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# Analyzing Attribute Control Charts for Defectives Based on Intuitionistic Fuzzy Sets

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**Abstract:**
Control charts (CCs) are one of the most used Statistical Quality Control (SQC) techniques to determine the process' situation is under control or not. The CCs can be classified into two groups based on the quality characteristics such as "variable" or "attribute". Two well–known attribute control charts (ACCs) named $p$ and $np$ control charts are designed to measure the defectives during the manufacturing stages. If the process is deal with the number of defectives, then $np$ control chart is used. Similarly, if the process deals with the defective rate, the $p$ control chart is used. In the traditional CCs, one of the most important issues is to represent the available data with the highest rate. Since the handled data may consist of uncertain information, ordinary $p$ and $np$ CCs have remained incapable of the ability to reflect the data. Moreover, the operators or the observers of the system can be hesitant while measuring these values during the data gathering process. Therefore, dealing with these problems can be realized by extending the ordinary CCs with useful tools. In the literature, classical fuzzy sets have been used to extend $p$ and $np$ control charts. This paper aims to extend these CCs by using Intuitionistic fuzzy sets (IFSs). Comparing with the available studies, the usage of IFSs enables to represent the hesitancy in $p$ and $np$ control charts' design stages. For this aim, two types of ACCs have been re-designed based on IFSs to improve their sensitiveness and flexibility. In this paper, the extensions of $p$ and $np$ control charts with IFs are proposed and the design of these CCs based on IFs has also been represented in detail. Additionally, control limits and center lines have been re-formulated by using IFs. Moreover, a descriptive example is introduced to analyze the applicability of the proposed method.

**Keywords:** Fuzzy logic, Intuitionistic fuzzy sets, $np$ control chart, $p$ control chart.

## 1 Introduction

In a competitive marketing, quality of the products is one of the most vital criteria, which is a necessity for any industry while reaching the most desirable product level. To do this, statistical process control (SPC) is one of the most efficient methodology for inspection, evaluation, handling and controlling the process' performances. Moreover, implementing SPC to the production plants enables to improve production quality by decreasing process variation based on the unnatural events. Control charts (CCs) are one of the most used statistical quality control techniques, which enable the controllers can measure the product specifications in the manufacturing process to determine the process is whether or not in the accepted limits [1]. If the measured characteristics are outside of the determined limits, CCs notice the system to take necessary precautions to keep the process at the desired level. The CCs can be classified based on the quality characteristics, which are measurable on numerical scales or not, and they can be classified into two groups based on data as "variable" or "attribute" [2]. Two well–known attribute control charts (ACCs) named $p$ and $np$ control charts are designed to measure not only the defectives during the manufacturing stages but also observe the defections in the service sectors. If the process is deal with the number of defectives, then $np$ control charts are used. Similarly, if the process deals with the defective rate, the $p$ control charts are used.

In the traditional CCs, one of the most important issues is to represent the available data with the highest rate. Since the handled data may consist of uncertain information, ordinary $p$ and $np$ CCs have remained incapable of the ability to reflect the uncertainty. Moreover, the operators or the observers of the system can be hesitant while measuring these values during the data gathering process. Therefore, dealing with these problems can be realized by extending the ordinary CCs with useful tools. It is essential to represent the available data with the highest rate while the observed system is analyzed. Since the handled data may consist of uncertain information, ordinary control charts have remained incapable of the ability to reflect the data. Moreover, the operators or the observers of the system can be hesitant while measuring these values during the data gathering process. The fuzzy set theory (FST) is a tool, which enables to the representation of uncertainty by assigning membership function to an element by indicating the level of its belongingness to a set [3]. In the literature, classical fuzzy sets are used to extend $p$ an $np$ control charts in many studies to increase the models' data representation and interpretation [4–7]. Classic rules of statistical control consider a process as out of control when one of its samples is out of the control limits of the used chart. However, the values represented by the samples as well as the control limits are considered deterministic when they are essentially stochastic [8]. Studies made by [9]-[8] present the effects of considering control limits as value ranges with probability distributions associated, achieving better results in comparison to traditional charts.

In case of uncertain environments, where the data has types of uncertainty and the decision maker or the expert has intuitive evaluations, the ordinary CCs are incapable of representing these properties in the mathematical representations. Therefore, new adaptive charts are needed to represent both uncertainty and the hesitancy. Since the FST are incapable of representing hesitancy, the mentioned ordinary fuzzy models are not practical to include hesitancy during the calculations. Intuitionistic fuzzy sets (IFSs), which is an extension of FST is a way of representing not only the uncertainty in the data but also the hesitancy of the decision-makers [24]. It is a useful tool to represent attribute information by using scales corresponded with intuitionistic fuzzy numbers (IFNs) during constructing the system structure. Comparing with the existed studies, the usage of IFSs enables the quality engineers to represent the hesitancy in their control charts. For this aim, in this paper, two types of ACCs, $p$ and $np$ control charts, have been re-designed based on IFSs to improve their sensitiveness and flexibility. For this aim, control limits and center lines have been re–formulated by using the proposed extension. Moreover, a descriptive example is introduced to check the applicability of the proposed method.

The rest of this paper has been organized as follows: Section 2 includes a brief information related with Attribute Control Charts (ACCs) and literature review. Section 3 details Intuitionistic fuzzy sets (IFSs). Attribute Control Charts based on IFSs have been analyzed into Section 4. A descriptive example has been added to explain the proposed approach. The obtained results and future research directions have been discussed into Section 5.

## 2 Attribute Control Charts

Control charts (CCs) are one of the most used statistical quality control (SQC) techniques, which enable the controllers can measure the product specifications in the manufacturing process to determine the process is whether or not in the control limits [1]. CCs have been considered as a powerful statistical techniques in industry to improve process' quality. A control chart contains lower and upper control limits, central value and plots a suitable statistic for each of subgroups.The process is indicated as out of control if any plotting data is located out of the control limits. If the measured value are located at outside of the determined limits, CCs notice the system to take necessary precautions to keep the process at the desired level. The CCs can be classified based on the quality characteristics as measurable on numerical scales or not [2], which are grouped as "variable" and "attribute". Based on the aim, the process is deal with the number of defectives, then $np$ control charts is used. Similarly, if the process deals with the defective rate, the $p$ control charts is used.

Attribute data, or data taking the form of counts, play a vital role in quality improvement. Many quality characteristics cannot be conveniently represented numerically, but can be represented as conforming or nonconforming [11]. The $p$ control chart is used to monitor the fraction nonconforming that is the ratio of number of defectives to the total number of items. Moreover, to increase its ability to represent the data, several extensions are introduced to the literature. Amirzadeh et al. developed trapezoidal fuzzy $p$ control chart and conducted to an automotive segment control process for a manufacturing company [16]. Shu and Wu developed a fuzzy $p$ control chart using LR type fuzzy numbers [4]. In addition, they obtained the membership degrees of the lower and upper control limits with the spectral resolution theorem. Sogandi et al. developed a triangular fuzzy $p$ control chart using the alpha level fuzzy midrange approach [7]. Erginel has developed triangular and trapezoidal fuzzy $p$ and $np$ control charts for fixed and changeable sample size that aims to make a decision by creating a rule base instead of transformation processes at the decision stage [10].

Let $\bar{p}$ is the estimation of the long-term process mean established during the inspection. The limits in $p$ control charts are calculated by using Eqs. (1–3) as follows:

$$CL = \frac{\sum_{n=1}^{k} p_n}{m} \tag{1}$$

where $p_n$ is the defective rate of $n^{th}$ sample, and $m$ is the number of the samples.

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \tag{2}$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}. \tag{3}$$

In similar way, the limits of the ordinary $np$ control chart are calculated by using Eqs. (4–6) as follows:

$$CL = n\bar{p}, \tag{4}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \tag{5}$$

where LCL is the lower control limit.

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \tag{6}$$

where UCL is the upper control limit.

Based on the SCOPUS database, there are 741 studies, which related with the attribute control charts by using the following search pattern:

TITLE-ABS-KEY ( *attribute* AND *control* AND *charts* )

Among them, 32 articles are reviewed in terms of their application fields. In Figure 1, the application areas are presented.

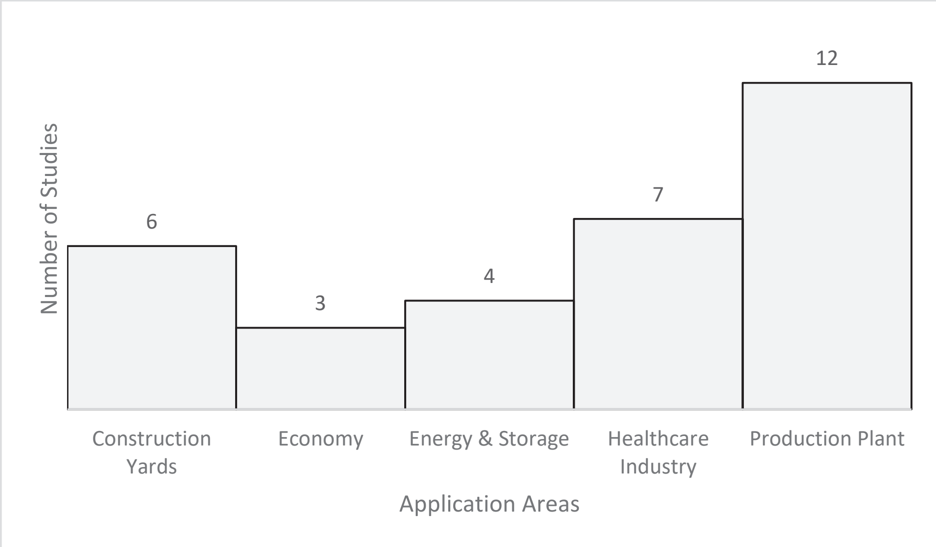


**Fig. 1**: Application areas of the attribute control charts

As in Figure 1, in many fields such as construction yards [12], economy [14], energy & storage [17], healthcare industry [15], production plants [13], attribute control charts are used for the solution.

In case of ordinary CCs are incapable of representing uncertainty, they are extended with the fuzzy set theory (FST) to handle it. FST is a tool, which enables to the representation of uncertainty by assigning membership function to an element by indicating the level of belongingness to a set [3]. In the literature, classical fuzzy sets are used to extend $p$ an $np$ control charts in many studies to increase the models' data representation and interpretation such as [4–7, 18–23]. Since the FST are incapable of representing hesitancy, the models are not practical to include hesitancy during the calculations. Different of the literature, attribute control charts, which is limited with the $p$ and $np$ control charts are extended with IFSs to increase their ability to reflect the available data based on the linguistic terms under uncertain environments.

## 3 The Proposed Approach Based on IFSs

In this section, details of the proposed approach and preliminaries of the intuitionistic fuzzy sets are introduced.

### *3.1 Intuitionistic Fuzzy Sets*

Atanassov introduces intuitionistic fuzzy sets (IFSs) to express experts' evaluations in a larger domain area than ordinary fuzzy sets by using not only membership functions but also non-membership functions of the elements in a fuzzy set [24]. It can be in many forms such as single–valued, interval–valued, triangular, or trapezoidal. During the study, single–valued forms are used to design of ACCs. IFSs provide a theoretical basis for managing the knowledge of hesitation by people when evaluating the questions [25]. Mathematical representation of a single–valued IFS is given in Eq. (7) as below [24]:

$$\tilde{A} = \{x, \mu_{\tilde{A}}, v_{\tilde{A}}; x \in X\} \tag{7}$$

where $0 \leq \mu_{\tilde{A}} + v_{\tilde{A}} \leq 1$ for every $x \in X$.

By assigning two functions, IFSs enable to represent the hesitancy of the decision makers. The mathematical formula of the hesitancy degree is given in Eq. (8) as below [24]:

$$\pi = 1 - \left(\mu_{\tilde{A}} + v_{\tilde{A}}\right). \tag{8}$$

Let $\tilde{A} = \left(\mu_{\tilde{A}}, v_{\tilde{A}}\right)$ and $\tilde{B} = \left(\mu_{\tilde{B}}, v_{\tilde{B}}\right)$ be two single–valued intuitionistic fuzzy numbers and $\lambda$ is a constant crisp number. Then the mathematical operations of these two IFNs are given as in Eqs. (9–12) as below [24]:

$$\tilde{A} \oplus \tilde{B} = \left(\mu_{\tilde{A}} + \mu_{\tilde{B}} - \mu_{\tilde{A}}\mu_{\tilde{B}}, v_{\tilde{A}}v_{\tilde{B}}\right), \tag{9}$$

$$\tilde{A} \otimes \tilde{B} = \left(\mu_{\tilde{A}}\mu_{\tilde{B}}, v_{\tilde{A}} + v_{\tilde{B}} - v_{\tilde{A}}v_{\tilde{B}}\right), \tag{10}$$

$$\lambda\tilde{A} = (1 - \left(1 - \mu_{\tilde{A}}\right)^{\lambda}, \nu_{\tilde{A}}^{\lambda}), \tag{11}$$

$$\tilde{A}^{\lambda} = (\mu_{\tilde{A}}^{\lambda}, 1 - \left(1 - v_{\tilde{A}}\right)^{\lambda}). \tag{12}$$

### *3.2 Attribute Control Charts Based on IFSs*

The proposed methodology consists of the $p$ and $np$ control charts with their intuitionistic fuzzy extensions. The notations of the proposed methodology are given as follows:

Let $\tilde{p}_i$ be the $i^{th}$ sample, which is checked for the number of defectives. Then, the proposed intuitionistic p control chart's sample is represented as in Eq. (13).

$$\tilde{p}_i = (p_a, p_b, p_c, p_d; \min(\mu_A(x)), \max(v_A(x))) \tag{13}$$

where $p_{a_i}$ is the lower bound of $\tilde{p}_i$, $p_{b_i}$ is the left peak point of $\tilde{p}_i$, $p_{c_i}$ is the right peak point of $\tilde{p}_i$, $p_{d_i}$ is the upper bound of $\tilde{p}_i$, $\mu_A(x)$ is the membership function of $\tilde{p}_i$, and $v_A(x)$ is the non–membership function of $\tilde{p}_i$ .

Based on the Eq. (13), the proposed intuitionistic $p$ control chart's average is calculated as in Eq. (14).

$$\tilde{\bar{p}} = \left( \frac{\sum_1^m p_{a_i}}{m}, \frac{\sum_1^m p_{b_i}}{m}, \frac{\sum_1^m p_{c_i}}{m}, \frac{\sum_1^m p_{d_i}}{m}; \min(\mu_A(x)), \max(v_A(x)) \right) \tag{14}$$

where $m$ is the number of samples.

### 3.3 Intuitionistic Fuzzy $p$ Control Chart

By using the average value given in Eq. (14), intuitionistic $p$ control chart's limits are calculated by using the Eqs. (15–17).

$$C\tilde{L}_p = \bar{p} = (\overline{p_a}, \bar{p}_b, \bar{p}_c, \bar{p}_d; \min(\mu_A(x)), \max(v_A(x))), \tag{15}$$

$$U\tilde{C}L_p = \left( \overline{p_a} + 3\sqrt{\frac{\overline{p_a}(\overline{q_a})}{n}}, \overline{p_b} + 3\sqrt{\frac{\overline{p_b}(\overline{q_b})}{n}}, \overline{p_c} + 3\sqrt{\frac{\overline{p_c}(\overline{q_c})}{n}}, \overline{p_d} + 3\sqrt{\frac{\overline{p_d}(\overline{q_d})}{n}}; \min(\mu_A(x)), \max(v_A(x)) \right), \tag{16}$$

$$\begin{aligned} L\tilde{C}L_p = (&\max(0, \overline{p_a} - 3\sqrt{\frac{p_d(\overline{q_d})}{n}}), \max(0, \overline{p_b} - 3\sqrt{\frac{p_c(\overline{q_c})}{n}}), \\ &\max(0, \overline{p_c} - 3\sqrt{\frac{\overline{p_b}(\overline{q_b})}{n}}), \max(0, \overline{p_d} - 3\sqrt{\frac{p_a(\overline{q_a})}{n}}); \min(\mu_A(x)), \max(v_A(x))). \end{aligned} \tag{17}$$

### 3.4 Intuitionistic Fuzzy $np$ Control Chart

Similar to $p$ control chart, average of the sample is identified by using the Eq. (18).

$$n\tilde{\bar{p}} = (\overline{np}_a, \overline{np_b}, \overline{np}_c, \overline{np_d}; \min(\mu_A(x)), \max(v_A(x))) \tag{18}$$

where $n$ is the sample size.

By using the average value given in Eq. (18), intuitionistic $np$ control chart's limits are calculated by using the Eqs. (19–21).

$$C\tilde{L}_{np} = n\tilde{p} = (n\bar{p}_a, n\bar{p}_b, n\bar{p}_c, n\bar{p}_d; \min(\mu_A(x)), \max(v_A(x))) \tag{19}$$

$$U\tilde{C}L_{np} = \left( n\overline{p_a} + 3\sqrt{n\overline{p_a}(\overline{q_a})}, n\overline{p_b} + 3\sqrt{n\overline{p_b}(\overline{q_b})}, \overline{p_c} + 3\sqrt{n\overline{p_c}(\overline{q_c})}, \overline{p_d} + 3\sqrt{n\overline{p_d}(\overline{q_d})}; \min(\mu_A(x)), \max(v_A(x)) \right) \tag{20}$$

$$\begin{aligned} L\tilde{C}L_{np} = (&\max(0, n\bar{p}_a - 3\sqrt{np_d(q_d)}), \max(0, \overline{p_b} - 3\sqrt{n\bar{p}_c(\overline{q_c})}), \\ &\max(0, \overline{p_c} - 3\sqrt{np_b(\overline{q_b})}), \max(0, \overline{p_d} - 3\sqrt{n\bar{p}_a(\overline{q_a})}); \min(\mu_A(x)), \max(v_A(x))) \end{aligned} \tag{21}$$

## 4 A Descriptive Example

Let consider a contract manufacturer for an automobile company who decided to monitor its process with the statistical charts. At the end of the production process, a batch, which consists 60 products is sent for the quality controls to the operator. Operator, which controls the batch, realize this process by assigning the below linguistic terms with the determined rules. 30 samples are taken for the analysis. The operator assigned the linguistic terms for each sample. During the inspection, the scale given in Table 1 is used.

| Linguistic terms | Corresponded intuitionistic fuzzy number |
|---|---|
| Totally Defected Rate - HKL | [(1, 1, 1, 1), (1, 0)] |
| Very High Defected Rate- CYK | [(0.85, 0.9, 0.94, 1), (0.9, 0.1)] |
| High Defected Rate - YK | [(0.7, 0.77, 0.82, 0.9), (0.75, 0.25)] |
| Above Average Defected Rate - OU | [(0.55, 0.62, 0.67, 0.75), (0.6, 0.4)] |
| Average Defected Rate - OK | [(0.4, 0.47, 0.52, 0.6), (0.7, 0.5)] |
| Below Average Defected Rate - OA | [(0.25, 0.32, 0.37, 0.45), (0.4, 0.6)] |
| Low Defected Rate - AK | [(0.1, 0.17, 0.22, 0.30), (0.25, 0.75)] |
| Very Low Defected Rate - CAK | [(0, 0.03, 0.06, 0.10), (0.1, 0.9)] |
| Totally Flawless - HKS | [(0, 0, 0, 0), (0, 1)] |

**Table 1** Linguistic terms and their corresponded intuitionistic fuzzy numbers

After the operator's inspection, observations given in Table 2 are obtained.

| Sample No | Defected Rate on Sample | Sample No | Defected Rate on Sample |
|---|---|---|---|
| **1** | [(0.375; 0.425; 0.462; 0.52), (0, 1)] | **16** | [(0.45; 0.494; 0.527; 0.579), (0, 1)] |
| **2** | [(0.493; 0.54; 0.576; 0.631), (0, 1)] | **17** | [(0.381; 0.434; 0.474; 0.535), (0, 1)] |
| **3** | [(0.451; 0.488; 0.516; 0.559), (0, 1)] | **18** | [(0.465; 0.519; 0.557; 0.619), (0, 1)] |
| **4** | [(0.441; 0.49; 0.527; 0.584), (0, 1)] | **19** | [(0.465; 0.519; 0.557; 0.619), (0, 1)] |
| **5** | [(0.375; 0.428; 0.466; 0.526), (0, 1)] | **20** | [(0.364; 0.412; 0.449; 0.505), (0, 1)] |
| **6** | [(0.464; 0.516;0.554; 0.614), (0, 1)] | **21** | [(0.389; 0.434; 0.468; 0.52), (0, 1)] |
| **7** | [(0.408; 0.452; 0.485; 0.536), (0, 1)] | **22** | [(0.47; 0.5; 0.524; 0.56), (0, 1)] |
| **8** | [(0.384; 0.429; 0.463; 0.515), (0, 1)] | **23** | [(0.509; 0.562; 0.602; 0.664), (0, 1)] |
| **9** | [(0.486; 0.535; 0.57; 0.625), (0, 1)] | **24** | [(0.443; 0.495; 0.533; 0.594), (0, 1)] |
| **10** | [(0.436; 0.493; 0.534; 0.599), (0, 1)] | **25** | [(0.376; 0.427; 0.465; 0.524), (0, 1)] |
| **11** | [(0.394; 0.443; 0.479; 0.536), (0, 1)] | **26** | [(0.451; 0.498; 0.533; 0.588), (0, 1)] |
| **12** | [(0.468; 0.513; 0.546; 0.599), (0, 1)] | **27** | [(0.431; 0.477; 0.511; 0.564), (0, 1)] |
| **13** | [(0.476; 0.529; 0.568; 0.629), (0, 1)] | **28** | [(0.421; 0.467; 0.502; 0.555), (0, 1)] |
| **14** | [(0.371; 0.415; 0.449; 0.5), (0, 1)] | **29** | [(0.373; 0.422; 0.46; 0.518), (0, 1)] |
| **15** | [(0.524; 0.569; 0.602; 0.654), (0, 1)] | **30** | [(0.443; 0.494; 0.532; 0.591), (0, 1)] |

**Table 2** Analyse report based on the operator inspection

By using the Eq. (15) for each inspection presented in Table 2, the center line is calculated as follows:
$CL_{\tilde{p}} = [(0.433, 0.481, 0.516, 0.572), (0, 1)]$.
Based on the center line, by using Eq. (16– 17), upper point and lower limits are calculated as follows:
$UCL = [(0.73, 0.78, 0.816, 0.869), (0, 1)]$,
$LCL = [(0.135, 0.181, 0.216, 0.275), (0, 1)]$.
Based on the results and the inspections, the intuitionistic fuzzy $p$ control chart is constructed as shown in Figure 2.

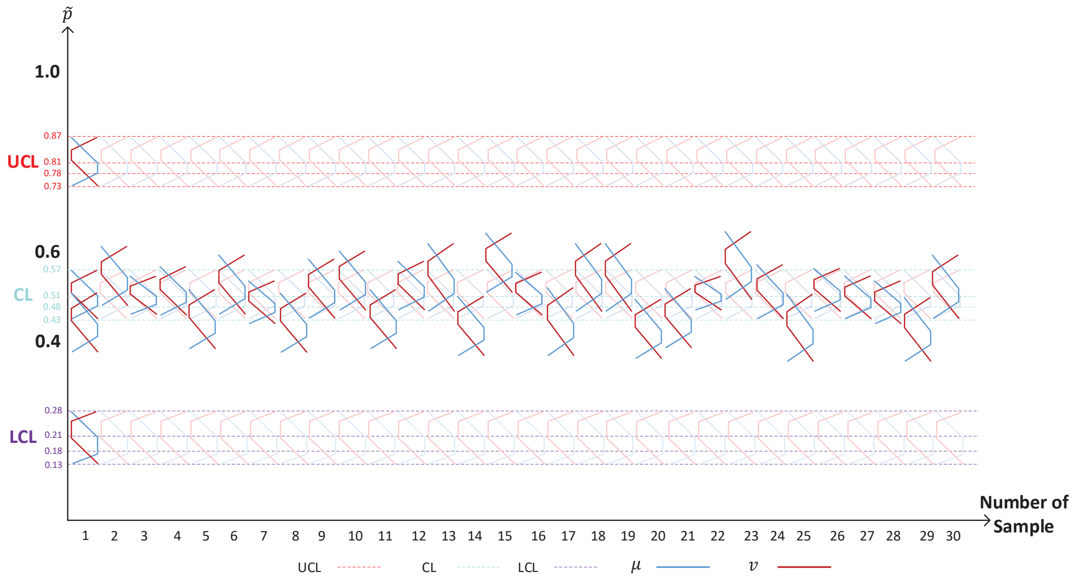


**Fig. 2**: Constructed intuitionistic fuzzy $p$ control chart

For the interpretation, the obtained results shown in Figure 2 are defuzzified and the chart given in Figure 3 is constructed.

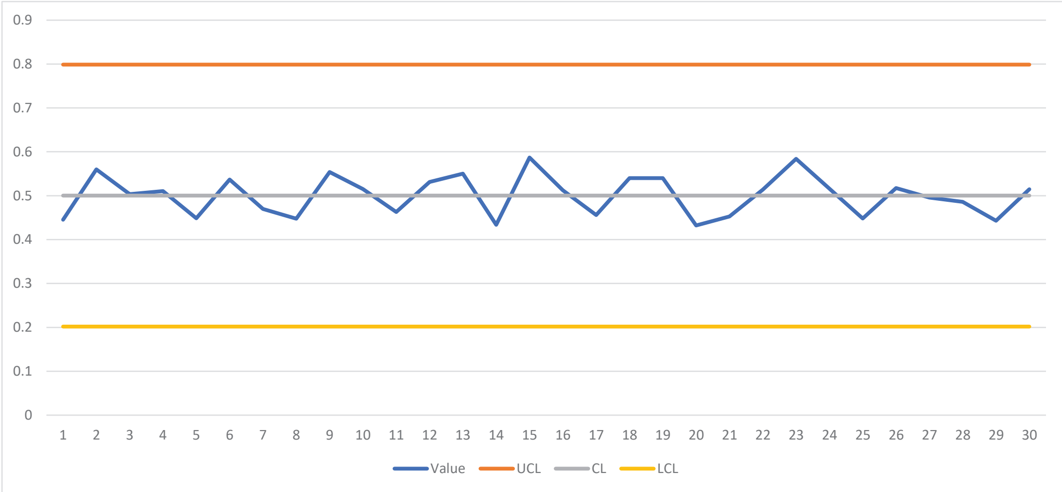


**Fig. 3**: Constructed $p$ control chart based on defuzzified values

According to Figure 3, the process is in statistical control. Figure 3 shows there is no out of control point and the process' variation is under control. Moreover, when Figure 2 and Figure 3 are compared, the results of the two methods are similar. But, the representation of the defectives and the information about the process are remarkably increased in the IF $p$ control chart. This enables to researchers or the experts to make more meaningful interpretations about the system.

## 5 Conclusions

CCs are one of the most critical statistical quality control techniques used to monitor the variability of process and to determine whether the process is under control or not. In order to design the CCs effectively, quality characteristics should be clearly defined. Due to the problems of the accuracy of the measuring operator, the measuring instrument, or the measuring method, the observation values may not be defined accurately. To overcome these problems, the fuzzy set theory has been introduced in CCs to use its ability to handle impreciseness. Additionally, the extensions of fuzzy sets are applied to extend CCs, which improves the ability of representing types of uncertainty such as hesitancy and indeterminacy. Through that, in this study, attribute control charts for defectives based on IFSs are introduced. For this aim, extensions of $p$ and $np$ control charts with IFs are proposed and the design of these control charts based on IFs has been analyzed. The control limits and center lines have been re–formulated by using the proposed extension.

As a result, it is determined that IFSs provide effective results in designing and evaluating CCs in order to overcome the uncertainty. Based on the comparison with the classical $p$ control chart, the proposed method presents meaningful and detailed outcomes. For the future studies, the study should be implemented with a real case application data to compare the obtained results. Moreover, other types of fuzzy sets, which able to use both for representing uncertainty and hesitancy can be conducted for the comparison.

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