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Highway geometric design using bézier curve approximation

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Highlights

- Bezier curve as a mathematical function could be used for highway design
- Curve radius is obtained up to 150% greater on bezier curve designed highway
- Centrifugal force is decreased up to 96%

Abstract

Traditional highway design procedure is a trial and error process where the system manager spends a lot of time. To improve this manual process, a new method to design highways is investigated. A mathematical function called Bézier Curve is used. Highway designs have been made with both traditional and Bézier Curve methods on a level type terrain. According to results, Bézier Curve gives a shorter highway than the traditional method. And, the curve radius is also larger than traditional method which helps improve highway safety and drivers' comfort. So, the highway designed with Bézier Curve reduces overall roadway length and larger curve radius than traditionally designed highway.

Keywords: *bézier curve, control points, comfort, highway design, safety*

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1. Introduction

Nowadays, vehicle speed increases the transportation requirements. Although increased speed is a desired feature because the value of time is further increasing, high speeds raise safety problems. Drivers are adapting themselves to the geometric design of the highway in a short time. To do that, they use the experience in previous travels where the highway characteristics are close to each other [1]. Considering this information, it is necessary to design a highway where it is easy to get adapted and able to drive safer at high speeds. To meet this expectation, geometric standards should be examined in detail by design process. One of the major accident occurrence reason can be the lack of geometric design consistency which causes critical driving maneuvers [2]. Also, a consistent highway design provides consecutive highway elements which produce a highway without any surprising areas [3]. Traditionally designed highways have straight parts which are followed with some curves with different radius. In most cases, drivers drive directly from the straight parts to these curves. They didn't know how much to steer the wheel before they drive in the curve. This problem could be eliminated using smooth and continuous curvature on design step where all the drivers get informed about the next geometric part of the highway, and how it looks. Because driving on a continuous curve, they will adapt themselves to the curves more easily.

Traditional design methods are a decision-making process of system manager which means that the process is more subjective than objective. There are some studies [1, 4-11] about new highway design methods such as various programming methods, mathematical equations and mathematical spline types like B-Splines. These method objectives are the minimization of the parameters such as highway length, earthwork and grade while improving safety and comfort. As a result, construction and vehicle operation costs and travel times are optimised.

In this paper, a new highway is designed with traditional design method and spline approximation. Splines have been used as mathematical tools since 1960's to

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interpolate or to approximate functions. In real life, they have been used to smoothing curves, designing ship hulls and car bodies [1]. To use the splines in an industrial area, spline energy is optimised. A spline has three types of energy which are stretch, strain and jerk. The stretch energy measures the length of the curve while strain energy measures how much the curve bent [12]. The stretch energy of a curve is calculated with Equation 1 [13]. The stretch energy of the curve is the length of the curve. So, in our case, it is the length of the highway.

$$\tilde{E}_{stretch}(x)=\int_a^b \|x'(t)\|^2 dt \quad (1)$$

The used spline in this paper is Bézier Curve (BC). BC drew most appropriate and continuous curve in line with the control points. Splines are used infrequently as an alternative to traditional highway design but when they become mathematically simpler the use becomes more frequent.

Walton and Meek [14] discussed the use of BC instead of clothoid which is used traditionally in highway design. As result, BC is accepted as a reasonable alternative to the clothoid for highway and railway applications. Bosurgi and D'Andrea [15] used a fifth-degree polynomial curve to obtain a continuous alignment. They have studied semi-direct and inner loop connections with this method. As result of the study, using the polynomial curve, they fully meet the practical needs of design, improves the safety and comfort, and provides the better visual perception of the curve compared to straights and clothoid solution. However, a computer program is needed for solving the various design cases which are not necessary for BC approach. Designing the whole alignment with a continuous curve, determining the control points according to the vertical alignment and drawing just a highway plan in accordance with the control points and so designing three-dimensional are the novelties of this paper.

This paper's structure is as follows. After the introduction, BC is exposed. Then the case study on a level type terrain is presented. On this presentation, the pros and cons of the BC are explained. Finally, obtained conclusions are listed.

2. Bézier Curves

Before the 1950's, smooth shape design was very difficult. In 1959, two men who called Paul de Faget de Casteljau and Pierre Bézier, working both on two different automobile companies, worked on smoothing the intersection by iterating affine combinations of polygons. Both versions produce the same curve but because Pierre Bézier was the first to write a public paper, so the curve is called BC [16].

A BC is formed in line with the control points $(P_0, P_1, P_2, \dots, P_n)$. The curve passes through first and last control points while approximating the remaining control points. However, Arikoglu and Yucesan [17], have studied about the minimization of the jerk energy, which means that the approximated curve could be redrawn to offset the highway from the surrounding utilities or connecting to existing networks, junctions, bridges, etc. if needed with a few calculations of new control point(s).

To draw the BC, it is needed to draw a weighting function which demonstrates the influence of the control points. Hereby, Bernstein polynomials are used as weighting function and the definition is [18]:

$$B_{i,n}(u)=\frac{n!}{i!(n-i)!}u^i(1-u)^{n-i} \quad (2)$$

where P is the control point, and n is the degree of the curve. Bernstein polynomials are the basis function of the BC. The most useful degree is 3. When n<3 there are not many possibilities to work and if n>3, the curve becomes hard to work.

The polynomials are symmetric and sum to unity [16]. It is important that the basis functions always sum to unity. And, a BC of degree n is defined as follows:

$$C(u)=\sum_{i=0}^{n} P_i B_{i,n}(u)\ ,\ u\in[0,1] \quad (3)$$

Figure 1 is an example of a BC of degree 3 with 4 control points. The straight lines between control points are called control polygons.

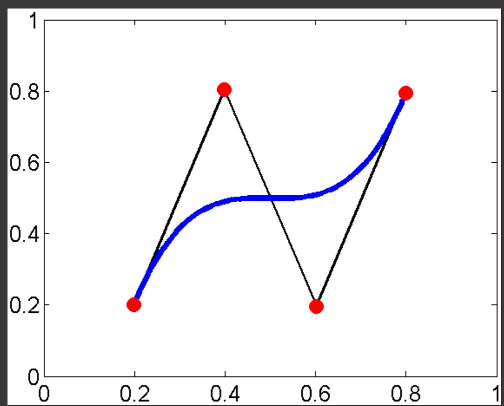

Figure 1. BC of degree 3 and control polygon saw on coordinate system

As seen in Figure 1 BC starts with the first control point and ends with last control point. And, the BC is drawn with the intermediate control points. The increase in the control point's number brings the BC closer to control polygon.

To demonstrate the approach, let A is a control polygon which has control points $P(x_i, y_i)$. Including $A \subset \mathbb{R}^2$ and $x, y \in [0,1]$, the control polygon is defined as follows:

$$A=\left\{(x,y)\ \middle|\ \begin{matrix}(0,y) \text{ if } x=0\\(x,1) \text{ if } x\neq 0\end{matrix}\right\} \quad (4)$$

Now 5 control points and 15 control points on control polygon A are selected. Figure 2 shows the difference between 5 and 15 control points.

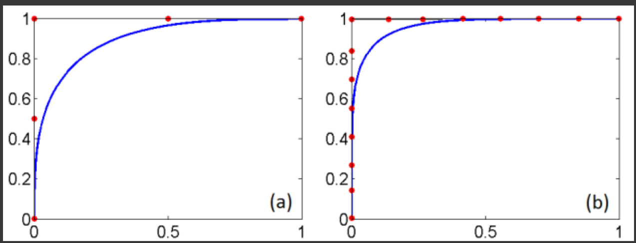

Figure 2. Difference between BC seen on coordinate system with 5 control points (a) and 15 control points (b)

From Figure 2 as long as the number of control points increases on the same control polygon, the BC gets closer to the control polygon.

3. Comparison of the Traditional Design with Bézier Curve

Highways are three-dimensional objects. Although horizontal and vertical alignment is thought separately from one another, there is a relationship between the two alignments. The coordination of the horizontal and vertical alignment is important for aesthetic, economic and safe driving. So horizontal and vertical alignments should be designed together. By design process, the designer should avoid both designing a highway with steep grades to obtain minimal curvature and designing a highway with massive horizontal curvature to obtain a level vertical alignment [19].

3.1. Traditional highway design

Traditional design usually defines the horizontal alignment with a succession of straights, and curves while defining the vertical alignment with a succession of straights and parabolas. To design traditionally, system manager determines firstly whether the area where the highway goes through is level, rolling or mountainous. After the terrain type is determined, maximum allowed grade is selected according to design speed of the highway. The maximum allowed grades for rural arterial specified in AASHTO "Green Book" [20] is shown in Table 1.

Table 1 Maximum grade allowed according to AASHTO

Terrain Type*	Max. Grade (%) for Specified Design Speed (km/h) 60	70	80	90	100	110	120	130
L	5	5	4	4	3	3	3	3
R	6	6	5	5	4	4	4	4
M	8	7	7	6	6	5	5	5

*L: Level; R: Rolling; M: Mountainous

Drawing a possible route with a constant grade would help to design the whole highway three-dimensional. Therefore, a null polyline is used to draw a route with constant grade value in accordance with Table 1. A null polyline is the guide route for the final highway which is drawn with the selected standards (speed and grade). The null polyline is the merging of the lines which are placed between the consecutive contour lines and have the same length. Because of all the lines which are creating the null polyline, are of the same length, the grades of all the lines are also the same. So, when the obtained null polyline is used as the highway's horizontal route, which is almost impossible, the longitudinal section of this route would have one vertical alignment with a constant grade where the earthwork is minimum. However, it is almost impossible to use the null polyline directly for the highway's route design, it could be used as guide route where the vertical alignment with minimum earthwork is determined. So would the highway design have made as three-dimensional.

At first, to draw the null polyline, the merging lines' length which is called compasses clearance, should be calculated using the selected grade value for specified speed. The maximum allowable grade value is changing in accordance with the terrain type. For example, when the specified design speed is 100 km/h, the grade value for calculation would be 3 for level, 4 for rolling and 6 for the mountainous type. So, the drawn null polyline is depended on with the terrain types. The calculation is made with the help of the tangent formulation. The elevation difference value between the two consecutive contours divided by the selected grade level has the result of the compasses clearance (Equation 5).

$$CC=\frac{\Delta_h}{g} \quad (5)$$

where CC is compasses clearance (m), Δ_h is the difference between the two contour lines and g is the selected grade value (%). With the compasses clearance value, a null polyline is drawn between two destination points. Drawn null polyline creates a highway plan where the grade is fixed to the selected value. However, the highway plan is folded. From this point, the designer must decide the best alignment. The designer draws more than one highway plan according to null polyline and decides which plan is optimum. Drawn highway plan is formed with straights and curves. Selection of the correct alignment is a trial and error process. Final selection is based on costs and environment. The procedure for obtaining the final highway plan is described as follows:

i. Select the max grade according to terrain type and design speed.
ii. Calculate the compasses clearance using Equation 5.
iii. Draw the null polyline with the compasses clearance (Figure 3a).

iv. Highlight some points of horizontal curves and draw straights between this some points (Figure 3b).

v. Insert a curve with appropriate and specification providing radius between the both tangents (Figure 3c).

vi. Check if the drawn highway is acceptable, if not go back to *Step 4* and draw a new highway plan until an acceptable highway is drawn.

In Figure 3(a), blue lines are null polyline. The short straights which create the null polyline have a common feature. They are all the same lengths. Because the contour line's altitude difference is same, the grade for all short straights is the same. In Figure 3(b), green points have selected some points of the horizontal curves and the red lines are drawn as alignment between the curve's some points. In Figure 3(c), most suitable curve radius is selected with trial and error process.

Drawing the null polyline first helps to find the vertical alignment with the fixed grade value. Because of the null polyline is not smooth and there are many horizontal curves, it is almost impossible to use the null polyline directly. Therefore, the designer draws a route according to a null polyline with straights and curves to obtain a highway where the driver can drive comfortable and safe. And also, the construction cost is reduced while drawing according to a null polyline. With the help of null polyline, the highway is planned three-dimensional. Figure 4 shows the whole horizontal alignment drawn by the designer.

Figure 3. Highway design procedure is seen on coordinate system, (a) null polyline, (b) highway route, (c) most suitable curve selection

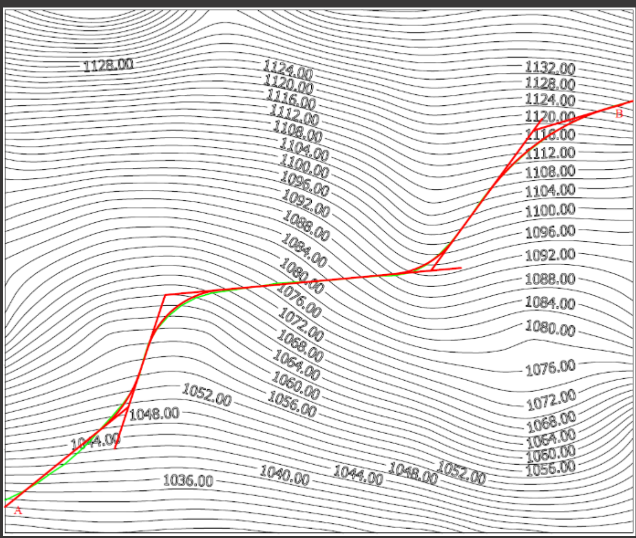

Figure 4. Drawn highway plan by designer saw on coordinate system

Drawn horizontal alignment showing in Figure 4 consists of 5 straight and 4 circular curves from point A to B with radius 300, 200, 200, 550 m, respectively.

3.2. Bézier curve design

The same highway is designed with BC. As explained in Section 2, Bézier Curves (BC) need control points to be drawn. To design the highway, the drawn null polyline is used as control polygon. Therefore, the breaking points which are occurred by merging the compasses clearances are defined as control points. So, 48 control points are obtained. Then with the help of this control points, a BC is drawn. The procedure for obtaining the final highway plan using BC approximation is described as follows:

i. Select the max grade according to terrain type and design speed.

ii. Calculate the compasses clearance using Equation 5.

iii. Draw the null polyline with the compasses clearance.

iv. Determine the merging points of the compasses clearance as "Control Points".

v. Represent the control points on a coordinate system.

vi. Draw straights between the control points P_0P_i on coordinate system and obtain "Control Polygon"

vii. Draw a new control polygon between the control point (P_0) and a previous drawn control polygons control point (P_{i-1}).

viii. Repeat *Step 7* until control polygon which is a straight line between the control points P_0P_1 is drawn on the same coordinate system and obtained $(i-1)$ overlapping control polygons.

ix. Move all the control polygons over a previously drawn control polygon by moving all the control points P_i from moving control polygons to the control points P_{i+1} from moved control polygon on the same coordinate system.

The drawn BC with 48 control points is shown in Figure 5. Drawn BC with the help of null polyline shown in Figure 5 consists of only one smooth curve without any straight alignments. However, there is almost no curvature on the transition areas between reverse curves. So, the safety criteria are provided. When the curve is needed to define in transition section and circular curve form then the BC consists of 7 transition sections and 4 circular curves from point A to B with radius 750, 350, 450, 650 m, respectively.

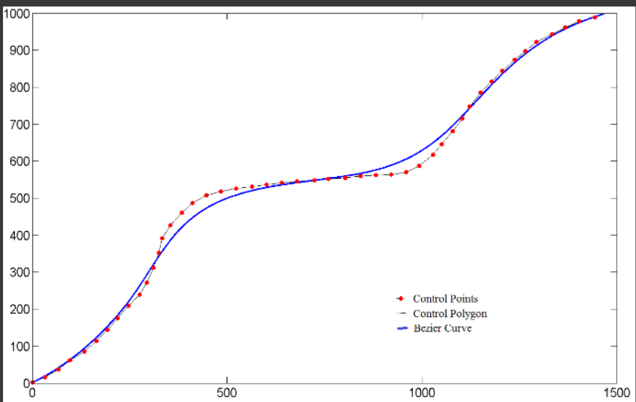

Figure 5. Drawn highway plan with BC seen on coordinate system

3.3. Comparison the both design methods

To compare the two methods, first, a highway is designed with the traditional method. Then the same highway is designed with BC and both highways are compared with road length and curve radius. When the both highway plans overlaid on the same terrain map then Figure 6 appears.

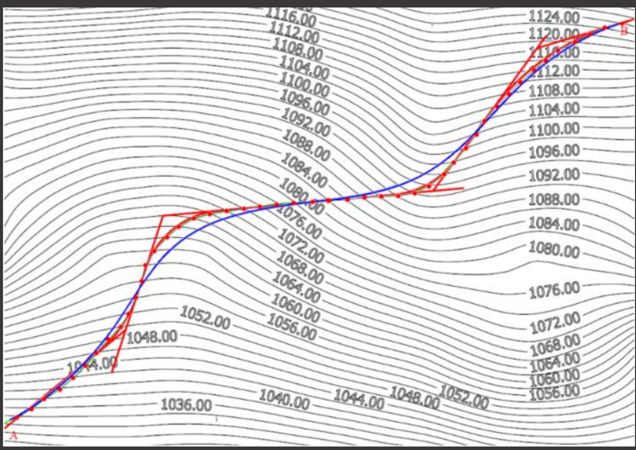

Figure 6. Overlaid BC and traditional highway design is seen on coordinate system

As seen in Figure 6, the null polyline is used as control polygon for BC. BC follows the same leaning as a traditionally designed highway. The total highway length is on traditional design ~1902 meter long and the length is decreased to ~1861 meters on BC.

Curve radius is also changed on BC. Highway drawn with BC has larger radius values than traditional designed one. The increase in horizontal curve radius decreases the lateral jerk value as calculated with Equation 6.

$$p' = \frac{V^3}{46{,}7 * R * L_g} - \frac{V * d}{36{,}7 * L_g} \quad (6)$$

where, p' is lateral jerk value (m/s^3), V is the speed of the vehicle (km/h), R is the curvature radius (m), d is the superelevation value (%) and L_g is the transition section length (m)

The lateral jerk value is very important for comfortable driving. The increase on the lateral jerk decreases the driver's and passenger's comfort [21, 22]. So, the increase in the curve radius decreases the lateral jerk value which increases the comfort of drivers and passengers. Table 2 shows the change in the radius and lateral jerk between traditional designed highway and BC designed highway. Although the whole highway is designed as a continuous curve, just the minimum radius values are listed in Table 2. The reason is that the sections with minimum curve radius are critical than sections with larger curve radius for safety. In addition, the curve radius change on the whole highway is very smooth where the driver wouldn't make any sudden manoeuvre.

Table 2 Change of the lateral jerk

Radius Values			Lateral Jerk Values		
Traditional Designed Highway (m)	Bezier Curve Designed Highway (m)	Change of Radius (%)	Traditional Designed Highway (m/s^3)	Bezier Curve Designed Highway (m/s^3)	Change of Lateral Jerk (%)
300	750	+150	0,2161	0,0079	-96,31
200	350	+75	0,3895	0,1665	-57,25
200	450	+125	0,3895	0,1004	-74,21
550	650	+18	0,0584	0,0293	-49,82

To calculate the lateral jerk and compare both design method, vehicle speed, superelevation value and transition section length are accepted as 90 km/h, 8% and 150 m, respectively. As seen in Table 2 the curve radius values are increased up to 150% while the lateral jerk values are decreased up to 96%. The observations show that lateral jerk is noticeable from 0,3 m/s^3 and uncomfortable from 0,4 m/s^3 [23]. According to this information, while two curves from the traditionally designed highway are comfortable, all curves are comfortable in BC designed highway.

According to AASHTO "Green Book" [20], the designed highway should ensure minimum curve radius values for a safe drive. The minimum radius value is calculated using Equation 7.

$$R_{min} = \frac{V^2}{127(0{,}01 e_{max} + f_{max})} \quad (7)$$

where R_{min} is the allowable minimum curve radius (m), V is design speed (km/h), e_{max} is max superelevation value (%) and f_{max} is the side friction value. With design speed as 90 km/h and maximum superelevation value as 8%, minimum calculated curve radius is 304 meters [20]. All the curve radius values obtained with BC ensures the minimum curve radius from "Green Book".

Ben-Arieh et al. [24] used B-Splines to define the highway alignment. The difference between the BC and B-Splines is that BC is a single polynomial while B-Spline is defined piecewise by a polynomial. So, Ben-Arieh et al. [24] procedure have a disadvantage of the need for dividing the alignment into sections. However, by BC design method the highway alignment doesn't need to be divided.

In addition to these results, there is no noteworthy change in vertical alignment between traditional designed and BC designed horizontal highway plans. So, the earthwork is almost the same. Therefore, it is not considered necessary to make a cost analyse.

4. Conclusion

In this paper, traditional highway design is compared to highway design by using a mathematical curve type called BC. Therefore, a horizontal alignment is designed with traditional methods. Then the same highway is designed using BC. The results can be listed as;

Traditional highway design method is a trial and error process. So, the result is more subjective than objective. Whereas, BC is a mathematical function and offers one result after determining the null polyline. For this reason, BC can be a faster solution.

Highway designed with BC is in accordance with a null polyline. So, horizontal alignment is drawn accordance with vertical alignment. And, designed horizontal alignment is very smooth where the driver can drive fast and safe. Also being so smooth reduces the risk of encountering a surprising highway section.

Total highway length is decreased with BC against traditional highway design while remaining faithful to the specification limit values and considering of the construction costs.

By decreasing the total length of the highway, the curve radius is obtained up to 150% larger on BC method. The increase of radius decreases the centrifugal force which increases safety and the lateral jerk is decreased up to 96% which increases the comfort of the drivers and passengers.

This study was only a case study and should be tested on rolling and mountainous terrain types. And, BC can also be used on vertical alignment where it seems useful as well.

Declaration of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contribution Statement

E.Eriskin: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Draft writing – **S. Karahancer:** Conceptualization, Investigation, Resources, Visualization, Draft writing – **S. Terzi:** Conceptualization, Investigation, Methodology, Supervision, Validation, Visualization Review&Editing – **M. Saltan:** Conceptualization, Investigation, Supervision, Visualization, Review&Editing

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