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Research Article

A novel differential evolution algorithm approach for estimating the parameters of Gamma distribution: An application to the failure stresses of single carbon fibres

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Abstract

Three-parameter (3-p) Gamma distribution is widely used to model for skewed data in the reliability field. Thus, the problem of parameter estimation for the Gamma distribution has remained significant and interesting in all times. The maximum likelihood (ML) and the least square (LS) are the most popular methods in the parameter estimation. In this study, a novel Differential Evolution (DE) algorithm is proposed for the ML and LS estimation of the parameters of the 3-p Gamma distribution. This approach overcomes the problem of how to determine the search space of the DE by utilizing a new search space based on the confidence interval. The modified maximum likelihood and the profile likelihood methods are considered to constitute the confidence interval. In order to examine the performance of the proposed approach, an extensive Monte Carlo simulation study and a real data application are performed. The results show that this proposed approach is effective for estimating the parameters of the 3-p Gamma distribution with respect to mean square error and deficiency criteria.

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1. Introduction

Three parameters (3-p) Gamma distribution is one of the commonly used distributions for modeling skewed data in the field of reliability, hydrology, and finance [3, 6, 12, 18]. The probability density function and cumulative distribution function of the 3-p Gamma distribution are defined by

$$f(x;\sigma,\beta,\mu) = \frac{1}{\Gamma(\sigma)\beta^{\sigma}}(x-\mu)^{\sigma-1}\exp\left(-\frac{x-\mu}{\beta}\right), \quad x \geq \mu, \sigma > 0, \beta > 0. \tag{1.1}$$

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and

$$F(x;\sigma,\beta,\mu) = \frac{1}{\Gamma(\sigma)}\gamma\left(\sigma, \frac{x-\mu}{\beta}\right), \quad x \geq \mu, \sigma > 0, \beta > 0 \tag{1.2}$$

respectively. In Equations (1.1) and (1.2), σ is the shape parameter, β is the scale parameter, μ is the location parameter, and γ indicates the incomplete Gamma function [3, 11].

3-p Gamma distribution contains three special cases according to the values of the shape parameter σ. 3-p Gamma distribution is "J" shaped for $\sigma \leq 1$ and bell-shaped for $\sigma > 1$. The distribution becomes an exponential distribution if $\sigma = 1$. Therefore, it is one of the common distributions used in the statistical modeling [4, 18].

The successful applications of the 3-p Gamma distribution depend on having acceptable statistical estimates of its parameters. However, parameter estimation for the 3-p Gamma distribution is a quite difficult problem. There are various studies for the parameter estimation of the 3-p Gamma distribution in the literature. Cohen and Whitten [6] proposed a modification of the maximum likelihood (ML) and moment estimators for parameters of the 3-p Gamma distribution and they revealed that the new estimators are better than both the ML and the moment estimators in terms of bias, variance, and ease of calculation. Cohen and Whitten [7] presented modified moment estimators by replacing the third moment with a function of the first-order statistic and they showed that these estimators are computationally simpler than ML even when samples are small. Hirose [10] suggested a new ML parameter estimation scheme by utilizing the reparametrized distribution function and predictor-corrected method and the effectiveness of the proposed method was showed by a Monte Carlo simulation study. Balakrishnan and Wang [3] presented some simple efficient estimators by using the first few order statistic and they indicated that these estimators are more efficient than Cohen and Whitten [7]'s method. Tzavelas [27] introduced a program in Mathematica to obtain the solutions of the likelihood equations in the ML parameter estimation for the 3-p Gamma distributions. Lakshmi and Vaidyanathan [12] proposed a novel approach for the ML, the least square (LS) and the maximum product spacing (MPS) methods to estimate the parameters of the 3-p Gamma distribution by using an extensive search over three-dimensional parameter space.

The ML and the LS are the two most commonly used methods for parameter estimation. Due to the fact that these methods don't provide explicit estimators for the parameters of 3-p Gamma distributions, iterative methods are needed [1].

In this study, differential evolution (DE) algorithm, which is one of the most powerful evolutionary algorithms (EAs) for the continuous parameter spaces in recent times, is used to obtain the ML and LS estimators of the parameters of 3-p Gamma distribution. To the best of our knowledge, this is the first study to use the DE for estimating the parameters of the 3-p Gamma distribution. In addition, there is no study using another metaheuristic method for this in the literature.

The performance of the DE is highly affected by initial search space. Arbitrary determination of the initial search space can cause some following problems. (i) if the search space is determined too wide, the algorithm may provide unacceptable results, or (ii) if the search space is determined too narrow, this search space may not contain the actual values which are sought. To overcome these problems, we propose a new approach to construct search space for the parameters of the 3-p Gamma distribution by utilizing confidence intervals based on the modified maximum likelihood (MML) estimators and the profile likelihood (PL) method. The MML method was proposed by Tiku and Akkaya [26] and MML estimators can be written in explicit form as a function of observations for the parameters of β and μ when σ is known. In this case, the PL method can be used for estimating the shape parameter σ. Thus, confidence intervals can be obtained through the MML and the PL methods.

The rest of this paper is organized as follows: In Section 2, the ML and the LS estimations process for the parameters of the 3-p Gamma distribution are introduced and the proposed DE approach is presented. In Section 3, an extensive Monte Carlo simulation study is conducted, and the results of the simulation are given. In Section 4, a real-life application is examined. Conclusions are presented in the final section.

2. Parameter estimation for the parameters of Gamma distribution by DE algorithm

In this section, the ML and the LS estimators of parameters of the 3-p Gamma distribution by the DE algorithm are discussed. After the DE algorithm is introduced briefly, the notions of the ML and LS estimators of the 3-p Gamma distribution parameters are given and the proposed approach is presented by explaining the Modified Maximum Likelihood (MML) and Profile Likelihood (PL) methods.

2.1. The DE algorithm

The Differential Evolution (DE), which is proposed by Storn [23] and Storn and Price [24], is one of the most powerful evolutionary algorithms which are mainly inspired from the biological process of evolution [2]. Due to its easy implementation, strong global search ability and fast convergence speed, DE has become one of the most popular optimization techniques and it has been widely used for solving the nonlinear and complex problems in recent years [9, 13, 15-17, 22, 32].

Although the DE algorithm has only three control parameters which are scaling factor, crossover factor and population size, it performs remarkably in terms of accuracy, computation speed and robustness while optimizing various objective functions [9].

The DE algorithm consists of three basic steps: mutation, crossover and selection. After initialization, the DE enters an evolutionary process including mutation, crossover, and selection operations as described as following [19, 21]:

Initialization

For a D-dimensional optimization problem, the DE algorithm searches the global optimum point by using the vector $\vec{x}_i^g = [x_{1,i}^g, x_{2,i}^g, ..., x_{D,i}^g]$ which is represented the vector i of the population at current generation g. Each vector, also known as genome/chromosome, forms a candidate solution to the multidimensional optimization problem.

The initial population is generated by using the prescribed minimum and maximum parameter bounds ($x_{j,\min}$) and ($x_{j,\max}$), which are known as search space. The initial value of the jth component of the vector i at $g = 0$ is generated by following:

$$x_{j,i}^0 = x_{j,\min} + rand_{ij}[0,1]\{x_{j,\max} - x_{j,\min}\} \tag{2.1}$$

In Equation (2.1), $rand_{ij}[0,1]$ represents a uniformly distributed random variable in the range $[0,1]$ [25, 31].

Mutation

The DE applies the mutation operator to create a donor vector ($\vec{v}_i^g$), after initialization. There are different mutation strategies in the literature. The general notation used for naming the various mutation strategies is DE/x/y/z, where DE stands for differential evolution, x specifies the vector to be mutated, and y is the number of difference vectors, z denotes the type of crossover used (exp: exponential and bin: binomial) [25, 31]. In this study, the standard variant of the DE, i.e. DE/rand/1/bin, has been used.

For each vector of the population, three distinct vectors $r_1, r_2, r_3; r_1 \neq r_2 \neq r_3 \neq i \in (0, NP)$ is randomly chosen and a new donor vector ($\vec{v}_i^g$) by mutation scheme is generated as follows:

$$\vec{v}_i^g = \vec{x}_{r1}^g + F\left(\vec{x}_{r2}^g - \vec{x}_{r3}^g\right) \tag{2.2}$$

In Equation (2.2), F is the scaling factor which is used to scale the difference vectors.

Crossover

Crossover operation is applied to each pair of the target vector $\vec{x}_i^g$ and its corresponding mutant vector $\vec{v}_i^g$ to generate a trial vector, after generating the new donor vector. A random index $j_{rand} \in [0, D]$ is generated and the crossover operator is applied for increasing the diversity of the population by following:

$$\vec{u}_{j,i}^{g+1} = \begin{cases} \vec{v}_{j,i}^g\,, & if \;\; rand_{i,j}[0,1] \leq CR \;\; or \;\; j = j_{rand} \\ \vec{x}_{j,i}^g\,, & otherwise \end{cases} \tag{2.3}$$

In Equation (2.3), $rand_{i,j}[0,1]$ represents a uniformly distributed random variable in the range $[0,1]$. $CR \in [0,1]$ is a crossover rate which is a user-defined constant, $j_{rand} \in [1, 2, ..., D]$ is randomly chosen index used to ensure that at least one of the components comes from $\vec{v}_i^g$.

Selection

Selection operator is used to determine target vector ($\vec{x}_i^g$) and trial vector ($\vec{u}_i^g$) which will be transferred to the next generation. The selection scheme given by Equation (2.4) is applied for determining the solutions to be transferred to the next generation.

$$\vec{x}_i^{g+1} = \begin{cases} \vec{u}_i^g, & f(\vec{u}_i^g) \geq f(\vec{x}_i^g) \\ \vec{x}_i^g, & f(\vec{u}_i^g) < f(\vec{x}_i^g) \end{cases} \tag{2.4}$$

In Equation (2.4), f is the objective function to be maximized. The vector transferred to the next generation is better than the competing vector, and such a selection guarantees that the global best of the population is transferred to the next generation all the time. In other words, the elitist strategy is implemented as the DE selection strategy.

2.2. Maximum likelihood estimation

Let $x_1, x_2, ..., x_n$ be a random variable sample drawn from the Gamma distribution expressed as $Gamma(\sigma, \beta, \mu)$. The log-likelihood function, $\log L(\theta/\underline{x})$ of the gamma distribution is given by

$$\log L(\theta/\underline{x}) = (\sigma - 1)\sum_{i=1}^{n} \log(x_i - \mu) - \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\beta}\right) - n \log \Gamma(\sigma) - n\sigma \log \beta \tag{2.5}$$

where $\theta = (\sigma, \beta, \mu)$ is a vector of representing unknown parameters [4, 12]. To obtain the ML estimates of the parameters σ, β and μ, the likelihood function (or the log likelihood function) is maximized as follows:

$$\begin{gathered}\max_{\theta=(\sigma,\beta,\mu)} \log L(\theta = (\sigma, \beta, \mu)/\underline{x}) \\ \mu \leq \min(\underline{x}) \\ \sigma \geq 0, \ \beta \geq 0\end{gathered} \tag{2.6}$$

2.3. Least square estimation

Let $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$ be the observed values of the order statistics taken from population which have $Gamma(\sigma, \beta, \mu)$ distribution.

$$F(x_{(i)}) = \frac{1}{\Gamma(\sigma)} \gamma \left(\sigma, \frac{x_{(i)} - \mu}{\beta} \right), \qquad x_{(i)} \geq \mu, \sigma > 0, \beta > 0 \tag{2.7}$$

If the empirical distribution function is written instead of $F(x_{(i)})$ in Equation (2.7), the model can be written as follows:

$$\frac{i}{n+1} = F(x_{(i)}) + \varepsilon_i, i = 1, 2, ..., n \tag{2.8}$$

Sum of squares of error terms ε_i is minimized to obtain the LS estimates of the parameters σ, β and μ as follows:

$$\begin{gathered}\min_{\theta=(\sigma,\beta,\mu)} \textstyle\sum_{i=1}^{n} \varepsilon_i^2 = \min_{\theta=(\sigma,\beta,\mu)} \sum_{i=1}^{n} \left(\frac{i}{n+1} - F(x_{(i)}) \right)^2 \\ \mu \leq \min(\underline{x}) \\ \sigma \geq 0, \ \beta \geq 0\end{gathered} \tag{2.9}$$

Such problems given in Equation (2.9) can be solved by taking the first partial derivatives of the objective function according to unknown parameters and equalizing to zero. Because the equations that contain nonlinear function cannot be solved, it is recommended that using metaheuristic algorithms to solve the optimization problems in Equation (2.6) and (2.9). Therefore, a novel DE algorithm is proposed to find the ML and LS estimates of the unknown parameters of the 3-p Gamma distribution in this study. The DE is chosen in this study because it is one of the well-known and the most effective evolutionary algorithms for optimization problems and it has obvious advantages including simplicity to implementation, robustness, reliability [13, 14, 19]. Furthermore, Özsoy *et al.* [20] showed DE gives better performance than other metaheuristic methods such as PSO, SA, GA in terms of bias, variance, MSE values of the parameter estimations.

2.4. A novel DE algorithm approach based on modified maximum likelihood and Profile Likelihood

The DE algorithm is considered for solving the optimization problems to find the ML and the LS estimates of the parameters of the 3-p Gamma distribution in this study. However, the performance of this algorithm is highly influenced by search space similar to other meta-heuristic methods. Previously, Örkcü, Aksoy [19] used the DE algorithm by arbitrarily determining the search space for estimating the parameters of the 3-p Weibull distribution.

In this study, we proposed a novel method to construct confidence intervals for each parameter which will be used as search space of the DE algorithm, by using Modified Maximum Likelihood (MML) and Profile Likelihood (PL) methods. The MML method is handled since the MML estimators for the 3-p Gamma distribution can be obtained in a clear form, namely written as a function of the observations.

The methodologies of the MML and the PL are given below.

Modified maximum likelihood for the 3-p Gamma distribution

To obtain the MML estimates of the parameters for the 3-p Gamma distribution, the following steps are used:

Step 1: The standardized observations $z_i = (x_i - \mu)/\beta$ are sorted in ascending order of magnitude as $z_{(1)} \leq z_{(2)} \leq \cdots \leq z_{(n)}$.

Step 2: The likelihood equations are written using the rank statistics.

$$\begin{aligned} \frac{\partial \log L}{\partial \sigma} &= \textstyle\sum_{i=1}^{n} \log(z_{(i)}) - \frac{n}{\beta}\psi(\sigma) - \frac{n}{\beta}\log\beta = 0, \\ \frac{\partial \log L}{\partial \beta} &= \textstyle\frac{1}{\beta}\sum_{i=1}^{n} z_{(i)} - \frac{\sigma-1}{\beta}\sum_{i=1}^{n} z_{(i)} w(z_{(i)}) - \frac{n}{\beta} = 0. \\ \frac{\partial \log L}{\partial \mu} &= \textstyle -\frac{(\sigma-1)}{\beta}\sum_{i=1}^{n} w(z_{(i)}) + \frac{n}{\beta} = 0. \end{aligned} \tag{2.10}$$

In Equation (2.10), $w(z) = z^{-1}$, $z_{(i)} = \left(x_{(i)} - \mu\right)/\beta$ and $\sum_{i=1}^{n} z_{(i)} = \sum_{i=1}^{n} z_i$.

Step 3: Let $t_{(i)} = E(z_{(i)})$be the expected value of ordered standardized variable $z_{(i)}$. A linear approximation of the nonlinear function $w(z_{(i)})$ using the first two terms of Taylor series expansion around $t_{(i)}$ is as follows:

$$\begin{aligned} w(z_{(i)}) &\cong w(t_{(i)}) + (z_{(i)} - t_{(i)})\left\{\frac{\partial}{\partial z}w(z)\right\}_{z=t_{(i)}} \\ &= \lambda_i - \gamma_i z_{(i)},\ i = 1, 2, ..., n \end{aligned} \tag{2.11}$$

In Equation (2.11);

$$\gamma_i = -\frac{\partial}{\partial z}w(z)|_{z=t_{(i)}} = -\frac{\partial}{\partial z}z^{-1}|_{z=t_{(i)}} = \frac{1}{t_{(i)}^2}, \tag{2.12}$$

$$\lambda_i = w(t_{(i)}) + t_{(i)}\gamma_i = \frac{1}{t_{(i)}} + t_{(i)}\frac{1}{t_{(i)}^2} = \frac{2}{t_{(i)}}. \tag{2.13}$$

Despite the fact that the exact values of $t_{(i)}$ are not available, approximate values can be obtained from the solution of equation $\int_{-\propto}^{t_{(i)}} f(z)dz = \frac{i}{n+1}$, $1 \le i \le n$. $f(.)$ indicates the probability density function of the 3-p Gamma distribution.

Step 4: The modified likelihood equations are obtained by inserting Equation (2.11) into the Equation (2.10) as follows:

$$\frac{\partial \log L}{\partial \sigma} = \sum_{i=1}^{n} \log(z_{(i)}) - \frac{n}{\beta}\psi(\sigma) - \frac{n}{\beta}\log\beta = 0,$$

$$\frac{\partial \log L}{\partial \beta} = \frac{1}{\beta}\sum_{i=1}^{n} z_{(i)} - \frac{\sigma-1}{\beta}\sum_{i=1}^{n} z_{(i)}(\lambda_i - \gamma_i z_{(i)}) - \frac{n}{\beta} = 0. \tag{2.14}$$

$$\frac{\partial \log L}{\partial \mu} = -\frac{(\sigma-1)}{\beta}\sum_{i=1}^{n}(\lambda_i - \gamma_i z_{(i)}) + \frac{n}{\beta} = 0.$$

Step 5: The MML estimators are obtained by solving the modified likelihood equations with regard to the parameters of μ and β, as follows, respectively:

$$\hat{\mu}_{MML} = K + D\hat{\sigma}_{MML} \tag{2.15}$$

$$\hat{\beta}_{MML} = \frac{-B + \sqrt{B^2 + 4nC}}{2n} \tag{2.16}$$

In Equations (2.15) and (2.16), $K = \frac{1}{m}\sum_{i=1}^{n}\gamma_i y_{[i]}$, $m = \sum_{i=1}^{n}\gamma_i$, $D = \frac{1}{m}\sum_{i=1}^{n}\lambda_i$, $B = \sum_{i=1}^{n}\lambda_i(y_i - K)$, and $C = \sum_{i=1}^{n}\gamma_i\left(y_{(i)} - K\right)^2 = \sum_{i=1}^{n}\gamma_i {y_{(i)}}^2 - mK^2$.

Because the MML estimators given in Equations (2.15) and (2.16) are the functions of the sample observations, it is easy to compute the MML estimators. Furthermore, the MML estimators are robust, unbiased and minimum variance bound estimators under certain mild regularity conditions [29].

As the MML estimators asymptotically equal to the ML estimators, the asymptotic variance covariance matrix $I^{-1}(\mu,\beta)$ is equal to inverse of the Fisher information matrix $I(\mu,\beta)$. Thus, asymptotic variances of $\hat{\mu}_{MML}$ and $\hat{\beta}_{MML}$ for $\sigma > 2$ are given as follows, respectively [26]

$$V(\hat{\mu}_{MML}) = \frac{\sigma(\sigma-2)\beta^2}{2n} \tag{2.17}$$

$$V(\hat{\beta}_{MML}) = \frac{\beta^2}{2\,(n-3)} \tag{2.18}$$

Confidence intervals with asymptotic $(1-\alpha)$ significance level for the position and the scale parameters are defined by following, respectively

$$\hat{\mu}_{MML} - z_{\alpha/2}\sqrt{V(\hat{\mu}_{MML})} < \mu < \hat{\mu}_{MML} - z_{\alpha/2}\sqrt{V(\hat{\mu}_{MML})} \tag{2.19}$$

$$\hat{\beta}_{MML} - z_{\alpha/2}\sqrt{V(\hat{\beta}_{MML})} < \beta < \hat{\beta}_{MML} - z_{\alpha/2}\sqrt{V(\hat{\beta}_{MML})} \tag{2.20}$$

The MML estimators have three desirable properties which are best asymptotically normality, almost fully efficiency for small samples, and explicit functions of sample observations for easy computation [5, 26].

The MML estimators can be obtained under the assumption that the shape parameter σ is known. Therefore, the Profile Likelihood (PL) method is used to estimate the shape parameter σ [28, 30].

Profile likelihood method for the 3-p Gamma distribution

The steps of the PPL method for this study are given below.

Step 1: Calculate the $\hat{\mu}_{MML}$ and $\hat{\beta}_{MML}$ for a given value of σ by using the Equation (2.15) and (2.16), respectively.

Step 2: Calculate the log-likelihood function value $\log L(\theta = (\sigma, \hat{\mu}_{MML}, \hat{\beta}_{MML})/\underline{x})$ given Equation (2.5).

Step 3: Repeat the Step 1 and Step 2 for different σ values.

Step 4: The value of σ that maximizes the log likelihood function is determined as the possible estimate for the shape parameter σ.

Confidence interval with asymptotic $(1-\alpha)$ significance level for shape parameter σ is constituted as follows:

$$\hat{\sigma} - z_{\alpha/2}\sqrt{V(\hat{\sigma})} < \sigma < \hat{\sigma} + z_{\alpha/2}\sqrt{V(\hat{\sigma})} \tag{2.21}$$

The search space of the location, scale and shape parameters of the 3-p Gamma distribution are constructed by Equations (2.19), (2.20) and (2.21), respectively.

The flowchart for the proposed DE approach is illustrated in Figure 1.

3. Simulation study

In this section, Monte-Carlo simulation study is conducted to investigate the performance of the proposed approach for the methods of ML and the LS. Actual parameter values for the 3-p Gamma distribution are thought to be $\theta = (3, 1, -2)$, $\theta = (4, 0.5, 10)$ and $\theta = (2.56, 0.75, 8)$. In addition, sample sizes 10, 30, 50, 100, 250, 500, and 1000 are considered for each parameter in order to investigate the effect of sample size on parameter estimation process.

A scaling factor $F \in (0, 1]$ and a crossover factor $C_r = 2$ are considered for the DE algoithm. Population sizes (pop) are considered as 10, 25 and, 50. In the proposed approach, search space is obtained using confidence intervals via the MML and PL methods. In addition, the effectiveness of estimates for different search spaces are compared by using the arbitrarily specified search spaces shown in Table 3.1.

Parameter estimations are obtained by simulations. Each simulation structure is repeated 1000 times, and $3 \times 7 \times 3 \times 1000 = 63.000$ independent experiments are performed

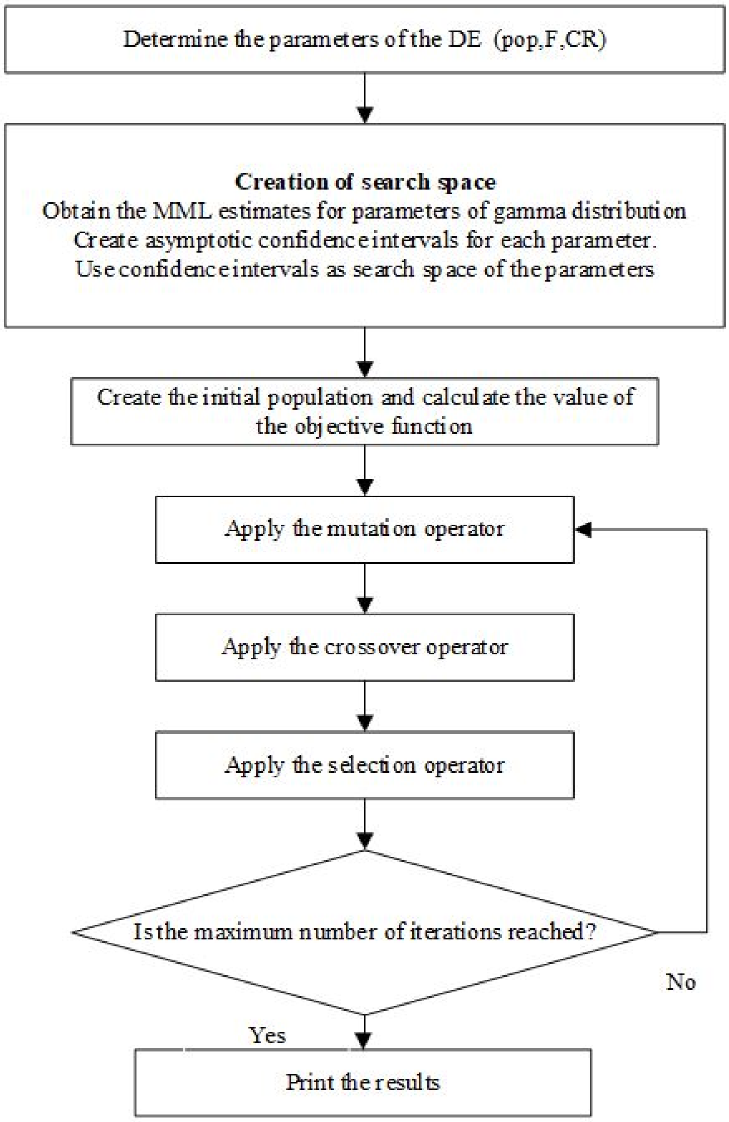

Figure 1. Flowchart for the proposed DE algorithm approach

Table 1. Search spaces for differential evolution algorithm.

Proposed Search Space (SSP_ML and SSP_LS):	95% confidence interval based on the MML estimates
Search Space 1 (SS1_ML and SS1_LS):	$[0,20]$ for σ, $[0,20]$ for β, $[-10,10]$ for μ
Search Space 2 (SS2_ML and SS2_LS):	$[0,100]$ for σ, $[0,100]$ for β, $[-50,50]$ for μ

when all cases are considered. Both the ML and LS estimates of the parameters for the 3-p Gamma distribution are calculated using the DE algorithm. The simulated mean, Mean Square Error (MSE) and deficiency (DEF) values are given in Tables 2-10. The DEF values are calculated as the sum of the MSE as follows:

$$DEF(\hat{\sigma},\hat{\beta},\hat{\mu}) = MSE(\hat{\sigma}) + MSE(\hat{\beta}) + MSE(\hat{\mu}) \quad (3.1)$$

Table 2. Simulation results for the estimation of the parameters $\theta = (3, 1, -2)$ for pop=10.

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	4.1081	6.3603	0.8273	0.1188	-1.9823	0.7847	**7.2639**
	SS1_ML	2.9587	13.4602	1.9513	2.7452	-1.7448	1.9343	18.1397
	SS2_ML	10.2772	296.1017	2.1231	3.9120	-9.9916	233.4482	533.462
	SSP_LS	4.9978	8.1338	0.8744	0.0855	-3.0801	2.9397	11.1590
	SS1_LS	4.7277	20.6180	1.4697	1.1587	-3.2869	6.5435	28.3202
	SS2_LS	13.1472	528.8568	5.3684	82.8275	-16.4342	451.4205	1063.1049
30	SSP_ML	3.1466	0.8031	0.9679	0.0333	-1.9414	0.1927	**1.02916**
	SS1_ML	3.4958	9.9920	1.2642	0.4509	-2.1178	1.3599	11.8028
	SS2_ML	10.6030	300.9108	1.7162	2.3138	-10.1954	245.0608	548.2855
	SSP_LS	3.5873	1.2773	0.9647	0.0307	-2.3604	0.5391	1.8470
	SS1_LS	4.5447	18.5068	1.3150	0.6667	-3.0188	4.9930	24.1665
	SS2_LS	10.5735	310.4276	4.5812	41.1483	-18.2958	539.2399	890.8158
50	SSP_ML	2.8464	0.4929	1.0392	0.0279	-1.8731	0.1243	**0.6452**
	SS1_ML	3.6563	9.3872	1.1394	0.2531	-2.2046	1.2856	10.9259
	SS2_ML	10.5346	294.4134	1.7056	3.3944	-10.0279	230.6749	528.4827
	SSP_LS	3.1328	0.5080	1.0199	0.0217	-2.1103	0.2035	0.7331
	SS1_LS	4.9827	22.8831	1.2133	0.4902	-3.1541	5.6937	29.0670
	SS2_LS	12.9022	507.8048	4.4482	52.1437	-15.4235	403.6760	963.6245
100	SSP_ML	2.8215	0.2478	1.0430	0.01457	-1.9027	0.0619	**0.3244**
	SS1_ML	3.5953	7.9865	1.1034	0.1841	-2.2270	1.2396	9.4103
	SS2_ML	10.4232	285.6879	1.6719	3.3319	-10.4079	245.7482	534.768
	SSP_LS	2.9650	0.2430	1.0336	0.0106	-2.0246	0.1106	0.3642
	SS1_LS	5.0581	22.0390	1.1738	0.4621	-3.3092	6.0832	28.5843
	SS2_LS	12.4918	468.1017	4.6259	57.5204	-15.2758	407.4964	933.1186
250	SSP_ML	2.8218	0.1047	1.0462	0.0067	-1.9367	0.0205	**0.1320**
	SS1_ML	3.6613	6.8480	1.0542	0.1115	-2.2907	1.0608	8.0204
	SS2_ML	10.9778	285.3779	1.5901	5.3516	-10.7245	251.501	542.2306
	SSP_LS	2.8453	0.1041	1.0480	0.0063	-1.9614	0.0376	0.1480
	SS1_LS	4.8449	19.0609	1.1654	0.4484	-3.1308	4.7909	24.3002
	SS2_LS	10.6644	318.6986	4.5701	47.9942	-17.8700	520.3152	887.0080
500	SSP_ML	2.8825	0.0503	1.0290	0.0029	-1.9559	0.0102	**0.0634**
	SS1_ML	3.5845	6.3440	1.0549	0.1533	-2.2392	0.8191	7.3166
	SS2_ML	9.8410	481.4533	1.6262	3.0355	-10.4271	246.2865	481.4533
	SSP_LS	2.8758	0.0558	1.0327	0.0030	-1.9561	0.0192	0.0780
	SS1_LS	4.8133	19.5092	1.1741	0.3769	-3.1835	5.1652	25.0513
	SS2_LS	10.8120	332.7709	4.9663	73.5282	-18.4075	529.6455	935.9445
1000	SSP_ML	2.9398	0.0242	1.0149	0.0013	-1.9777	0.0044	**0.0300**
	SS1_ML	3.5932	5.8670	1.0566	0.1903	-2.2932	1.0644	7.1218
	SS2_ML	10.8903	301.1794	1.6159	3.3527	-11.0935	272.0797	576.6119
	SSP_LS	2.9421	0.0282	1.0156	0.0012	-1.9801	0.0104	0.0398
	SS1_LS	5.1798	21.4202	1.1375	0.3950	-3.3210	5.6725	27.4877
	SS2_LS	11.4837	402.5591	4.5409	49.0188	-17.7199	497.7178	949.2957

The simulation results show that SSP and SS1 give better estimation values than SS2 in all cases. SS2 search space gives unreliable results because of very large. Moreover, SSP produces better results than SS1 in both the ML and LS methods according to the DEF criteria for most cases. As the number of sample size increased, a significant increase in the performance of the SSP search space has also observed. It is concluded that SSP is more suitable for estimating the parameters of the 3-p Gamma distribution except

Table 3. Simulation results for the estimation of the parameters $\theta = (3, 1, -2)$ for pop=25 .

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	3.8700	7.2861	0.8814	0.1202	-1.8610	0.8559	**8.2622**
	SS1_ML	2.1969	11.6152	2.4064	3.7625	-1.3813	1.2610	16.6388
	SS2_ML	3.4367	73.7615	2.3816	3.5016	-2.2581	15.281	92.5445
	SSP_LS	4.8712	8.9178	0.9023	0.1034	-2.9515	2.7041	11.7253
	SS1_ LS	3.8317	15.1783	1.5499	1.6589	-2.3889	3.0184	19.8557
	SS2_ LS	8.1162	306.1603	3.7872	37.8444	-6.8014	130.0211	474.0258
30	SSP_ML	3.0062	0.8182	0.9973	0.0352	-1.8827	0.1956	**1.0490**
	SS1_ML	2.8752	5.7905	1.3248	0.4839	-1.8127	0.5362	6.8106
	SS2_ML	4.4656	88.1826	1.4980	0.7484	-2.8087	21.236	110.1678
	SSP_LS	3.5372	1.4636	0.9762	0.0375	-2.3149	0.5231	2.0242
	SS1_ LS	3.9305	12.5226	1.1817	0.3005	-2.3609	1.9107	14.7339
	SS2_ LS	7.9032	240.2697	3.3291	21.8602	-8.8312	195.4372	457.5670
50	SSP_ML	2.7374	0.5794	1.0685	0.0348	-1.8285	0.1363	**0.7506**
	SS1_ML	2.8597	2.2978	1.1620	0.1810	-1.8561	0.2551	2.7340
	SS2_ML	4.7705	82.6319	1.3311	0.4161	-2.9714	19.716	102.7642
	SSP_LS	3.0971	0.6096	1.0294	0.0282	-2.0775	0.2051	0.8429
	SS1_ LS	3.7560	10.4265	1.1414	0.2191	-2.2635	1.4418	12.0874
	SS2_ LS	8.7670	297.4878	2.6827	17.7751	-6.9580	132.6581	447.9209
100	SSP_ML	2.7481	0.2872	1.0616	0.0188	-1.8734	0.0650	**0.3710**
	SS1_ML	2.8744	1.2812	1.0826	0.0651	-1.9029	0.1257	1.4721
	SS2_ML	4.2958	66.2131	1.2218	0.2086	-2.7249	15.6844	82.1062
	SSP_LS	2.9168	0.2829	1.0432	0.0149	-1.9899	0.1053	0.4032
	SS1_ LS	3.9077	10.3111	1.0583	0.1248	-2.3261	1.3128	11.7487
	SS2_ LS	8.8412	317.7574	2.7961	24.0806	-6.5267	107.2595	449.0975
250	SSP_ML	2.8023	0.1177	1.0514	0.0079	-1.9292	0.0214	**0.1471**
	SS1_ML	2.8975	0.4320	1.0485	0.0263	-1.9525	0.0446	0.5030
	SS2_ML	4.2272	58.8406	1.1664	0.1259	-2.8007	16.7065	75.6786
	SSP_LS	2.8301	0.1250	1.0500	0.0076	-1.9475	0.0388	0.1714
	SS1_ LS	3.8379	9.0928	1.0346	0.0893	-2.3108	1.1798	10.3619
	SS2_ LS	8.9133	306.1155	3.1061	28.2233	-8.3519	172.9180	507.2569
500	SSP_ML	2.8726	0.0577	1.0315	0.0033	-1.952	0.01093	**0.07206**
	SS1_ML	2.9328	0.3180	1.0319	0.0152	-1.9692	0.03217	0.3654
	SS2_ML	4.3728	65.1725	1.1248	0.1014	-2.7968	15.1136	80.3876
	SSP_LS	2.8743	0.0630	1.0328	0.0035	-1.9529	0.0196	0.0861
	SS1_ LS	3.9218	9.9909	1.0106	0.0727	-2.3333	1.1863	11.2498
	SS2_ LS	9.6920	335.7865	2.6505	11.6950	-9.0366	197.3591	544.8407
1000	SSP_ML	2.9329	0.0263	1.0165	0.0014	-1.9747	0.0045	**0.0323**
	SS1_ML	2.9586	0.2512	1.0225	0.0094	-1.9849	0.0283	0.2890
	SS2_ML	4.1003	45.1303	1.1123	0.08952	-2.7366	13.6658	58.8857
	SSP_LS	2.9375	0.0327	1.0158	0.0015	-1.9753	0.0105	0.0447
	SS1_ LS	4.0281	9.4578	0.9909	0.0728	-2.3915	1.1256	10.6562
	SS2_ LS	9.6719	337.4216	2.6890	10.6083	-9.4090	200.6948	548.7248

for a few cases, for example $\theta = (4, 0.5, 10)$,$n = 10$, *pop* = 25 and 50 for the LS and $\theta = (2.56, 0.75, 8)$,$n = 10$,*pop* = 25for the ML and LS. Therefore, it can be said that the proposed DE approach is preferable than the others.

Table 4. Simulation results for the estimation of the parameters $\theta = (3, 1, -2)$ for pop=50 .

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	3.8313	7.8802	0.9015	0.1225	-1.8383	0.9051	**8.9079**
	SS1_ML	2.1334	12.4083	2.5724	4.3032	-1.3570	1.3122	18.0238
	SS2_ML	1.8889	25.7838	2.6660	4.3152	-1.3236	1.8228	31.9219
	SSP_LS	4.9744	10.3985	0.9064	0.1129	-2.9611	2.8361	13.3474
	SS1_ LS	3.8160	14.0347	1.5306	1.5668	-2.3218	2.5635	18.1650
	SS2_ LS	6.2842	270.3826	3.5075	39.5510	-3.4243	42.3600	352.2937
30	SSP_ML	2.9827	0.8296	1.0037	0.0365	-1.8740	0.1959	**1.0622**
	SS1_ML	2.9501	6.3661	1.3269	0.5097	-1.8234	0.5480	7.4239
	SS2_ML	2.6886	13.7491	1.4812	0.6489	-1.8020	1.3787	15.7768
	SSP_LS	3.5093	1.5774	0.9852	0.0418	-2.2947	0.5237	2.1429
	SS1_ LS	3.7228	11.4252	1.1991	0.3329	-2.2041	1.2908	13.0490
	SS2_ LS	6.3778	215.3560	2.4797	8.1826	-4.1700	51.4928	275.0314
50	SSP_ML	2.7237	0.6010	1.0734	0.0362	-1.8239	0.1404	**0.7777**
	SS1_ML	2.9369	3.1182	1.1523	0.1777	-1.8685	0.2878	3.5838
	SS2_ML	2.9647	17.7667	1.2830	0.2768	-1.9249	1.3972	19.4408
	SSP_LS	3.0960	0.6808	1.0342	0.0323	-2.0742	0.2098	0.9229
	SS1_ LS	3.5481	8.3221	1.1285	0.1844	-2.1361	0.8902	9.3967
	SS2_ LS	5.7246	189.0630	2.1140	6.0979	-3.4530	39.9505	235.1114
100	SSP_ML	2.7385	0.2973	1.0647	0.0196	-1.8702	0.0665	**0.3835**
	SS1_ML	2.9109	1.1020	1.0716	0.0635	-1.9112	0.1157	1.2813
	SS2_ML	3.0300	14.8081	1.1771	0.1315	-1.9786	1.3586	16.2983
	SSP_LS	2.9116	0.3159	1.0463	0.0170	-1.9853	0.1098	0.4427
	SS1_ LS	3.3758	5.7175	1.0838	0.1075	-2.0881	0.6062	6.4312
	SS2_ LS	6.6015	253.7575	2.0297	5.3945	-3.6425	35.3657	294.5177
250	SSP_ML	2.7993	0.1199	1.0523	0.0081	-1.9283	0.0215	**0.1496**
	SS1_ML	2.9006	0.3569	1.0398	0.0216	-1.9505	0.0328	0.3569
	SS2_ML	2.9673	12.9489	1.1166	0.0587	-1.9987	1.1020	14.1097
	SSP_LS	2.8270	0.1342	1.0514	0.0084	-1.9458	0.0395	0.1821
	SS1_ LS	3.3329	3.9782	1.0389	0.0561	-2.0954	0.4253	4.4597
	SS2_ LS	7.3622	259.7597	2.0636	7.4142	-4.8842	64.1653	331.3391
500	SSP_ML	2.8716	0.0583	1.0319	0.0034	-1.9517	0.0109	**0.0728**
	SS1_ML	2.9383	0.1532	1.0220	0.0106	-1.9664	0.0175	0.1814
	SS2_ML	2.8774	2.7291	1.1009	0.0482	-1.9938	0.7305	3.5079
	SSP_LS	2.8762	0.0690	1.0327	0.0038	-1.9534	0.0206	0.0934
	SS1_ LS	3.2780	2.5897	1.0209	0.0385	-2.0835	0.3227	2.9509
	SS2_ LS	7.7825	289.9620	2.1047	7.0621	-5.1289	75.5625	372.5865
1000	SSP_ML	2.9316	0.02677	1.0169	0.0015	-1.9743	0.0045	**0.0328**
	SS1_ML	2.9559	0.0862	1.0145	0.0054	-1.9788	0.0088	0.1005
	SS2_ML	3.0109	6.9185	1.0835	0.0310	-2.0760	2.8762	9.8258
	SSP_LS	2.9351	0.0359	1.0166	0.0017	-1.9743	0.0107	0.0483
	SS1_ LS	3.3312	2.8849	1.0073	0.0322	-2.1094	0.3209	3.2381
	SS2_ LS	7.0331	241.5506	2.1044	6.2384	-4.7036	59.3074	307.0964

Table 5. Simulation results for the estimation of the parameters $\theta = (4, 0.5, 10)$ for pop=10 .

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	4.4927	6.7053	0.4729	0.0309	10.1617	0.3673	**7.1036**
	SS1_ML	2.9676	17.8085	1.2607	1.2206	10.0444	3.6257	22.6548
	SS2_ML	11.1550	319.2682	1.8011	4.8615	-0.9353	397.4822	721.6119
	SSP_LS	5.3598	6.9735	0.4910	0.0235	9.5633	0.8881	7.8851
	SS1_ LS	2.8564	6.7200	1.1727	0.9955	9.6895	3.0230	10.7385
	SS2_ LS	12.3061	480.4698	6.6717	121.3014	-8.6757	717.2849	1319.0561
30	SSP_ML	3.7075	1.8374	0.5353	0.0138	10.1215	0.1454	**1.9968**
	SS1_ML	3.5933	15.4422	0.8556	0.3576	9.7663	4.6108	20.41078
	SS2_ML	11.253	308.6472	1.6642	6.5920	-0.8054	389.1014	704.3407
	SSP_LS	4.3131	2.0663	0.5222	0.0096	9.8407	0.2614	2.3373
	SS1_ LS	3.1978	6.2303	0.9400	0.4778	9.7678	2.0474	8.7555
	SS2_ LS	11.1477	358.8691	6.2790	99.5005	-12.2756	899.8214	1358.1909
50	SSP_ML	3.5310	1.0042	0.5493	0.0099	10.1216	0.0823	1.0965
	SS1_ML	3.5576	12.0486	0.7767	0.1927	9.8686	2.3052	14.5466
	SS2_ML	11.0921	298.8128	1.5044	5.4673	-0.2465	369.8867	674.1669
	SSP_LS	3.8387	0.8791	0.5435	0.0073	9.9704	0.1160	**1.0024**
	SS1_ LS	3.2123	5.7100	0.9441	0.5337	9.7317	1.9430	8.1867
	SS2_ LS	12.2297	446.1462	6.0194	101.1597	-8.7481	737.6848	1284.9906
100	SSP_ML	3.5612	0.5612	0.5414	0.0050	10.0988	0.0381	0.6044
	SS1_ML	3.9300	13.3435	0.7165	0.1502	9.7671	2.6466	16.1404
	SS2_ML	10.8695	265.7512	1.4271	3.2553	-0.2312	362.0822	631.0888
	SSP_LS	3.6642	0.5171	0.5420	0.0044	10.0441	0.0586	**0.5801**
	SS1_ LS	3.3446	5.2311	0.8681	0.3477	9.7217	1.7741	7.3529
	SS2_ LS	12.3581	465.5964	6.2125	121.4982	-8.4911	719.5454	1306.6400
250	SSP_ML	3.7576	0.2075	0.5208	0.0015	10.0513	0.0138	**0.2230**
	SS1_ML	3.9972	11.4852	0.7153	0.1929	9.5811	4.4080	16.0862
	SS2_ML	10.7623	283.2713	1.3509	2.1354	0.07588	340.723	626.1298
	SSP_LS	3.7763	0.2252	0.5220	0.0015	10.0398	0.0245	0.2512
	SS1_ LS	3.6330	5.3033	0.8368	0.3401	9.6270	1.8655	7.5089
	SS2_ LS	11.3665	333.8431	5.9330	89.5090	-12.8759	927.8001	1351.1522
500	SSP_ML	3.8447	0.1145	0.5142	0.0008	10.0300	0.0067	**0.1222**
	SS1_ML	4.0225	10.4033	0.6983	0.2223	9.6520	3.1474	13.7731
	SS2_ML	11.9374	329.7972	1.3486	2.5385	-1.0968	404.5862	736.9218
	SSP_LS	3.8486	0.1256	0.5155	0.0007	10.0247	0.0131	0.1395
	SS1_ LS	3.5365	4.9741	0.8676	0.3499	9.5582	2.2155	7.5395
	SS2_ LS	10.9713	309.7106	5.9936	85.7278	-12.9568	962.6655	1358.1039
1000	SSP_ML	3.9552	0.0490	0.5037	0.0003	10.0101	0.0031	**0.0525**
	SS1_ML	4.1133	9.7414	0.6776	0.1691	9.6052	3.0781	12.9887
	SS2_ML	9.9975	223.1348	1.3682	2.4225	0.2815	336.5732	562.1306
	SSP_LS	3.9766	0.0602	0.5036	0.0002	10.0003	0.0070	0.0675
	SS1_ LS	3.6416	4.8782	0.8607	0.3823	9.5207	2.1090	7.3695
	SS2_ LS	11.7555	366.2718	6.4386	109.4302	-13.2351	948.0137	1423.7158

Table 6. Simulation results for the estimation of the parameters $\theta = (4, 0.5, 10)$ for pop=25 .

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	4.3236	8.4890	0.5014	0.036	10.2192	0.4208	8.9463
	SS1_ML	1.8934	12.0569	1.3959	1.3241	10.5883	0.6502	14.0313
	SS2_ML	3.2666	75.1719	1.5312	1.5971	9.4402	29.0729	105.8421
	SSP_LS	5.3234	8.0966	0.5010	0.0308	9.6205	0.8489	8.9764
	SS1_ LS	2.6049	5.9594	1.0289	0.7892	10.2932	0.5848	**7.3333**
	SS2_ LS	8.5574	333.5715	5.5630	119.0918	2.4118	263.4666	716.1299
30	SSP_ML	3.4952	2.1289	0.5595	0.0190	10.1714	0.1568	**2.3048**
	SS1_ML	2.9972	6.6971	0.7622	0.1864	10.2651	0.2802	7.1638
	SS2_ML	4.1640	78.4705	1.0280	0.5166	9.1412	28.4627	107.4544
	SSP_LS	4.1747	2.1961	0.5310	0.0134	9.9012	0.2386	2.4480
	SS1_ LS	2.7366	4.5384	0.7907	0.1807	10.2745	0.3122	5.0313
	SS2_ LS	8.8231	280.2917	4.7460	74.0181	-0.8255	375.4819	729.7918
50	SSP_ML	3.3617	1.2959	0.5689	0.0140	10.1615	0.0956	1.4057
	SS1_ML	3.0441	4.7673	0.6879	0.0883	10.2237	0.1930	5.04875
	SS2_ML	4.6207	89.5504	0.9251	0.3488	9.0328	27.4662	117.3655
	SSP_LS	3.7785	1.0971	0.5480	0.0096	10.0017	0.1240	**1.2306**
	SS1_ LS	2.9883	3.8932	0.7219	0.1146	10.2106	0.2675	4.2753
	SS2_ LS	8.8556	338.8390	4.2220	75.1658	2.4458	260.7591	674.7639
100	SSP_ML	3.5018	0.7172	0.5491	0.0064	10.1111	0.0422	0.7172
	SS1_ML	3.2427	2.9336	0.6238	0.0388	10.1610	0.1059	3.0784
	SS2_ML	4.5396	85.4658	0.8147	0.2051	9.1501	24.4157	110.0868
	SSP_LS	3.6452	0.5737	0.5424	0.0052	10.0578	0.0579	**0.6369**
	SS1_ LS	3.0900	3.1419	0.6774	0.0741	10.1776	0.1891	3.4051
	SS2_ LS	8.9699	329.9158	3.8824	43.5022	2.7742	227.4570	600.8750
250	SSP_ML	3.7249	0.2483	0.5243	0.0020	10.0581	0.0152	**0.2656**
	SS1_ML	3.3996	1.6394	0.5831	0.01836	10.1176	0.0573	1.7152
	SS2_ML	4.6783	69.9571	0.7591	0.1640	8.9209	28.6090	98.7302
	SSP_LS	3.7555	0.2492	0.5229	0.0017	10.0495	0.0242	0.2752
	SS1_ LS	3.3399	2.5821	0.6300	0.0482	10.1170	0.1309	2.7611
	SS2_ LS	9.6891	347.1285	4.5036	60.4417	-0.1056	334.9611	742.5313
500	SSP_ML	3.8337	0.1268	0.5152	0.0009	10.0324	0.0072	**0.1350**
	SS1_ML	3.4737	1.3998	0.5692	0.0116	10.0970	0.0463	1.4578
	SS2_ML	5.0636	83.8007	0.7383	0.1671	8.7998	30.8414	114.8093
	SSP_LS	3.8474	0.1332	0.5150	0.0008	10.0279	0.0127	0.1468
	SS1_ LS	3.4531	2.4743	0.6183	0.0423	10.0766	0.1374	2.6539
	SS2_ LS	10.1780	353.7534	4.1843	57.8931	-1.0371	397.2929	808.9394
1000	SSP_ML	3.9431	0.0559	0.5048	0.0003	10.0124	0.0034	**0.0597**
	SS1_ML	3.5021	0.7336	0.5578	0.0078	10.0886	0.0249	0.7664
	SS2_ML	4.8574	69.9133	0.7310	0.1649	8.8523	29.7498	99.8281
	SSP_LS	3.9699	0.0699	0.5039	0.0003	10.0035	0.0070	0.0772
	SS1_ LS	3.5570	2.2414	0.6052	0.0399	10.0488	0.1247	2.4061
	SS2_ LS	10.2309	327.2825	3.9128	46.5160	-1.7804	410.7385	784.5371

Table 7. Simulation results for the estimation of the parameters $\theta = (4, 0.5, 10)$ for pop=50 .

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	4.2683	9.1652	0.5131	0.0395	10.2371	0.4414	9.6462
	SS1_ML	2.0592	12.3012	1.3890	1.3435	10.5647	0.6424	14.2872
	SS2_ML	1.5514	17.4401	1.5498	1.5544	10.5852	1.5447	20.5392
	SSP_LS	5.3369	9.2060	0.5087	0.0355	9.6356	0.8885	10.1299
	SS1_ LS	2.7455	5.3813	0.9579	0.6875	10.2874	0.5161	**6.5850**
	SS2_ LS	5.1117	167.9172	4.5674	87.4712	7.5098	91.2558	346.6442
30	SSP_ML	3.4435	2.2750	0.5673	0.0210	10.1831	0.1639	**2.4600**
	SS1_ML	3.1289	5.7201	0.7248	0.1661	10.2510	0.2466	6.1329
	SS2_ML	2.8543	17.3835	0.9123	0.3210	10.1711	1.9764	19.6809
	SSP_LS	4.1580	2.5269	0.5374	0.0159	9.9092	0.2488	2.7916
	SS1_ LS	2.8942	3.9780	0.7349	0.1350	10.2719	0.2396	4.3526
	SS2_ LS	6.8806	251.1246	4.1015	65.9729	6.0875	108.5129	425.6103
50	SSP_ML	3.3449	1.3629	0.5731	0.0150	10.1667	0.0990	1.4770
	SS1_ML	3.1075	3.3861	0.6546	0.0651	10.2183	0.1484	3.5997
	SS2_ML	2.8689	13.2331	0.8136	0.1861	10.1837	0.9180	14.3374
	SSP_LS	3.7505	1.2272	0.5530	0.0113	10.0114	0.1276	**1.3662**
	SS1_ LS	2.9810	3.4078	0.6900	0.0811	10.2375	0.1966	3.6856
	SS2_ LS	5.4397	166.3668	2.9944	26.5011	7.8219	66.6044	259.4723
100	SSP_ML	3.4897	0.6931	0.5507	0.0068	10.1134	0.0433	0.7433
	SS1_ML	3.3902	2.0872	0.5885	0.0251	10.1443	0.08131	2.1936
	SS2_ML	3.0705	12.1302	0.7413	0.1235	10.1258	1.0299	13.2837
	SSP_LS	3.6350	0.6271	0.5443	0.0059	10.0620	0.0597	**0.6927**
	SS1_ LS	3.1200	2.4466	0.6346	0.0434	10.2113	0.1353	2.6253
	SS2_ LS	5.2318	151.6501	2.8139	24.9800	8.2256	47.3678	223.9978
250	SSP_ML	3.7187	0.2564	0.5250	0.0020	10.0593	0.0156	**0.2742**
	SS1_ML	3.5323	1.0121	0.5555	0.0101	10.0989	0.0387	1.0610
	SS2_ML	3.2166	6.2031	0.6667	0.06084	10.0771	0.6837	6.9477
	SSP_LS	3.7413	0.2735	0.5244	0.0020	10.0533	0.0248	0.3003
	SS1_ LS	3.3449	2.0583	0.5943	0.0242	10.1601	0.0936	2.1761
	SS2_ LS	7.2647	242.9537	2.9801	22.7486	5.3469	142.4554	408.1577
500	SSP_ML	3.8315	0.1298	0.5155	0.0009	10.0328	0.0073	**0.1381**
	SS1_ML	3.5890	0.6914	0.5472	0.0067	10.0791	0.0259	0.7241
	SS2_ML	3.2630	4.9462	0.6421	0.04873	10.0961	0.3291	5.3241
	SSP_LS	3.8459	0.1501	0.5155	0.0009	10.0280	0.0136	0.1647
	SS1_ LS	3.5045	1.7132	0.5734	0.0164	10.1136	0.0783	1.8079
	SS2_ LS	8.5492	337.4011	2.9457	27.7306	5.1378	139.5766	504.7083
1000	SSP_ML	3.9408	0.0570	0.5050	0.0003	10.0129	0.0034	**0.0609**
	SS1_ML	3.6291	0.4379	0.5391	0.0040	10.0702	0.01640	0.4583
	SS2_ML	3.2683	2.7705	0.6251	0.0391	10.1020	0.101541	2.7705
	SSP_LS	3.9646	0.0768	0.5044	0.0004	10.0048	0.0072	0.0844
	SS1_ LS	3.5715	1.3889	0.5644	0.0133	10.0867	0.0628	1.4651
	SS2_ LS	7.5491	243.1289	3.0848	36.1980	5.3303	136.3073	415.6343

Table 8. Simulation results for the estimation of the parameters $\theta = (2.56, 0.75, 8)$ for pop=10 .

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	3.7809	7.8255	0.6332	0.0673	7.9465	0.4281	**8.3210**
	SS1_ML	2.6556	15.9085	1.5197	1.3546	7.8879	3.3865	20.6497
	SS2_ML	13.0416	530.8435	6.5157	126.7316	-10.3124	684.8742	1342.449
	SSP_LS	5.0851	11.7824	0.614522	0.0570	7.0941	1.8480	13.6875
	SS1_ LS	4.4148	24.8730	1.8853	3.7670	5.2944	27.1118	55.7519
	SS2_ LS	12.4602	513.540	6.5240	127.3491	-9.41877	651.4303	1292.32
30	SSP_ML	2.8026	0.5396	0.6999	0.0186	8.0349	0.0567	**0.6150**
	SS1_ML	3.1277	12.9576	1.0299	0.3168	7.7246	2.5073	15.7819
	SS2_ML	11.0555	333.3878	1.6511	3.9034	-2.5283	375.5359	712.8272
	SSP_LS	3.1224	0.8023	0.7014	0.0174	7.7921	0.1799	0.9996
	SS1_ LS	4.6547	26.5247	1.4689	1.6912	5.2650	27.4967	55.7126
	SS2_ LS	10.8520	343.2408	6.0828	110.6547	-13.3322	846.8889	1300.7845
50	SSP_ML	2.6326	0.4722	0.7460	0.0141	8.0412	0.0488	**0.5352**
	SS1_ML	3.2429	11.8193	0.9713	0.2713	7.6195	2.9579	15.0486
	SS2_ML	11.0309	332.7216	1.5581	3.3208	-1.90314	336.1126	672.1551
	SSP_LS	3.0319	0.7745	0.7201	0.0107	7.8191	0.1475	0.9327
	SS1_ LS	4.7735	27.6503	1.4198	1.4677	5.2535	26.7401	55.8581
	SS2_ LS	11.7158	441.8068	5.4833	75.6122	-9.4618	647.0782	1164.4972
100	SSP_ML	2.4995	0.0793	0.7672	0.0053	8.0469	0.0125	0.09731
	SS1_ML	3.3625	12.7229	0.9061	0.1880	7.5609	3.3879	16.2989
	SS2_ML	10.7708	316.6808	1.5829	3.7033	-2.6695	379.1355	699.5196
	SSP_LS	2.5638	0.0584	0.7694	0.0056	7.9946	0.0198	**0.0839**
	SS1_ LS	4.7279	25.7711	1.3796	1.4570	5.2982	26.7773	54.0055
	SS2_ LS	11.7170	441.7485	5.8468	102.3679	-9.4784	646.5951	1190.7115
250	SSP_ML	2.4912	0.0575	0.7737	0.0030	8.0318	0.0057	0.0663
	SS1_ML	3.4480	13.0785	0.8621	0.1109	7.5427	2.9737	16.1631
	SS2_ML	11.14087	338.5419	1.5376	4.8033	-2.51524	365.159	708.5043
	SSP_LS	2.5116	0.0522	0.7746	0.0028	8.0164	0.0098	**0.0648**
	SS1_ LS	4.8735	27.0486	1.3414	1.3300	5.2525	26.6252	55.0038
	SS2_ LS	10.7525	338.3633	5.8998	95.2945	-13.4803	844.3087	1277.9664
500	SSP_ML	2.5296	0.0308	0.7658	0.0014	8.0187	0.0028	**0.0351**
	SS1_ML	3.3828	10.3804	0.8375	0.1095	7.5703	2.5217	13.0116
	SS2_ML	11.2535	369.99	1.4355	2.4073	-2.4308	368.8813	741.2870
	SSP_LS	2.5535	0.0298	0.7640	0.0013	8.0068	0.0061	0.0372
	SS1_ LS	4.8373	25.7278	1.3230	1.1450	5.2124	26.3145	53.1874
	SS2_ LS	10.8576	335.1701	5.9294	96.0040	-13.1429	828.6418	1259.8158
1000	SSP_ML	2.5587	0.0160	0.7589	0.0007	8.0118	0.0013	**0.01815**
	SS1_ML	3.3996	010.1453	0.8314	0.1074	7.5603	2.6359	12.8887
	SS2_ML	10.9587	334.4893	1.5165	4.0428	-2.2191	354.4369	692.9637
	SSP_LS	2.5666	0.0174	0.7584	0.0006	8.0078	0.0031	0.0211
	SS1_ LS	4.8971	26.7149	1.3392	1.3551	5.1878	27.2078	55.2777
	SS2_ LS	10.7706	337.3329	5.8955	97.0441	-13.1495	827.6200	1261.9970

Table 9. Simulation results for the estimation of the parameters $\theta = (2.56, 0.75, 8)$ for pop=25 .

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	3.2213	7.8948	0.7138	0.0615	8.1012	0.4388	**8.3951**
	SS1_ML	1.5156	6.4465	1.7864	1.8210	8.4117	0.4597	8.7274
	SS2_ML	3.0231	58.2342	1.8345	1.9019	7.1528	27.9849	88.1210
	SSP_LS	4.8160	12.7239	0.6559	0.0631	7.2409	1.6786	14.4656
	SS1_ LS	2.7116	9.3748	1.3902	1.4248	7.9054	1.2715	12.0711
	SS2_ LS	5.0092	200.6225	4.6330	81.3634	6.1252	53.3122	335.2982
30	SSP_ML	2.6957	0.5435	0.7177	0.0186	8.0675	0.0581	**0.6203**
	SS1_ML	2.2893	3.2782	1.0215	0.2746	8.1479	0.1658	3.7187
	SS2_ML	4.0812	89.1004	1.2096	0.4759	6.9153	28.4666	118.043
	SSP_LS	3.0385	0.8608	0.7156	0.0200	7.8307	0.1638	1.0446
	SS1_ LS	2.9435	10.5749	1.1391	0.5477	7.7234	2.6769	13.7995
	SS2_ LS	7.0028	264.3197	3.5858	49.3927	3.8524	120.6242	434.3366
50	SSP_ML	2.4833	0.4581	0.7738	0.0156	8.0823	0.0467	**0.5205**
	SS1_ML	2.3215	1.5162	0.9160	0.1202	8.1114	0.0821	1.7186
	SS2_ML	4.2731	85.8568	1.0870	0.2848	6.6770	32.4816	118.6233
	SSP_LS	2.9340	0.7428	0.7332	0.0135	7.8671	0.1209	0.8772
	SS1_ LS	3.0642	10.4554	1.0741	0.4427	7.6395	2.7570	13.6551
	SS2_ LS	8.5323	329.8226	3.5367	36.6483	1.6463	192.3202	558.7911
100	SSP_ML	2.4758	0.0937	0.7720	0.0061	8.0547	0.0131	0.1131
	SS1_ML	2.3935	0.7622	0.8405	0.0435	8.0688	0.0372	0.8430
	SS2_ML	4.0214	74.7848	1.0036	0.1822	6.9634	22.8519	97.8190
	SSP_LS	2.5601	0.0737	0.7685	0.0065	8.0024	0.0193	**0.0995**
	SS1_ LS	3.2307	11.1256	0.9765	0.2974	7.6192	2.6028	14.0258
	SS2_ LS	8.1413	286.0229	3.6583	44.3889	1.8357	185.2244	515.6362
250	SSP_ML	2.4794	0.0677	0.7764	0.0034	8.0353	0.0064	0.0776
	SS1_ML	2.4537	0.3922	0.8036	0.0164	8.0382	0.0214	0.4300
	SS2_ML	4.0328	73.4884	0.9312	0.1048	6.8150	32.9088	106.5021
	SSP_LS	2.5126	0.0596	0.7736	0.0032	8.0196	0.0095	**0.0723**
	SS1_ LS	3.3395	10.4881	0.9539	0.3893	7.5744	2.6145	13.4920
	SS2_ LS	9.5241	343.5391	4.1394	66.7236	-2.1932	343.7817	754.0444
500	SSP_ML	2.5187	0.0351	0.7680	0.0017	8.0217	0.0029	**0.0397**
	SS1_ML	2.4714	0.4105	0.7909	0.0095	8.0317	0.01942	0.4396
	SS2_ML	4.3226	93.7741	0.9101	0.1215	6.9341	22.8579	116.7536
	SSP_LS	2.5480	0.0346	0.7642	0.0016	8.0114	0.0059	0.0421
	SS1_ LS	3.3428	10.1838	0.9380	0.2983	7.5713	2.3045	12.7866
	SS2_ LS	9.9664	351.4923	3.6560	42.3463	-2.5675	349.9287	743.7673
1000	SSP_ML	2.5509	0.0173	0.7601	0.0007	8.0142	0.0013	**0.0195**
	SS1_ML	2.4914	0.1088	0.7795	0.0053	8.0262	0.0055	0.1198
	SS2_ML	4.0531	70.6981	0.8856	0.0891	6.9009	25.7387	96.5267
	SSP_LS	2.5654	0.0198	0.7583	0.0007	8.0092	0.0031	0.0237
	SS1_ LS	3.3702	9.4645	0.9322	0.3697	7.5570	2.2224	12.0566
	SS2_ LS	9.7969	351.3180	4.0083	57.9936	-2.2125	337.2107	746.5223

The Kruskal-Wallis test is used to examine the performance of the algorithms. While there is no significant difference in SSP search space at 95% confidence level for both methods (p value for the ML=0.865, p-value for the LS= 0.869), significant differences are

Table 10. Simulation results for the estimation of the parameters $\theta = (2.56, 0.75, 8)$ for pop=50.

		$\hat{\sigma}$		$\hat{\beta}$		$\hat{\mu}$		
n	Method	Mean	MSE	Mean	MSE	Mean	MSE	DEF
10	SSP_ML	3.1279	9.2329	0.7495	0.0650	8.15072	0.5079	9.8059
	SS1_ML	1.5355	7.1735	1.8501	1.9754	8.4207	0.4459	**9.5948**
	SS2_ML	1.4821	15.9981	1.9248	2.0580	8.3486	1.8519	19.9082
	SSP_LS	4.8225	12.6982	0.6552	0.0620	7.2393	1.6450	14.4052
	SS1_ LS	2.6733	8.7975	1.4083	1.4035	7.9239	1.1689	11.3698
	SS2_ LS	5.6894	262.1404	4.4139	60.5915	5.8439	62.7841	385.5159
30	SSP_ML	2.3751	0.6331	0.7802	0.01831	8.1369	0.0839	**0.7354**
	SS1_ML	2.3001	2.6873	1.0011	0.2861	8.1567	0.1367	3.0922
	SS2_ML	2.1957	9.7988	1.1591	0.3827	8.1245	0.5877	9.7988
	SSP_LS	3.0579	0.9761	0.7168	0.0219	7.8276	0.1690	1.1670
	SS1_ LS	2.6126	6.7054	1.0275	0.2962	8.0268	0.5004	7.5020
	SS2_ LS	6.8505	263.5265	3.4646	46.1029	4.0231	122.3993	432.0287
50	SSP_ML	2.4573	0.4694	0.7805	0.0164	8.0882	0.0474	**0.5333**
	SS1_ML	2.3858	1.8817	0.8917	0.1064	8.1053	0.0860	2.0742
	SS2_ML	2.3464	11.0274	1.0355	0.20514	8.0683	0.9778	12.2104
	SSP_LS	2.9005	0.8080	0.7423	0.0155	7.8799	0.1227	0.9462
	SS1_ LS	2.6290	5.6831	0.9679	0.1805	7.9976	0.5036	6.3672
	SS2_ LS	5.4041	177.1065	2.5312	14.1883	5.8558	47.2345	238.5293
100	SSP_ML	2.4753	0.0947	0.7722	0.0062	8.0548	0.0131	0.1141
	SS1_ML	2.4240	0.4930	0.8194	0.0345	8.0674	0.02801	0.5555
	SS2_ML	2.5523	9.6974	0.9308	0.1081	7.988	0.7037	10.5093
	SSP_LS	2.5587	0.0806	0.7692	0.0068	8.0035	0.0200	**0.1073**
	SS1_ LS	2.7013	5.5013	0.9165	0.1350	7.9880	0.3356	5.9720
	SS2_ LS	5.4807	186.1740	2.4491	17.8620	6.1878	40.8659	244.9019
250	SSP_ML	2.4780	0.0686	0.7767	0.0035	8.0355	0.0064	**0.0786**
	SS1_ML	2.4750	0.1706	0.7859	0.0105	8.0377	0.0098	1.1911
	SS2_ML	2.3214	1.0703	0.8753	0.0456	8.0336	0.1808	1.2987
	SSP_LS	2.5088	0.0670	0.7750	0.0035	8.0206	0.0099	0.0804
	SS1_ LS	2.7496	4.3696	0.8596	0.0817	7.9559	0.2698	4.7210
	SS2_ LS	6.2232	203.8677	2.8442	26.2443	4.2665	100.9938	331.1058
500	SSP_ML	2.5180	0.0354	0.7683	0.0017	8.0218	0.0029	**0.0401**
	SS1_ML	2.5088	0.0966	0.7748	0.0059	8.0259	0.0048	0.1074
	SS2_ML	2.6461	21.0011	0.8529	0.03401	7.8962	3.8415	24.8766
	SSP_LS	2.5482	0.0374	0.7644	0.0017	8.0114	0.0061	0.0451
	SS1_ LS	2.8341	4.2734	0.8312	0.0648	7.9349	0.2437	4.5819
	SS2_ LS	6.9904	253.8847	2.8179	19.0555	3.9053	121.2519	394.1921
1000	SSP_ML	2.5505	0.0175	0.7602	0.0008	8.0142	0.0013	**0.0197**
	SS1_ML	2.5262	0.0571	0.7678	0.0033	8.0213	0.0028	0.0633
	SS2_ML	2.50836	5.0090	0.8324	0.02712	7.9489	2.2257	7.2619
	SSP_LS	2.5657	0.0215	0.7583	0.0008	8.0092	0.0032	0.0255
	SS1_ LS	2.9471	4.8092	0.8151	0.0656	7.8984	0.2945	5.1692
	SS2_ LS	7.2016	258.1254	2.6375	16.7555	3.8897	115.2599	390.1408

determined in SS1 and SS2 search spaces (p-value=0.000 for all cases). Furthermore, significant differences are found between pop=10 and pop=25, between pop=10 and pop=50 in SS1, and also between all population sizes in SS2.

Mann Whitney-U test is also conducted for testing whether there is a significant difference between the ML and LS estimates using the SSP search space. The p values for the sample sizes n=10, 30, 50, 100, 250, 500, and 1000 have been obtained as 0.004, 0.136, 0.258, 0.863, 0.666, 0.340, and 0.258, respectively. It is found that, there is significantly difference between the ML and LS estimates only for $n = 10$.

4. Application

In this section, the implementation of the proposed DE approach on the actual data set is investigated. The data set taken from experiments which are conducted in the Department of Materials Science and Engineering, the University of Surrey includes failure stresses of single carbon fibers and it is given below [8]:

2.247; 2.640; 2.908; 3.099; 3.126; 3.245; 3.328; 3.355; 3.383; 3.572; 3.581; 3.681; 3.726; 3.727; 3.72; 3.783; 3.785; 3.786; 3.896; 3.912;3.964; 4.050; 4.063; 4.082; 4.111; 4.118; 4.141; 4.246; 4.251; 4.26; 4.326; 4.402; 4.457; 4.466; 4.519; 4.542; 4.555; 4.614; 4.632; 4.634; 4.636; 4.678; 4.698; 4.738; 4.83; 4.924; 5.043; 5.099; 5.134; 5.359; 5.473; 5.571; 5.684; 5.721; 5.998; 6.060

To test the suitability of this data set for 3-p Gamma distribution, Kolmogorov Smirnov (K-S) goodness of fit test is used. For K-S test, null hypothesis and alternative hypothesis are given as follows:

$$\begin{aligned} H_o &: F(x) = F_0(x) \\ H_1 &: F(x) \neq F_0(x) \end{aligned} \tag{4.1}$$

In this study, F_0 represents the distribution function for certain parameter values of 3-p Gamma distribution. The test statistic of KS is described by

$$D = \sup(|F_n(x) - F_0(x)|) \tag{4.2}$$

In Equation (4.2), $F_n(x)$ and $F_0(x)$ show the experimental and theoretical distribution functions at the sampling points of the Gamma distribution, respectively. At the confidence level α, the hypothesis H_o is rejected if the statistic D value is greater than or equal to the critical value d_α, or if the calculated p value is less than α. In this study, $\alpha = 0.05$ is taken.

The values of $\log L$ and Akaike Information Criteria (AIC) have been used to show the performance of the proposed approach. The AIC is defined as follows:

$$AIC = -2 \log L + 2k \tag{4.3}$$

Here, n is the number of recorded measurements, and k is the number of estimated parameters.

95 % confidence intervals for the parameters of the 3-p Gamma distribution based on the MML and PL methods are obtained as $6.5998 < \sigma < 12.7820$, $0.2315 < \beta < 0.3391$ and $1.0188 < \mu < 1.9463$. These confidence intervals are used as the search space for each parameter. The ML and LS estimations of the parameters for the 3-p Gamma distribution are obtained by the DE algorithm with various search spaces. Parameter estimation values,$\log L$, AIC, K-S test statistics, and p-values for the considered methods are given in Table 11.

Table 11. Parameter estimation values, logL, AIC, and K-S test statistics, and p-values for carbon fibre dataset.

	$\hat{\sigma}$	$\hat{\beta}$	$\hat{\mu}$	logL	AIC	K-S	p-value
SSP_ML	12.7813	0.2481	1.0648	-0.0441	6.0882	0.0750	0.9013
SS1_ ML	8.2790	0.3475	1.5679	-0.1765	6.3530	0.1344	0.2544
SS2_ ML	5.9780	0.5236	1.7295	-1.3667	8.7334	0.2731	0.0004
SSP_LS	12.3016	0.2421	1.3047	**-0.0255**	**6.0510**	0.0620	0.9804
SS1_ LS	9.4874	0.2850	1.6761	-0.0900	6.1800	0.0980	0.6370
SS2_ LS	8.1813	3.2249	-1.5607	-18.7883	43.5766	0.9976	0.0000

According to the results of the K-S test, the distribution of the carbon fibre dataset fit the 3-p Gamma distribution since the p-values are greater than α except for the SS2_ML and the SS2_LS.

It is clear from Table 11, SSP has the largest log L and the smallest AIC values for both the ML and LS. Thus, the SSP provides the most reliable results and the proposed approach should be preferred.

The histogram and the fitted density for carbon fibre data set for the ML and LS according to search space SSP and SS1, is shown in Figure 2 that supports the above results. In addition, it should be noted that SS1 can be used for the data set. However, it is more logical to use the SSP, since the arbitrarily determined SS1 may not include the actual parameter value for other datasets to be examined.

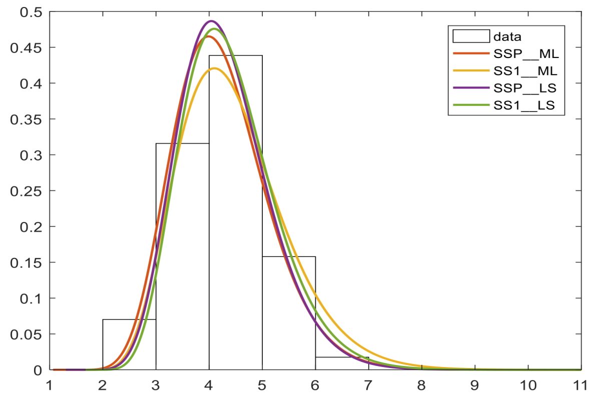

Figure 2. Histogram and fitted densities for the carbon fibre data set

5. Conclusion

This study discusses the ML and LS estimation problems of the parameters of the 3-p Gamma distribution. Because parameter estimation for the 3-p Gamma distribution is a quite difficult problem, it is wise to use a powerful metaheuristic method. Thus, the DE algorithm is used to find the ML and LS estimates of the parameters of the 3-p Gamma distribution. As with all metaheuristic methods, determination of the search space in DE is a very important problem. The performance of the DE is greatly affected by its search space in terms of both solution quality and computation time. In the literature, arbitrary search spaces have been traditionally used. However, the arbitrary search space may not contain the actual values and the global optimal solution cannot be found. To overcome this situation, a novel DE approach proposed in this study. In the proposed approach,confidence intervals based on the MML and PL methods are utilized for creating a search space for the DE algorithm which is narrow and has the actual values.

To test the performance of this proposed new approach, an extensive Monte Carlo simulation study is performed. Simulation results show that the proposed DE algorithm approach gives significantly better results in both the ML and the LS estimation of the parameters. In addition, a real data set analysis is performed to demonstrate the applicability of the proposed DE approach. The superiority of the proposed approach is also supported by real data set.

As a future research, the proposed DE approach can be applied to estimate the parameters of different statistical distributions. Furthermore, different metaheuristic methods can be used for estimating the parameters of the 3-p Gamma distribution in order to compare the solutions of the proposed DE approach.

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