IJEIR
International Journal of Engineering & Innovative Research

International Journal of Engineering and Innovative Research

http://dergipark.gov.tr/ijeir

A STUDY OF THIN PLATE VIBRATION USING HOMOTOPY PERTUBATION ALGORITHM

Falade Kazeem Iyanda[1*], Tiamiyu Abd'gafar Tunde[2], Tolufase Emmanuel[3]

[1*]Kano University of Science and Technology, Department of Mathematics, Wudil Kano State, Nigeria.
[2] Federal University of Technology, Department of Mathematics, Minna Niger State, Nigeria.
[3] Nigeria Police Academy, Department of Physics, Faculty of Science, Wudil Kano State, Nigeria.

[*]Corresponding Author: faladekazeem2016@kustwudil.edu.ng
(**Received:** 12.05.2020; **Revised**: 02.06.2020; **Accepted:** 16.06.2020)

ABSTRACT: In this paper, we apply homotopy perturbation method for the numerical solution of three dimensional second-order partial differential equation which occurred in a plate vibration behavious. The MAPLE 18 Mathematical software was used to develop a four steps algorithm based on homotopy perturbation method (HPM). Three test cases are considered to verify the reliability and efficiency of the method. The suggested algorithm is quite efficient and practically well suited for use in these problems. The approximated solutions are in good agreement with analytical solutions for the tested problems Moreover, the approximate solutions obtained proved that the proposed method is easy, efficient, and accurate.

Keywords: Second-Order Partial Differential Equation, Homotopy Perturbation Method, MAPLE 18 Mathematical Software, Algorithm, Analytical Solutions.

1. INTRODUCTION

Modal analysis of plate vibration is important engineering analysis that relates to the safe design of this type of structures because many such structures are expected to survive cyclic load applications. Such situation is vulnerable to structural failures in resonant vibration should the frequency of the applied cyclic loads coincides with any natural frequencies of the plate found in the modal analysis. Solutions for these natural frequencies of plates of given geometry and material properties requires the solution of the shape of the deformed plates at various modes, and it will also provide engineers with possible shapes of the plate under each of these modes of vibration. Solids of plane geometry, such as thin plates are common appearance in machines and structures. Thin plates can be as small as printed electric circuit boards with micrometers in size or as large as floors in building structures. Like flexible cables, thin flexible plates are normally flexible and be vulnerable to transverse vibration. In some cases, these plates may rupture due to resonant vibrations, resulting in significant loss of property, and even human lives.

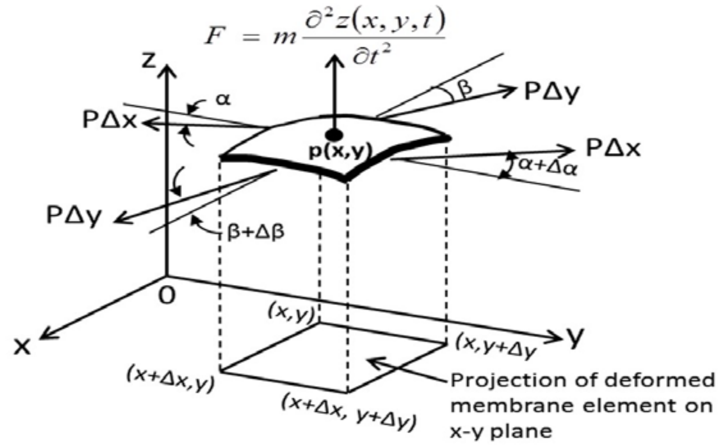


Figure 1. A free-body diagram of forces in an element of vibrating membrane

This section will derive appropriate partial differential equations (PDEs) that allow engineers to assess the amplitudes in free vibration of thin plates that are flexible enough to be simulated to thin membranes Ran [1]

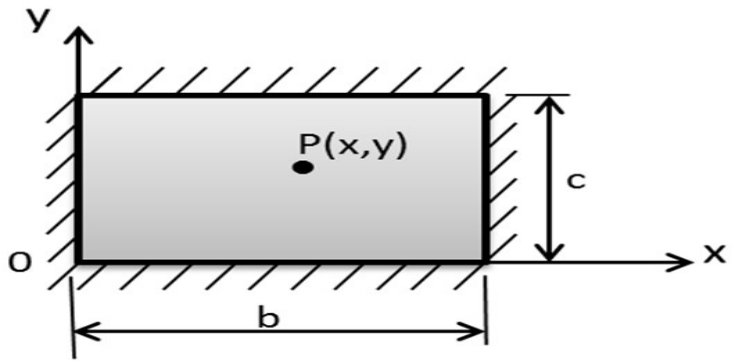


Figure 2. Plan view of a flexible thin plate undergoing a transverse vibration

The magnitudes of a transverse vibrating thin plate such as a computer mouse pad, induced by a slight instantaneous disturbance in the z-direction in Figure 2. We will have the following PDE and the given appropriate initial condition for the solution of the magnitudes of the vibrating plate at given time t, i.e. $z(x,y,t)$ as second order partial differential equations which describe rate of change in three coordinate directions of function $z(x,y,t)$ of the form:

$$\frac{\partial^2 z(x,y,t)}{\partial t^2} = \lambda^2 \left[\frac{\partial^2 z(x,y,t)}{\partial x^2} + \frac{\partial^2 z(x,y,t)}{\partial y^2}\right] \tag{1}$$

subject to initial conditions

$$\begin{cases} z(x,y,t)_{t=0} = f(x,y) \\ \left.\frac{\partial z(x,y,t)}{\partial t}\right|_{t=0} = g(x,y) \end{cases} \tag{2}$$

Where $\lambda = \sqrt{\frac{Pg}{m}}$ in which P = tension in the cable and m = mass per unit length which needs to be computed with given conditions. The mass per unit length of the cable is $m = \frac{M}{L}$ where M = total mass of the cable with M = ρV with V being the volume of the cable. Function $f(x,y)$

describes the cable at the initial position before the vibration take place while $g(x, y)$ describes the velocity of the plate across the plane of the plate at the inception of the vibration.

Many researchers published some works in solving this type of classical differential equation, Irvine [2] describes the in-plane and out-of-plane small amplitude free vibration of a suspended elastic cable with small sag, Leonard [3] present the finite element method has also been used for forced vibration response analysis, Rega [4] parametric analysis of large amplitude free vibrations of a suspended cable was considered, Ni et all. [5] developed a hybrid pseudo-force/Laplace transform method for transient response of suspended cables. Homotopy Perturbation Method (HPM) has been a promising numerical technique in solving partial differential equations which describe different fields of science, physical phenomena, engineering, mechanics and soon. homotopy perturbation method was proposed by He [6] for solving linear and nonlinear differential equations and integral equations. Many researchers used HPM to approximate the solutions of partial differential equations and integral equations. Vahidi et all. [7] solved nonlinear DEs, which yields the Maclaurin series of the exact solution, Chang and Liou [8] developed a third-order explicit approximation to find the roots of the dispersion relation for water waves that propagate over dissipative media, Zhou and Wu [9] solved the nonlinear PB equation describing spherical and planar colloidal particles immersed in an arbitrary valence and mixed electrolyte solution, Özi and Akçı [10] obtained periodic solutions for certain non-smooth oscillators using iterated homotopy perturbation method combined with modified Lindstedt Poincaré technique, Yazdi [11] solved nonlinear vibration analysis of functionally graded plate. Al-Saif and Abood [12] used homotopy perturbation method for solving K (2, 2) equations, Aswhad and Jaddoa [13] obtained the approximate solutions of Newell whitehead segel and fisher equations using the Adomian decomposition Method, Babolian et all. [14] solved advection problem, vibrating beam equation linear and nonlinear PDEs and the system of nonlinear PDEs and Adil et all.[15] studied general second-order partial differential equations using homotopy perturbation method. However, the fact that the HPM solves many applied mathematical problems without any transformation, discretization or restrictive assumptions can be considered as a clear advantage of this technique over the some numerical methods was estaiblished by Mohyud and Noor [16]. Moreover, several techniques including the method of characteristic, Riemann invariants, combination of waveform relaxation and multi-grid, periodic multi-grid wave form, variational iteration e.t.c encounter the inbuilt deficiencies and involve huge computational work. Thus, the homotopy perturbation algorithm was formulated to address the computational shortcoming while efficiency and accuracy are still mentained He [17].

In this work, we examimed the feasibility of employing the HPM to formulate four steps algorithm for the numerical solution of three dimentional second-order PDE that occurred in a plate vibration and to investigate the behavious of functions $f(x, y)$ which describe the cable at the initial position couple with $g(x, y)$ that describes the velocity of the plate across the plane of the plate at the inception of the vibration.

2. HOMOTOPY PERTURBATION METHOD (HPM)

In this section, we present a brief description of the HPM, to illustrate the basic ideas of the homotopy perturbation method, this method is a coupling between the traditional perturbation method and homotopy, which is a highly interesting and useful concept in topology, and deforms continuously to a simple problem which is easily solved. We consider the following differential equation employed in Chun and Sakthivel [18].

$$A(u) - f(\gamma) = 0 \qquad \gamma \in \Omega \tag{3}$$

with boundary conditions:

$$B\left(u, \frac{\partial u}{\partial \gamma}\right) = 0 \qquad \gamma \in \Omega \tag{4}$$

where A is general differential operator, B is a boundary operator, $f(\gamma)$ a known analytic function and ∂u is the boundary of the domain Ω. The operator A can be generally divided into two parts of L and N where L is linear part, while N is the nonlinear part in the DE, Therefore Eq.(3) can be rewritten as follows:

$$L(u) + N(u) - f(\gamma) = \tag{5}$$

By using homotopy technique, one can construct a homotopy

$$Z(\gamma, m): \Omega \text{ X } [0,1] \mapsto R \tag{6}$$

which satisfies

$$H(z,m) = (1-m)[L(z) - L(u_0)] + m[L(z) + N(z) - f(\gamma)] = 0 \tag{7}$$

or

$$H(z,m) = L(z) - L(u_0) + mL(u_0 + m[N(z) - f(\gamma)]) = 0 \tag{8}$$

where m ∈ [0, 1], τ ∈ Ω and m is called homotopy parameter and u_0 is an initial approximation for the solution of Eq.(3) which satisfies the boundary conditions obviously, using Eq.(7) or Eq.(8), we have the following equation:

$$H(z,0) = L(z) - L(u_0) = 0 \tag{9}$$

$$H(z,1) = L(z) + N(z) - f(\gamma) = 0 \tag{10}$$

Assume that the solution of (7) or (8) can be expressed as a series in m as follows:

$$Z = z_0 + mz_1 + m^2 z_2 + m^3 z_3 + \cdots = \sum_{i=0}^{\infty} m^i z_i \tag{11}$$

set m → 1 results in the approximate solution of (3).

Consequently,

$$u(\gamma) = \lim_{m \to 1} Z = z_0 + z_1 + z_2 + z_3 + \cdots = \sum_{i=0}^{\infty} z_i \tag{12}$$

It is worth to note that the major advantage of He's homotopy perturbation method is that the perturbation equation can be freely constructed in many ways and approximation can also be freely selected

It is well known that series (12) is convergent for most of the cases and also the rate of convergence is dependent on $L(z)$. The comparisons of equal powers of m give solutions of various orders. In sum, according to Liu [19], He's HPM considers the solution $u(x)$ of the homotopy equation in a series of m as

$$z(x) = \sum_{i=0}^{\infty} m^i z_i = z_0 + m z_1 + m^2 z_2 + m^3 z_3 + \cdots \tag{13}$$

and the method considers the nonlinear $N(z)$ as

$$N(u) = \sum_{i=0}^{\infty} m^i H_i = H_0 + m H_1 + m^2 H_2 + m^3 H_3 + \cdots \tag{14}$$

Where H_n are the so-called He's polynomials [17] which can be calculated by using the formula

$$H_n(z_0, z_1 \dots z_n) = \frac{1}{n!}\frac{\partial^n}{\partial m^n}\left(N\left(\sum_{i=0}^{\infty} m^i z_i\right)\right)_{m=0} \tag{15}$$

where $n = 0,1,2,3, \dots$

2.1. Homotopy Perturbation Algorithm (HPA)

In this section, we consider Eq. (1) and Eq. (2) couple with Eq. (15). We formulate a four steps algorithm using MAPLE 18 Mathematical software command to solve Eq. (1) as follows:

restart:
$\boldsymbol{Step\ 1}$: $z(x,y,0) \coloneqq f(x,y)$;
$z_t(x,y,0) \coloneqq g(x,y)$;
$z_0 \coloneqq z(x,y,0) + t * z_t(x,y,0)$;
$N \coloneqq \mathbb{R}^+$;
$\lambda \coloneqq \sqrt{\frac{P}{m}}$;
$\boldsymbol{Step\ 2}$: $for\ i\ from\ 0\ to\ 0\ do$
$A \coloneqq \left(Diff(z_0,[t,t]) + \lambda * \left(Diff(z_0,[x,x]) + Diff(z_0,[y,y])\right)\right)$;
$B \coloneqq -Int(A,t,t)$; (16)
$z_{i+1} \coloneqq value(B)$;
end do
$\boldsymbol{Step\ 3}$: $for\ i\ from\ 1\ to\ N\ do$
$A \coloneqq \left(\lambda * \left(Diff(z_i,[x,x]) + Diff(z_i,[y,y])\right)\right)$;
$B \coloneqq -Int(A,t,t)$;
$z_{i+1} \coloneqq value(B)$;
end do
$\boldsymbol{Step\ 4}$: $Z \coloneqq sum(z_j, j = 0,1, \dots N+1)$;
$z(x,y) \coloneqq eval(Z, t=0)$;
$z(x,t) \coloneqq eval(Z, y=0)$;
$z(t,y) \coloneqq eval(Z, x=0)$;
output: see tables (1,2,3)

where N is the computational length.

3. NUMERICAL EXPERIMENT

In order to assess the accuracy of the formulated algorithm (16) for the numerical solution of the three dimentional second-order PDE that occurred in a plate vibration, we have introduced three test cases to investigate the nature of function $f(x,y)$ and subject to initial condition in [1] when vibration of the pad induced by a slight instantaneous disturbance lateral to the pad from a static equilibrium condition (i.e.,zero velocity) in which $g(x,y)=0$ and compare the approximate solutions with the analytical solutions.

$$\frac{\partial^2 z(x,y,t)}{\partial t^2} = \lambda^2 \left[\frac{\partial^2 z(x,y,t)}{\partial x^2} + \frac{\partial^2 z(x,y,t)}{\partial y^2}\right] \tag{17}$$

subject to initial conditions

$$\begin{cases} z(x,y,t)_{t=0} = f(x,y) \\ \left.\dfrac{\partial z(x,y,t)}{\partial t}\right|_{t=0} = g(x,y) \end{cases} \tag{18}$$

when the pad has fixed edges with initial sagging that can be described by functions

$$f(x,y) = \begin{cases} (10-x^2)(5-y^2) \\ 10\sin(x) + 5\cos(y) \\ e^{(10x+5y)} \end{cases} \tag{19}$$

vibration of the pad induced by a slight instantaneous disturbance lateral to the pad from a static equilibrium condition (i.e., zero velocity)

$$g(x,y) = 0 \tag{20}$$

and

$$\lambda = \sqrt{\frac{Pg}{m}} = \sqrt{\frac{(0.5lb_t)\left(\frac{32.0ft}{s^2}\right)}{\frac{0.00155lb_m}{in^2}}\left(\frac{12in}{ft}\right)} = 353.05\ in/s \tag{21}$$

Compute the above parameters into algorithm (16), we have the following numerical results tables:

Table 1. $f(x,y) = (10-x^2)(5-y^2)$, $g(x,y)=0$, $\lambda = 353.05$ $N=3$ **Case 1**

$z(x,y)$.		When $t=0$		$z(x,t)$.		When $y=0$		$z(t,y)$.		When $x=0$	
x	y	Analytical Solution	HPA Solution	x	t	Exact Solution	HPA Solution	t	y	Exact Solution	HPA Solution
0	0	50.00000	50.00000	0	0	50.000000	50.000000	0	0	50.000000	50.000000
0.1	0.1	49.85010	49.85010	0.1	0.1	107.02700	107.02700	0.1	0.1	106.97780	106.97780
0.2	0.2	49.40160	49.40160	0.2	0.2	327.54200	327.54210	0.2	0.2	327.34200	327.34200
0.3	0.3	48.65810	48.65810	0.3	0.3	859.84750	859.84740	0.3	0.3	859.39741	859.39741
0.4	0.4	47.62560	47.62560	0.4	0.4	1951.1134	1951.1133	0.4	0.4	1950.3130	1950.3130
0.5	0.5	46.31250	46.31250	0.5	0.5	947.37817	947.37812	0.5	0.5	3946.1281	3946.1283
0.6	0.6	44.72960	44.72960	0.6	0.6	7293.5485	7293.5486	0.6	0.6	7291.7485	7291.7486
0.7	0.7	42.89010	42.89010	0.7	0.7	12533.399	12533.399	0.7	0.7	12530.949	12530.949
0.8	0.8	40.80960	40.80960	0.8	0.8	20309.573	20309.573	0.8	0.8	20306.372	20306.372
0.9	0.9	38.50610	38.50610	0.9	0.9	31363.580	31363.580	0.9	0.9	31359.530	31359.530

1.0	1.0	36.00000	36.00000	1.0	1.0	46535.800	46535.800	1.0	1.0	46530.800	46530.800

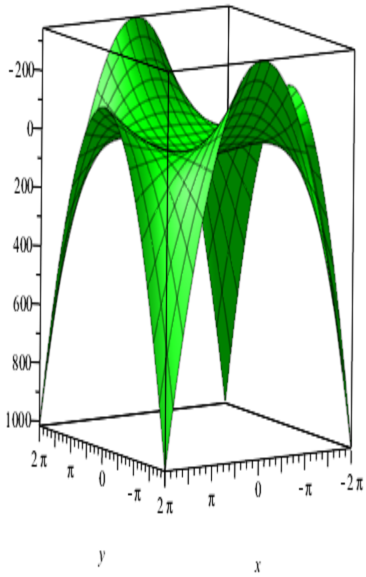
Table 2. $f(x,y) = 10\sin(x) + 5\cos(y)$, $g(x,y) = 0$ and $\lambda = 353.05$ $\boldsymbol{N = 10}$ **Case 2**

$z(x,y)$.		When $t = 0$		$z(x,t)$.		When $y = 0$		$z(t,y)$.		When $x = 0$	
X	y	Analytical Solution	HPA Solution	X	t	Exact Solution	HPA Solution	t	y	Exact Solution	HPA Solution
0	0	5.0000000	5.0000000	0	0	5.0000000	5.000000	0	0	5.000000	5.000000
0.1	0.1	5.9733549	5.9733549	0.1	0.1	20.092790	20.09279	0.1	0.1	16.66497	16.66497
0.2	0.2	6.8870261	6.8870262	0.2	0.2	49.804244	49.80424	0.2	0.2	105.0698	105.0698
0.3	0.3	7.7318845	7.7318845	0.3	0.3	1116.0813	1116.081	0.3	0.3	670.1484	670.1484
0.4	0.4	8.4994883	8.4994883	0.4	0.4	8168.9733	8168.973	0.4	0.4	4229.799	4229.798
0.5	0.5	9.1821681	9.1821681	0.5	0.5	58887.972	58887.97	0.5	0.5	26382.33	26382.33
0.6	0.6	9.7731028	9.7731029	0.6	0.6	4.191-E05	4.182-E05	0.6	0.6	1.62-E05	1.63-E05
0.7	0.7	10.266387	10.266388	0.7	0.7	2.931-E06	2.931-E06	0.7	0.7	9.79-E05	9.78-E05
0.8	0.8	10.657094	10.657095	0.8	0.8	2.002-E07	2.002-E07	0.8	0.8	5.72-E06	5.72-E06
0.9	0.9	10.941318	10.941319	0.9	0.9	1.313-E08	1.313-E08	0.9	0.9	3.18-E07	3.18-E07
1.0	1.0	11.116222	11.116223	1.0	1.0	8.117-E08	8.117-E08	1.0	1.0	1.63-E08	1.63-E08

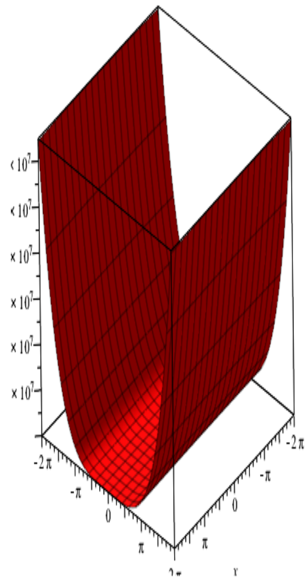
Table 3. $f(x,y) = e^{(10x+5y)}$, $g(x,y) = 0$ and $\lambda = 353.05$ $\boldsymbol{N = 15}$ **Case 3**

$z(x,y)$.		When $t = 0$		$z(x,t)$.		When $y = 0$		$z(t,y)$.		When $x = 0$	
X	y	Analytical Solution	HPA Solution	X	t	Exact Solution	HPA Solution	t	y	Exact Solution	HPA Solution
0	0	1.000000	1.000000	0	0	1.000000	1.000000	0	0	1.000000	1.000000
0.1	0.1	4.481689	4.481689	0.1	0.1	6.182-E08	6.182-E08	0.1	0.1	3.75 -E06	3.76 E06
0.2	0.2	20.08553	20.08554	0.2	0.2	1.571-E17	1.571-E17	0.2	0.2	5.77-E16	5.78-E16
0.3	0.3	90.01713	90.01714	0.3	0.3	2.331-E23	2.331-E23	0.3	0.3	5.20-E22	5.20-E22
0.4	0.4	403.4288	403.4289	0.4	0.4	6.933-E27	6.933-E27	0.4	0.4	9.38-E26	9.39-E26
0.5	0.5	1808.042	1808.042	0.5	0.5	2.492-E31	2.492-E31	0.5	0.5	2.04-E30	2.05-E30
0.6	0.6	8103.084	8103.084	0.6	0.6	2.376-E34	2.377-E34	0.6	0.6	1.18-E33	1.19-E33
0.7	0.7	36315.50	36315.51	0.7	0.7	9.107-E36	9.108-E36	0.7	0.7	2.75-E35	2.76-E35
0.8	0.8	1.628-E05	1.628-E05	0.8	0.8	1.794-E39	1.795-E39	0.8	0.8	3.29-E37	3.28-E37
0.9	0.9	7.294-E05	7.294-E05	0.9	0.9	2.129-E41	2.128-E41	0.9	0.9	2.36-E39	2.35-E39
1.0	1.0	3.269-E06	3.269-E06	1.0	1.0	1.694-E43	1.695-E43	1.0	1.0	1.14-E41	1.14-E41

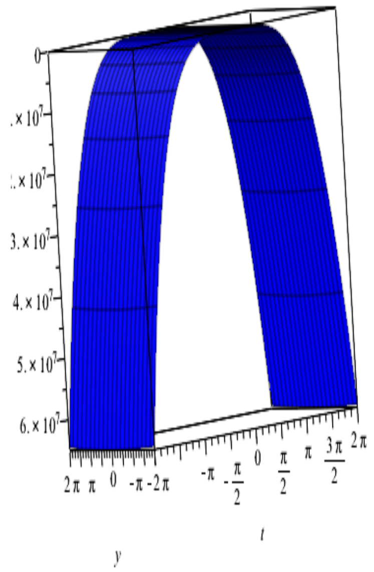
4. GRAPHS REPRESENTATION



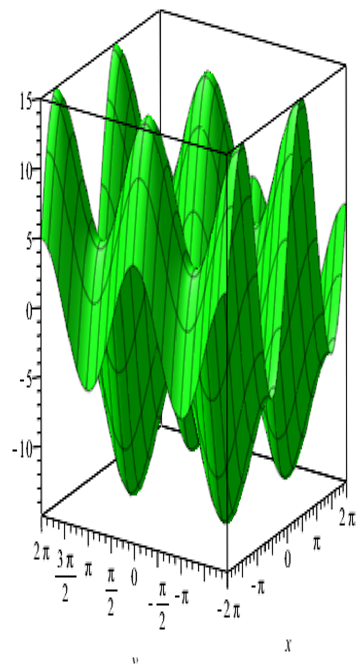
Graph 1. Numerical solution at $z(x,y)$ for $f(x,y) = (10 - x^2)(5 - y^2)$.

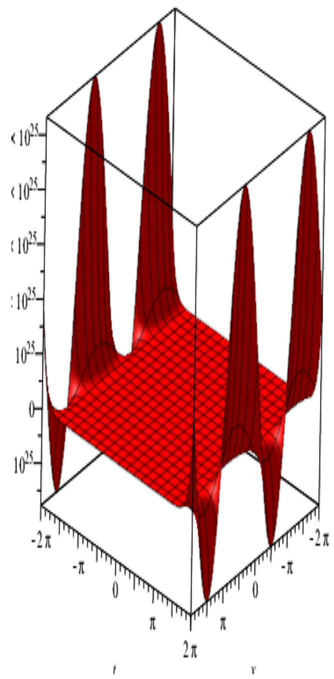


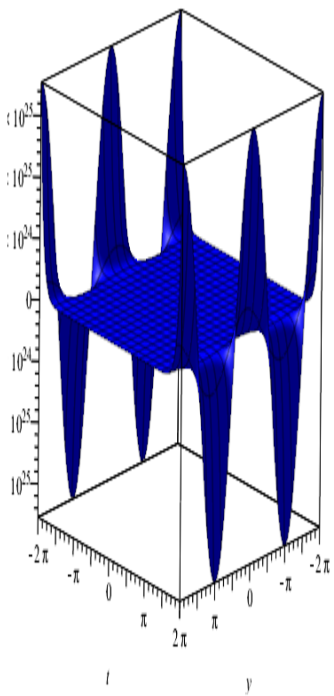
Graph 2. Numerical solution at $z(x,t)$ for $f(x,y) = (10 - x^2)(5 - y^2)$.



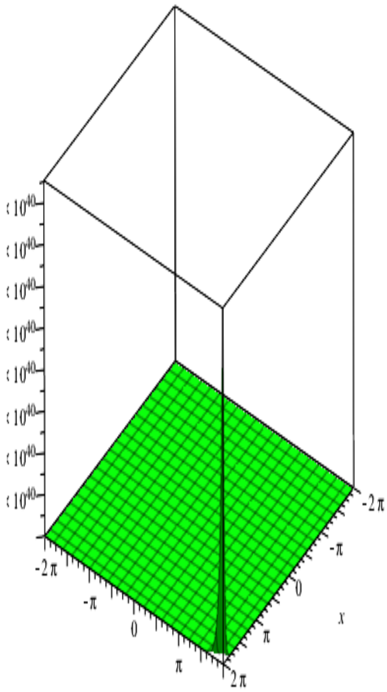
Graph 3. Numerical solution at $z(t,y)$. for $f(x,y) = (10 - x^2)(5 - y^2)$.



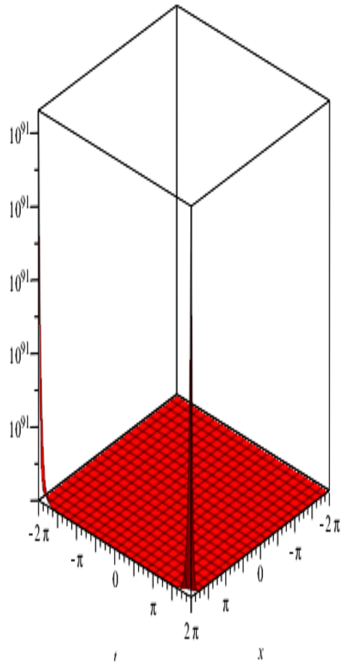




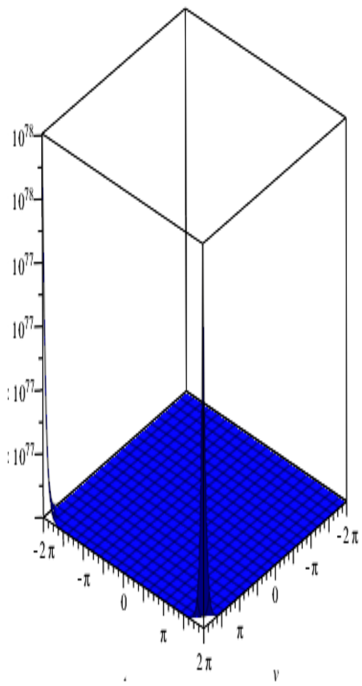
Graph 4. Numerical solution at $z(x,y)$. **Graph 5.** Numerical solution at $z(x,t)$. **Graph 6.** Numerical solution
at $z(t,y)$.for $f(x,y) = 10\sin(x) + 5\cos(y)$. for $f(x,y) = 10\sin(x) + 5\cos(y)$ for $f(x,y) =$
$10\sin(x) + 5\cos(y)$



Graph 7. Numerical solution at $z(x, y)$. for $f(x, y) = e^{(10x+5y)}$



Graph 8. Numerical solution at $z(x, t)$. for $f(x, y) = e^{(10x+5y)}$



Graph 9. Numerical solution at $z(t, y)$. for $f(x, y) = e^{(10x+5y)}$

5. DISCUSSION AND CONCLUSIONS

5.1. Discussion

Modal shapes of thin plates require numerical solutions of $Z(x, y, t)$ which is a very tedious job. It will also be a great deal of laborious efforts to obtain numerical solution to graphical representations of these shapes. Consequently, the objective of this study is to formulate fast,easy, and accuraty algorithm to simulate the task ahead.Thus, the objective was accomplished through the formulated algorithm using MAPLE 18 matheamtical software inwhich the plots are shown in graphs 1 to 9 for the geometric behaviours of thin plates vibration when functions $f(x, y)$ (algebric, trigonometric and exponential) at a static equilibrium condition $g(x, y) = 0$ which was examined. The corresponding numerical solutions for the three test cases are presented in Tables1 to 3. Moreover, the graphs modal shapes also indicate where the peak amplitudes of vibration of the flexible plate or the nature of frequency of the wave would occur, from which the design engineers should take precaution for not placing delicate attachments at these locations to avoid possible damages due to excessive deformation of the plate structure.

5.2. Conclusion

In this paper, we formulate a four steps algorithm using homotopy perturbation method for the numerical solution of three dimentional second-order PDE which occured in a plate vibration. The method is applied in a direct way without any transformation, discretization or restrictive assumptions. Also, we have tested the HPA on three different implementations which are shown the efficiency and accuracy of the proposed method. The approximate solutions are in good agree with analytical solutions (see tables 1 to 3). It may be concluded that the HPA is powerful and efficient in finding the numeric-analytic solutions for a wide class of problems in applied sciences and engineering.

REFERENCES

[1] Ran Hsu Tai (2018) "Applied Engineering Analysis", published by John Wiley & Sons, (ISBN 9781119071204) Applied Engineering Analysis pp:71-78.
[2] Irvine M, (1981) "Cable Structures", The MIT Press, Cambridge, New York pp: 23-45.
[3] Leonard J.W, (1988) "Tension Structures", McGraw-Hill Book Company, New York.
[4] Rega .G, Vestroni F and Benedettini F, (1984) "Parametric analysis of large amplitude free vibrations of a suspended cable",International Journal of Solids and Structures20(2), pp:95–105.
[5] Ni.Y.Q, Lou.W.J and Ko,J.M, (2000) "A hybrid pseudo force/Laplace transform method for non-linear transient response of a suspended cable", Journal of Sound and Vibration238(2), pp:189–214.
[6] He, J.-H. (1999), "Homotopy perturbation technique", Computer methods in applied mechanics and engineering, 178, 257
[7] Vahidi, A., Babolian, E., & Azimzadeh, Z. (2011), "An improvement to the homotopy perturbation method for solving nonlinear Duffing's equations", Bulletin of the Malaysian Mathematical Sciences Society pp: 23-34
[8] Chang, H.-K., and Liou, J.-C. (2006), "Solving wave dispersion equation for dissipative media using homotopy perturbation technique", Journal of waterway, port, coastal, and ocean engineering, 132.
[9] Zhou, S., and Wu, H. (2012) "Analytical solutions of nonlinear Poisson–Boltzmann equation for colloidal particles immersed in a general electrolyte solution by homotopy perturbation technique", Colloid and Polymer Science, 290, 1165.
[10] Özi¸s, T., and Akçı, C. (2011) "Periodic solutions for certain non-smooth oscillators by iterated homotopy perturbation method combined with modified Lindstedt Poincaré technique", Meccanica, 46, 341.
[11] Yazdi, A. A. (2013) "Homotopy perturbation method for nonlinear vibration analysis of functionally graded plate", Journal of Vibration and Acoustics, 135, 021012.
[12] Al-Saif, A. and Abood, D. A. (2011) "The Homotopy Perturbation Method for Solving K (2, 2) Equation", J, Basrah Researches (Sciences).
[13] Aswhad, A. A., & Jaddoa, A. F. (2016) "The Approximate Solution of Newell Whitehead Segel and Fisher Equations Using The Adomian Decomposition Method", arXiv preprint arXiv: 1602.04084.
[14] Babolian, E., Azizi, A., & Saeidian, J. (2009) "Some notes on using the homotopy perturbation method for solving time-dependent differential equations", Mathematical and Computer Modelling, 50, 213.
[15] Adil M. Al-Rammahi, and Ghassan A. Al-Juaifri (2017) "A Study of General Second-Order Partial Differential Equations Using Homotopy Perturbation Method", Global Journal of Pure and Applied Mathematics. ISSN 0973-1768 Volume 13, Number 6, pp. 2471–2492
[16] Mohyud-Din S.T and Noor M.A (2009) "The HPM for Solving Partial Differential Equations" Downloaded from PubFactory at 09/06/2016 12:45:36AM via ReadCube/Labtiva.
[17] He,J.H, (2004) "Comparison of homotopy perturbation method and homotopy analysis method", Applied Mathematics and Computation 156, pp527-539.
[18] Chun, C. and Sakthivel, R. (2010) "Homotopy perturbation technique for solving twopoint boundary value problems. Comparison with other methods", Computer Physics Communications, 181, 1021.
[19] Liu H.-K, (2011) "Application of homotopy perturbation methods for solving systems of linear equations," Applied Mathematics and Computation, vol.217, no.12, pp.5259–5264.