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Internal bremsstrahlung spectrum of $^{86}_{37}$Rb for forbidden beta transition

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ABSTRACT

Many researchers have worked on to explain the theory of internal bremsstrahlung process released along with a beta particle and a neutrino from the beta-decaying radioactive nuclei. According to the original Knipp, Uhlenbeck and Bloch (KUB) theory, IB is a low-intensity continuous spectrum of electromagnetic radiation which accompanies all types of beta decay. Former experimental studies on the internal bremsstrahlung emission from the forbidden beta transitions have shown marked deviations from the theoretical calculations. We took the more theoretical calculations for the IB probability problem. The analytical expressions proposed by the work of Chang and Falkoff for forbidden transitions were used for the IB spectrum. We have also calculated the Coulomb effects of IB spectrum from the study by Lewis and Ford who first addressed to these phenomena. We have handled and analyzed the data of IB emissions of $^{86}_{37}$Rb beta emitting isotope that its transition is classified as forbidden. In addition to the analytical calculation, IB spectrum was also obtained by applying Monte Carlo Method to IB problem. The results were compared with the IB spectrum results which are calculated from the analytical expressions.

1. Introduction

The disintegration of the nucleus by emitting electrons or positron, or the capture of one of the electrons around the nucleus orbits, is called beta decay. There is no change in the mass number of the nucleus, but there is always a change in the nuclear charge after the disintegration. During this disintegration, a beta particle (electron or positron) and a neutrino with daughter nucleus is released into the medium. Beta particles emitted from radioactive nuclei have a continuous energy distribution. Beta particles have kinetic energy from zero to a maximum value with various possibilities. The energy of disintegration is shared among the beta, the recoil nuclei and neutrino/antineutrino. Therefore, in beta decay, it has been shown that the sharing of disintegration energy between the beta particle and neutrino or antineutrino is random [1]. Thus, the beta particles have a continuous energy spectrum from E = 0 to a maximum energy value E = E_m. The quantum mechanical theory of beta spectrum was developed by Fermi [2, 3]. The total energy of a beta particle having an energy E in the unit of electron rest mass energy is;

$$W = \frac{E}{mc^2} + 1 \tag{1}$$

The number of beta particles in the energy range W and W +dW, is given by [4, 1, 2, 3];

$$N(W)dW = \frac{|P|^2}{\tau_0} F(Z,W)(W^2-1)^{1/2}(W_0-W)^2 W dW \tag{2}$$

Here $|P|^2$ is the square of the matrix element for the transition. τ_0 is the time constant and F(Z, W) is a complex function defined as the electron density ratio. This study was performed for β^- particles and β^+ particles were not considered.
The speed of the electron In terms of the speed of light, depending on W can be written as

$$\beta = (W^2-1)^{1/2} \tag{3}$$

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The following expression is obtained for electrons, i.e. beta particles, if the Eq.(3) of beta is written as in the equation given by Almaz [5],

$$F(Z,W) \approx 2\,\pi\,\alpha Z\,W/(W^2-1)^{1/2} \quad (4)$$

If this expression is used in Eq.(2), the energy spectrum of beta particles can be expressed as,

$$N(W) = \frac{|P|^2}{\tau_0}\,2\pi\,\alpha\,Z\,(W_0 - W)^2\,W^2 \quad (5)$$

There are some differences in the basic properties of beta disintegration. The smaller the square of the transition matrix $|P|^2$ seen in the Eq.(5) (the greater the f t), the more impossible the transition is considered, i.e. prohibited. Such transitions are called forbidden transitions. The greater the $|P|^2$ (the smaller the f t), the more the transition is possible which means allowed. Such transitions are called allowed transitions. The value of $|P|^2$ is proportional to the amount of superposition of the wave functions of the mother nucleus and daughter nucleus. The more the wave functions overlap, the greater the value $|P|^2$ and approach the value 1. The f t values for the various beta transitions range from 10^3 (super-allowed transitions) to 10^{23} (most forbidden transitions).

The magnitude of P depends on the selection rules as in the gamma decay and the magnitude of the L_β orbital angular momentum which is taken away by the $\beta^-,\bar{\nu}\ /\beta^+,\nu$ beta pair from the daughter nucleus. When L_β increases by one degree, $|P|^2$ becomes smaller by $10^{-2}-10^{-4}$, therefore, the probability of transition is reduced. For the most allowed transitions, the angular momentum carried by the pair $\beta^-,\bar{\nu}\ /\beta^+,\nu$ is zero. Therefore the transitions are allowed for $L_\beta = 0$. Transitions for $L_\beta = 1$ are first forbidden and for $L_\beta = 2$ are second forbidden.

The theoretical energy spectrum obtained from Eq. (5) for Rb-86 β^- source with Z value of 37 is given in Figure 1.

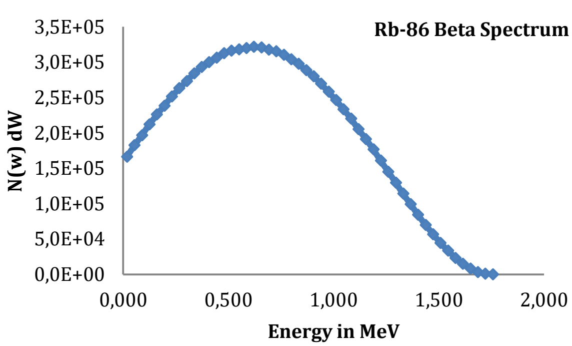

Figure 1. Energy spectrum of beta distribution of Rb-86 isotope

2. Internal Bremsstrahlung (IB) Spectrum from Rb-86 Isotope

There is a certain likelihood of a photon emission simultaneously along with the beta particle and neutrino/anti-neutrino in beta decay mechanism. IB photons have a distinguishing distribution and they are in the form of a continuous photon spectrum as well. The IB photon is caused by the sudden change of electric charge in the nucleus at the moment of release of the beta particle emitted from the radioactive source. for the reason of that this nuclear event is called the Internal Bremsstrahlung (IB) for the sake of distinguishing it from the External Bremsstrahlung (EB) caused by the loss of radiative energy with other particles rather than the nucleus from which the electron emerges. Knipp, Uhlenbeck [6] and independently Bloch [7] presented the IB theory for allowed beta transitions, without taking into account Coulomb effects. Wang Chang and Falkoff [8] have extended The KUB theory for the first and second forbidden transitions. Nilsson [9], Lewis and Ford [10] and Spruch and Gold [11] have calculated the IB spectra considering the Coulomb effects of the nucleus. For beta decays of forbidden transitions, IB spectra were first discussed by Lewis and Ford [10]. The solutions of the first proposed KUB theory were found to be incompatible with experimental data at the midpoint and high energy region of the spectrum. Further theoretical calculations were made by Ford and Martin [12] for the possibility of IB that the detour transitions, a distinctive behavior accompanying the forbidden transitions of the nucleus, were taken into consideration in their studies. As a result, some improvements have been achieved in the agreement between the theory developed by Ford and Martin [12] and the experiments.

In this study, the analytical solutions proposed by Chang and Falkoff [8] for the purpose of having the IB photon spectra for first forbidden beta transitions were calculated as well as Monte Carlo Method for IB problem in computer environment. Thus the IB spectrum of Rb-86 has calculated via this way.

2.1. IB Spectrum by Monte Carlo Simulation Technique

The β^- particle energy is sampled between a cutoff energy value of $E_c = 10$ keV and the endpoint energy of E_m. For the sampling process, it is not possible to find an analytical expression by applying the basic Monte Carlo principle to the energy distribution of β^- particles given by Eq. (5). Therefore, the Rejection Method is used for sampling the distribution [13, 15]. In the execution of the Rejection Method, the rectangular rejection function was used. Detailed calculations related to Monte Carlo method were applied as given in Almaz and Almaz and Cengiz [5, 13]. The IB spectrum obtained by the Monte Carlo Method for Rb-86 was compared with the IB spectrum of the same isotope obtained for forbidden transitions.

2.2. IB spectrum for Allowed and Forbidden Transitions

The distribution theory of the IB spectrum for allowed transitions is in the notation of Chang and Falkoff [10] if x = W_0-k:

$$S(k) = \frac{\alpha G^2}{8\pi^4}\,\frac{2}{15}\left|R_{\alpha\beta}\right|^2\frac{1}{k}\Big\{\Big[\frac{58}{63}x^3 - \frac{86}{21}W_0x^8 + \frac{488}{63}W_0^2x^7 - \frac{364}{45}W_0^3x^6 + \frac{151}{30}W_0^4x^5 - \frac{11}{6}W_0^5x^4 + \frac{1}{3}W_0^6x^3\Big]\,ln2x - \Big[\frac{151553}{79380}x^9 - \frac{72661}{8820}W_0x^8 + \frac{33142}{2205}W_0^2x^7 - \frac{1138}{75}W_0^3x^6 + \frac{16577}{1800}W_0^4x^5 - \frac{239}{72}W_0^5x^4 + \frac{11}{18}W_0^6x^3\Big]\Big\} \quad (6)$$

IB spectrum in scalar interaction for isotopes with first forbidden beta transitions is given by,

$$S(k) = \frac{\alpha G^2|M|^2}{12\pi^4}\frac{1}{k}\Big\{\Big[W_0^4\left(\frac{2}{3}x^3+x\right)+W_0^3\left(-\frac{7}{3}x^4-3x^2+\frac{1}{8}\right)+ W_0^2\left(\frac{18}{5}x^5+5x^2\right)+W_0\left(-\frac{13}{5}x^6-\frac{14}{3}x^4-\frac{15}{8}x^2-\frac{5}{24}\right)+ \left(\frac{76}{105}x^7+\frac{5}{3}x^5+\frac{11}{6}x^3+\frac{3}{8}x\right)\Big]\,ln[x+(x^2-1)^{1/2}] - \Big[W_0^4\left(\frac{11}{9}x^2+\frac{4}{9}\right)+W_0^3\left(-\frac{151}{36}x^3-\frac{73}{72}x\right)+W_0^2\left(\frac{163}{25}x^4+\frac{152}{75}x+\frac{4}{75}\right)+W_0\left(-\frac{4303}{900}x^5-\frac{5783}{1800}x^3-\frac{2441}{1800}x\right)+\left(\frac{14{,}741}{11{,}025}x^6+\frac{8133}{4900}x^4+\frac{135{,}853}{88{,}200}x^2+\frac{136}{2205}\right)\Big](x^2-1)^{\frac{1}{2}}\Big\} \tag{7}$$

IB spectrum in tensor interaction for isotopes with first forbidden beta transitions is;

$$S(k) = \frac{\alpha G^2|M|^2}{12\pi^4}\frac{1}{k}\Big\{\Big[W_0^4\left(\frac{2}{3}x^3+x\right)+W_0^3\left(-2x^4-2x^2+\frac{1}{4}\right)+ W_0^2\left(\frac{43}{15}x^5+\frac{8}{3}x^3-\frac{3}{8}x\right)+W_0\left(-\frac{31}{15}x^6-\frac{7}{3}x^4-\frac{3}{8}x^2-\frac{1}{12}\right)+ \left(\frac{64}{105}x^7+\frac{8}{5}x^5+\frac{1}{12}x\right)\Big]\,ln\left(x+(x^2-1)^{1/2}\right)-\Big[W_0^4\left(\frac{11}{9}x^2+\frac{4}{9}\right)+W_0^2\left(-\frac{7}{2}x^3-\frac{1}{4}x\right)+W_0^2\left(\frac{4481}{900}x^4+\frac{97}{600}x^2+\frac{4}{225}\right)+ W_0\left(-\frac{363}{100}x^5-\frac{1337}{1800}x^3-\frac{437}{900}x\right)+\left(\frac{1306}{1225}x^6-\frac{769}{22{,}050}x^4+\frac{7733}{44{,}100}x^2+\frac{24}{1225}\right)\Big](x^2-1)^{\frac{1}{2}}\Big\}. \tag{8}$$

Where, G is a constant that determines the strength of the coupling of the electron neutrino field with nuclei. M is the matrix element. α is the fine structure constant with a value of 1/137, W_0 is the total energy in the rest mass energy unit and k is the energy value of the photon.

3. Results and Discussion

The radionuclide $^{86}_{37}$Rb has a branch with two different maximum energies. The first branch decay into the ground level of (2-> 0+) transition having an E_{m1} = 1774.4 keV endpoint energy and a decay probability of η1 = 0.912. This transition is classified as a unique first-forbidden transition. The other branch, decays into the first excited state of (2- > 2+) transition having an E_{m2} = 697.0 keV endpoint energy and a η2 = 0.088 disintegration probability. This forbidden transition state is labeled as the first non-unique. IB calculations with KUB theory were performed separately for the two decay branches. In order to obtain the total spectrum, the decay rates of the respective decay branches were added together.

IB spectra obtained for $^{86}_{37}$Rb radionuclide are given in Figure 2. Some abbreviations have been made for the spectra obtained in the given graph. Accordingly, MC specifies the Monte Carlo solution. For allowed transitions, LF and LF2 show, Z = 0 and Z ≠ 0 respectively, LF3 and LF4 show the IB spectra containing Coulomb corrections for the forbidden transitions in the case of Z = 0 and Z ≠ 0, respectively. Abbreviations for SCA and TNS show the IB spectrum solutions for forbidden transitions by Bolgiano et al. [14]. Finally, the abbreviation KUB represents the IB spectrum obtained by the analytical solution given by Chang and Falkoff [10].

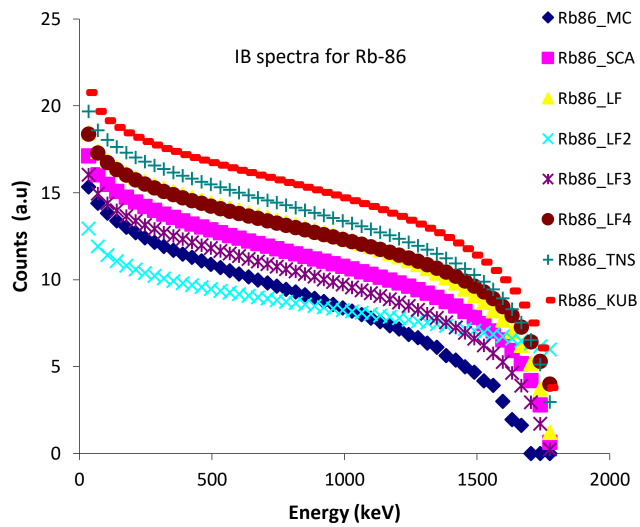

Figure 2. IB spectra obtained by the solutions from the Monte Carlo Method, KUB theory, tensor and scalar calculations for forbidden transition and Coulomb corrected allowed and forbidden transition for $^{86}_{37}$Rb

All solutions given are parallel to each other except for the LF2 solution. In the high energy region, the MC solution drops faster than other solutions. While the LF and LF4 are very compatible with each other, the LF4 solution deviated slightly upwards in the region from 1200 keV to endpoint energy. KUB, TNS and SCA solutions have been parallel to each other.

4. Conclusions

Within the framework of these graphical data, the phenomenon of internal bremsstrahlung released into the environment by the sudden change of nuclear charge during beta disintegration due to bremsstrahlung event, which is an important issue in nuclear physics, was examined in this study with theoretical and Monte Carlo method. We compared the IB spectra from theoretical calculation to the IB spectrum by Monte Carlo method for the Rb-86 isotope. In the comparison, we took into account the agreements in low energy, medium energy and high energy regions. When we look at IB studies historically, positive deviations in experimental values between experimental and theoretical values always occur especially in high energy region. The value of this study is that the IB spectrum can be predicted by the Monte Carlo method as well as the theoretical calculations and the option of comparing it with the experimental data can be presented.

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