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ON FUZZY b-I-CONTINUOUS FUNCTIONS

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Abstract: The concepts of fuzzy b-I-open sets and fuzzy b-I-continuity in fuzzy ideal topological spaces are investigated and some of their properties are obtained. Also we study these in relation to some other types of sets and functions.

Key words: Fuzzy b-I-open sets, fuzzy b-I-continuous

AMS Mathematics Subject Classification (2000): 54C08, 54A05

BULANIK b-I-SÜREKLİ FONKSİYONLAR ÜZERİNE

Özet: Bulanık ideal topolojik uzaylarda bulanık b-I-süreklilik ve b-I-açık küme kavramları araştırıldı ve bunların bazı özellikleri elde edildi. Ayrıca, kümeler ve fonksiyonların bazı diğer tiplerle ilişkisi çalışıldı.

Anahtar kelimeler: Bulanık b-I-açık kümeler, bulanık b-I-süreklilik

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1. INTRODUCTION

The fundemental concept of a fuzzy set was introduced by ZADEH (1965). Subsequently, CHANG (1968) defined the notion of fuzzy topology. An alternative definition of fuzzy topology was given by LOWEN (1976). YALVAC (1987) introduced the concepts of fuzzy set and function on fuzzy spaces. In general topology, by introducing the notion of ideal, KURATOWSKI (1966), VAIDYANATHASWAMY (1945 & 1960) and several other authors carried out such analysis. There has been an extensive study on the importance of ideal in general topology in the paper of JANKOVIĆ & HAMLET (1990). SARKAR (1997) introduced the notions of fuzzy ideal and fuzzy local function in fuzzy set theory. MAHMOUD (1997 & 2002) investigated one application of fuzzy set theory. MALAKAR (1992) introduced the concepts fuzzy semi-irresolute and strongly irresolute functions. HATIR

& JAFARI (2007) and NASEF & HATIR (2007) defined fuzzy semi-I-open set and fuzzy pre-I-open set via fuzzy ideal.
In this paper, we define fuzzy b-I-open set and obtain several characterizations of fuzzy b-I-continuous functions. Moreover, we introduce the concept of fuzzy b-I-open functions and their properties in fuzzy ideal topological spaces.

2. PRELIMINARIES

Through this paper, X represents a nonempty fuzzy set and fuzzy subset A of X, denoted by $A \leq X$, then is characterized by a membership function in the sense of ZADEH (1965). The basic fuzzy sets are the empty set, the whole set the class of all fuzzy sets of X which will be denoted by 0_X, 1_X and I^X, respectively. A subfamily τ of I^X is called a fuzzy topology due to CHANG (1968). Moreover, the pair (X,τ) will be meant by a fuzzy topological space, on which no separation axioms are assumed unless explicitly stated. The fuzzy closure, the fuzzy interior and the fuzzy complement of any set in A in (X,τ) are denoted by Cl(A), Int(A) and 1_X-A, respectively. A fuzzy set which is a fuzzy point WONG (1974) with support $x \in X$ and the value $\lambda \in (0,1]$ will be denoted by x_λ. The value of a fuzzy set A for some $x \in X$ will be denoted by A(x). Also, for a fuzzy point x_λ and a fuzzy set A we shall write $x_\lambda \in A$ to mean that $\lambda \leq A(x)$. For any two fuzzy sets A and B in (X,τ), $A \leq B$ if and only if $A(x) \leq B(x)$ for all $x \in X$. A fuzzy set in (X,τ) is said to be quasi–coincident with a fuzzy set B, denoted by AqB, if there exists $x \in X$ such that A(x)+B(x)>1 (PAO-MING & YING-MING 1980). A fuzzy set V in (X,τ) is called a q-neigbourhood (q-nbd, for short) of a fuzzy point x_λ if and only if there exists a fuzzy open set U such that $x_\lambda qU \leq V$ (PAO-MING & YING-MING 1980, CHANKRABORTY & AHSANULLAH 1991). We will denote the set of all q-nbd of x_λ in (X,τ) by $N_q(x_\lambda)$. A fuzzy subset A of a fuzzy topological space (X,τ) is said to be fuzzy α-open set (BIN SHAHANA 1991) (resp. fuzzy pre-open set (BIN SHAHANA 1991), fuzzy semi-open set (AZAD 1981), fuzzy β-open set (FATH 1984)) if $A \leq Int(Cl(Int(A)))$ (resp. $A \leq Int(Cl(A))$, $A \leq Cl(Int(A))$, $A \leq Cl(Int(Cl(A)))$). A nonempty collection of fuzzy sets I of a set X is called a fuzzy ideal (MAHMOUD 2002, SARKAR 1997) on X if and only if (1) $A \in I$ and $B \leq A$, then $B \in I$ (heredity), (2) if $A \in I$ and $B \in I$, then $A \vee B \in I$ (finite additivity). The triple (X,τ,I) means fuzzy ideal topological space with a fuzzy ideal I and fuzzy topology τ. For (X,τ,I) the fuzzy local function of $A \leq X$ with respect to τ and I is denoted by $A^*(\tau,I)$ (briefly A^*) (SARKAR 1997). The fuzzy local function $A^*(\tau,I)$ of A is the union of all fuzzy points x_λ such that if $U \in N_q(x_\lambda)$ and $E \in I$ then there is at least one $y \in X$ for which U(y)+A(y)-1>E(y) (SARKAR 1997). Fuzzy closure operator of a fuzzy set A in (X,τ,I) is defined as $Cl^*(A)=A \vee A^*$ (SARKAR 1997). In (X,τ,I), the collection $\tau^*(I)$ means an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta=\{U-E:U \in \tau, E \in I\}$ as a base (SARKAR 1997). A fuzzy subset A of a fuzzy ideal topological space (X,τ,I) is said to be fuzzy I-open (NASEF & MAHMOUD 2002) (resp. fuzzy α-I-open set (YUKSEL et al. 2008), fuzzy pre-I-open set (NASEF & HATIR 2007), fuzzy semi-I-open set (HATIR & JAFARI

2007), fuzzy β-I-open set (YUKSEL et al. 2008)) if $A \leq Int(A^*)$ (resp. $A \leq Int(Cl^*(Int(A)))$, $A \leq Int(Cl^*(A))$, $A \leq Cl^*(Int(A))$, $A \leq Cl(Int(Cl^*(A)))$). The family of all fuzzy I-open (resp. fuzzy α-I-open, fuzzy pre-I-open, fuzzy semi-I-open, fuzzy β-I-open) sets is denoted by FIO(X) (resp. FαIO(X), FPIO(X), FβIO(X)). The complement of a fuzzy I-open set (resp. fuzzy α-I-open set, fuzzy pre-I-open set, fuzzy semi-I-open set, fuzzy β-I-open set) is said to be fuzzy I-closed set (resp. fuzzy α-I-closed set (YUKSEL et al. 2008), fuzzy pre-I-closed set (NASEF & HATIR 2007), fuzzy semi-I-closed set (HATIR & JAFARI 2007), fuzzy β-I-closed set (YUKSEL et al. 2008).

3. FUZZY b-I-OPEN SETS

Definition 3.1. A fuzzy subset A of a fuzzy ideal topological space (X,τ,I) is said to be fuzzy b-I-open if $A \leq Cl^*(Int(A)) \vee Int(Cl^*(A))$. The family of all fuzzy b-I-open sets in (X,τ,I) is denoted FbIO(X).

Theorem 3.1. In a fuzzy ideal topological space (X,τ,I), the following statements hold:

a) Every fuzzy I-open set is fuzzy b-I-open,
b) Every fuzzy open set is fuzzy b-I-open,
c) Every fuzzy α-I-open set is fuzzy b-I-open,
d) Every fuzzy semi-I-open set is fuzzy b-I-open,
e) Every fuzzy pre-I-open set is fuzzy b-I-open.

Proof. This is obvious.
Converse of the above need not be true as seen in the following examples.

Example 3.1. Let X={a,b,c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,1 A(b)=0,3 A(c)=0,1
B(a)=0,3 B(b)=0,5 B(c)=0,7

We put $\tau = \{0_X, 1_X, A\}$. If we take I={0_X}, then B is fuzzy b-I-open but not fuzzy I-open.

Example 3.2. In Example 3.1, B is fuzzy b-I-open but not fuzzy open.

Example 3.3. In Example 3.1, B is fuzzy b-I-open but not fuzzy α-I-open.

Example 3.4. Let X={a,b,c}and A, B be fuzzy subsets of X defined as follows:

A(a)=0,2 A(b)=0,8 A(c)=0,5
B(a)=0,6 B(b)=0,5 B(c)=0,4

We put $\tau = \{0_X, 1_X, A\}$. If we take I={0_X}, then B is fuzzy b-I-open but not fuzzy semi-I-open.

Example 3.5. In Example 3.1, B is fuzzy b-I-open but not fuzzy pre-I-open.

Definition 3.2. A fuzzy subset A of a fuzzy topological space (X,τ) is said to be fuzzy b-open if A$\leq$ Cl(Int(A))$\vee$Int(Cl (A)).The family of all fuzzy b-open sets in (X,τ) is denoted FbO(X).

Theorem 3.2. Every fuzzy b-I-open set is fuzzy b-open.

Proof. This is obvious.

Example 3.6. Let X={a,b,c} and A, B be fuzzy subsets of X defined as follows:

A(a)=0,6 A(b)=0,5 A(c)=0,2

B(a)=0,8 B(b)=0,6 B(c)=0,5

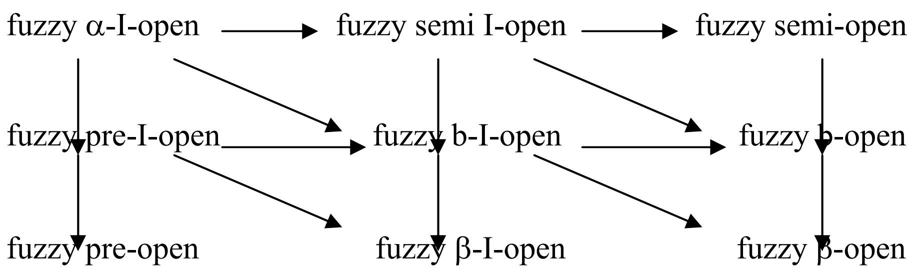
We put $\tau = \{0_X, 1_X, A\}$. If we take I=P(X), then B is fuzzy b-open but not fuzzy b-I-open.

Theorem 3.3. Every fuzzy b-I-open set is fuzzy β-I-open.

Proof. It is obvious.

Converse of the above need not be true as seen in the following example.

Example 3.7. In Example 3.6**,** B is fuzzy β-I-open but not fuzzy b-I-open.
The above discussions with, we have the following diagram for fuzzy subsets of a fuzzy ideal topological space.



Definition 3.3. A fuzzy subset A of a fuzzy ideal topological space (X,τ,I) is said to be fuzzy *-perfect if A=A^*.

Theorem 3.4. For a fuzzy ideal topological space (X,τ,I) and a fuzzy subset A of X, we have

a) If I={0_X} and A$\in$FPO(X) then A$\in$FbIO(X),
b) If I=P(X) and A$\in$FbIO(X), then A=Int(A),
c) If Int(A)= 0_X and A$\in$FbIO(X), then A$\in$FPIO(X),
d) If A is fuzzy *-perfect and A$\in$FbIO(X), then A$\in$FSIO(X).

Proof. (a) We know that if I={0_X}, then A^*=Cl(A). Let A be a fuzzy pre-open set. Therefore, A$\leq$Int(Cl(A))=Int(A^*)$\leq$Int(Cl^*(A))$\leq$Int(Cl^*(A))$\vee$$Cl^*$(Int(A)). Hence A is fuzzy b-I-open.

(b) It is clear that if I=P(X), then $A^* = 0_X$ and $Cl^*(A)=A$. Let A be a fuzzy b-I-open set. Then $A \le Cl^*(Int(A)) \vee Int(Cl^*(A)) = Int(A) \vee Int(A) = Int(A)$. Hence A is fuzzy open.
(c) and (d) are obvious.

Theorem 3.5. In a fuzzy ideal topological space (X,τ,I) ,the union of two fuzzy b-I-open sets are fuzzy b-I-open.

Proof. Let A and B be fuzzy b-I-open sets in (X,τ,I), then

$$\begin{aligned} A \vee B &\le [Cl^*(Int(A)) \vee Int(Cl^*(A))] \vee [Cl^*(Int(B)) \vee Int(Cl^*(B))] \\ &= [Cl^*(Int(A)) \vee Cl^*(Int(B))] \vee [Int(Cl^*(A)) \vee Int(Cl^*(B))] \\ &\le [Cl^*(Int(A) \vee Int(B))] \vee [Int(Cl^*(A) \vee Cl^*(B))] \\ &\le Cl^*(Int(A \vee B)) \vee Int(Cl^*(A \vee B)) \end{aligned}$$

Remark 3.1. In a fuzzy ideal toplogical space (X,τ,I), the intersection of two fuzzy b-I-open sets need not be fuzzy b-I-open as shown by the following example.

Example 3.8. Let X={a,b,c} and A, B fuzzy subsets of X defined as follows:

A(a)=0,3 A(b)=0,5 A(c)=0,6
B(a)=0,8 B(b)=0,6 B(c)=0,4

We put $\tau = \{0_X, 1_X, A\}$. If we take I={0_X}, then A and B are fuzzy b-I-open sets but $A \wedge B$ is not fuzzy b-I-open.

Lemma 3.1 Let A and B be fuzzy subsets of a fuzzy ideal topological space (X,τ,I). Then we have

a) If $A \le B$, then $A^* \le B^*$,
b) If $U \in \tau$, then $U \wedge A \le (U \wedge A)^*$,
c) A^* is fuzzy closed in (X,τ,I)

(MAHMOUD 2002, SARKAR 1997).

Theorem 3.6. Let (X,τ,I) be a fuzzy ideal topological space and A, B be fuzzy subsets of X. Then the following properties hold:

a) If $U_\alpha \in$ FbIO(X) for each $\alpha \in \Delta$, then $\vee\{U_\alpha : \alpha \in \Delta\} \in$ FbIO(X,τ),
b) If $A \in$ FbIO(X) and $B \in \tau$, then $A \wedge B \in$ FbIO(X,τ),
c) If $A \in$ FαIO(X) and $B \in$ FbIO(X), then $A \wedge B \in$ FbIO(X).

Proof. a) Since $U_\alpha \in$ FbIO(X), we have

$$U_\alpha \le Cl^*(Int(U_\alpha)) \vee Int(Cl^*(U_\alpha))$$

for each $\alpha \in \Delta$. Thus by the Lemma 3.1, we obtain

$$\begin{aligned} \bigvee_{\alpha\in\Delta} U_\alpha &\le \bigvee_{\alpha\in\Delta} [Cl^*(Int(U_\alpha)) \vee Int(Cl^*(U_\alpha))] \\ &= \bigvee_{\alpha\in\Delta} [(Int(U_\alpha) \vee (Int(U_\alpha))^*) \vee Cl^*(Int(U_\alpha))] \\ &\le [\bigvee_{\alpha\in\Delta} (Int(U_\alpha))^*] \vee Int(\bigvee_{\alpha\in\Delta} U_\alpha) \vee Int(\bigvee_{\alpha\in\Delta} (Cl^*(U_\alpha))) \\ &\le (Int(\bigvee_{\alpha\in\Delta} U_\alpha))^* \vee Int(\bigvee_{\alpha\in\Delta} U_\alpha) \vee Int(Cl^*(\bigvee_{\alpha\in\Delta} U_\alpha)) \end{aligned}$$

$$= Cl^*(Int(\vee_{\alpha\in\Delta} U_\alpha))\vee Int(Cl^*(\vee_{\alpha\in\Delta} U_\alpha))$$

and hence $\vee_{\alpha\in\Delta} U_\alpha \in$ FbIO(X).

b) Let $A\in$ FbIO(X) and $B\in\tau$, then

$$\begin{aligned}
A\wedge B &\leq [Cl^*(Int(A))\vee Int(Cl^*(A))]]\wedge B\\
&= [Cl^*(Int(A))\wedge B]\vee[Int(Cl^*(A))\wedge B]\\
&= [(Int(A)\vee((Int(A))^*)\wedge B] \vee [Int(A\vee A^*)\wedge B]\\
&= [(Int(A)\wedge B)\vee((Int(A))^*\wedge B)]\vee[Int(A\vee A^*)\wedge Int(B)]\\
&\leq [(Int(A)\wedge B)\vee(Int(A)\wedge B)^*]\vee Int[(A\vee A^*)\wedge B]\\
&= [(Int(A)\wedge Int(B))\vee(Int(A)\wedge Int(B))^*]\vee Int[(A \wedge B)\vee(A^*\wedge B)]\\
&\leq [Int(A\wedge B)\vee(Int(A\wedge B))^*]\vee Int[(A\wedge B)\vee(A\wedge B)^*]\\
&= Cl^*(Int(A\wedge B))\vee Int(Cl^*(A\wedge B)).
\end{aligned}$$

This shows that $A\wedge B\in$ FbIO(X).

c) Straightforward.

Lemma 3.2. Let (X,τ,I) be a fuzzy ideal topological space and A, B fuzzy subset of X, such that $B\leq A$. Then $B^*(\tau|A,I|A)=B^*(\tau,I)\wedge A$ (NASEF & MAHMOUD 2002).

Theorem 3.7. Let (X,τ,I) be a fuzzy ideal topological space. If $U\in\tau$ and $A\in$ FbIO(X), then $U\wedge A\in$ FbIO(U, τ|U, I|U).

Proof. We have $Int_U(V) = Int(V)$ for any fuzzy subset V of U, since $U\in\tau$. Thus, by using this fact and Lemma 3.2, we have the result.

Definition 3.4. Let (X,τ,I) be a fuzzy ideal topological space. A fuzzy subset of X is called fuzzy b-I-closed if its complement is fuzzy b-I-open. The family of all fuzzy b-I-closed sets in (X,τ,I) is denoted by FbIC(X).

Theorem 3.8. If a fuzzy subset A of a fuzzy ideal topological space (X,τ,I) is fuzzy b-I-closed then $Cl^*(Int(A))\wedge Int(Cl^*(A))\leq A$.

Proof. Since $A\in$ FbIC(X), 1_X -$A\in$ FbIO(X). Thus,

$$\begin{aligned}
1_X - A &\leq Cl^*(Int(1_X - A))\vee Int(Cl^*(1_X - A))\\
&\leq Cl(Int(1_X - A))\vee Int(Cl(1_X - A))\\
&=(1_X - (Int(Cl(A))))\vee(1_X - (Cl(Int(A))))\\
&\leq(1_X - Int(Cl^*(A))))\vee (1_X - (Cl^*(Int(A)))).
\end{aligned}$$

Hence we obtain $Cl^*(Int(A))\wedge Int(Cl^*(A))\leq A$.

Remark 3.2. For fuzzy subset A of (X,τ,I), we have

$$1_X - Int(Cl^*(A)) \neq Cl^*(Int(1_X - A))$$

as seen in the following example.

Example 3.9. In Example 3.1., if we get I=P(X) then the fuzzy set B shows that the above property holds.

Corollary 3.1. Let A be a fuzzy subset in (X,τ,I) such that

$$1_X\text{-Int}(Cl^*(A))=Cl^*(\text{Int}(1_X\text{-}A)).$$

Then A is fuzzy b-I-closed if and only if $Cl^*(\text{Int}(A))\wedge\text{Int}(Cl^*(A))\leq A$.

Corollary 3.2. Let (X,τ,I) be a fuzzy ideal topological space.

a) If $A\in$ FbIC(X) and $B\in\tau^t$, then $A\vee B\in$ FbIC(X),

b) If $A\in$ FbIC(X) and $B\in$ FαIC(X), then $A\vee B\in$ FbIC(X).

Proof. It is clear from Theorem 3.6 and Definition 3.4.

4. FUZZY b-I-CONTINUOUS FUNCTIONS

Definition 4.1. A function f:(X,τ,I)→(Y,φ) is called fuzzy b-I-continuous if the inverse image of each fuzzy open set in Y is fuzzy b-I-open in (X,τ,I).

Theorem 4.1. A function f:(X,τ,I)→(Y,φ) is fuzzy b-I-continuous if and only if for each fuzzy point x_λ in X and each fuzzy open set V≤Y containing $f(x_\lambda)$, there exists $W\in$ FbIO(X) containing x_λ such that $f(W)\leq V$.

Proof. Necessity. Let $x_\lambda\in X$ and V be any fuzzy open set in Y containing $f(x_\lambda)$. Set $W=f^{-1}(V)$, then since f fuzzy b-I-continuous by Definition 4.1, W is fuzzy b-I-open set containing x_λ and $f(W)\leq V$.

Sufficiency. Let V be any fuzzy open set in Y containing $f(x_\lambda)$. Then by hypothesis there exists W_{x_λ} fuzzy b-I-open such that

$$f(W_{x_\lambda})\leq V\Rightarrow W_{x_\lambda}\leq f^{-1}(V).$$

Let $\vee W_{x_\lambda}=f^{-1}(V)$. Therefore $f^{-1}(V)$ is fuzzy b-I-open by Theorem 3.6 (a). This shows that f fuzzy b-I-continuous.

Remark 4.1. Every fuzzy continuous function is fuzzy b-I-continuous.
Converse of the above is not true as seen in the following example.

Example 4.1. Let X={a,b,c}, Y={x,y,z} and A, B be fuzzy subsets defined as follows:

A(a)=0,3 A(b)=0,7 A(c)=0,5
B(x)=0,7 B(y)=0,9 B(z)=0,2

Let $\tau=\{0_X,1_X,A\}$, $\varphi=\{0_Y,1_Y,B\}$ and I={0_X}. Then the function f:(X,τ,I)→(Y,φ) defined by f(a)=x, f(b)=y and f(c)=z is fuzzy b-I-continuous but not fuzzy continuous.

Definition 4.2. A function f:(X,τ)→(Y,φ) is called fuzzy b-continuous if the inverse image of each fuzzy open set in Y is fuzzy b-open in (X,τ).

Remark 4.2. Every fuzzy b-I-continuous function is fuzzy b-continuous.
But the converse is not true as shown in the following example.

Example 4.2. Let X={a,b,c}, Y={x,y,z} and A, B be fuzzy subsets defined as follows:

A(a)=0,4 A(b)=0,6 A(c)=0,2
B(x)=0,1 B(y)=0,8 B(z)=0,4

Let $\tau = \{0_X, 1_X, A\}$, $\varphi = \{0_Y, 1_Y, B\}$ and I=P(X). Then the function f:(X,τ,I)→(Y,φ) defined by f(a)=x, f(b)=y and f(c)=z is fuzzy b-continuous but not fuzzy b-I-continuous.

Theorem 4.2. A function f:(X,τ,I)→(Y,φ) is fuzzy b-I-continuous if the graph function g:X→X×Y of f is fuzzy b-I-continuous.

Proof. Let V be a fuzzy open set in Y. Then $1_X \times$V is fuzzy open set in X×Y. Since g is fuzzy b-I-continuous $g^{-1}(1_X \times V) \in$ FbIO(X). Thus

$$f^{-1}(V) = 1_X \wedge f^{-1}(V) = g^{-1}(1 \times V) \;,\quad f^{-1}(V) \in \mathrm{FbIO}(X).$$

Hence f is fuzzy b-I-continuous.

Theorem 4.3. Let f:(X,τ,I)→(Y,φ) be a fuzzy b-I-continuous function and U∈τ. Then the restriction f|U is fuzzy b-I-continuous.

Proof. Let V∈φ, then f^{-1}(V) is fuzzy b-I-open in X since f is fuzzy b-I-continuous. By Theorem 3.6 (b), f^{-1}(V)∧U is fuzzy b-I-open in U. Therefore $(f|U)^{-1}$(V)=U∧ f^{-1}(V) is fuzzy b-I-open in U. Hence we obtain that f|U is fuzzy b-I-continuous.

Theorem 4.4. If f:(X,τ,I)→(Y,φ) is fuzzy b-I-continuous and g:(Y,φ,J)→(Z,ψ) is fuzzy continuous, then $g \circ f$:(X,τ,I)→(Z,ψ) is fuzzy b-I-continuous.

Proof. Let W be any fuzzy open set in Z. Since g is fuzzy continuous, g^{-1}(W) is fuzzy open in Y. Since f is fuzzy b-I-continuous, $f^{-1}(g^{-1}(W))$ is fuzzy b-I-open in X. Hence $g \circ f$ is fuzzy b-I-continuous.

Remark 4.3. Composition of two fuzzy b-I-continuous functions need not be fuzzy b-I-continuous as shown by the following example.

Example 4.3. Let X={a,b,c}, Y={x,y,z}, Z={k,l,m} and A, B, C be fuzzy subsets defined as follows:

A(a)=0,5 A(b)=0,6 A(c)=0,3
B(x)=0,6 B(y)=0,4 B(z)=0,5
C(k)=0,2 C(l)=0,4 C(m)=0,6

Let $\tau=\{0_X,1_X,A\}$, $\varphi=\{0_Y,1_Y,B\}$, $\psi=\{0_Z,1_Z,C\}$, I={0_X} and J={0_Y}. If the function f:(X,τ,I)→(Y,φ) defined by f(a)=x, f(b)=y and f(c)=z and the function g:(Y,φ,J)→(Z,ψ) defined by g(x)=k, g(y)=l, g(z)=m, then f and g are fuzzy b-I-continuous but $g \circ f$ is not fuzzy b-I-continuous.

Definition 4.3. A function f:(X,τ,I)→(Y,φ) is called fuzzy b-I-irresolute if the inverse image of each fuzzy b-open set of Y is fuzzy b-I-open in (X,τ,I).
Remark 4.4. Every fuzzy b-I-irresolute function is fuzzy b-I-continuous.
Converse of the above theorem is not true as shown in the following example.

Example 4.4. Let X={a,b,c},Y={x,y,z} and A, B be fuzzy subsets defined as follows:

A(a)=0,7 A(b)=0,4 A(c)=0,8

B(x)=0,2 B(y)=0,5 B(z)=0,4

Let τ={0,1,A},φ={0,1,B} and I={0}. Then the function f:(X,τ,I)→(Y,φ) defined by f(a)=x, f(b)=y and f(c)=z is fuzzy b-I-continuous but for a fuzzy b-open set in Y such that E(x)=0,2, E(y)=0,4, E(z)=0,1 whose inverse image is not fuzzy b-I-open in X so f is not fuzzy b-I-irresolute.

Definition 4.4. A function f:(X,τ)→(Y,φ) is called fuzzy b-irresolute if the inverse image of each fuzzy b-open set of Y is fuzzy b-open in (X,τ).

Theorem 4.5. If f:(X,τ,I)→(Y,φ) is fuzzy b-I-irresolute and g:(Y,φ,J)→(Z,ψ) is fuzzy b-irresolute, then $g \circ f$:(X,τ,I)→(Z,ψ) is fuzzy b-I-irresolute.

Proof. Let W be any fuzzy b-open set in Z. Since g is fuzzy b-irresolute, g^{-1}(W) is fuzzy b-open in Y. Since f is fuzzy b-I-irresolute, f^{-1}(g^{-1}(W)) is fuzzy b-I-open in X. Hence $g \circ f$ is fuzzy b-I-irresolute.

Theorem 4.6. Let f:(X,τ,I)→(Y,φ) be a function, then the following statements are equivallent:

a) f is fuzzy b-I-irresolute,
b) For each fuzzy point x_λ in X and each fuzzy b-open set V in Y containing $f(x_\lambda)$, there exists a fuzzy b-I-open set U containing x_λ such that f(U)≤V,
c) f^{-1}(V)≤Cl^*(Int(f^{-1}(V)))∨Int(Cl^*(f^{-1}(V))) for every fuzzy b-open set V in Y,
d) f^{-1}(F) is fuzzy b-I-closed in X for every fuzzy b-closed set F in Y.

Proof. (a)⇒(b) Let $x_\lambda \in X$ and V be any fuzzy b-open set in Y containing $f(x_\lambda)$.By assumption, f^{-1}(V) is fuzzy b-I-open in X. Set U=f^{-1}(V), then U is a fuzzy b-I-open in X containing x_λ such that f(U)≤V.

(b)⇒(c) Let V be any fuzzy b-open set in Y and $x_\lambda \in f^{-1}$(V). By (b) there exists a fuzzy b-I-open set U of X containing x_λ such that f(U)≤V. Thus we obtain,

$$x_\lambda \in U \le Cl^*(Int(U)) \vee Int(Cl^*(U))$$

$$\leq Cl^*(Int(f^{-1}(V)))\vee Int(Cl^*(f^{-1}(V)))$$

and hence

$$x_\lambda \in Cl^*(Int(f^{-1}(V)))\vee Int(Cl^*(f^{-1}(V))).$$

This shows that for every fuzzy b-open set V of Y,

$$f^{-1}(V) \leq Cl^*(Int(f^{-1}(V)))\vee Int(Cl^*(f^{-1}(V)))$$

holds.

(c)⇒(d) Let F be any fuzzy b-closed subset of Y and V=1_Y-F. Then V is fuzzy b-open in Y. By (c),

$$f^{-1}(V)\leq Cl^*(Int(f^{-1}(V)))\vee Int(Cl^*(f^{-1}(V))).$$

This shows that f^{-1}(F)= 1_X -f^{-1}(V) is fuzzy b-I-closed in X.

(d)⇒(a) Let V be any fuzzy b-open set in Y and F=1_Y-V. Then by (d),

$$f^{-1}(F)= 1_X - f^{-1}(V)$$

is fuzzy b-I-closed in X. Hence f^{-1}(V) is fuzzy b-I-open in X and f is fuzzy b-I-irresolute.

5. FUZZY b-I-OPEN AND FUZZY b-I-CLOSED FUNCTIONS

Definition 5.1. A function f:(X,τ)→(Y,φ,J) is called fuzzy b-I-open (resp. fuzzy b-I-closed) if the image of each open (resp. closed) set of X is fuzzy b-I-open (resp. fuzzy b-I-closed) set in (Y,φ,J).

Remark 5.1. Every fuzzy open function is fuzzy b-I-open function.
But the converse is not true as seen in the following example.

Example 5.1. Let X={a,b,c}, Y={x,y,z}and A, B be fuzzy subsets defined as follows:

A(a)=0,5 A(b)=0,7 A(c)=0,3
B(x)=0,3 B(y)=0,8 B(z)=0,1

Let $\tau=\{0_X,1_X,A\}$, $\varphi=\{0_Y,1_Y,B\}$ and J={0_Y}. Then the function f:(X,τ)→(Y,φ,J) defined by f(a)=x, f(b)=y and f(c)=z is fuzzy b-I-open but not fuzzy open function.

Definition 5.2. A function f:(X,τ)→(Y,φ) is called fuzzy b-open (resp. fuzzy b-closed) if the image of each open (resp. closed) set of X is fuzzy b-open (resp. fuzzy b-closed) set in (Y,φ,J).

Remark 5.2. Every fuzzy b-I-open function is fuzzy b-open function.
But the converse is not true as seen in the following example.

Example 5.2. Let X={a,b,c}, Y={x,y,z}and A, B be fuzzy subsets defined as follows:

A(a)=0,3 A(b)=0,5 A(c)=0,6
B(x)=0,3 B(y)=0,4 B(z)=0,8

Let $\tau=\{0_X,1_X,A\}$, $\varphi=\{0_Y,1_Y,B\}$ and J=P(Y). Then the function f:(X,τ)→(Y,φ,J) defined by f(a)=x, f(b)=y and f(c)=z is fuzzy b-open but not fuzzy b-I-open function.

Theorem 5.1. If a function f:(X,τ)→(Y,φ,J) is a fuzzy b-I-open, then for each $x_\lambda \in X$ and each fuzzy open set U containing x_λ, there exists a fuzzy b-I-open set W containing $f(x_\lambda)$ such that W≤ f(U).

Proof. Let $x_\lambda \in$ X and U be any fuzzy open set containing x_λ. Since f is fuzzy b-I-open, f(U)∈FbIO(Y). Put W= f(U), then $f(x_\lambda) \in$ W where W is fuzzy b-I-open such that W≤ f(U).

Theorem 5.2. Let f:(X,τ)→(Y,φ,J) be a fuzzy b-I-open function. If W ≤Y and F≤X is fuzzy closed set containing f^{-1}(W), then there exists a fuzzy b-I-closed set H ≤Y containing W such that f^{-1}(H)≤F.

Proof. Let F be a fuzzy closed set in X. Then G= 1_X-F is fuzzy open in X. Since f is fuzzy b-I-open function, f(G) is fuzzy b-I-open inY. Hence H=1_Y-f(G) is fuzzy b-I-closed in Y and f^{-1}(H)= f^{-1}(1_Y-f(G))= 1_X-f^{-1}(f(G))≤ 1_X-G=F.

Theorem 5.3. For any bijective function f:(X,τ,I)→(Y,φ,J), the following statements are equivalent:
a) f^{-1}:(Y,φ,J) →(X,τ) is fuzzy b-I-continuous,
b) f is fuzzy b-I-open,
c) f is fuzzy b-I-closed.

Proof. This is obvious.

Theorem 5.4. Let f:(X,τ,I)→(Y,φ,J) and g:(Y,φ,J)→(Z,ψ,K) be two functions where I, J, K are ideals on X, Y, Z respectively. The followings hold:
a) If f is fuzzy open and g is fuzzy b-I-open, then $g \circ f$ is fuzzy b-I-open,
b) If $g \circ f$ is fuzzy open and g is fuzzy b-I-continuous, then f is fuzzy b-I-open.

Proof. This is obvious.

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