiconductor lasers Goedgebuer *et al.* (1998); Rogister *et al.* (2001); Argyris *et al.* (2005) and vertical-cavity surface-emitting semiconductor lasers Scire *et al.* (2003); Lee *et al.* (2004) with optical feedback. There is few studies published on chaos synchronization and communication in semiconductor lasers with direct modulation to date Kouomou and Woafo (2003); Kingni *et al.* (2012a).

Chaos synchronization and communication based on unidirectional coupled SRLs with direct modulation has not yet been studied. In recent years, SRLs have received ever more attention Krauss *et al.* (1990), since they are ideal candidates for the photonic integrated circuits as main components Hill *et al.* (2004).

This device shows different operating regions characterized by bidirectional-continuous waves to alternate oscillations and bistability Sorel *et al.* (2002); Van der Sande *et al.* (2008). From the application point of view, the bistable unidirectional regime opens the possibility of using SRL in systems for all-optical switching and optical memories Van der Sande *et al.* (2008); Liang *et al.* (1997). The authors of Ref. Kingni *et al.* (2012b) demonstrated asymptotically and numerically that SRLs with direct modulation exhibits chaotic behaviors for specific values of the frequency and the amplitude of modulation.

In this paper, the chaotic synchronization characteristics of unidirectional coupled SRLs with direct modulation and its application to encode communications are numerically analysed. In section 2, the unidirectional coupled SRLs with direct modulation is described and modelled. In section 3, the synchronization properties and message transmission in unidirectional coupled SRLs with direct modulation are investigated. Section 4 concludes the paper.

MODELLING OF UNIDIRECTIONAL COUPLED SEMICONDUCTOR RING LASERS WITH DIRECT MODULATION

The schematic diagram of unidirectional coupled SRLs with direct modulation is shown in Fig. 1.

In Figure 1 the coupling is done by optically injected the CW and CCW modes of the master. Since synchronization between the modes intensities of the master and the slave has not found when

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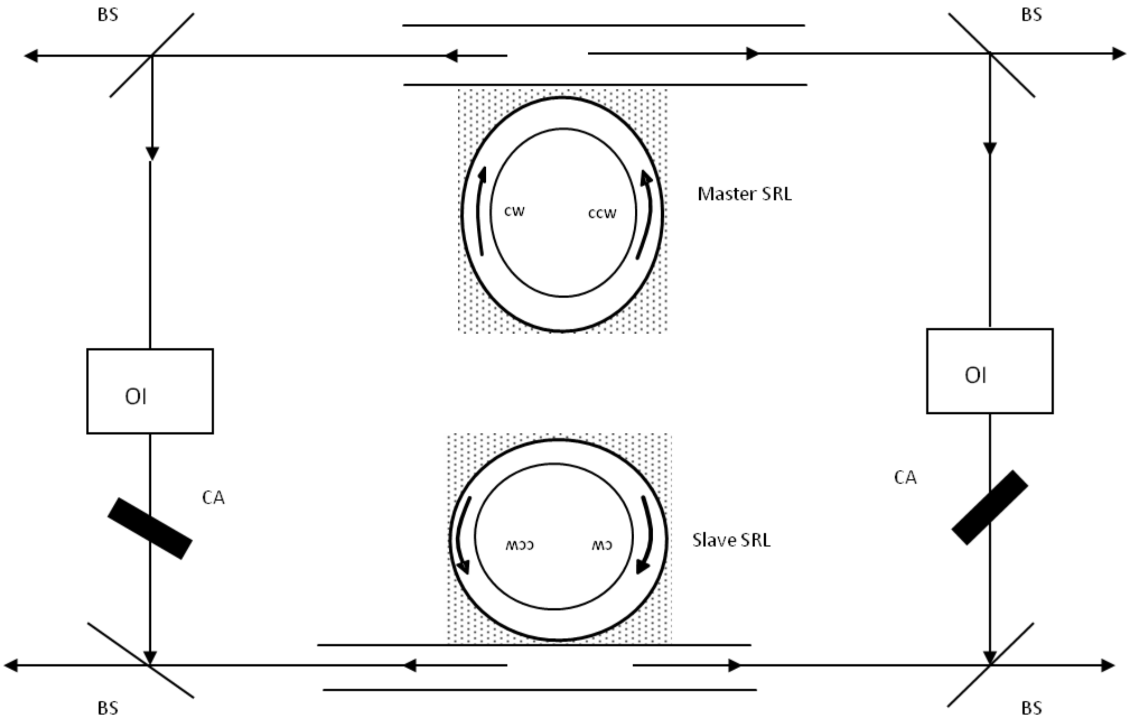

Figure 1 Schematic representation of unidirectional coupled SRLs with direct modulation. BS: Beam spitter, OI: Optical isolator, CA: Coupling attenuator.

one of the modes of the master SRL into one of the modes of the slave SRL is optically injected. The couplings attenuator enable the percentage of the master powers fed into the slave to be controlled. It is assumed in the following that the detuning parameter between the two SRLs is equal to $0\,GHz$. The rate equations describing the master and slave system are given by :

$$\frac{dE_{ccw}}{dt} = (1+i\alpha)\left[N\left(1-s|E_{ccw}|^2-c|E_{cw}|^2\right)-1\right]E_{ccw}- (k_d+i\,k_c)\,E_{cw}, \tag{1a}$$

$$\frac{dE_{cw}}{dt} = (1+i\alpha)\left[N\left(1-s|E_{cw}|^2-c|E_{ccw}|^2\right)-1\right]E_{cw}- (k_d+i\,k_c)\,E_{ccw}, \tag{1b}$$

$$\frac{dN}{dt} = \gamma\left[\begin{array}{l}\mu_{dc}+\mu_m\,\sin\left(2\pi f_m\,t\right)-N-N\left(1-s|E_{ccw}|^2-c|E_{cw}|^2\right)\\ |E_{ccw}|^2-N\left(1-s|E_{cw}|^2-c|E_{ccw}|^2\right)|E_{cw}|^2\end{array}\right], \tag{1c}$$

$$\frac{d\tilde{E}_{ccw}}{dt} = (1+i\alpha)\left[\tilde{N}\left(1-s|\tilde{E}_{ccw}|^2-c|\tilde{E}_{cw}|^2\right)-1\right]\tilde{E}_{ccw}- (k_d+i\,k_c)\,\tilde{E}_{cw}+\eta E_{ccw}, \tag{1d}$$

$$\frac{d\tilde{E}_{cw}}{dt} = (1+i\alpha)\left[\tilde{N}\left(1-s|\tilde{E}_{cw}|^2-c|\tilde{E}_{ccw}|^2\right)-1\right]\tilde{E}_{cw}- (k_d+i\,k_c)\,\tilde{E}_{ccw}+\eta E_{cw}, \tag{1e}$$

$$\frac{d\tilde{N}}{dt} = \gamma\left[\begin{array}{l}\mu_{dc}+\mu_m\,\sin\left(2\pi f_m\,t\right)-\tilde{N}-\tilde{N}\left(1-s|\tilde{E}_{ccw}|^2-c|\tilde{E}_{cw}|^2\right)\\ |\tilde{E}_{ccw}|^2-\tilde{N}\left(1-s|\tilde{E}_{cw}|^2-c|\tilde{E}_{ccw}|^2\right)|\tilde{E}_{cw}|^2\end{array}\right], \tag{1f}$$

where the tilde $(\sim)$ indicates the slave SRL variables and η the scalar coupling parameter. The variable t is the time, E_{cw} and E_{ccw} are the slowly varying complex amplitudes of the counter-propagating waves and N is the carrier density. The internal parameters of the SRL are: $\gamma = 0.002$ is the ratio of photon lifetime $\tau_p = 10\,\text{ps}$ to carrier lifetime τ_s, the self-saturation coefficient $s = 0.005$, the cross-saturation coefficients $c = 0.01$ and the linewidth enhancement factor $\alpha = 3.5$ Sorel *et al.* (2002). The operating parameters are: the dissipative backscattering component $k_d = 0.000327$, the conservative backscattering component $k_c = 0.0044$, the dc bias injection current $\mu_{dc} = 1.704$, the modulation amplitude μ_m and the modulation frequency f_m. The value of μ_{dc} is chosen such that the free-running SRL operates in the bistable unidirectional regime.

The uncoupled SRL with direct modulation exhibits some regions of chaotic dynamics as shown in Ref. Kingni *et al.* (2012b). For each value of μ_m and f_m displaying chaotic behavior, the complexity of the generated chaos by using the well-known Kolmogorov-Sinai Entropy (not shown) are calculated. The time series of the two counter-propagating modes and the corresponding Poincaré section (for $\mu_m = 1.704$, $\mu_m = 0.233$ and $f_m = 240MHz$) in a robust chaotic regime is presented In Fig. 2.

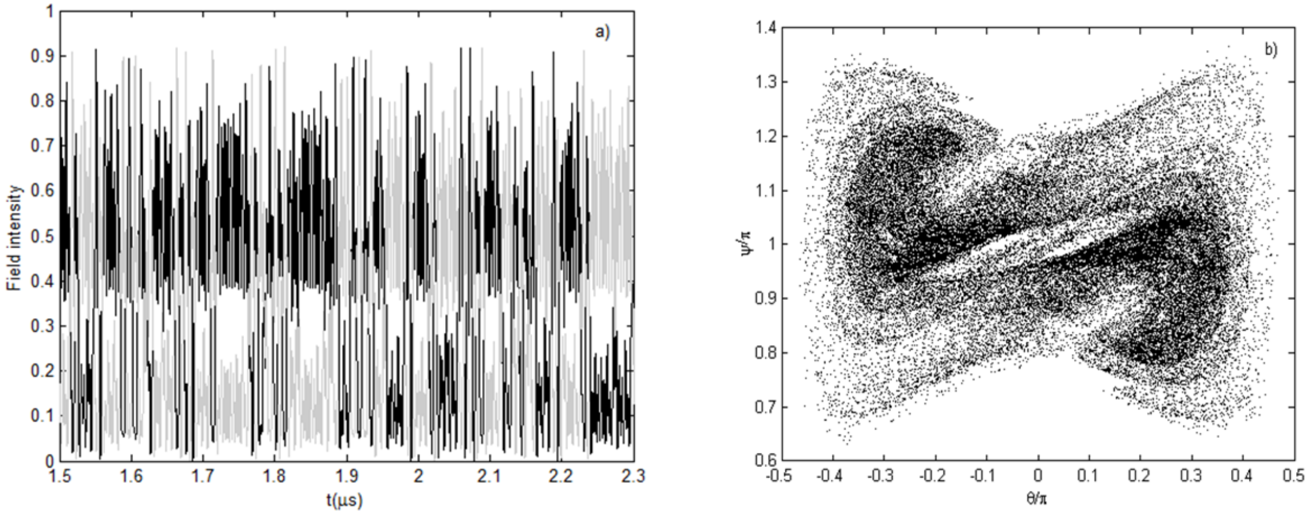

Figure 2 (a) Time series of $I_{\mathrm{CCW}} = |E_{ccw}|^2$ and $I_{\mathrm{CW}} = |E_{cw}|^2$ and (b) the corresponding Poincaré section for $\mu_m = 1.704$, $\mu_m = 0.233$ and $f_m = 240MHz$ (Chaotic behavior). In (a), black (grey) line indicates time trace of $I_{\mathrm{CCW}}(I_{\mathrm{CW}})$. The variable $\theta = 2\arctan\left(\frac{|E_{cw}|}{|E_{ccw}|}\right) - \frac{\pi}{2} \in \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$ represents the relative modal intensity and the variable $\psi \in [0, 2\pi]$ is the phase difference between the counterpropagating modes.

SYNCHRONIZATION PROPERTIES AND MESSAGE TRANSMISSION

The quality of synchronization between the master and slave s estimated by Takougang Kingni *et al.* (2012). THere, it is firstly assumed that the two coupled SRLs with direct modulation are identical, i.e., the parameters are the same with different initial conditions. Figure 3 shows the results obtained for the cross-correlation coefficient between the time series intensities of the master and the slave in the parameter space $(\eta, k_s = \tilde{k})$

In Fig. 3 (a) (C_{ccw_M,ccw_S}) and Fig. 3 (b) (C_{cw_M,cw_S}), the region of good synchronization is localized in a strip around the parameter conditions $k = \tilde{k}$ and $0.19\,ns^{-1} \leq \eta \leq 0.44\,ns^{-1}$. When the system operates out of this optimal conditions ($k = \tilde{k}$ and $0.19\,ns^{-1} \leq \eta \leq 0.44\,ns^{-1}$), a strong degradation of the synchronization occurs. Similar results had already been obtained in the case of unidirectional coupled semiconductor lasers subjected to coherent optical feedback Mirasso *et al.* (2004). Whereas in Fig. 3 (c) (C_{tot_M,tot_S}), the domain of good synchronization is localized for any values of $\tilde{k}$ and for $0.0\,ns^{-1} \leq \eta \leq 0.44\,ns^{-1}$ but for $\eta \succ 0.44\,ns^{-1}$, the appearance of a violent degradation of the synchronization is noted. The CCW/CW mode of master SRL and CW/CCW mode of slave SRL respectively (see Fig. 3 (d) and Fig. 3 (e) show some degree of anticorrelation in large range of parameters η. The region of good synchronization ($k = \tilde{k}$) is further detailed in Fig. 4 which depicts the average synchronization error (left panel/panel 1) and the cross-correlation coefficient (right panel/panel 2) between the outputs of the master and slave as function of the coupling parameter η for $k = \tilde{k}$.

Figure 4 (a1) and (a2) show a good level of synchronization between the total intensities in master and slave SRLs for the coupling parameter $0.0\,ns^{-1} \leq \eta \leq 0.44\,ns^{-1}$ ($C_{tot_M,tot_S} \approx 0.9999$). In Figures 4 (b1), 4 (b2), 4 (c1) and 4 (c2), a better synchronization qualility between the CCW injected light intensity dynamics and the dynamics in the slave SRL CCW mode and then between the CW mode intensity in master and the CW mode intensity in slave is noted for $0.0\,ns^{-1} \leq \eta \leq 0.44\,ns^{-1}$ with the cross-correlation coefficients $C_{ccw_M,ccw_S} \approx 0.9999$ and $C_{cw_M,cw_S} \approx 0.9999$. It is important to underline that by injecting optically the CW and CCW modes of the master into the CCW and CW modes of the slave respectively.

We have also found better quality of synchronization as above between the total intensities in master and slave SRLs then between the CCW mode intensity in master and the CW mode intensity in slave finally between the CW mode intensity in master and the CCW mode intensity in slave. However, the high-good quality of synchronization mentioned above can solely be obtained for the ideal condition. In practice perfect parameter matching is difficult to realize. It is important to study the efffect of the mismatch of the two SRLs on the quality of synchronization. The external parameters of SRL such as μ_{dc}, μ_m and f_m is easly controlled.

Therefore, it is necessary to check the quality of synchronization when the internal parameters of the SRLs fluctuate. The typical internal SRL parameters are α, γ, s and c. We define parameter mismatch as $\Delta x = (x_S - x_M)/x_M$, where x represents α, γ, s and c. Firstly, the effects of the various mismatches are considered separately, all the others are set to zero. For $\eta = 0.3\,ns^{-1}$, the average synchronization error (left panel) and the cross-correlation coefficients (right panel) for different single parameter mismatches are illustrated in Fig. 5.

In Figure 5, the good quality of the synchronization decreases with the increase of mismatched value. We can also note that the parameter mismatch on the cross-saturation coefficient c significantly affects the quality of synchronization. In contrast, the parameter mismatch on parameter γ has much less effect on the quality of synchronization. For a mismatch in parameters α, s and c the average synchronization error seems to grow linearly with the mismatch. Positive and negative mismatches do not have similar effects. The case where all the mismatches are simultaneously considered is presented in Fig. 6.

When all the internal parameters of reciever SRL are simultaneously changed with the same relative amount, the synchronization quality become worse than changing only one of them (see Fig. 6). The cross-correlation coefficients C_{cw_M,cw_S} and C_{ccw_M,ccw_S}

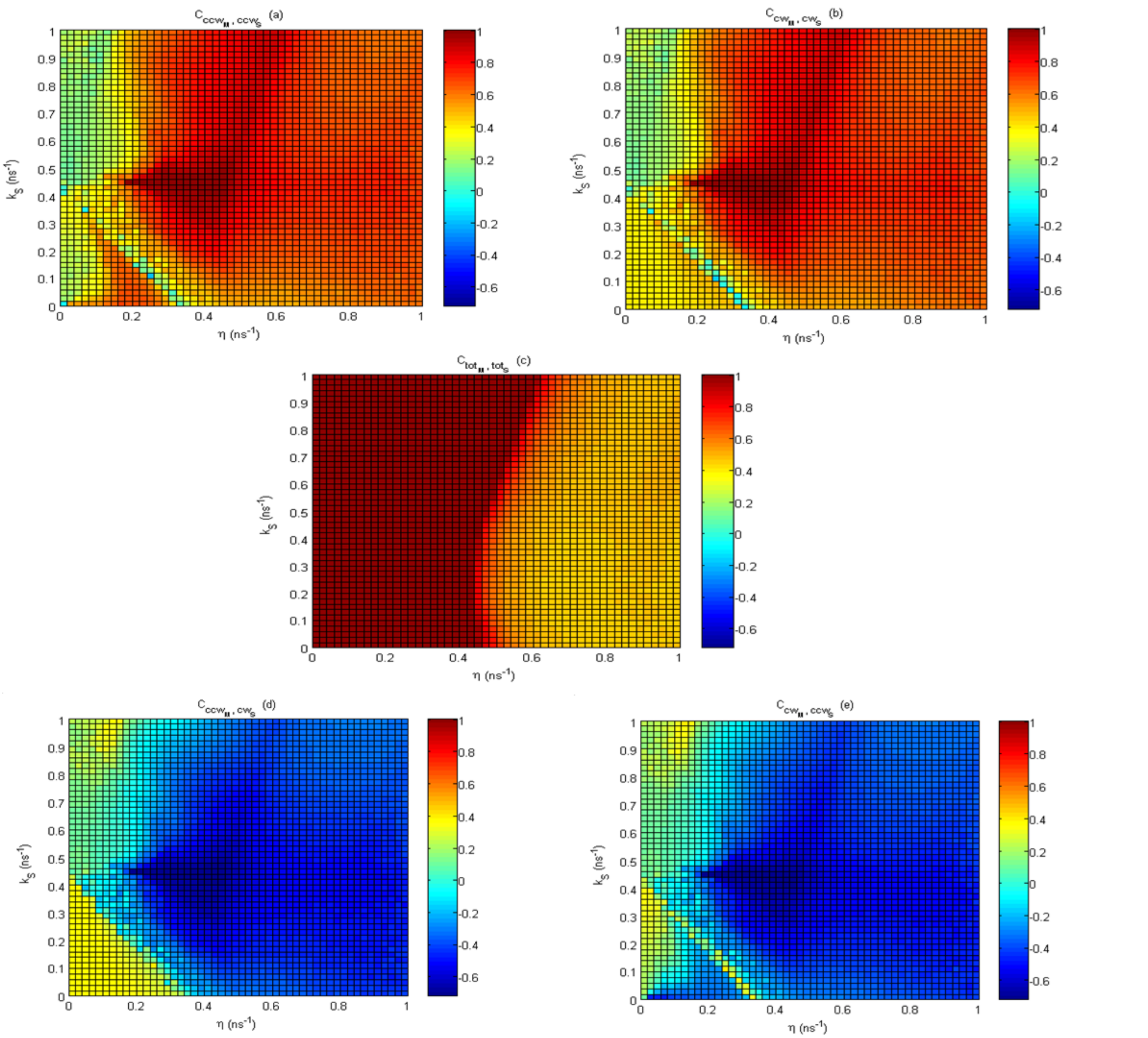

Figure 3 Mapping of correlation coefficient between the outputs of the master and slave in the parameter space spanned by the coupling parameter η and amplitude of the backscattering in slave SRL $k_s = \tilde{k}$ for $\gamma = 0.2\text{ns}^{-1}$, $\text{s} = 0.005$, $\text{c} = 0.01$, $\alpha = 3.5$, $\text{k}_\text{d} = 0.0327\text{ns}^{-1}$, $\text{k}_\text{c} = 0.44\text{ns}^{-1}$, $\mu_{dc} = 1.704$,, $\mu_m = 0.233$, $f_m = 240MHz$, $k = \sqrt{k_d^2 + k_c^2}$ and $\phi_k = \tilde{\phi}_k = \arctan\left(k_c/k_d\right)$.

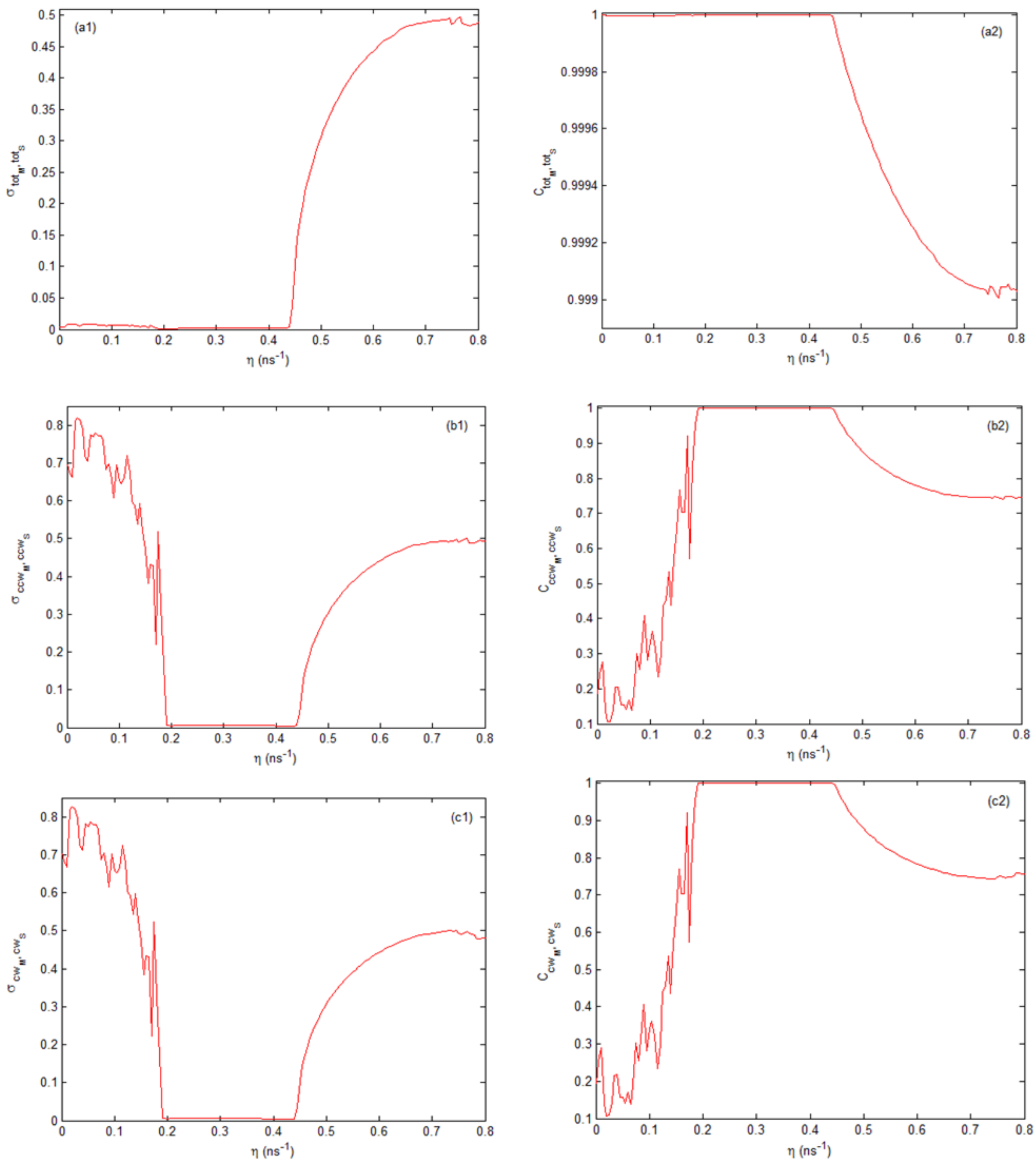

Figure 4 In panel (1), we plot the average synchronization error σ_{tot_M,tot_S} (a1), σ_{ccw_M,ccw_S} (b1), σ_{cw_M,cw_S} (c1) while in panel (2), we depit the corresponding cross-correlation coefficient C_{tot_M,tot_S} (a2), C_{ccw_M,ccw_S} (b2), C_{cw_M,cw_S} (c2) between the outputs of the master and slave as function of the coupling parameter. Parameters are specified in the text.

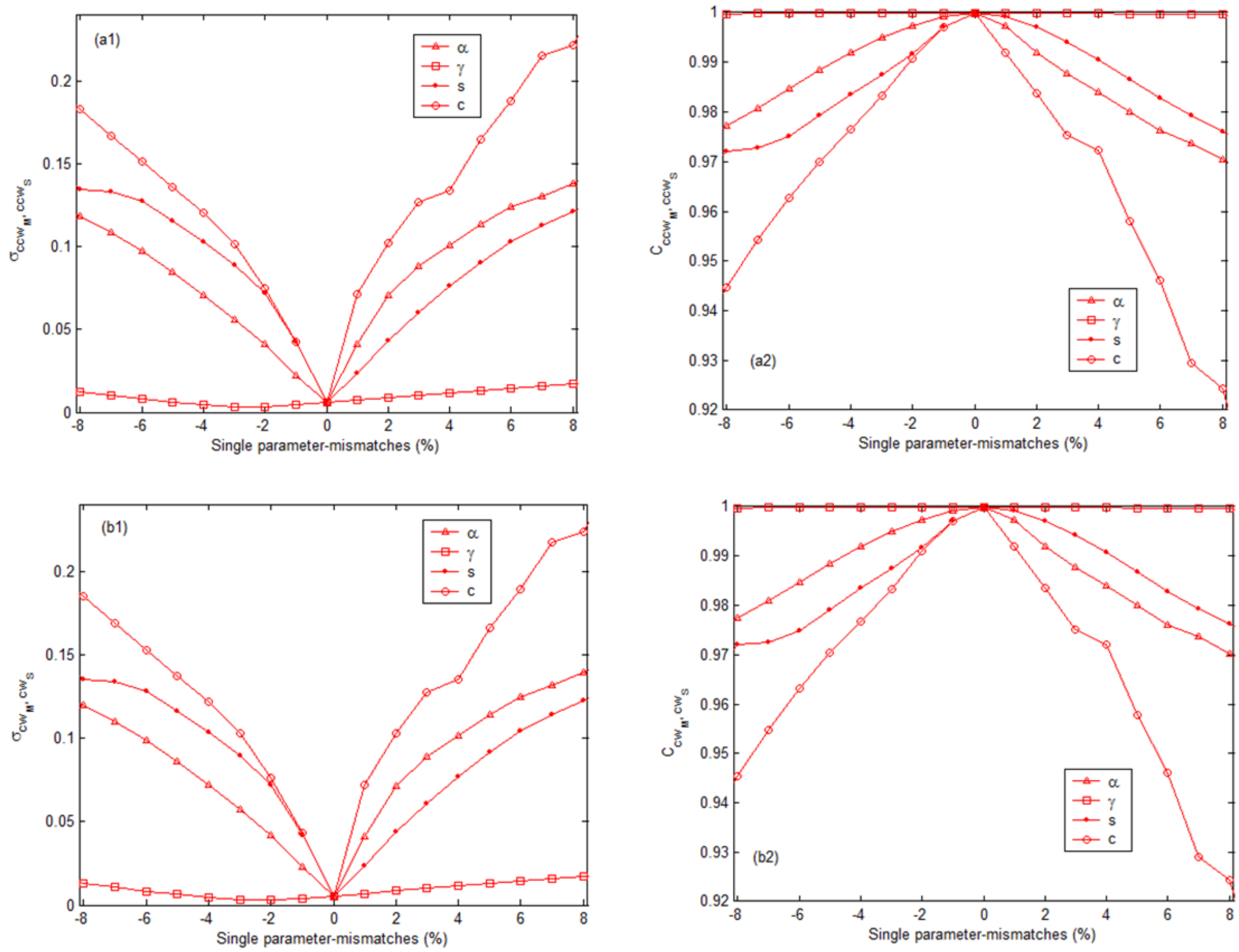

Figure 5 Variation of the average synchronization error (left panel/panel 1) and the corresponding cross-correlation coefficient (right panel/panel 2) as a function of single parameter mismatches for $\eta = 0.3\,ns^{-1}$ and the remaining parameters are given in the text.

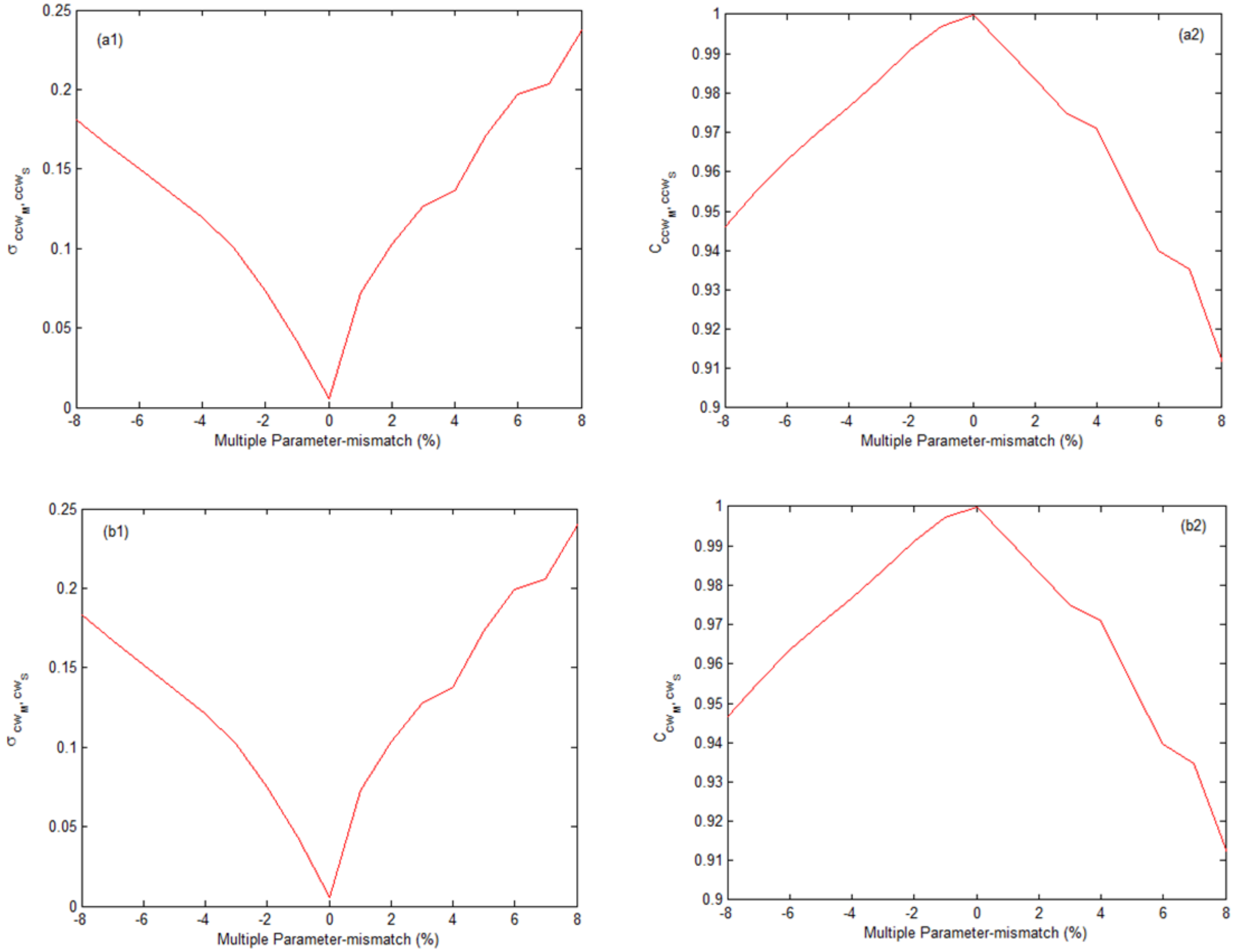

Figure 6 Variation of the average synchronization error (left panel/panel 1) and the corresponding cross-correlation coefficient (right panel/panel 2) as a function of multiple parameter-mismatch for $\eta = 0.3\, ns^{-1}$ and the remaining parameters are given in the text.

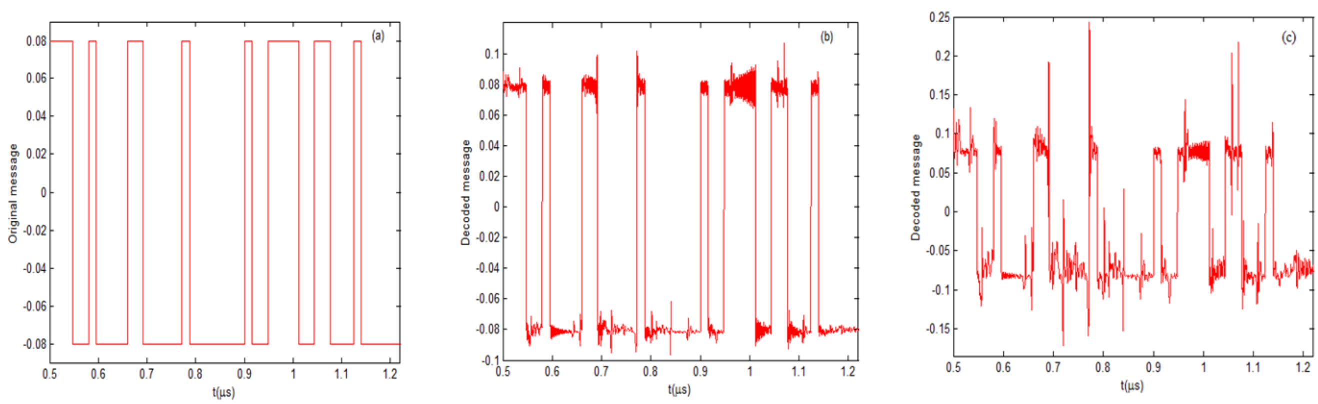

Figure 7 Encoding and decoding messages: (a) original message, (b) decoded message without parameter mismatches and (c) decoded message with $0.5\,^0/_0$ mismatch of typical internal parameters (α, γ, s and c) for $\eta = 0.42\, ns^{-1}$. The remaining parameters are given in the text.

remains greater than 0.9 as shown in Fig. 6. Therefore, this system is robust to the mismatched parameters. This robustness to the mismatched parameters is probably due to the type of coupling applied between the two directly modulated SRLs.

Finally, the communication performances of unidirectionally coupled directly modulated SRLs is examined. Here, the CMO technique Sanchez-Diaz *et al.* (1999) is considered because its implementation is simple. In chaos modulation method, the signal transmitted to the receiver is $E^{'}_{ccw,cw}(t) = E_{ccw,cw}(t)(1 + m_{ccw,cw}(t))$ Sanchez-Diaz *et al.* (1999). The receiver output $\tilde{E}_{ccw,cw}(t)$ to the transmitter carrier field $E_{ccw,cw}(t)$ rather than to the transmitted signal $E^{'}_{ccw,cw}(t)$, help to decode the encrypted message. The message can be recovered in real time by comparing $|E_{ccw,cw}(t)|^2$ and $|\tilde{E}_{ccw,cw}(t)|^2$. The decoding message is obtained as:

$$m^{'}_{ccw,cw}(t) = \sqrt{|E_{ccw,cw}(t)|^2 / |\tilde{E}_{ccw,cw}(t)|^2} - 1. \qquad (2)$$

Two different sequences ($m_{CCW}(t)$ and $m_{CW}(t)$) of a non-return-to-zero (NRZ) pseudorandom digital bits at 62.5 $Mbit.s^{-1}$ encoded respectively to the outputs of the CCW and CW emitted fields of the master. To check the communication performances between each pair of counter-propagating mode of two SRLs, it is calculated for different values of the coupling strength η the correlation coefficients between each original message and the corresponding decoded message (decoding efficiency of message). The maximum value of the variation of decoding efficiency of messages ($m_{CCW}(t)$ and $m_{CW}(t)$) versus the coupling strength η obtained between each pair of counter-propagating mode is less than 0.735. This result demonstrates that the encoded of two different messages respectively to the outputs of the CW and CCW modes of the master are not recovered for both of the counter-propagating modes of the slave. While by encoding the same message in each outputs of the counter-propagating emitted fields of the master is possible to extract successfully the original message for both of the counter-propagating modes of the slave as shown in Fig. 7.

The transmission of a 62.5 $Mbit.s^{-1}$ NRZ pseudorandom message is presented in Fig. 7. Figure 7 (a) shows the input message. Figure 7 (b) depicts the decoded message without parameter mismatches, as can be seen from this figure, the decoded message agrees well with the input message without using any auxiliary optical or electronic filter. Nevertheless, the recovered message contains some noises Liu and Tsimring (2006). Fig. 7 (c) exhibits the decoded message for a 0.5 $^0/_0$ mismatch of typical internal parameters (α, γ, s and c). From this figure, it is noted that the decoding performance is degraded but the message is still recovered.

CONCLUSION

This paper dealt with the numerical investigation of synchronization characteristics of unidirectional coupled chaotic SRLs with direct modulation and its applications to cryptography communications. The results show that unidirectional coupled SRLs with direct modulation can achieve high-quality of synchronization if the operating parameters are adequately chosen. The internal parameter mismatches between the master and the slave SRLs have influenced on the quality of synchronization. It was found that the most critical one is the cross-saturation coefficient. a good quality of synchronization was found for small mismatches parameters. The encoding message can be recorved easily without any filter. Other coupling schemes can be considered and then why not to consider an array of current modulated SRLs with global coupling.

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Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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