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A Solution Method for Longitudinal Vibrations of Functionally Graded Nanorods

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Abstract

In the present study, a nonlocal finite element formulation of free longitudinal vibration is derived for functionally graded nano-sized rods. Size dependency is considered via Eringen's nonlocal elasticity theory. Material properties, Young's modulus and mass density, of the nano-sized rod change in the thickness direction according to the power-law. For the examined FG nanorod finite element, the axial displacement is specified with a linear function. The stiffness and mass matrices of functionally graded nano-sized rod are found by means of interpolation functions. Functionally graded nanorod is considered with clamped-free boundary condition and its longitudinal vibration analysis is performed.

Keywords: Nonlocal elasticity theory, Functionally graded materials, Nanorod, Finite element method, Vibration

1. Introduction

One of the popular structures of recent times is functionally graded (FG) composite materials. The difference of these materials which are usually a combination of metal and ceramic from traditional laminated composites is that the smooth changing of material properties. In functionally graded materials, the material properties like Young's modulus, density, shear modulus etc. change according to a certain rule continuously along at least one direction. Thanks to this smooth property changing, functionally graded materials have been precious for many applications such as biomedical, chemistry, electronics, optics, aircraft, space vehicles and biology etc. [1,2]. In addition, functionally graded structures have attracted considerable attention in models of nano/micro mechanics. The studies on functionally graded nano/micro structures such as FG nanoplate [3-7], FG nanobeam [8-14], FG nanorod [14-18] have been presented by researchers in recent years.

In addition to the analytical solution [19,20], many other methods like discrete singular convolution method [21,22], polynomial differential quadrature method [23], finite difference method, finite element method [24] etc. have been used by researchers to solve a problem. In this study, a finite element formulation for free longitudinal vibration behavior of functionally graded nanorod is presented. Small-scale effect of the functionally graded nanorod is discussed based on the nonlocal elasticity theory. The nonlocal elasticity theory has an additional small-scale parameter (nonlocal parameter) and thanks to this nonlocal parameter the small-scale effects occurring in nano/micro-sized structures can be evaluated. The nonlocal elasticity theory has become a frequently performed theory in nanomechanics and micromechanics, as it allows the consideration of small-scale effects. In addition, articles using finite element method to examine the behavior of size-dependent microstructures/nanostructures such as vibration [25-30], buckling [29-32] and bending [29-30, 33-35] are also found in the literature.

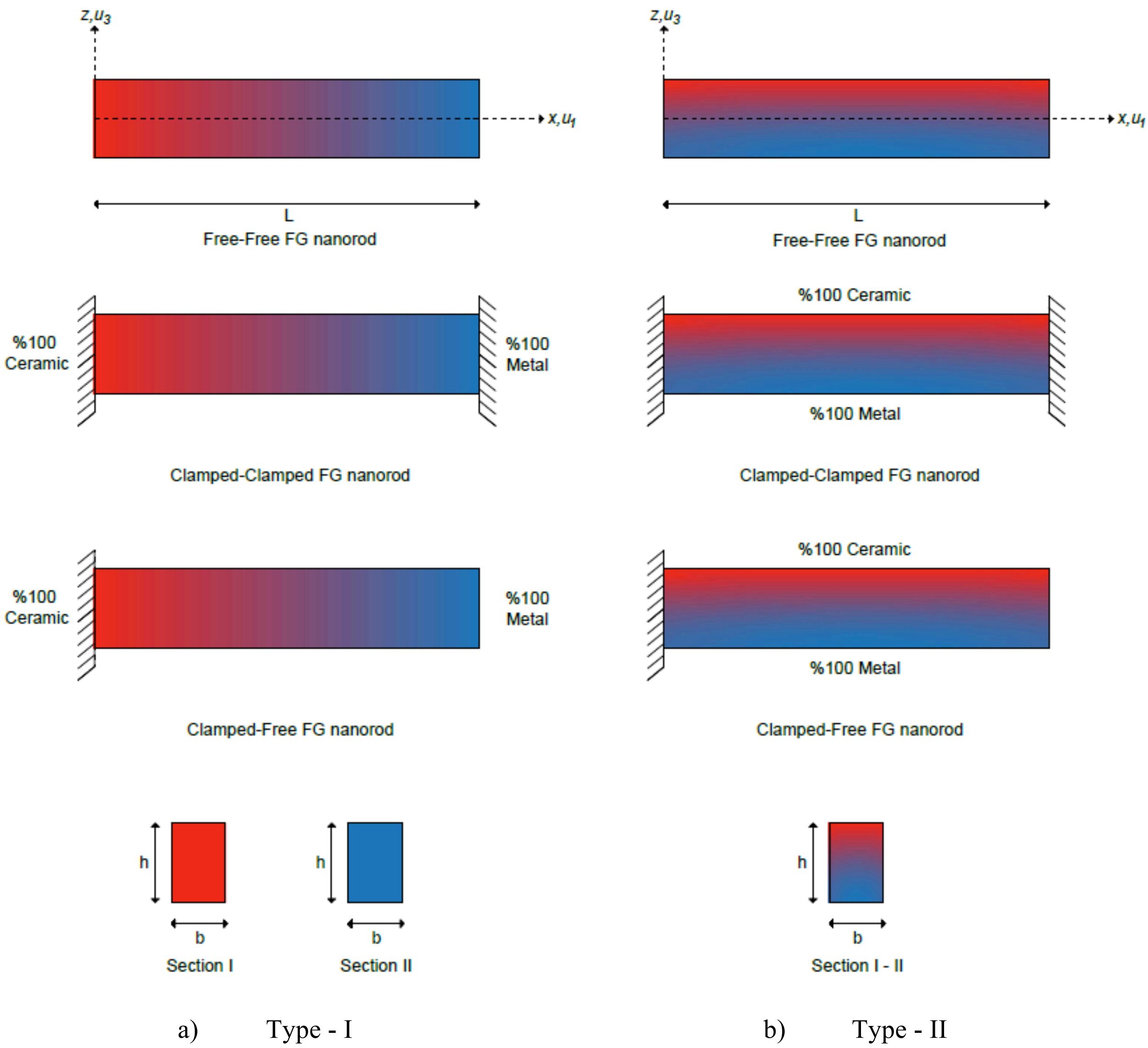

Fig. 1. Functionally graded nanorods with various boundary conditions

2. Functionally Graded Rod

FG nanorods with various boundary conditions like free-free, clamped-clamped and clamped-free are illustrated in Figure 1. *L*, *b* and *h* represent the length, width and thickness of the FG rod, respectively. Type I (Fig. 1a) and Type II (Fig. 1b) represent the FG nanorods whose material properties vary continuously in the axial direction and thickness direction, respectively. The material properties such as Young's modulus, density etc. change of the rod according to a power-law. If the changing of material properties of the rod is assumed in the thickness direction, the effective material properties of rod can be defined as [11, 13]

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_m \tag{1}$$

Where, *P* represents the effective material property, while *k* represents the non-negative power-law exponent. The subscripts *c* and *m* indicate the ceramic and metal materials, respectively. u_1, u_2 and u_3 are the displacements of the FG rod in the *x*, *y*, *z* directions, respectively, and may be written as follow

$$u_1(x,z,t) = u(x,t), \quad u_2(x,z,t) = 0, \quad u_3(x,z,t) = 0 \tag{2}$$

u, and *t* denote the axial displacement of any point on the neutral axis and time, respectively. Stress (σ) and normal force (*N*) expressions for the FG rod are written as follows

$$\sigma_{xx} = E(z)\varepsilon_{xx} \tag{3}$$

$$N = \int_A \sigma_{xx}(z)\, dA \tag{4}$$

Here, ε and *A* are strain and cross-section area, respectively. The equations of motions of FG nano-sized rod can be obtained by means of the Hamilton's principle [36]

$$\int_{t_1}^{t_2} \left(\delta K - \delta U + \delta W\right) dt = 0 \tag{5}$$

Where *U, K* and *W* are the strain energy, kinetic energy and work done by external forces, respectively. The external loads can be encountered as elastic foundation, axial compressive force, thermal loading etc. However, there are no external forces in this vibration problem of FG nanorod and so *W* is set to zero. The first variations of the strain energy and kinetic energy are given as follows

$$\delta\int_{t_1}^{t_2} U dt = \int_{t_1}^{t_2}\int_0^L N\delta\left(\frac{\partial u}{\partial x}\right) dxdt \tag{6}$$

$$\delta\int_{t_1}^{t_2} K dt = \int_{t_1}^{t_2}\int_0^L I_0 \frac{\partial u}{\partial t}\delta\left(\frac{\partial u}{\partial t}\right) dxdt \tag{7}$$

Here, I_0 is expressed as

$$I_0 = \int_A \rho(z) dA, \tag{8}$$

By substituting equations (6) - (7) into equation (5) and after some mathematical arrangements, we obtain the equation of motion of the rod as follows

$$\delta u : \frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} \tag{9}$$

3. Size-Dependent Finite Element Formulation

The nonlocal constitutive formulation is [37]

$$\left[1-\left(e_0 a\right)^2 \nabla^2\right]\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{10}$$

Where σ_{ij} is the stress tensor, C_{ijkl} is the fourth-order Young's modulus tensor, ε_{kl} is the strain tensor, e_0a is the nonlocal parameter. The Equation (10) can be rewritten as

$$\sigma_{xx} - \left(e_0 a\right)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(z)\varepsilon_{xx} \tag{11}$$

Integrating Eq. (11) over the cross-section area, we obtain the axial force-strain relation as Eq. (12)

$$N - (e_0 a)^2 \frac{\partial^2 N}{\partial x^2} = A_1 \frac{\partial u}{\partial x} \tag{12}$$

Here, A_1 is expressed as

$$A_1 = \int_A E(z) dA, \tag{13}$$

Differentiating Equation (9) with respect to x, then substituting into Equation (12) we obtain Equation (14).

$$N = A_1 \frac{\partial u}{\partial x} + (e_0 a)^2 I_0 \frac{\partial^3 u}{\partial x \partial t^2} \tag{14}$$

By substituting Equation (14) into Equation (9), the equation of the motion of FG nanorod is obtained as

$$A_1 \frac{\partial^2 u}{\partial x^2} + (e_0 a)^2 I_0 \frac{\partial^4 u}{\partial x^2 \partial t^2} - I_0 \frac{\partial^2 u}{\partial t^2} = 0 \tag{15}$$

In this study, a rod finite element is considered has two nodes. ϕ is the interpolation (or shape) functions matrix of a rod finite element and expressed as below

$$[\phi] = \begin{bmatrix} 1 - \frac{x}{L} & \frac{x}{L} \end{bmatrix} \tag{16}$$

The stiffness matrix, classical mass and nonlocal mass matrices are obtained using Eqs. (15) - (16) as follows

$$K = \int_0^L A_1 \left([\phi]' \right)^T [\phi]' \, dx \tag{17}$$

$$M_{cl} = \int_0^L I_0 \left([\phi] \right)^T [\phi] dx \tag{18}$$

$$M_{nl} = (e_0 a)^2 \int_0^L I_0 \left([\phi]' \right)^T [\phi]' \, dx \tag{19}$$

In the above Equations, superscript T represents the transpose operator. The subscripts cl and nl are used to indicate the classical and nonlocal theories, respectively. The frequencies of FG nano-sized rod are found as follows

$$\left| K - \omega_n^2 \left(M_{nl} + M_{cl} \right) \right| = 0 \tag{20}$$

Here ω_n and the subscript n indicate the circular frequency and mode number.

4. Numerical Results

In this section, comparison studies and numerical examples are performed. Comparison studies are presented by Xu et al. [38] and Numanoğlu et al. [39]. Table 1 is presented to compare the validity of the method and to show the compatibility with each other. Comparisons of non-dimensional frequencies for the first four modes of clamped-free homogeneous nanorods are shown in Table 1. Also, this Table demonstrates the effect of the number of finite element (N) on convergence. As can be seen, the number of finite elements is an important issue for the convergence of frequency values. The appropriate number of elements should be chosen to ensure desired convergence. As can be seen, low number of finite elements provides the desired convergence for low modes. However, it may be necessary to increase the number of finite elements as the mode number increase. Dimensionless parameters used in the comparison studies are defined as follows

$$\bar{\omega}_n = \omega_n L\sqrt{\rho / E}, \quad \bar{\mu} = e_0 a / L \tag{21}$$

Table 1. Comparison of dimensionless frequencies of homogeneous nanorod

$\bar{\mu}$	$\bar{\omega}_n$	Xu et al. [38]	Numanoğlu et al. [39]	Present study (N=200)	Present study (N=100)	Present study (N=50)	Present study (N=20)
0.0	*n=1*	1.57080	1.57080	1.5708	1.5708	1.5709	1.5712
	n=2	4.71239	4.71239	4.7125	4.7128	4.7141	4.7233
	n=3	7.85398	7.85398	7.8545	7.8560	7.8621	7.9045
	n=4	10.99557	10.99557	10.9970	11.0011	11.0177	11.1345
0.1	*n=1*	1.55177	1.55177	1.5518	1.5518	1.5518	1.5522
	n=2	4.26279	4.26279	4.2629	4.2631	4.2641	4.2709
	n=3	6.17668	6.17668	6.1769	6.1777	6.1806	6.2012
	n=4	7.39805	7.39805	7.3985	7.3997	7.4048	7.4399
0.2	*n=1*	1.49858	1.49858	1.4986	1.4986	1.4986	1.4989
	n=2	3.42933	3.42933	3.4294	3.4295	3.4300	3.4335
	n=3	4.21782	4.21782	4.2179	4.2181	4.2191	4.2256
	n=4	4.55152	4.55152	4.5516	4.5519	4.5531	4.5612

In this section, effects of power-law exponent and the nonlocal parameter on the free vibration response of functionally graded nanorod are investigated. In the numerical calculations, the number of finite elements for FG nanorod is chosen as 200. Functionally graded nanorod is considered composed of aluminum and alumina and with clamped-free boundary condition. The top and bottom surfaces of the nanorod are composed of pure alumina (ceramic) and aluminum (metal), respectively. Mechanical properties of functionally graded nanorod constituents are given as [40]: E_m=70 GPa, ρ_m=2700 kg/m^3 for aluminum and E_c=393 Gpa, ρ_c=3960 kg/m^3 for alumina. The following dimensionless frequency parameter is used

$$\lambda_n = \omega_n L\sqrt{\rho_c / E_c} \tag{22}$$

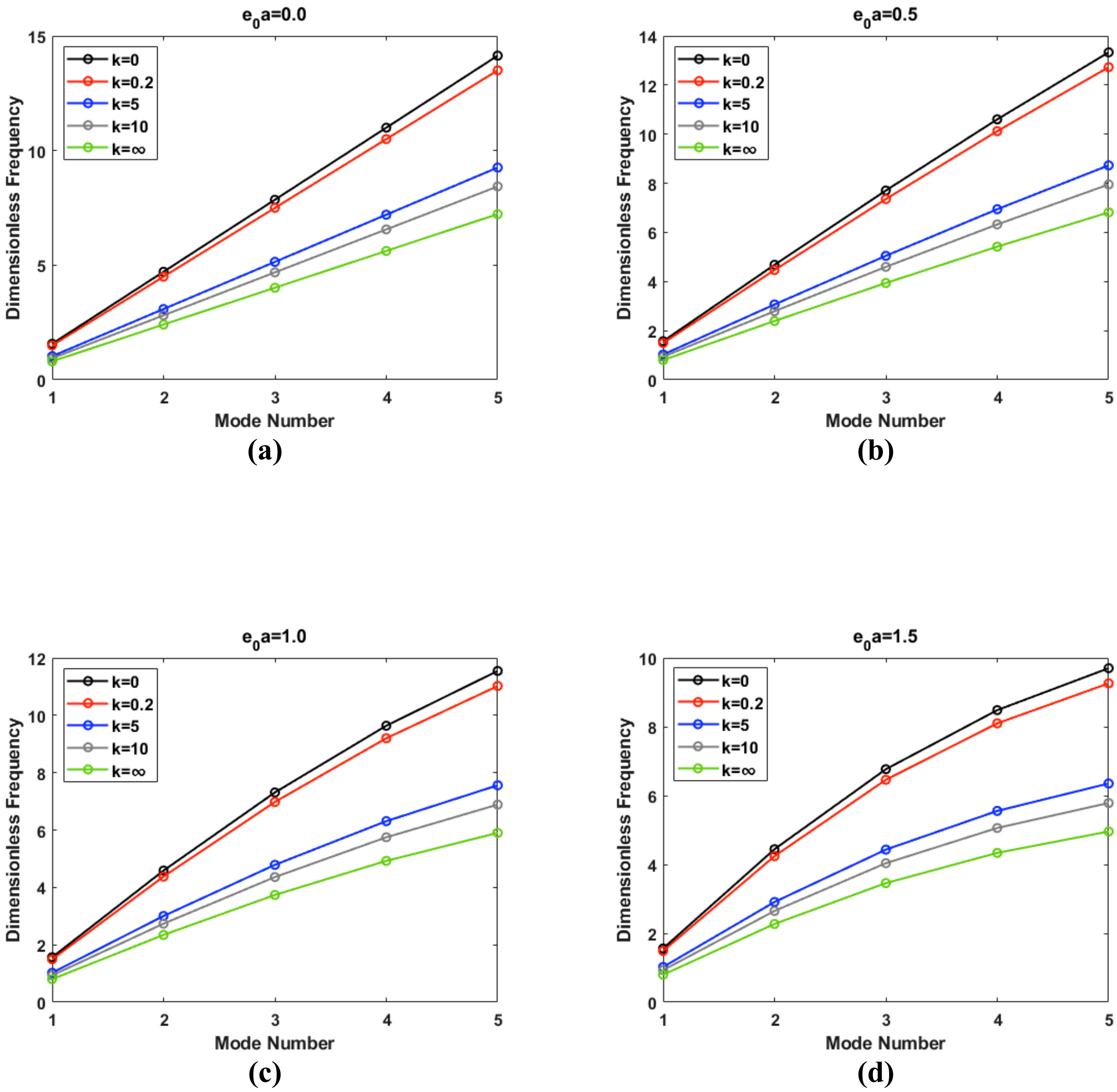

Fig. 2. Variation of dimensionless frequencies of FG nanorod

Figure 2 displays the variation of dimensionless frequencies of functionally graded nanorod with respect to mode numbers for various power-law exponent (k) and nonlocal parameter (e_0a) values. The Figure 2 is plotted from the analyses of FG nanorod with various nonlocal parameters ranging from 0 to 1.5 and various power-law exponents ranging from 0 to ∞. It is concluded from the Figure that the increasing values of power-law exponent and nonlocal parameter lead to a decrease in the dimensionless frequencies of FG nanorods. It should be noted that when the power-law exponent set to zero (k=0), the results give the frequencies of alumina (pure ceramic). If the power-law exponent sets to infinity (k=∞), the frequencies of aluminum (pure metal) are obtained. Also, if the nonlocal parameter e_0a set to zero, the frequencies of the classical theory are obtained.

5. Conclusions

In the present study, the nonlocal finite element formulation of functionally graded nanorod is proposed in conjunction with Eringen's nonlocal elasticity theory. The stiffness and mass matrices essential to the vibration response of functionally graded nanorod are found using interpolation functions. Finally, an eigenvalue problem is defined with the obtained matrices and ω_n, and the eigenvalues ω_n are found by setting the determinant of the coefficient matrix to zero. A numerical example for clamped-free boundary condition is given to investigate the influences of some parameters on frequencies of FG nanorod. The main results obtained in this study can be summarized as follows: When the nonlocal effect is ignored, that is when the e_0a value is taken as zero, the frequencies of the FG nanorod have the highest values. It is understood from that the nonlocal effect causes a reduction in the frequency of the FG nanorod. In addition, it is seen that with the increase of the power-law exponent value, that is with the transition of material properties from ceramic to metal, there is a decrease in frequencies.

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