IKONION JOURNAL OF MATHEMATICS

YEAR: 2020 VOLUME:2 ISSUE: 2

ISSN: 2687-6531

ANALYTICAL SOLUTIONS OF CONFORMABLE BOUSSINESQ-DOUBLE-SINH-GORDON AND FIRST BOUSSINESQ-LIOUVILLE EQUATIONS WITH THE HELP OF AUXIULARY EQUATION METHOD

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(Received: 07.09.2020, Accepted: 02.02.2021, Published Online: 02.02.2021)

Abstract

In this article, the analytical solutions of nonlinear fractional order Boussinesq-Double-Sinh-Gordon equation and first Boussinesq-Liouville equation are obtained with the aid of auxiulary equation method where the fractional derivatives are in conformable sense. Both equations were first converted to non-linear ordinary derivative differential equations with the help of wave transformation. auxiliary equation method was used to find analytical solutions of these ordinary derivative equations. Three dimensional graphics of the obtained results for nonlinear fractional order Boussinesq-Double-Sinh-Gordon equation and first Boussinesq-Liouville equation are given.

Keywords: Conformable Fractional Partial Differential Equations; Auxiulary Equation Method; Conformable Boussinesq-Double-Sinh-Gordon; Conformable First Boussinesq-Liouville.

1. Introduction

Obtaining analytical solutions of fractional order nonlinear partial differential equations is crucial for understanding the physical behavior and change process of the event under consideration.

Because the nonlinear partial differential equation models containing integer-order derivatives do not correspond exactly to events in nature, while the differential equations containing fractional order derivatives in which parameters are present correspond exactly. That is, the nonlinear partial differential that occurs when modeling a physical event according to the fractional computation integer computation it helps to express the equation more clearly. Analytical and numerical solutions of fractional order nonlinear partial differential equations including Riemann-Liouville, Caputo and conformable fractional derivative approach, which are frequently encountered in the literature, were obtained using various methods. Some of these methods are (G '/ G) expansion [9], first integral [7], exponential function [10], jacobi elliptical [12], homotopy analysis [13], finite elements [5], finite difference [6], functional change [3], auxiliary equation [2] Alhakim and Moussa, Methods such as tangent hyperbolic [4], separation of variables [11] were used.

In this article, non-linear Boussinesq-Double-Sinh-Gordon which contains conformable fractional order derivatives and analytical solutions of First Boussinesq-Liouville equations with auxiliary equation method was obtained.

2. Metarial and Method

In 2003, the auxiliary equation method was first used by S. Jion and Sirendaoreji to obtain complete solutions of nonlinear partial differential equations [8]. S. Jion and Sirendaoreji to obtain the exact solutions of the partial differential equations discussed in this study with the auxiliary equation method

$$\left(\frac{dz}{d\xi}\right)^2 = az^2(\xi) + bz^3(\xi) + cz^4(\xi) \tag{1}$$

they benefited from the solutions of ordinary differential equations. Then, in 2008, M.A. in his study, Abdou [1] gave a wider class of the solutions of his equation, Schrödinger, the nonlinear partial differential equation, obtained analytical solutions of the Whitham – Broer – Kaup and generalized Zakharov equations [1]. The following steps are followed to solve the analytical solution of the fractional order partial differential equation by the auxiliary equation method [8].

I. **Step:** General form of a partial differential equation containing a conformable fractional order derivative according to the nonlinear time variable

$$P\left(\frac{\partial^p u}{\partial t^p}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \dots\right) = 0 \tag{2}$$

it can be written as. Here P is a nonlinear function, $p \in (0,1)$ ve $\frac{\partial^p u}{\partial t^p}$ derivative, means the p-order conformable fractional derivative of the function $u(x,t)$.

II. Step: (2) to show the wave velocity of w, $u(x,t) = U(\xi)$, $\xi = x + w\frac{t^p}{p}$

if conversion is used (2) partial differential equation

$$G(U, U', U'', U''', \dots) = 0 \tag{3}$$

is transformed into ordinary differential differential equation.

III. Step: (3) given the ordinary differential equation

$$U(\xi) = \sum_{i=0}^{n} a_i z^i(\xi) \tag{4}$$

search for analytical solution. Here $a_i(i = 0,1, \dots, n))$ are the coefficients to be determined later, while the function $z(\xi)$ is the solutions of the differential equation (1). The positive n value in the equation given by (4) is found with the help of homogeneous balance. For this, for the term derivative which is linear from the highest digit in the equation given by (3)

$$\mathcal{O}\left(\frac{d^r U}{d\xi^r}\right) = n + r, \ r = 0,1,2, \dots$$

and for the highest non-linear term

$$\mathcal{O}\left(u^q \frac{d^r U}{d\xi^r}\right) = qn + r, \qquad r = 0,1,2,\ldots, \qquad q = 0,1,2,\ldots$$

the formulas written are synchronized [11].

IV. Step: As a last step, by writing the equation (4)and the necessary derivatives in the equation (3), an equation containing the forces of the expression $z(\xi)$ is obtained. The resulting equation is arranged according to the forces of the expression $z(\xi)$ and then a coefficient system of algebraic equation is created by synchronizing the coefficients of the forces of the expression $z(\xi)$ to zero. This algebraic system of equations containing a, b, c, w, a_i coefficients is solved with the help of the Mathematica program and coefficients are found. Analytical solutions of fractional order partial differential equation are obtained by using the results obtained by solving this system and using the formulas given in Table 1.

No	$z(\xi)$	Condition
1	$\dfrac{-absech^2\left(\frac{\sqrt{a}}{2}\xi\right)}{b^2 - ac\left(1-\varepsilon\tanh\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2}$	$a > 0$
2	$\dfrac{abcsch^2\left(\frac{\sqrt{a}}{2}\xi\right)}{b^2 - ac\left(1+\varepsilon\coth\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2}$	$a > 0$
3	$\dfrac{2ab\text{sech}(\sqrt{a}\xi)}{\varepsilon\sqrt{\Delta} - bsech(\sqrt{a}\xi)}$	$a > 0, \Delta > 0$
4	$\dfrac{2ab\sec(\sqrt{-a}\xi)}{\varepsilon\sqrt{\Delta} - bsec(\sqrt{-a}\xi)}$	$a < 0, \Delta > 0$
5	$\dfrac{2ab\text{csch}(\sqrt{a}\xi)}{\varepsilon\sqrt{-\Delta} - bcsch(\sqrt{a}\xi)}$	$a > 0, \Delta < 0$
6	$\dfrac{2ab\csc(\sqrt{-a}\xi)}{\varepsilon\sqrt{\Delta} - bcsc(\sqrt{-a}\xi))}$	$a < 0, \Delta > 0$
7	$\dfrac{-asech^2\left(\frac{\sqrt{a}}{2}\xi\right)}{b + 2\varepsilon\sqrt{ac}tanh\left(\frac{\sqrt{a}}{2}\xi\right)}$	$a > 0, c > 0$
8	$\dfrac{-asec^2\left(\frac{\sqrt{-a}}{2}\xi\right)}{b + 2\varepsilon\sqrt{-ac}tan\left(\frac{\sqrt{-a}}{2}\xi\right)}$	$a < 0, c > 0$
9	$\dfrac{-acsch^2\left(\frac{\sqrt{a}}{2}\xi\right)}{b + 2\varepsilon\sqrt{ac}coth\left(\frac{\sqrt{a}}{2}\xi\right)}$	$a > 0, c > 0$

10	$\dfrac{-acsc^2\left(\frac{\sqrt{-a}}{2}\xi\right)}{b+2\varepsilon\sqrt{-ac}cot\left(\frac{\sqrt{-a}}{2}\xi\right)}$	$a<0, c>0$
11	$-\frac{a}{b}\left(1+\varepsilon tanh\left(\frac{\sqrt{a}}{2}\xi\right)\right)$	$a>0, \Delta=0$
12	$-\frac{a}{b}\left(1+\varepsilon coth\left(\frac{\sqrt{a}}{2}\xi\right)\right)$	$a>0, \Delta=0$
13	$\dfrac{4ae^{\varepsilon\sqrt{a}\xi}}{\left(e^{\varepsilon\sqrt{a}\xi}-b\right)^2-4ac}$	$a>0$
14	$\dfrac{\pm 4ae^{\varepsilon\sqrt{a}\xi}}{1-4ace^{2\varepsilon\sqrt{a}\xi}}$	$a>0, b=0$

Table 1: Solutions of equation (1) with $\Delta = b^2 - 4ac$ and Ɛ = ± 1 [1]

3. Aplications of the Method

In this section, we will show the solution applications of fractional order partial differential equations using the auxiliary equation method.

Example 1: Fractional Order Boussinesq-Double-Sinh-Gordon Equation Boussinesq-Double-Sinh-Gordon equation with conformable fractional order derivative according to time variable

$$\frac{\partial^{2p}u}{\partial t^{2p}}-\alpha\frac{\partial^2 u}{\partial x^2}+\frac{\partial^4 u}{\partial x^4}=\sinh(u)+\frac{3}{2}\sinh(2u) \tag{5}$$

be considered as. Here, α is real constant and p∈ (0,1). (5) Conformable fractional order Boussinesq-Double-Sinh-Gordon equation

$$u(x,t)=u(\xi)$$

İncluding

$$\xi=x+w\frac{t^p}{p} \tag{6}$$

wave transformation is applied

$$(w^2-\alpha)u_{\xi\xi}+\ u_{\xi\xi\xi\xi}=\sinh(u)+\frac{3}{2}\sinh(2u) \tag{7}$$

ordinary differential system of differential equations is obtained. Here

$$v(\xi)=e^{u(\xi)} \tag{8}$$

is transformed and with the help of this transformation

$\sinh(u) = \dfrac{v - v^{-1}}{2}$

and

$\sinh(2u) = \dfrac{v^2 - v^{-2}}{2}$

by writing down the found equations in the equation given by (7)

$$-4w^2v^3v_{\xi\xi} + 4w^2v^2\left(v_\xi\right)^2 + 4\alpha v^3 v_{\xi\xi} - 4v_{\xi\xi\xi\xi}v^3 - 4\alpha v^2\left(v_\xi\right)^2 + 16v_{\xi\xi\xi}v_\xi v^2 + 24\left(v_\xi\right)^4$$
$$-48v_{\xi\xi}\left(v_\xi\right)^2 v + 12\left(v_{\xi\xi}\right)^2 v^2 + 3v^6 + 2v^5 - 2v^3 - 3v^2 = 0 \qquad (9)$$

equation is obtained. The equilibrium between the terms $v_{\xi\xi\xi\xi}v^3$and the highest order nonlinear v^6 containing the highest order derivative in the equation given by (9)

$n + 4 + 3n = 6n$

equality is obtained. This equation has the value of $n = 2$. So in the equation (9)

$$v(\xi) = a_0 + a_1 z(\xi) + a_2 z^2(\xi) \qquad (10)$$

In the form of an analytical solution is sought. If the equation (10) is substituted in the equation (9) ,an equation based on the forces of the expression $z(\xi)$ is obtained. By equating the coefficients of $z(\xi)$ and forces in this equation to zero;

$z^0(\xi):\ -3a_0^2 - 2a_0^3 + 2a_0^5 + 3a_0^6 = 0$

$z^1(\xi):\ -6a_0a_1 - 6a_0^2a_1 - 4a^2a_0^3a_1 + 10a_0^4a_1 + 18a_0^5a_1 - 4aa_0^3a_1w^2 + 4aa_0^3a_1\alpha = 0$

$z^2(\xi):\ -3a_1^2 - 6a_0a_1^2 + 16a^2a_0^2a_1^2 + 20a_0^3a_1^2 + 45a_0^4a_1^2 - 6a_0a_2 - 6a_0^2a_2 - 64a^2a_0^3a_2 + 10a_0^4a_2 + 18a_0^5a_2 - 30aa_0^3a_1b - 8aa_0^2a_1^2w^2 - 16aa_0^3a_2w^2 - 6a_0^3a_1bw^2 + 8aa_0^2a_1^2\alpha + 16aa_0^3a_2\alpha + 6a_0^3a_1b\alpha = 0$

$z^3(\xi):\ -2a_1^3 - 4a^2a_0a_1^3 + 20a_0^2a_1^3 + 60a_0^3a_1^3 - 6a_1a_2 - 12a_0a_1a_2 + 52a^2a_0^2a_1a_2 + 40a_0^3a_1a_2 + 90a_0^4a_1a_2 + 10aa_0^2a_1^2b - 260aa_0^3a_2b - 30a_0^3a_1b^2 - 80aa_0^3a_1c - 4aa_0a_1^3w^2 - 44aa_0^2a_1a_2w^2 - 14a_0^2a_1^2bw^2 - 20a_0^3a_2bw^2 - 8a_0^3a_1cw^2 + 4aa_0a_1^3\alpha + 44aa_0^2a_1a_2\alpha + 14a_0^2a_1^2b\alpha + 20a_0^3a_2b\alpha + 8a_0^3a_1c\alpha = 0$

$z^4(\xi):\ 10a_0a_1^4 + 45a_0^2a_1^4 - 6a_1^2a_2 - 32a^2a_0a_1^2a_2 + 60a_0^2a_1^2a_2 + 180a_0^3a_1^2a_2 - 3a_2^2 - 6a_0a_2^2 + 256a^2a_0^2a_2^2 + 20a_0^3a_2^2 + 45a_0^4a_2^2 - 10aa_0a_1^3b - 110aa_0^2a_1a_2b - 15a_0^2a_1^2b^2 - 210a_0^3a_2b^2 - 80aa_0^2a_1^2c - 480aa_0^3a_2c - 120a_0^3a_1bc - 32aa_0a_1^2a_2w^2 - 32aa_0^2a_2^2w^2 - 10a_0a_1^3bw^2 - 62a_0^2a_1a_2bw^2 - 20a_0^2a_1^2cw^2 - 24a_0^3a_2cw^2 + 32aa_0a_1^2a_2\alpha + 32aa_0^2a_2^2\alpha + 10a_0a_1^3b\alpha + 62a_0^2a_1a_2b\alpha + 20a_0^2a_1^2c\alpha + 24a_0^3a_2c\alpha = 0$

$z^5(\xi):\ 2a_1^5 + 18a_0a_1^5 - 4a^2a_1^3a_2 + 40a_0a_1^3a_2 + 180a_0^2a_1^3a_2 - 6a_1a_2^2 + 52a^2a_0a_1a_2^2 + 60a_0^2a_1a_2^2 + 180a_0^3a_1a_2^2 - 2aa_1^4b - 152aa_0a_1^2a_2b + 436aa_0^2a_2^2b - 12a_0a_1^3b^2 - 204a_0^2a_1a_2b^2 - 64aa_0a_1^3c - 608aa_0^2a_1a_2c - 144a_0^2a_1^2bc - 672a_0^3a_2bc - 96a_0^3a_1c^2 - 4aa_1^3a_2w^2 - 44aa_0a_1a_2^2w^2 - 2a_1^4bw^2 - 56a_0a_1^2a_2bw^2 - 44a_0^2a_2^2bw^2 - 16a_0a_1^3cw^2 - 80a_0^2a_1a_2cw^2 + 4aa_1^3a_2\alpha + 44aa_0a_1a_2^2\alpha + 2a_1^4b\alpha + 56a_0a_1^2a_2b\alpha + 44a_0^2a_2^2b\alpha + 16a_0a_1^3c\alpha + 80a_0^2a_1a_2c\alpha = 0$

$z^6(\xi):\ 3a_1^6 + 10a_1^4a_2 + 90a_0a_1^4a_2 + 16a^2a_1^2a_2^2 + 60a_0a_1^2a_2^2 + 270a_0^2a_1^2a_2^2 - 2a_2^3 - 64a^2a_0a_2^3 + 20a_0^2a_2^3 + 60a_0^3a_2^3 - 38aa_1^3a_2b + 94aa_0a_1a_2^2b - 3a_1^4b^2 - 156a_0a_1^2a_2b^2 + 150a_0^2a_2^2b^2 - 16aa_1^4c -$

$512a\mathrm{a}_0\mathrm{a}_1^2\mathrm{a}_2c + 160a\mathrm{a}_0^2\mathrm{a}_2^2c - 96\mathrm{a}_0\mathrm{a}_1^3bc - 1008\mathrm{a}_0^2\mathrm{a}_1\mathrm{a}_2bc - 144\mathrm{a}_0^2\mathrm{a}_1^2c^2 - 480\mathrm{a}_0^3\mathrm{a}_2c^2 - 8a\mathrm{a}_1^2\mathrm{a}_2^2w^2 - 16a\mathrm{a}_0\mathrm{a}_2^3w^2 - 14\mathrm{a}_1^3\mathrm{a}_2bw^2 - 74\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2bw^2 - 4\mathrm{a}_1^4cw^2 - 80\mathrm{a}_0\mathrm{a}_1^2\mathrm{a}_2cw^2 - 56\mathrm{a}_0^2\mathrm{a}_2^2cw^2 + 8a\mathrm{a}_1^2\mathrm{a}_2^2\alpha + 16a\mathrm{a}_0\mathrm{a}_2^3\alpha + 14\mathrm{a}_1^3\mathrm{a}_2b\alpha + 74\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2b\alpha + 4\mathrm{a}_1^4c\alpha + 80\mathrm{a}_0\mathrm{a}_1^2\mathrm{a}_2c\alpha + 56\mathrm{a}_0^2\mathrm{a}_2^2c\alpha = 0$

$z^7(\xi)$: $18\mathrm{a}_1^5\mathrm{a}_2 + 20\mathrm{a}_1^3\mathrm{a}_2^2 + 180\mathrm{a}_0\mathrm{a}_1^3\mathrm{a}_2^2 - 4a^2\mathrm{a}_1\mathrm{a}_2^3 + 40\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^3 + 180\mathrm{a}_0^2\mathrm{a}_1\mathrm{a}_2^3 - 2a\mathrm{a}_1^2\mathrm{a}_2^2b - 76a\mathrm{a}_0\mathrm{a}_2^3b - 42\mathrm{a}_1^3\mathrm{a}_2b^2 - 6\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2b^2 - 144a\mathrm{a}_1^3\mathrm{a}_2c - 272a\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2c - 24\mathrm{a}_1^4bc - 768\mathrm{a}_0\mathrm{a}_1^2\mathrm{a}_2bc - 48\mathrm{a}_0^2\mathrm{a}_2^2bc - 96\mathrm{a}_0\mathrm{a}_1^3c^2 - 864\mathrm{a}_0^2\mathrm{a}_1\mathrm{a}_2c^2 - 4a\mathrm{a}_1\mathrm{a}_2^3w^2 - 26\mathrm{a}_1^2\mathrm{a}_2^2bw^2 - 28\mathrm{a}_0\mathrm{a}_2^3bw^2 - 24\mathrm{a}_1^3\mathrm{a}_2cw^2 - 104\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2cw^2 + 4a\mathrm{a}_1\mathrm{a}_2^3\alpha + 26\mathrm{a}_1^2\mathrm{a}_2^2b\alpha + 28\mathrm{a}_0\mathrm{a}_2^3b\alpha + 24\mathrm{a}_1^3\mathrm{a}_2c\alpha + 104\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2c\alpha = 0$

$z^8(\xi)$: $45\mathrm{a}_1^4\mathrm{a}_2^2 + 20\mathrm{a}_1^2\mathrm{a}_2^3 + 180\mathrm{a}_0\mathrm{a}_1^2\mathrm{a}_2^3 + 10\mathrm{a}_0\mathrm{a}_2^4 + 45\mathrm{a}_0^2\mathrm{a}_2^4 - 18a\mathrm{a}_1\mathrm{a}_2^3b - 33\mathrm{a}_1^2\mathrm{a}_2^2b^2 - 30\mathrm{a}_0\mathrm{a}_2^3b^2 - 176a\mathrm{a}_1^2\mathrm{a}_2^2c - 160a\mathrm{a}_0\mathrm{a}_2^3c - 216\mathrm{a}_1^3\mathrm{a}_2bc - 504\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2bc - 24\mathrm{a}_1^4c^2 - 672\mathrm{a}_0\mathrm{a}_1^2\mathrm{a}_2c^2 - 240\mathrm{a}_0^2\mathrm{a}_2^2c^2 - 18\mathrm{a}_1\mathrm{a}_2^3bw^2 - 44\mathrm{a}_1^2\mathrm{a}_2^2cw^2 - 40\mathrm{a}_0\mathrm{a}_2^3cw^2 + 18\mathrm{a}_1\mathrm{a}_2^3b\alpha + 44\mathrm{a}_1^2\mathrm{a}_2^2c\alpha + 40\mathrm{a}_0\mathrm{a}_2^3c\alpha = 0$

$z^9(\xi)$: $60\mathrm{a}_1^3\mathrm{a}_2^3 + 10\mathrm{a}_1\mathrm{a}_2^4 + 90\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^4 - 4a\mathrm{a}_2^4b - 24\mathrm{a}_1\mathrm{a}_2^3b^2 - 128a\mathrm{a}_1\mathrm{a}_2^3c - 288\mathrm{a}_1^2\mathrm{a}_2^2bc - 192\mathrm{a}_0\mathrm{a}_2^3bc - 192\mathrm{a}_1^3\mathrm{a}_2c^2 - 576\mathrm{a}_0\mathrm{a}_1\mathrm{a}_2^2c^2 - 4\mathrm{a}_2^4bw^2 - 32\mathrm{a}_1\mathrm{a}_2^3cw^2 + 4\mathrm{a}_2^4b\alpha + 32\mathrm{a}_1\mathrm{a}_2^3c\alpha = 0$

$z^{10}(\xi)$: $45\mathrm{a}_1^2\mathrm{a}_2^4 + 2\mathrm{a}_2^5 + 18\mathrm{a}_0\mathrm{a}_2^5 - 6\mathrm{a}_2^4b^2 - 32a\mathrm{a}_2^4c - 192\mathrm{a}_1\mathrm{a}_2^3bc - 288\mathrm{a}_1^2\mathrm{a}_2^2c^2 - 192\mathrm{a}_0\mathrm{a}_2^3c^2 - 8\mathrm{a}_2^4cw^2 + 8\mathrm{a}_2^4c\alpha = 0$

$z^{11}(\xi)$: $18\mathrm{a}_1\mathrm{a}_2^5 - 48\mathrm{a}_2^4bc - 192\mathrm{a}_1\mathrm{a}_2^3c^2 = 0$

$z^{12}(\xi)$: $3\mathrm{a}_2^6 - 48\mathrm{a}_2^4c^2 = 0$

algebraic equation system is obtained. By solving this system of equations with the help of Mathematica program

$$\mathrm{a} = 1,\quad b = 2\sqrt{c},\quad w = -\sqrt{1+\alpha},\quad \mathrm{a}_0 = -1,\quad \mathrm{a}_1 = -4\sqrt{c},\quad \mathrm{a}_2 = -4c \tag{11}$$

$$\mathrm{a} = 1,\quad b = 2\sqrt{c},\quad w = -\sqrt{3+\alpha},\quad \mathrm{a}_0 = 1,\quad \mathrm{a}_1 = 4\sqrt{c},\quad \mathrm{a}_2 = 4c \tag{12}$$

$$\mathrm{a} = \frac{1}{4},\quad b = 0,\quad w = \sqrt{3+\alpha},\quad \mathrm{a}_0 = 1,\quad \mathrm{a}_1 = 0,\quad \mathrm{a}_2 = 4c \tag{13}$$

$$\mathrm{a} = \frac{1}{4},\quad b = 0,\quad w = \sqrt{1+\alpha},\quad \mathrm{a}_0 = -1,\quad \mathrm{a}_1 = 0,\quad \mathrm{a}_2 = -4c \tag{14}$$

solution sets are obtained.

The values given by (11) are substituted in the equations (6) and (10) ,and the solutions given in Table 1 and (8) using the transformation are given by the (5) conformable fractional order Boussinesq-double-sinh-Gordon Equation $u(x,t)$ analytical solutions

$$u_{1,2}(\mathrm{x},\mathrm{t}) = \log\left(1 - \frac{8\,\mathrm{sech}^2\left(\frac{1}{2}\xi\right)}{4 - \left(1 \pm \tanh\left(\frac{1}{2}\xi\right)\right)^2} + \frac{16\,\mathrm{sech}^4\left(\frac{1}{2}\xi\right)}{\left(4 - \left(1 \pm \tanh\left(\frac{1}{2}\xi\right)\right)^2\right)^2}\right)$$

$$u_{3,4}(\mathrm{x},\mathrm{t}) = \log\left(1 + \frac{8\,\mathrm{csch}^2\left(\frac{1}{2}\xi\right)}{4 - \left(1 \pm \coth\left(\frac{1}{2}\xi\right)\right)^2} + \frac{16\,\mathrm{csch}^4\left(\frac{1}{2}\xi\right)}{\left(4 - \left(1 \pm \coth\left(\frac{1}{2}\xi\right)\right)^2\right)^2}\right)$$

$$u_{5,6}(\mathrm{x,t})=\log\left(1-\frac{2\,\mathrm{sech}^2\left(\frac{1}{2}\xi\right)}{1\pm\tanh\left(\frac{1}{2}\xi\right)}+\frac{\mathrm{sech}^4\left(\frac{1}{2}\xi\right)}{\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)^2}\right)$$

$$u_{7,8}(\mathrm{x,t})=\log\left(1+\frac{2\,\mathrm{csch}^2\left(\frac{1}{2}\xi\right)}{1\pm\coth\left(\frac{1}{2}\xi\right)}+\frac{\mathrm{csch}^4\left(\frac{1}{2}\xi\right)}{\left(1\pm\coth\left(\frac{1}{2}\xi\right)\right)^2}\right)$$

$$u_{9,10}(\mathrm{x,t})=\log\left(1-2\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)+\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)^2\right)$$

$$u_{11,12}(\mathrm{x,t})=\log\left(1-2\left(1\pm\coth\left(\frac{1}{2}\xi\right)\right)+\left(1\pm\coth\left(\frac{1}{2}\xi\right)\right)^2\right)$$

$$u_{13,14}(\mathrm{x,t})=\log\left(1+\frac{16\sqrt{c}e^{\pm\xi}}{-4c+\left(-2\sqrt{c}+e^{\pm\xi}\right)^2}+\frac{64ce^{\pm2\xi}}{\left(-4c+\left(-2\sqrt{c}+e^{\pm\xi}\right)^2\right)^2}\right)$$

it is found as. Here $\xi=x-\sqrt{3+\alpha}\frac{t^p}{p}$.

The values given by (12) are substituted in the equations (6) and (10) ,and the solutions given in Table 1 and (8) using the transformation are given by the (5) conformable fractional order Boussinesq-double-sinh-Gordon Equation $u(x,t)$ analytical solutions

$$u_{15,16}(\mathrm{x,t})=\log\left(-1+\frac{8\,\mathrm{sech}^2\left(\frac{1}{2}\xi\right)}{4-\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)^2}-\frac{16\,\mathrm{sech}^4\left(\frac{1}{2}\xi\right)}{\left(4-\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)^2\right)^2}\right)$$

$$u_{17,18}(\mathrm{x,t})=\log\left(-1-\frac{8\,\mathrm{csch}^2\left(\frac{1}{2}\xi\right)}{4-\left(1\pm\coth\left(\frac{1}{2}\xi\right)\right)^2}-\frac{16\,\mathrm{csch}^4\left(\frac{1}{2}\xi\right)}{\left(4-\left(1\pm\coth\left(\frac{1}{2}\xi\right)\right)^2\right)^2}\right)$$

$$u_{19,20}(\mathrm{x,t})=\log\left(-1-\frac{2\,\mathrm{sech}^2\left(\frac{1}{2}\xi\right)}{1\pm\tanh\left(\frac{1}{2}\xi\right)}+\frac{\mathrm{sech}^4\left(\frac{1}{2}\xi\right)}{\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)^2}\right)$$

$$u_{21,22}(\mathrm{x,t})=\log\left(-1-\frac{2\,\mathrm{csch}^2\left(\frac{1}{2}\xi\right)}{1\pm\coth\left(\frac{1}{2}\xi\right)}-\frac{\mathrm{csch}^4\left(\frac{1}{2}\xi\right)}{\left(1\pm\coth\left(\frac{1}{2}\xi\right)\right)^2}\right)$$

$$u_{23,24}(\mathrm{x,t})=\log\left(-1+2\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)-\left(1\pm\tanh\left(\frac{1}{2}\xi\right)\right)^2\right)$$

$$u_{25,26}(\mathrm{x,t}) = \log\left(-1 + 2\left(1 \pm \coth\left(\frac{1}{2}\xi\right)\right) - \left(1 \pm \coth\left(\frac{1}{2}\xi\right)\right)^2\right)$$

$$u_{27,28}(\mathrm{x,t}) = \log\left(-1 - \frac{16\sqrt{c}e^{\pm\xi}}{-4c + \left(-2\sqrt{c} + e^{\pm\xi}\right)^2} - \frac{64ce^{\pm 2\xi}}{\left(-4c + \left(-2\sqrt{c} + e^{\pm\xi}\right)^2\right)^2}\right)$$

it is found as. Here $\xi = x - \sqrt{1+\alpha}\frac{t^p}{p}$.

The values given by (13) are substituted in the equations (6) and (10) ,and the solutions given in Table 1 and (8) using the transformation are given by the (5) conformable fractional order Boussinesq-double-sinh-Gordon Equation $u(x,t)$ analytical solutions

$$u_{29}(\mathrm{x,t}) = \log\left(1 - \mathrm{sech}^2\left(\frac{1}{2}\xi\right)\right)$$

$$u_{30}(\mathrm{x,t}) = \log\left(1 + \mathrm{csch}^2\left(\frac{1}{2}\xi\right)\right)$$

$$u_{31}(\mathrm{x,t}) = \log\left(1 + \frac{1}{4}\mathrm{csch}^2\left(\frac{1}{4}\xi\right)\mathrm{sech}^2\left(\frac{1}{4}\xi\right)\right)$$

$$u_{32,33}(\mathrm{x,t}) = \log\left(1 + \frac{4ce^{\pm\xi}}{(-c + e^{\pm\xi})^2}\right)$$

$$u_{34,35}(\mathrm{x,t}) = \log\left(1 + \frac{4ce^{\pm\xi}}{(1 - ce^{\pm\xi})^2}\right)$$

it is found as. Here $\xi = x + \sqrt{3+\alpha}\frac{t^p}{p}$.

The values given by (14) are substituted in the equations (6) and (10), and the solutions given in Table 1 and (8) using the transformation are given by the (5) conformable fractional order Boussinesq-double-sinh-Gordon Equation $u(x,t)$ analytical solutions

$$u_{36}(\mathrm{x,t}) = \log\left(-1 - \mathrm{sech}^2\left(\frac{1}{2}\xi\right)\right)$$

$$u_{37}(\mathrm{x,t}) = \log\left(-1 + \mathrm{csch}^2\left(\frac{1}{2}\xi\right)\right)$$

$$u_{38}(\mathrm{x,t}) = \log\left(-1 + \frac{1}{4}\mathrm{csch}^2\left(\frac{1}{4}\xi\right)\mathrm{sech}^2\left(\frac{1}{4}\xi\right)\right)$$

$$u_{39,40}(\mathrm{x,t}) = \log\left(-1 + \frac{4ce^{\pm\xi}}{\left(-c+e^{\pm\xi}\right)^2}\right)$$

$$u_{41,42}(\mathrm{x,t}) = \log\left(-1 + \frac{4ce^{\pm\xi}}{(1 - ce^{\pm\xi})^2}\right)$$

it is found as. Here $\xi = x + \sqrt{1+\alpha}\frac{t^p}{p}$.

The surfaces of some analytical solutions of the conformable fractional order equation given in the following figures 1-3 are given.

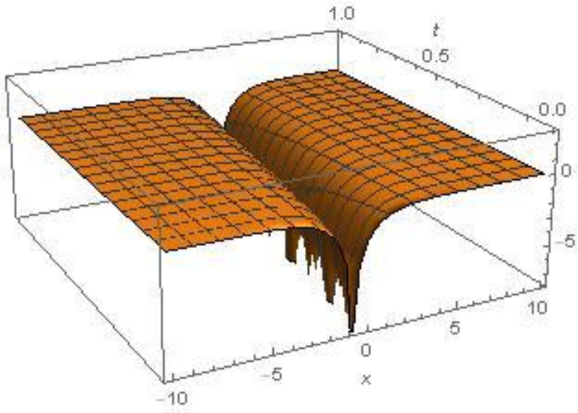

Figure 1: The surface of the solution $u_{15}(x,t)$ at $\alpha = 1, p = 0.75$

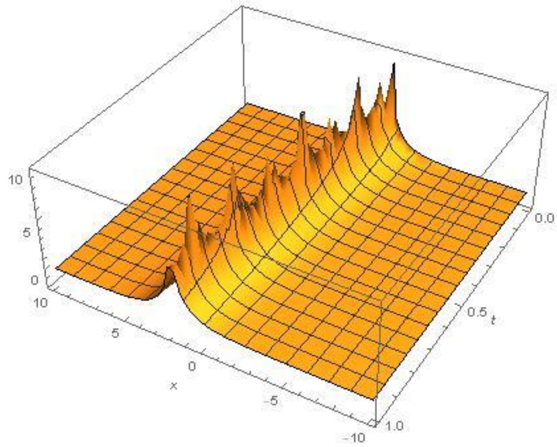

Figure 2: The surface of the solution $u_{17}(x,t)$ at $\alpha = 1, p = 0.75$

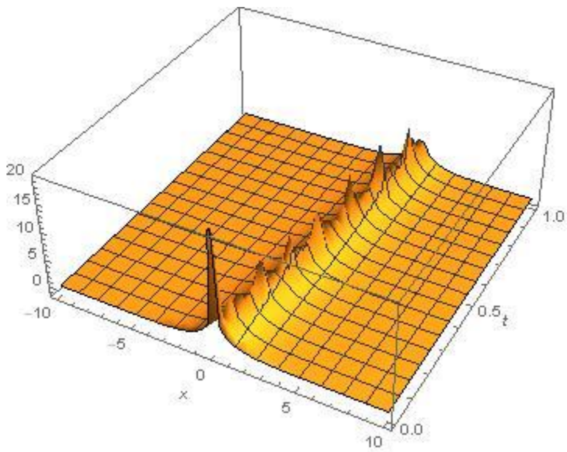

Figure 3: The surface of the solution $u_{21}(x,t)$ at $\alpha = 1, p = 0.75$

Example 2: *Fractional Order First Boussinesq-Liouville Equation*

Fractional Order First Boussinesq-Liouville Equation;

$$\frac{\partial^{2p}u}{\partial t^{2p}}-\alpha\frac{\partial^2 u}{\partial x^2}+\frac{\partial^4 u}{\partial x^4}=e^u+\frac{3}{4}e^{2u} \qquad (15)$$

be considered as. Here, α is real constant and $p\in(0,1)$. Conformable fractional order given by (15) to the First Boussinesq-Lioville equation

$$u(x,t)=u(\xi)$$

İncluding

$$\xi=x+w\frac{t^p}{p} \qquad (16)$$

wave transformation is applied;

$$(w^2-\alpha)u_{\xi\xi}+\ u_{\xi\xi\xi\xi}=e^u+\frac{3}{4}e^{2u} \qquad (17)$$

ordinary differential differential equation is obtained. In the equation (17)

$$v(\xi)=e^{u(\xi)}$$

if transform is applied;

$$-4w^2v^3v''+4w^2v^2(v')^2+4\alpha v^3v''-4v^{(4)}v^3-4\alpha v^2(v')^2+16v'''v'v^2-48v''(v')^2v$$
$$+24(v')^4+12(v'')^2v^2+3v^6+4v^5=0 \qquad (18)$$

differential equation is obtained. The value of $v^{(4)}v^3$, which contains the highest order derivative in the equation given by (18), and $n=2$ value from the homogeneous balance between the highest order nonlinear term and v^6. Thus, in the equation (18),

$$u(\xi)=\mathrm{a}_0+\mathrm{a}_1z(\xi)+\ \mathrm{a}_2z^2(\xi) \qquad (19)$$

In the form of a full solution is sought. If the equation (19) and the necessary derivatives are substituted in the equation (18), an equation based on the forces of the expression $z(\xi)$ is obtained.
With this equation the coefficients of $z(\xi)$ and their forces equal to zero.

$z^0(\xi)$: $4\mathrm{a}_0^5+3\mathrm{a}_0^6=0$

$z^1(\xi)$: $-4a^2\mathrm{a}_0^3\mathrm{a}_1+20\mathrm{a}_0^4\mathrm{a}_1+18\mathrm{a}_0^5\mathrm{a}_1-4a\mathrm{a}_0^3\mathrm{a}_1w^2+4a\mathrm{a}_0^3\mathrm{a}_1\alpha=0$

$z^2(\xi)$: $16a^2\mathrm{a}_0^2\mathrm{a}_1^2+40\mathrm{a}_0^3\mathrm{a}_1^2+45\mathrm{a}_0^4\mathrm{a}_1^2-64a^2\mathrm{a}_0^3\mathrm{a}_2+20\mathrm{a}_0^4\mathrm{a}_2+18\mathrm{a}_0^5\mathrm{a}_2-30a\mathrm{a}_0^3\mathrm{a}_1b-8a\mathrm{a}_0^2\mathrm{a}_1^2w^2-16a\mathrm{a}_0^3\mathrm{a}_2w^2-6\mathrm{a}_0^3\mathrm{a}_1bw^2+8a\mathrm{a}_0^2\mathrm{a}_1^2\alpha+16a\mathrm{a}_0^3\mathrm{a}_2\alpha+6\mathrm{a}_0^3\mathrm{a}_1b\alpha=0$

$z^3(\xi)$: $-4a^2\mathrm{a}_0\mathrm{a}_1^3+40\mathrm{a}_0^2\mathrm{a}_1^3+60\mathrm{a}_0^3\mathrm{a}_1^3+52a^2\mathrm{a}_0^2\mathrm{a}_1\mathrm{a}_2+80\mathrm{a}_0^3\mathrm{a}_1\mathrm{a}_2+90\mathrm{a}_0^4\mathrm{a}_1\mathrm{a}_2+10a\mathrm{a}_0^2\mathrm{a}_1^2b-260a\mathrm{a}_0^3\mathrm{a}_2b-30\mathrm{a}_0^3\mathrm{a}_1b^2-80a\mathrm{a}_0^3\mathrm{a}_1c-4a\mathrm{a}_0\mathrm{a}_1^3w^2-44a\mathrm{a}_0^2\mathrm{a}_1\mathrm{a}_2w^2-14\mathrm{a}_0^2\mathrm{a}_1^2bw^2-20\mathrm{a}_0^3\mathrm{a}_2bw^2-8\mathrm{a}_0^3\mathrm{a}_1cw^2+4a\mathrm{a}_0\mathrm{a}_1^3\alpha+44a\mathrm{a}_0^2\mathrm{a}_1\mathrm{a}_2\alpha+14\mathrm{a}_0^2\mathrm{a}_1^2b\alpha+20\mathrm{a}_0^3\mathrm{a}_2b\alpha+8\mathrm{a}_0^3\mathrm{a}_1c\alpha=0$

$z^4(\xi)$: $20a_0a_1^4 + 45a_0^2a_1^4 - 32a^2a_0a_1^2a_2 + 120a_0^2a_1^2a_2 + 180a_0^3a_1^2a_2 + 256a^2a_0^2a_2^2 + 40a_0^3a_2^2 + 45a_0^4a_2^2 - 10aa_0a_1^3b - 110aa_0^2a_1a_2b - 15a_0^2a_1^2b^2 - 210a_0^3a_2b^2 - 80aa_0^2a_1^2c - 480aa_0^3a_2c - 120a_0^3a_1bc - 32aa_0a_1^2a_2w^2 - 32aa_0^2a_2^2w^2 - 10a_0a_1^3bw^2 - 62a_0^2a_1a_2bw^2 - 20a_0^2a_1^2cw^2 - 24a_0^3a_2cw^2 + 32aa_0a_1^2a_2\alpha + 32aa_0^2a_2^2\alpha + 10a_0a_1^3b\alpha + 62a_0^2a_1a_2b\alpha + 20a_0^2a_1^2c\alpha + 24a_0^3a_2c\alpha = 0$

$z^5(\xi)$: $4a_1^5 + 18a_0a_1^5 - 4a^2a_1^3a_2 + 80a_0a_1^3a_2 + 180a_0^2a_1^3a_2 + 52a^2a_0a_1a_2^2 + 120a_0^2a_1a_2^2 + 180a_0^3a_1a_2^2 - 2aa_1^4b - 152aa_0a_1^2a_2b + 436aa_0^2a_2^2b - 12a_0a_1^3b^2 - 204a_0^2a_1a_2b^2 - 64aa_0a_1^3c - 608aa_0^2a_1a_2c - 144a_0^2a_1^2bc - 672a_0^3a_2bc - 96a_0^3a_1c^2 - 4aa_1^3a_2w^2 - 44aa_0a_1a_2^2w^2 - 2a_1^4bw^2 - 56a_0a_1^2a_2bw^2 - 44a_0^2a_2^2bw^2 - 16a_0a_1^3cw^2 - 80a_0^2a_1a_2cw^2 + 4aa_1^3a_2\alpha + 44aa_0a_1a_2^2\alpha + 2a_1^4b\alpha + 56a_0a_1^2a_2b\alpha + 44a_0^2a_2^2b\alpha + 16a_0a_1^3c\alpha + 80a_0^2a_1a_2c\alpha = 0$

$z^6(\xi)$: $3a_1^6 + 20a_1^4a_2 + 90a_0a_1^4a_2 + 16a^2a_1^2a_2^2 + 120a_0a_1^2a_2^2 + 270a_0^2a_1^2a_2^2 - 64a^2a_0a_2^3 + 40a_0^2a_2^3 + 60a_0^3a_2^3 - 38aa_1^3a_2b + 94aa_0a_1a_2^2b - 3a_1^4b^2 - 156a_0a_1^2a_2b^2 + 150a_0^2a_2^2b^2 - 16aa_1^4c - 512aa_0a_1^2a_2c + 160aa_0^2a_2^2c - 96a_0a_1^3bc - 1008a_0^2a_1a_2bc - 144a_0^2a_1^2c^2 - 480a_0^3a_2c^2 - 8aa_1^2a_2^2w^2 - 16aa_0a_2^3w^2 - 14a_1^3a_2bw^2 - 74a_0a_1a_2^2bw^2 - 4a_1^4cw^2 - 80a_0a_1^2a_2cw^2 - 56a_0^2a_2^2cw^2 + 8aa_1^2a_2^2\alpha + 16aa_0a_2^3\alpha + 14a_1^3a_2b\alpha + 74a_0a_1a_2^2b\alpha + 4a_1^4c\alpha + 80a_0a_1^2a_2c\alpha + 56a_0^2a_2^2c\alpha = 0$

$z^7(\xi)$: $18a_1^5a_2 + 40a_1^3a_2^2 + 180a_0a_1^3a_2^2 - 4a^2a_1a_2^3 + 80a_0a_1a_2^3 + 180a_0^2a_1a_2^3 - 2aa_1^2a_2^2b - 76aa_0a_2^3b - 42a_1^3a_2b^2 - 6a_0a_1a_2^2b^2 - 144aa_1^3a_2c - 272aa_0a_1a_2^2c - 24a_1^4bc - 768a_0a_1^2a_2bc - 48a_0^2a_2^2bc - 96a_0a_1^3c^2 - 864a_0^2a_1a_2c^2 - 4aa_1a_2^3w^2 - 26a_1^2a_2^2bw^2 - 28a_0a_2^3bw^2 - 24a_1^3a_2cw^2 - 104a_0a_1a_2^2cw^2 + 4aa_1a_2^3\alpha + 26a_1^2a_2^2b\alpha + 28a_0a_2^3b\alpha + 24a_1^3a_2c\alpha + 104a_0a_1a_2^2c\alpha = 0$

$z^8(\xi)$: $45a_1^4a_2^2 + 40a_1^2a_2^3 + 180a_0a_1^2a_2^3 + 20a_0a_2^4 + 45a_0^2a_2^4 - 18aa_1a_2^3b - 33a_1^2a_2^2b^2 - 30a_0a_2^3b^2 - 176aa_1^2a_2^2c - 160aa_0a_2^3c - 216a_1^3a_2bc - 504a_0a_1a_2^2bc - 24a_1^4c^2 - 672a_0a_1^2a_2c^2 - 240a_0^2a_2^2c^2 - 18a_1a_2^3bw^2 - 44a_1^2a_2^2cw^2 - 40a_0a_2^3cw^2 + 18a_1a_2^3b\alpha + 44a_1^2a_2^2c\alpha + 40a_0a_2^3c\alpha = 0$

$z^9(\xi)$: $60a_1^3a_2^3 + 20a_1a_2^4 + 90a_0a_1a_2^4 - 4aa_2^4b - 24a_1a_2^3b^2 - 128aa_1a_2^3c - 288a_1^2a_2^2bc - 192a_0a_2^3bc - 192a_1^3a_2c^2 - 576a_0a_1a_2^2c^2 - 4a_2^4bw^2 - 32a_1a_2^3cw^2 + 4a_2^4b\alpha + 32a_1a_2^3c\alpha = 0$

$z^{10}(\xi)$: $45a_1^2a_2^4 + 4a_2^5 + 18a_0a_2^5 - 6a_2^4b^2 - 32aa_2^4c - 192a_1a_2^3bc - 288a_1^2a_2^2c^2 - 192a_0a_2^3c^2 - 8a_2^4cw^2 + 8a_2^4c\alpha = 0$

$z^{11}(\xi)$: $18a_1a_2^5 - 48a_2^4bc - 192a_1a_2^3c^2 = 0$

$z^{12}(\xi)$: $3a_2^6 - 48a_2^4c^2 = 0$

algebraic equation system is obtained. By solving this system of equations with the help of Mathematica program

$$a = 2 - w^2 + \alpha, b = \frac{a_1}{2}, c = -\frac{a_1^2}{16(w^2 - 2 - \alpha)}, \ a_0 = 0, \ a_2 = -\frac{a_1^2}{4(w^2 - 2 - \alpha)} \tag{20}$$

$$a = \frac{1}{4}(2 - w^2 + \alpha), b = 0, c = \frac{a_2}{4}, a_0 = 0, a_1 = 0 \tag{21}$$

solutions are obtained. $u(x,t)$ analytical solutions of the conformable fractional order first Boussinesq-Liouville equation given by using the solutions given in Table 1 with the values given in (20) in the equations (16) and (19).

$$u_{1,2}(\mathrm{x},\mathrm{t}) = \log\left(-\frac{8a\,\mathrm{sech}^2\left(\frac{\sqrt{a}}{2}\xi\right)}{4-\left(1\pm+\tanh\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2} + \frac{16a\,\mathrm{sech}^4\left(\frac{\sqrt{a}}{2}\xi\right)}{\left(4-\left(1\pm\tanh\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2\right)^2} \right), a>0$$

$$u_{3,4}(\mathrm{x},\mathrm{t}) = \log\left(\frac{8a\,\mathrm{csch}^2\left(\frac{\sqrt{a}}{2}\xi\right)}{4-\left(1\pm\coth\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2} - \frac{16a\,\mathrm{csch}^4\left(\frac{\sqrt{a}}{2}\xi\right)}{\left(4-\left(1\pm\coth\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2\right)^2} \right), a>0$$

$$u_{5,6}(\mathrm{x},\mathrm{t}) = \log\left(-\frac{2a\,\mathrm{sech}^2\left(\frac{\sqrt{a}}{2}\xi\right)}{1\pm\tanh\left(\frac{\sqrt{a}}{2}\xi\right)} + \frac{a\,\mathrm{sech}^4\left(\frac{\sqrt{a}}{2}\xi\right)}{\left(1\pm\tanh\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2} \right), a>0$$

$$u_{7,8}(\mathrm{x},\mathrm{t}) = \log\left(\frac{2a\,\mathrm{csch}^2\left(\frac{\sqrt{a}}{2}\xi\right)}{1\pm\coth\left(\frac{\sqrt{a}}{2}\xi\right)} + \frac{a\,\mathrm{csch}^4\left(\frac{\sqrt{a}}{2}\xi\right)}{\left(1\pm\coth\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2} \right), a>0$$

$$u_{9,10}(\mathrm{x},\mathrm{t}) = \log\left(-2a\left(1\pm\tanh\left(\frac{\sqrt{a}}{2}\xi\right)\right) + a\left(1\pm\tanh\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2 \right), a>0$$

$$u_{11,12}(\mathrm{x},\mathrm{t}) = \log\left(-2a\left(1\pm\coth\left(\frac{\sqrt{a}}{2}\xi\right)\right) + a\left(1\pm\coth\left(\frac{\sqrt{a}}{2}\xi\right)\right)^2 \right), a>0$$

$$u_{13,14}(\mathrm{x},\mathrm{t}) = \log\left(\frac{4a\mathrm{a}_1 e^{\pm\sqrt{a}\xi}}{\left(-\frac{\mathrm{a}_1}{2}+e^{\pm\sqrt{a}\xi}\right)^2-\frac{\mathrm{a}_1{}^2}{4}} + \frac{4a\mathrm{a}_1{}^2 e^{\pm 2\sqrt{a}\xi}}{\left(\left(-\frac{\mathrm{a}_1}{2}+e^{\pm\sqrt{a}\xi}\right)^2-\frac{\mathrm{a}_1{}^2}{4}\right)^2} \right), a>0$$

it is found as. Here $\xi = x + w\frac{t^p}{p}$ ve a $= 2 - w^2 + \alpha$.

$u(x,t)$ analytical solutions of the conformable fractional order first Boussinesq-Liouville equation given by using the solutions given in Table 1 with the values given in (21) in the equations (16) and (19).

$$u_{15,16}(\mathrm{x},\mathrm{t}) = \log\left(\pm\mu\,\mathrm{sech}^2\left(\frac{\sqrt{-\mu}}{2}\xi\right)\right), \mu<0$$

$$u_{17,18}(\mathrm{x},\mathrm{t})=\log\left(\pm\mu\sec^2\left(\frac{\sqrt{\mu}}{2}\xi\right)\right),\mu>0$$

$$u_{19,20}(\mathrm{x},\mathrm{t})=\log\left(\pm\mu\,\mathrm{csch}^2\left(\frac{\sqrt{-\mu}}{2}\xi\right)\right),\mu<0$$

$$u_{21,22}(\mathrm{x},\mathrm{t})=\log\left(\pm\mu\csc^2\left(\frac{\sqrt{\mu}}{2}\xi\right)\right),\mu>0$$

$$u_{23}(\mathrm{x},\mathrm{t})=\log\left(-\frac{1}{4}\mu\,\mathrm{csch}^2\left(\frac{\sqrt{-\mu}}{4}\xi\right)\mathrm{sech}^2\left(\frac{\sqrt{-\mu}}{4}\xi\right)\right),\mu<0$$

$$u_{24}(\mathrm{x},\mathrm{t})=\log\left(\frac{1}{4}\mu\csc^2\left(\frac{\sqrt{\mu}}{4}\xi\right)\mathrm{sech}^2\left(\frac{\sqrt{\mu}}{4}\xi\right)\right),\mu>0$$

$$u_{25,26}(\mathrm{x},\mathrm{t})=\log\left(\frac{\mathrm{a}_2\mu^2e^{\pm\sqrt{-\mu}\xi}}{\left(e^{\pm\sqrt{-\mu}\xi}+\frac{\mathrm{a}_2\mu}{4}\right)^2}\right),\mu<0$$

$$u_{27,28}(\mathrm{x},\mathrm{t})=\log\left(\frac{\mathrm{a}_2\mu^2e^{\pm\sqrt{-\mu}\xi}}{\left(1+\frac{\mathrm{a}_2\mu}{4}e^{\pm\sqrt{-\mu}\xi}\right)^2}\right),\mu<0$$

it is found as. Here $\xi=x+w\frac{t^p}{p}$ ve $\mu=w^2-2-\alpha$.

The surfaces of some analytical solutions of the conformable fractional order equation given in the following figures 4-6 are given.

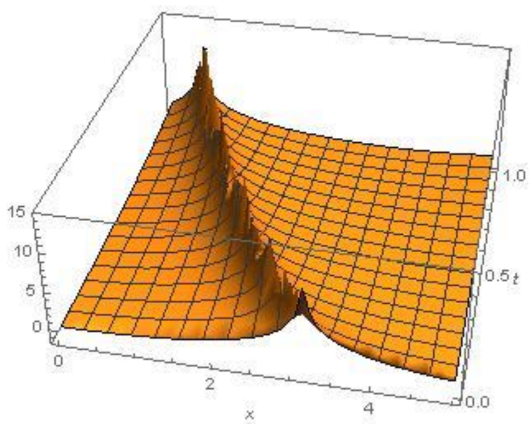

Figure 4: The surface of the solution $u_{17}(x,t)$ at $w=2,\alpha=1,p=0.75$

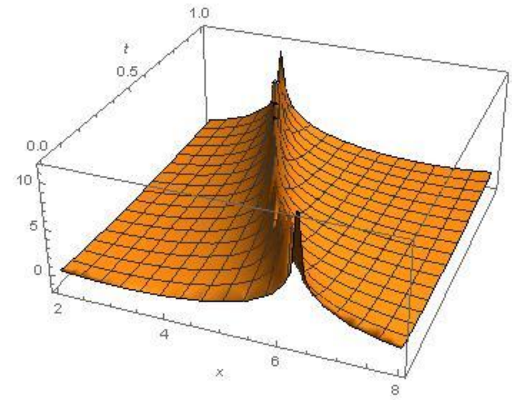

Figure 5: The surface of the solution $u_{19}(x,t)$ at $w = 2, \alpha = 3, p = 0.75$

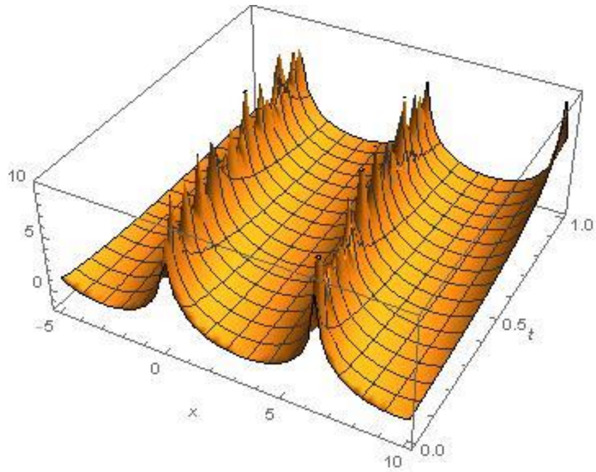

Figure 6: The surface of the solution $u_{21}(x,t)$ at $w = 2, \alpha = 1, p = 0.75$

4. Results and Discussions

In this article, the Boussinesq-Double-Sinh-Gordon and First Boussinesq-Liouville equations, which are non-linear fractional order partial differential equations containing conformable fractional derivatives based on time, are discussed. Both equations were first converted to non-linear ordinary derivative differential equations with the help of wave transformation. auxiliary equation method was used to find analytical solutions of these ordinary derivative equations. For this, $z(\xi)$ consists of the forces of the expression

$$\sum_{i=0}^{n} a_i z^i(\xi)$$

searched for analytical solution in form. Substituting this analytical solution form into ordinary differential equations, an equation containing the powers of the expression $z(\xi)$ was found. The solutions of the algebraic equation system obtained by equating the coefficients of the forces of the $z(\xi)$ expression in this equation to zero were found with the help of the Mathematica program. Analytical solutions of the equations dealt with with the help of these values were found. As a result; it was seen by using Mathematica program that all analytical solutions obtained.

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