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Katibe Gizem Yığ[1], Zeynep Sonay Ay[2]
[1]Burdur Mehmet Akif Ersoy University
[2]Hacettepe University

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An Analysis of the Qualities of the Problems Posed by the Students in a Seventh Grade Mathematics Course Assisted by the Problem Posing Approach

Katibe Gizem Yığ[1*], Zeynep Sonay Ay[2]
[1]Burdur Mehmet Akif Ersoy University
[2]Hacettepe University

Abstract

In this study, it is primarily aimed to determine the qualities of the problems posed by the students in a mathematics class delivered through the problem-posing approach and to examine the mean scores of the students obtained from these qualifications. The linear equations topic at the seventh grade was taught using the problem-posing approach. The study was designed as a case study and involved twenty students as participants. The data were collected using thirteen problem-posing tasks. At the first step of the study, a problem-posing evaluation rubric was developed. The rubric involved the following criteria: clarity, mathematical accuracy, contextual originality, originality in terms of mathematical relations, complexity level and pertinence to situation qualifications. Then, this rubric was used to identify the qualities of these problems. It was also employed to determine whether or not the mean scores of the participants significantly differed based on the objectives stated. The findings of the study suggest that in parallel to the participants' improvement on the objectives, their mean scores on contextual originality, originality in terms of mathematical relations, and complexity also improved. It is concluded that the integrity of the problem-posing approach into the educational program will improve the qualities of the problems developed by the participants.

Key words: Problem posing, Linear equations, Problem posing evaluation rubric, Qualities of the problems

Introduction

Problem posing can be defined as introducing new problems or rearranging an existing problem and generating various mathematical problems from a given situation in the broadest terms (Leung, 1993; Silver, 1993). Problem posing is also expressed as interpreting concrete situations based on mathematical experience and formulating them as a meaningful mathematical problem and it is emphasized that this definition includes a meaning suitable for the addressing of problem-solving in the context of school mathematics in accordance with the aims of mathematics teaching (Stoyanova & Ellerton, 1996).

Problem posing is a skill the importance of which has been emphasized by scientists such as Einstein, Darwin and Werthemier in terms of mathematical and scientific inquiry (McDonald & Smith, 2020; Silver & Cai, 2005; Stoyanova, 2003). Einstein (Einstein & Infeld, 1938), expressed the importance of problem posing with the following phrase "To raise new questions, a new possibility, to regard old problems from a new angle, requires creative imagination and marks real advances in sciences" (as cited in McDonald & Smith, 2020, p.400). Research on problem posing has demonstrated that problem-posing is closely related to problem-solving and that problem-posing makes some contributions to the development of problem-solving skills (Silver & Cai, 1996). However, the positive effects of problem-posing are not limited to its contribution to problem-solving skills. Posed problems reflect the details of students' understanding of mathematics, their mathematical skills and beliefs so that teachers can also benefit from problem posing to learn about students' mathematical concepts and processes (Kwek, 2015; Klaassen & Doorman, 2015; Stoyanova, 2003; Toluk-Uçar, 2009). It is stated that problem-posing fosters flexible thinking, contributes to creativity and also provides opportunities for

* Corresponding Author: *Katibe Gizem YIĞ, kgizemyig@mehmetakif.edu.tr*
** This study is produced from the doctoral dissertation of the first author.

understanding and interpreting mathematical concepts (Bonotto & Santo, 2015; Canköy, 2014; Işık & Kar, 2012; Kwek, 2015).

Many studies on the algebra learning area have shown that students have difficulty in making sense of algebraic symbolism, coordinate system, linearity and linear equations and have some misconceptions about them. Some of the difficulties encountered in algebra are transforming verbal expressions into algebraic symbolism, providing transitions between different representations, and creating a graph of linear equations (Canadas, Molino & Rio, 2018; Çelik & Güneş, 2013; De Bock et al., 2002; Hattikudur et al., 2012; Nirhamati, Fatimah & Irma, 2020; Sezgin-Memnun, 2011). The necessity of studies to improve the teaching of algebra and geometry, including the topics of the coordinate system and linear relationship, transitions between different representations of a linear relationship and linear equations and to overcome difficulties encountered in the teaching of these subjects has also been emphasized in several previous studies (Birgin, Kutluca & Gürbüz, 2008; Erbas, Çetinkaya & Ersoy, 2009; Mielickey & Wiley, 2016; Sezgin-Memnun, 2011; Yenilmez & Yasa, 2008, Wilkie, 2016). In particular, the understanding of the subjects of the coordinate system and linear relationship forms the basis of the analytic geometry and the subject of function. Therefore, it is noted that lack of understanding in the subjects of the coordinate system and linear equations in middle school may cause misunderstandings about some subjects in the high school mathematics curriculum (e.g. functions, complex numbers, limits, derivatives, integrals) (Birgin & Kutluca, 2006; Turanlı, Keçeli & Türker, 2007). Therefore, it can be maintained that the topic of linear equations is a difficult subject, but it is very important for understanding many different mathematical topics.

In the current study, an instruction was designed with the support of the problem-posing approach considering its possible contributions to the understanding and interpretation of the subject of linear equations as it has been emphasized in many studies that it considerably contributes to the understanding and interpretation of mathematical concepts (Cai & Hwang, 2019; Ticha & Hospesova, 2013; Toluk-Uçar, 2009).

It has been seen in various studies that problem-posing tasks are used as an assessment tool to reveal students' verbal skills, representational skills, and knowledge and skills about the transition between them (Cai et al., 2013; Canadas et al, 2018). The problems posed by the students reflect their mathematical knowledge and skills. Therefore, in this study, it is aimed to examine the problems posed by the students who took linear equations courses supported by the problem-posing approach. For this aim, the qualification of the problems posed by students was explored.

Theoretical framework

Problem posing

The notion of problem-posing was described in various ways by different researchers. To illustrate, Duncaner (1945) expressed it as reformulating a given situation or creating a new problem (as cited in Stoyanova, 2003). Leung (1993) defined problem-posing as formulating a set of mathematical problems from a given situation. Likewise, Silver (1993) stated that problem-posing means both creating a new problem and reformulating the given problems. Accordingly, it can be derived from these definitions that problem-posing can occur before, during or after the solution of the problem.

Considering the opportunities it creates for mathematics education, it is important to include problem posing in teaching mathematics in schools (Singer, Ellerton & Cai, 2013). Being a major element of instruction, problem-posing has also been considered a component of measuring students' mathematical understanding (Cai et al., 2013; Cai & Hwang, 2019).

One of the commonly used classifications for problem-posing tasks belongs to Stoyanova and Ellerton (1996). Stoyanova and Ellerton categorized problem-posing activities as free problem posing, semi-structured problem posing and structured problem posing. According to this theoretical framework, students are asked to pose a problem for a general situation, such as a mathematical calculation problem, a problem they think is difficult in the content of the free problem-posing activity, in semi-structured problem-solving situations, students are given an incomplete problem situation. For example, a problem is created based on an equation or a shape. In structured activities, a well-structured problem and a problem solution are given to create a new problem for this situation (Stoyonova, 2003). It is seen that the structures of the tasks used in problem-posing studies are determined in line with the research purposes.

In some studies, it has been observed that the problems posed by students were evaluated in terms of achieving the related objectives or concepts and the difficulties encountered (Cai et al., 2013; Işık & Kar, 2012). In some other studies, it has emerged that the aim is to measure problem-posing skills and various tools that have been created for this purpose (Bonotto & Santo, 2015; Canköy, 2014; Kaba & Şengül, 2016; Kwek & Lye ,2008; Silver & Cai, 1996).

Different characteristics of the posed problems have been examined in the previous studies. For instance, Silver & Cai (1996) asked students to pose various arithmetic problems by giving them a problem situation. The problems posed by the students were first examined as to whether they were mathematical questions or not. If the posed problem was indeed a mathematical problem, in the next step, the problems were categorized into two groups as solvable and unsolvable questions. If there was not enough information to solve the posed problem, then these problems were included in the category of unsolvable questions. In the final stage, a semantic and linguistic analysis was conducted for the difficulty of the problems in the category of solvable questions. A semantic analysis was also conducted for the unsolvable questions (Silver & Cai, 1996). Several pieces of research were utilized in this analysis process (Bonotto & Santo, 2015; Crespo & Sinclair, 2008; Kwek & Lye, 2008).

In a different study, Crespo & Sinclair (2008) used the problem-posing tasks identified in the literature (Silver & Cai, 1996; Vacc, 1993) with pre-service teachers and analysed them according to the same analysis schemes and found similar results to those reported in the literature. Subsequently, they conducted group discussions with the pre-service teachers about the characteristics that a good problem should have, and they evaluated the problems posed by themselves. At the end of this study, they stated that broader categorizing schemes are needed to evaluate the problems and that aesthetic criteria (e.g. surprise, novelty, fruitfulness) of the problems can also be taken into consideration.

In another study, a detailed analysis of the complexity of the problem was addressed in three categories; namely, low complexity, moderate complexity and high complexity (Kwek & Kye, 2008; Kwek 2015). The problems with low complexity are directed to recognizing and recalling the previously learned information. Answers to problems with moderate complexity require more flexible thinking than problems with low complexity and generally involve more than one step. The problems with high complexity, on the other hand, require more abstract reasoning, creative thinking and association and analytical skills.

In their exploratory study, Bonotto and Santo (2015) also desired to determine the relationship between creativity and problem-posing activities based on real-life situations. Firstly, considering some analysis schemes in the literature (Silver & Cai, 1996; Yuan, 2008; cited in Bonotto & Santo, 2015), the problems were analyzed as to whether they were problems or not and if they included reasonable and adequate data. The problems including reasonable and adequate data were analysed concerning the sub-dimensions of creativity; flexibility, fluency and originality. In the problem-posing stage of this study carried out at three stages, the students were asked to pose various problems in a specified period on the basis of the mathematical situations in a brochure handed out to the students. Flexibility was determined by categorizing the posed problems according to different types of knowledge and problem types included in the reasonable problems; fluency was expressed with the means of the number of the problems posed by the students. Originality, on the other hand, was expressed as the answer to the posed problem being similar to fewer than 10% of the answers to all the problems.

More recently, Kaba & Sengül (2016) developed a rubric to evaluate the posed problems. It is observed that although there are items that show the difficulty level of the problem and the originality of the problem in the draft form of the rubric, these items are removed in the final form. The reason for the removing of the difficulty item is stated as the fact that all problems are problems posed in similar 2-3 steps and this item does not contain any distinguishing properties. The reason why the item of originality is not included in the final version of the rubric is that the students pose problems that are far from real life due to the concern of posing original problems and this item does not make a difference in terms of performance. The problem qualities in the last form of the rubric are the text of the problem (language and expression), the compatibility of the problem with the mathematical principles, the type/structure of the problem and the solvability of the problem.

Another study examining problem-posing skill according to the variables of attitude towards problem-solving, gender and success was carried out by Özgen, Aydın, Geçici, and Bayram (2017). In this research; seven criteria were determined as using mathematical language, grammar and expressiveness, suitability of the problem to the objectives, the amount and quality of the data in the problem, the solvability of the problem, the originality of the problem, and the level of the students are stated as the evaluation criteria for problem-posing skills. Each criterion was assessed on a four-point scale ranging from 1 to 4 levels (0 to 3 point). At the end of the study, it was found that students had difficulty posing problems. It was observed that there was no difference in problem-posing skills according to the gender variable. However, a significant difference was found in relation to problem-posing skills according to general academic success and mathematics course success.

As it can be seen from the review of the studies thus far, various problem-posing qualities emerge according to the feature focused on in problem-posing studies. Since problem-posing tasks are open-ended, it is probable for students to create a wide variety of problems with these problem-posing tasks. Although this variety is the desired situation in terms of teaching, it presents several difficulties as to measurement (Silver & Cai, 2005). Here, the problem of which criteria a teacher will base his/her assessment on while using problem-posing in the classroom arises. In this regard, Kwek (2015) stressed that before making decisions, teachers should take into account their teaching objectives and the potential of problem-posing tasks to provide evidence of these objectives. The characteristics of the tasks complying with the teaching objectives provide the teacher with ideas to determine the criteria for assessing student's achievement. In other words, it is emphasized that the characteristics of a problem-posing task are important in determining criteria for the assessment of problem-posing tasks.

Problem posing in algebra

Students' ability to provide transitions between various representations is very important to perform meaningful learning in algebra and these transitions have received focal attention in some problem-posing studies. Various difficulties in transitions between various representations with problem-posing tasks involving algebra topics have been revealed through problems (Cai et al., 2013 & Canadas et al., 2018).

Canadas et al. (2018) used problem-posing to determine students' understanding of algebraic expressions and the difficulties experienced according to the characteristics of the given algebraic statements. In the study, pre-service teachers were given free and semi-structured problem-posing tasks containing symbolic statements. The problems were examined in terms of syntactic structure and semantic structure. Thus, the problems were converted into algebraic symbolism to analyze the syntactic structure, the compatibility with the symbolic statement has been examined. Semantic analysis was discussed within the additive and multiplicative structure. Additive problems were classified into comparison, part part whole and change categories. In addition, multiplicative problems were classified as comparison, cartesian and equal grouping. Meaningless problems, or problems that do not require algebraic symbolism to be solved, were not taken in evaluation. The results have demonstrated that in most cases, students posed problems in a syntactic structure different from the given symbolic expressions, so it was inferred that students have difficulty giving meaning to a given statement. It was observed that additive structures can pose problems more easily than multiplicative problems, and the rate of problems in comparison type is lower in both multiplicative and additive structural.

Cai et al. (2013) aimed to measure the effect of secondary school curriculum on high school learning by using problem posing as a tool. This study aimed to determine the effects, similarities and differences of a standard-based curriculum and a more traditional curriculum on students' learning algebra. The problem-posing task used in the research consists of two parts. In the first task, a system of equations task was given and the participants were asked to find the x and y values by solving them. After that, it was asked to the participants to pose a problem that could be solved by using the given system of equations. In the second task, a graph was provided and the students were asked to write an equation that would produce the given graph. Afterwards, the students were asked to write a real-life situation that could be represented by this graph. At the end of the research, it was found that 22% of the students were able to solve the equation system and only 19% of them could write the correct equation for the graph task. Only 6.2% of the students were able to pose a problem for the system of equations task, and 16.6% could pose a valid problem for the graph task. When the conditions of the posed problems to meet the conditions given in the problem are examined, it baceame obvious that only 14.3% for the system of equations task and 18.3% for the graph task match at least one condition. As a result, it was concluded that a small proportion of the students were able to pose valid problems for both tasks.

In the current study, problem-posing is integrated into the linear equations instruction, which is considered a challenging subject for most students. It is aimed to examine the problems related to linear equations posed by the students taking this course. The qualifications of the posed problems were determined for this aim. Linear equations topic includes 3 separate subtopics: coordinate system, linear correlation and linear equation graphs (Mone, 2013).

In problem-posing researches, the difficulties with various representations, and transitions with these representations were indicated. It has been observed that representational changes within the same subject have a different level of difficulty (Cai et all, 2013). While different difficulties arise in different representations of the same concept, it was aimed to examine whether the qualification of the problems posed for the subtopics of a mathematics subject differ at a significant level.

From this point of view, this study aimed to determine whether the qualities of the problems in the mathematics teaching program ((MoNE, 2013), the sub-topics of the coordinate system, linear correlation and linear equation graphs, which are included in the 7th-grade linear equations topic, differ or not.

To this end, answers to the following research questions were sought throughout the extant study.

1. What are the qualities of the problems that the students posed in the linear equations courses supported by a problem-posing approach?
2. How do mean scores gotten from the problem qualities vary according to the objectives set in the curriculum?

Methodology

The current case study was conducted in the classes where the subject of linear equations was taught with the support of the problem-posing approach to 20 seventh grade students. The students were chosen by the typical sampling method to study the average ranking of successes. The mathematics teachers in a province in the Mediterranean region in Turkey were interviewed to determine the study sample. The students in the classroom of a teacher who volunteered to participate in the study were included in the study. To find answers to research problems, the problem-posing assessment rubric was developed by the researcher. Then, by using this rubric it was aimed to determine the qualities of the problems posed by the students and to investigate whether the means of the scores obtained from these qualities varied in terms of the objectives set in the math curriculum. The lesson contents prepared are related to the subject of linear equations taught in the seventh grade. The related objectives set in the Turkish 5^{th} – 8^{th} Math Curriculum are presented in Table 1 below (MoNE, 2013).

Table 1. Objectives set for the subject of linear equations

Objective No.	Short names to denote the objective	Objectives
1	Coordinate system	Knows the coordinate system with its features and shows ordered pair.
2	Linear correlation	Expresses how one of the two variables having a linear correlation between them changes depending on the other with tables, graphs and equations.
3	Linear equation graph	Draws the graph of linear equations

The problem-posing tasks were integrated into the course content. The course contents were prepared by the researchers with the support of an expert group constituted by math educators and the data collection process was conducted in continuous cooperation with the course teacher. In the 2015-2016 school year, a 12-hours pilot study was carried out to test the relevance of the course outlines and problem-posing tasks. Then required corrections for course contents and the problem-posing tasks were made for the final form of the course contents by the researchers. Incomprehensible points were determined in problem-posing tasks and some explanations were added and linguistic arrangements were made to clarify them. A 12 hour-class for the subject of the linear equation was conducted by the course teacher in their normal class hours in the fall term of the 2016-2017 school year. Before the linear equations lessons, 3 hours of problem-posing exercises were conducted with the students. It was ensured that the students were familiar with problem-posing. All three types of problem-posing tasks are integrated into linear equations lessons. The general purpose of the structured problem-posing activities is to reinforce newly learned knowledge and to understand in which contexts these mathematical concepts are needed. As noted by some scholars, such activities would be useful for students to start posing problems and to understand the structure of the problem (e.g. Stoyanova, 2003). The tasks used towards the end of the lessons are semi-structured and free problem-posing. By giving more general situations with semi-structured and free problem-posing tasks, students were guided to reveal their mathematical knowledge and skills in their new acquisition. A total of 13 problem-posing tasks integrated into the course were used as the data collection tool. These problem-posing tasks were designed in such a way to support mathematical knowledge and skills related to the concept. As proportional to the weight of the objectives in the curriculum, there are two tasks (one semi-structured and one free) for the coordinate system; six tasks (five semi-structured and one free) for linear correlation and five tasks (four semi-structured and one free) for the linear equation graphs objectives (Karaaslan, 2018, pp.241-261). Students individually posed problems for the tasks integrated into the course flow. Sufficient time is allowed for all students to complete the problem.

A sample problem-posing task integrated into the teaching of the subject of the coordinate system is given in Figure 1 below. Before the problem-posing task, to draw the attention of students to coordinate system objective, Descartes' anecdote about how to locate the position of a fly walking on the ceiling was told. A discussion was conducted with the students on the basis of this anecdote and then definitions of ordered pair and the features of the areas were discussed through the problem tasks. Following the problem-solving activities, the students were facilitated to pose similar problems through the structured problem-posing tasks and then they were asked to pose their own problems to foster their mathematical comprehension and problem-posing skills

PROBLEM ACTIVITY 4 - PROBLEM POSING: of the A, B, C and D points, two are on the axes and the other two are in another region. By using the information given, place the points in the coordinate system and by using these points, pose a different solvable problem including a real-life situation and then solve the problem you have posed.

Figure 1. The task of posing a sample problem in the subject of the coordinate system

Mathematical situations allowing them to verbally explain the mathematical patterns and relationships given for the subject of linear correlation and to relate them to real-life situations were preferred. Problem situations that require mathematical relations to be expressed in tables, graphs and algebraic expressions and transitions to be made between these representations were used. For the subject of graphs of linear equations, problem situations that enabled the creation of graphs and to relate them to other representations were used.

Data Collection and Analysis Procedures

In order to find an answer to the first research question "What are the qualities of the problems that the students posed in the linear equations courses supported by a problem-posing approach?" a problem-posing assessment rubric was developed by the researchers. On the other hand, to find an answer to the second research question "How do the scores gotten from the problem qualities vary in relation to objectives?", one of the non-parametric tests; the Friedman test was used. For the problem qualities for which a significant difference was observed, the non-parametric Wilcoxon signed ranks test was used to find the difference between the objectives.

Different methods were employed to increase the validity and reliability of the study. Throughout all the processes followed in the current study such as data collection, data analysis and interpretation, great care was taken to be consistent. A long-term interaction was carried out with the data sources and while evaluating the problems posed by the students, first all the problems were carefully examined and then the problems with similar qualities were coded into the categories in the rubric. In the evaluation process of the problems, besides the researchers, another expert was involved in the process for the expert confirmation. The stages followed in the rubric development process are given below in detail.

Development of a Problem-Posing Evaluation Rubric

As the problem-posing tasks were open-ended, it was possible to create a great variety of problems from these tasks. Although this variety is a desired situation in terms of teaching, it presents several difficulties in terms of measurement (Silver & Cai, 2005). Here, the problem of which criteria a teacher will base his/her assessment on while using problem posing in the classroom arises. In this regard, Kwek (2015) stated that before making decisions, teachers should take into account their teaching objectives and the potential of problem-posing tasks to provide evidence of these objectives. The characteristics of the tasks complying with the teaching objectives provide the teacher with ideas to determine the criteria for assessing students. In other words, it is emphasized that the characteristics of a problem-posing task are important in determining criteria for the assessment of problem-posing tasks.

Silver and Cai (2005) talked about three main criteria that might be used for the assessment of problem posing. These are quantity, originality and complexity. Quantity refers to the number of problems correctly posed in accordance with the problem-posing task. Here, the teacher can take into account the number of problems correctly posed by the student, as well as the number of correct answers that are different from each other. Constructing a large number of answers in a fluent manner has also been regarded as a measure of creativity (Guilford cited in Silver & Cai, 2005). When problem-posing tasks are used to measure creativity, the dimension of originality emerges as another evaluation criterion. Particularly in problem-posing tasks applied to a large number of people, the answer that is different from the typical answers given by the people is considered original (Silver & Cai, 2005). Another criterion of evaluation is complexity. Silver and Cai (2005) stated that complexity is a criterion that can be addressed in many respects. One way of evaluating complexity is to examine the scope of mathematical relationships within the problems posed by students. Kwek and Lye (2008), for example, discussed the complexity in terms of mathematical relationships involved in the problem. According to this approach, the problems of low complexity are considered to be problems for recalling and recognizing prior learning. Answers to problems with moderate complexity require more flexible thinking than the problems of low complexity, and these problems often involve more than one step. The problems with high complexity, on the other hand, require more abstract reasoning, creative thinking and association and analysis skills. Another way of evaluating complexity is to evaluate the difficulty level of problems (Silver & Cai, 2005). There is also linguistic complexity. The works of Silver and Cai (1996), Işık and Kar (2012) can be given as examples to the studies that accept linguistic complexity as a criterion.

In addition to the criteria of quantity, originality and complexity, there are also other evaluation criteria used. Vacc (1993), for example, evaluated the type of problem posed, and Canköy (2014) evaluated the solvability, logic, and structure of the problem. Kaba & Şengül (2016) looked at the language and expression of the problem, the compatibility of the problem with the mathematical principles, the type/structure of the problem and the solvability of the problem. The problem evaluation criteria discussed in the literature are summarised in Table 2.

Table 2. Major problem evaluation criteria

Researchers	Publication date	Problem evaluation criteria for characteristics
Kaba & Şengül	2016	The Text of the Problem (Language and Expression) The Compatibility of the Problem with the Mathematical Principles The Type/Structure of the Problem The Solvability of the Problem
Bonotto & Santo	2015	Flexibility Fluency Originality
Canköy	2014	Solvability Reasonability Mathematical Structure
Kwek & Lye	2008	Mathematical Complexity
Silver & Cai	2005	Quantity Originality Complexity
Silver & Cai	1996	Mathematical question/nonmathematical question/statement Solvability Mathematical Complexity Linguistic Complexity
Vacc	1993	Question Types

A closer inspection of the literature reveals that studies investigating problem-posing focused on different qualities depending on the research purpose. While developing the problem-posing assessment rubric in the current study, a detailed investigation of the problem qualities discussed in the literature was conducted (Bonotto & Santo, 2015; Canköy, 2014; Kaba & Şengül, 2016; Kwek & Lye, 2008; Silver & Cai, 1996; 2005; Vacc, 1993) and during this investigation, it was realized that a rubric to serve the purpose of the current study in particular needed. In line with the purpose of the study, the opinions of 8 math education experts were sought and thus the qualities to be taken into consideration in the current study were determined. The qualities taken into consideration in the current study are; clarity, mathematical accuracy, contextual originality, originality in terms of mathematical relations, complexity level and pertinence to the situation (Karaaslan, 2018, pp.265-280). While developing the rubric, rubric development stages proposed by Andrade (2000) were followed. Problem posing tasks left unanswered or incompletely answered were not taken into evaluation. Scores for the problems were calculated by the researcher and another expert math educator and the reliability coefficient calculated according to Miles & Huberman (2015, s.64) formula “reliability = the number of agreements / (the number of agreements + the number of disagreements)” was found to be 0.92. The problems on which the experts could not agree were discussed again and thus an agreement was reached and the problem qualities scores were calculated according to this.

Findings

Findings on the First Research Question: “What are the Characteristics of the Problems That the Students Developed in the Linear Equations Courses Supported by a Problem-Posing Approach?”

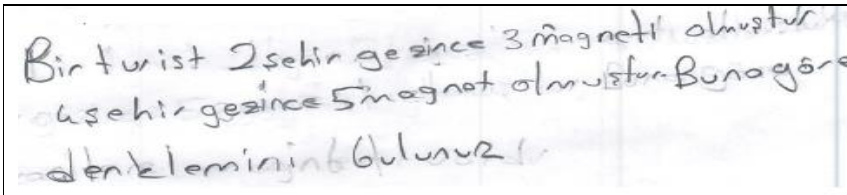
The problems posed by the students were evaluated with the problem-posing assessment rubric and the findings obtained in relation to the qualities of the problem are presented in Table 3.

Table 3. Distribution of the scores across the problem qualities

Score	Clarity		Mathematical accuracy		Contextual originality		Originality in terms of mathematical relationship		Complexity level		Pertinence to situation qualifications	
	f	%	f	%	f	%	f	%	f	%	f	%
0	5	%2,3	56	%25,2	65	%29,3	31	%14	11	%5	24	%10,8
1	92	%41,4	31	%14	75	%33,8	165	%74,3	95	%42,8	60	%27
2	78	%35,1	49	%22,1	34	%15,3	22	%10	110	%49,5	42	%18,9
3	47	%21,1	86	%38,7	48	%21,6	4	%1,8	6	%2,7	96	%43,2

When the scores obtained by the students from the problem qualities were examined, it was found that only 5 problems (2.3%) were given 0 point for clarity. When there was unnecessary information in the problem statement, when the problem statement did not have unity; that is, when the problem was unclear, it was given 0 point. The fact that 41.4% of the students got 1 point shows that many problems could be understood when they were examined together with the information external to the problem, such as notes written on the problem task, notes on the graph and if there was, the solution to the problem. Seventy-eight (35.1%) of the problems were assigned 2 points, which shows that what was intended to be asked was understood in general but there is also some linguistic unclarity. The problems in this category include one or more difficulties, such as incorrect use of affixes, use of some words with wrong meanings or missing elements in the sentence (e.g., it was asked to write the equation. However, which linear relation should be written was not clearly explained). Only in 21.1% of the problems, the problem statement was clear and understandable which clearly shows that the students experienced difficulty in posing linguistically understandable problems.

When the mathematical accuracy of the problems was examined, it was found that 56 (25.2%) of the problems included understandings completely erroneous from a mathematical point of view. For example, cases such as thinking that the points in the coordination system can be summed up, not understanding that a point in the coordination system is an ordered pair or not understanding the relationship between two variables and the interdependent variation were assigned 0 point in terms of mathematical correctness. 31 problems (14%) were assigned 1 point. These problems included non-systematic mistakes. The problems in this category included one or more of the cases, such as improper use of definitions (improper use of mathematical definitions and expressions related to coordinate system, point, ordinate, axis etc.), mistakes resulting from calculation errors or inattention while posing the problem (e.g., although the linear correlation; that is, interdependent variation of two variables was conceptually understood, there were calculation errors), incorrect use of mathematical notations (showing ordered pairs without putting them into parentheses). A sample problem assigned 1 point in terms of mathematical correctness is illustrated in Figure 2.


Bir turist 2 şehir gezince 3 magneti olmuştur 4 şehir gezince 5 magnet olmuştur. Buna göre denkleminin bulunuz!

When a tourist visited 2 cities, he/she had 3 magnets. When he/she visited 5 cities, he/she had 5 magnets. Find the correct equation for this.

Figure 2. A sample problem assigned 1 point in terms of mathematical correctness

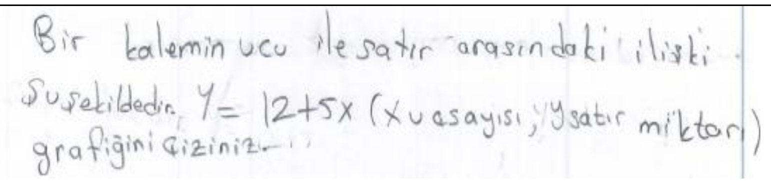
In this problem, it can be generally understood in respect of what is mathematically required. In the problem, it was intended to use the relationship between the number of cities visited and the number of magnets possessed. However, there is no expression showing that the relationship between the variables is linear. Moreover, it was not clearly explained between which variables the relationship that would constitute the equation was. Therefore, 1 point was assigned to it in terms of mathematical correctness. Forty-nine problems (22.1%) receiving 2 points for the mathematical accuracy included missing or unnecessary mathematical expressions. The problems in this category included one or more of the following cases: the values taken by the variables were exemplified with few numbers yet no expression indicating that the relationship persists as it is or that the relationship is linear was included in the problem; though it is understood that the graph related to the situation or the relationship between variables was asked to be drawn, it is not clear between which variables the relationship would be shown or which situation was asked to be shown on the coordinate system. Only 86 problems were assigned 3 points; that is, 38.7% of the problems included correct mathematical information, definitions, concepts and symbols.

When the problems were examined in terms of contextual originality, it was found that 65 problems (29.3%) drew on usually used contexts that are not original, used in the courses or the textbooks. For example, when one of the following contexts was used, 0 point was assigned: the use of the desks in the classroom or the cinema context for the coordinate system, the use of distance-time relationship, consumed full-time relationship and the growth of a sapling-time relationship for a linear relationship. Problems receiving 1 point from the contextual originality are problems that can be encountered in textbooks, include frequently used contexts, and in which no contribution by the student to context is observed.

For example, when contexts such as determination of the position of the houses on streets or cars in a car park for the coordinate system; the number of pages read and the time passed, money put into a moneybox and the time passed for a linear relationship and equation graphs were used, then 1 point was assigned for the 75 problems (33.8%). Thirty-four problems (15.3%) problems receiving 2 points from the contextual originality included contexts that could be found in textbooks, yet in which some original contributions were made to the context by the student. On the other hand, 48 problems (21.6%) were evaluated to be contextually original problems. These problems included a thoroughly original context that had never been encountered in classes or textbooks.

For the originality in terms of mathematical relationships, 31 problems (14%) were assigned 0 point. These problems included the same mathematical topics and situations as the ones found in the problems frequently studied in classes and textbooks. For instance, they included relationships, such as asking the coordinates of new points on the coordinate system by shifting one point on the coordinate system, asking the distance of the points to each other, giving one of the variables making up the linear relationship and asking the other. A total of 165 problems (74.3%) were assigned 1 point. Problems receiving 1 point are not original in nature, and include mathematical subjects and situations in the problems of linear equations in books. For example, the problems including the following cases were assigned 1 point: creating a closed area with certain points on the coordinate systems and asking the circumference of the area, asking the formation of the graph, equation and table for a linear relationship. A total of 22 problems (10%) received 2 points. If the problem included a mathematical topic or situation similar to the problems presented in classes or textbooks but if various contributions were made to the situation, the problem was assigned 2 points in terms of originality in mathematical relationships. For example, the problems including cases such as the following: instead of asking the area of the closed shape constructed by combining certain points given in the coordinate system, giving three points and asking the addition of the fourth point so that the required shape could be formed (for example, to form a rectangle) and then asking for the calculation of the area of the rectangle to be formed in this way were assigned 2 points. If there was a problem requiring the use of a mathematical topic that had not been encountered in the problems of linear equations in course or textbooks, then 3 points were assigned for this problem. Only four problems (1,8%) were in this category.

Eleven problems (5%) received 0 points from the complexity of the problem. Problems requiring the direct recall of the knowledge and not requiring any operations were assigned 0 points. Problems asking for showing an ordered pair on the coordinate system, giving a table or graphical representation of a linear relationship and asking the value of the other variable according to the value taken by one variable can be shown as examples of such problems. Ninety-five problems (42.8%) have a low level of complexity and they were assigned 1 point. Such problems required more than just recalling the prior knowledge. Nevertheless, they only required following the procedures and included one-step operations. For example, problems asking the distance between two points, requiring the formation of the next ordered pair in a problem in which table values of two variables are given, asking for the calculation of the value taken by the other variable in a relationship given with an algebraic expression in which the value of the one variable is known were included in this category. A total of 110 problems (49.5%) were found to have moderate complexity and they were assigned 2 points. The problems in this category require the problem solver to think more flexibly than the problems with low complexity, to decide what to do and to use information obtained from different representations. These are the problems generally solved in more than one step. Problems asking for the calculation of the area of the closed shape formed by combining many points given or of its circumference; asking for the verbal expression of a linear relationship and formation of the algebraic expression or graph suitable for this relationship; asking for the graph of a relationship on the coordinate system whose table values are given were included in this category. A sample problem receiving 2 points from the complexity is shown in Figure 3. The problem presented in Figure 3 requires a transition from an algebraic expression to a graphical representation.


Bir kalemin ucu ile satır arasındaki ilişki. Şu şekildedir. Y= 12+5x (X ucasayısı, Y satır miktarı) grafiğini çiziniz.

The relationship between the tip of a pencil and the line is as follows: y = 12+5x (x = the number of tips, y= the number of lines). Draw the related graph.

Figure 3: A sample problem receiving 2 points from the complexity quality

Only 6 problems (2.7%) received 3 points from the complexity. Students' not being able to form complex problems is another remarkable finding. Complex problems require more reasoning and include multiple steps and decision-making points. For example, a problem in which three points are given in the coordinate system and asking for the formation of the fourth point under certain conditions and then asking for the calculation of the area of the closed shape to be constructed by combining these four points was assigned 3 points. Three points were also assigned to the problems requiring the formation of a table of a linear relationship and then asking for the formation of another relationship on the basis of these values in their complexity quality.

While posing a problem, students are first expected to attempt to meet the stated conditions. There are 24 problems (10.8%) receiving 0 points from compliance with the condition's quality. If a problem did not meet any of the conditions stated in the problem-posing task, it was assigned 0 points. The problem-posing tasks used in the current study were developed to address 3 different objectives. Each problem-posing task was designed to meet more than one condition. For example, there are cases in which points were asked to meet certain conditions in the subject of the coordinate system and a real-life situation was asked to be used in the problem. There are some cases in the subject of a linear relationship in which the type of representation to be used in the problem situation or solution is known and in addition to this, a different type of representation is asked to be used for the linear relationship. There are cases in the subject of linear equation in which the type of representation to be involved in the problem situation or solution is known and/or in which the use of a real-life situation is asked and/or in which special conditions are asked for the equation. Sixty problems (27%) only meeting one of these conditions were assigned 1 point. When many of the required conditions were satisfied, these problems were assigned 2 points and 42 problems (18.9%) were found to be in this category. Ninety-six problems (43.2%) were found to meet all the conditions required in the problem-posing task. Although the students were told to meet all the conditions involved in the problem-solving task throughout the lessons, more than half of them posed a problem not complying with the conditions. Instead of meeting all the conditions in the problem-posing task, the students may have focused on few conditions through which they thought they would pose suitable problems.

Findings on the Second Research Question: How do the Mean Scores Obtained from the Problem Qualities Vary According to the Objectives Set in the Curriculum?

In order to determine whether there are differences between the mean scores obtained by the students for each of the three objectives, one of the non-parametric tests; the Friedman test, which is used to compare more than two measurements belonging to one group, was employed in the current study. Results are presented in Table 4.

Table 4. Results of the Friedman test for the objectives

	Objectives	N	Mean Rank	df	χ^2	P
Clarity	Coordinate system	20	1.93	2	5.787	.055
	Linear correlation	20	1.68			
	Linear equation graph	20	2.40			
Mathematical accuracy	Coordinate system	20	1.98	2	4.880	.087
	Linear correlation	20	1.68			
	Linear equation graph	20	2.35			
Contextual originality	Coordinate system	20	1.43	2	24.080	.000
	Linear correlation	20	1.73			
	Linear equation graph	20	2.85			
Originality in terms of mathematical relationship	Coordinate system	20	1.43	2	10.364	.006
	Linear correlation	20	2.25			
	Linear equation graph	20	2.33			
Complexity level	Coordinate system	20	1.55	2	19.795	.000
	Linear correlation	20	1.65			
	Linear equation graph	20	2.80			
Pertinence to situation qualifications	Coordinate system	20	2.73	2	16.545	.000
	Linear correlation	20	1.70			
	Linear equation graph	20	1.58			

As can be seen in Table 4, there is no statistically significant difference between the mean scores obtained for the clarity of the problem in relation to the objectives ($\chi 2$ = 5.787, p (0.055) > 0.05). Similarly, no significant difference was found between the mean scores obtained for the mathematical accuracy of the problem in relation to the objectives ($\chi 2$ = 4.880, p (0.087) > 0.05). Given that the time of the course was not very long, that 3 class hours were allocated for the first objective, 5 class hours were allocated for the objective of a linear relationship and 4 class hours for the objective of equation graphs, it can be said that this period was not enough for the students to develop the clarity of the problem which is related to their communication skills. The determination of whether this quality can be developed by conducting longer courses can make some contributions to the field.

It was also found that there is no statistically significant difference between the mean scores obtained for the mathematical accuracy quality. It should be taken into consideration that different mathematical knowledge was required for each of the objectives concerning the mathematical correctness quality. For instance, while the correctness of the mathematical knowledge related to ordered pairs and represent ordered pair on the coordinate system was investigated for the subject of the coordinate system, the correctness of the mathematical knowledge in the subjects of expression of the linear relationship in a manner suitable for different representations and the formation of the graph for the drawing of the equation graph were investigated in relation to the objective of the linear relationship. However, working for a longer period on the same objective is considered to create a significant difference in the mean scores obtained from the mathematical accuracy quality.

As can be seen in Table 4, there is a significant difference between the mean scores of the contextual originality of the problem in relation to the objectives ($\chi 2$ = 24.080, p < 0.05). In order to determine the source of difference, Wilcoxon signed ranks test was used and the results are presented in Table 5.

Table 5. Wilcoxon signed ranks test results for contextual originality

Contextual originality		N	Mean rank	Sum of ranks	z	p
Linear correlation-Coordinate system	Negative sequence	5	10	50	−.570[b]	.568
	Positive sequence	10	7	70		

	Equal	5				
Linear equation graph- Coordinate system	Negative sequence	1	1	1	-3.886^b	.000
	Positive sequence	19	11	209		
	Equal	-				
Linear equation graph– Linear correlation	Negative sequence	2	2	4	-3.771^b	.000
	Positive sequence	18	11.44	206		
	Equal					

b was arranged based on the negative sequence

In Table 5, the results show that there is no significant difference between the contextual originality mean scores in relation to the objectives of a linear relationship and coordinate system $(z = -0.570,\ p(0.568) > 0.05)$. On the other hand, a significant difference was found between the objectives of equation graphs and the coordinate system in favour of the objective of equation graphs $(z = -3.886,\ p < 0.05)$. When the difference between contextual originality mean scores for the subjects of equation graphs and the linear relationship was examined, a significant difference was found in favour of the subject of equation graphs $(z = -3.771, p < 0.05)$. As a result, it can be argued that the contextual originality means score obtained by the students from the problems they posed in the subject of equation graphs is higher than the contextual originality means scores obtained from the problems posed in the subjects of the linear relationship and coordinate system. That is, the students were able to pose more original problems for the objective largely addressed at the end of the study. Thus, it can be argued that the students' receiving an instruction supported with problem-posing throughout the lesson was influential in improving the contextual originality.

According to the results of the Friedman test, the originality in terms of mathematical relationships quality mean scores vary significantly by the objectives $(\chi^2 = 10.364,\ p < 0.05)$. In order to determine the source of this difference, Wilcoxon signed ranks test was run. The results of this test are shown in Table 6.

Table 6. The results for the originality in terms of mathematical relationship quality

Originality in terms of mathematical relationship		N	Mean Rank	Sum of ranks	z	p
Linear correlation-Coordinate system	Negative sequence	5	9.20	46	-2.207^b	.027
	Positive sequence	15	10.93	164		
	Equal	-				
Linear equation graph-Coordinate system	Negative sequence	3	10.67	32	-2.560^b	.010
	Positive sequence	16	9.88	158		
	Equal	1				
Linear equation graph–Linear correlation	Negative sequence	9	8.78	79	$-.284^b$	.776
	Positive sequence	9	10.22	92		
	Equal	2				

b was arranged based on the negative sequence

As can be seen in Table 6, there is a significant difference between the mean scores of the originality in mathematical relationships for the objectives of the linear relationship and coordinate system in favour of the objective of the linear relationship $(z = -2.207,\ p < 0.05)$. A significant difference was also found between the mean scores obtained from the originality in mathematical relationships for the objectives of equation graphs and the coordinate system in favour of the objective of equation graphs $(z = -2.560,\ p < 0.05)$. No significant difference emerged between the mean scores obtained from the originality in mathematical relationships for the objectives of the equation graphs and linear relationship $(z = -.284,\ p(0.776) > 0.05)$.

As a result, it seems that the problems posed for the objectives of the linear relationship and equation graphs are more original in terms of mathematical relationships than the problems posed for the coordinate system. There is no statistically significant difference between the mean scores obtained for the objectives of equation graphs and linear relationship. However, because mean scores for the objective of equation graphs are slightly higher than the mean scores for the objective of linear relationship, it can be argued that as the instruction process progressed, the students' scores acquired from the originality in mathematical relationships quality also increased. Thus, it can be said that the problem posing-enhanced instruction had positive effects on the originality in mathematical relationships quality. When the process progressed and the students posed more problems, they attempted to integrate more different mathematical relationships into their problems.

According to the results of the Friedman test, the complexity level quality mean scores vary significantly by the objectives $(\chi^2 = 19.795,\ p < .05)$. In order to determine the source of this difference, Wilcoxon signed ranks test was run. The results of this test are shown in Table 7.

Table 7. Wilcoxon signed ranks test results for the complexity level

Complexity level		N	Mean Rank	Sum of Ranks	z	p
Linear correlation-Coordinate system	Negative sequence	8	9.38	75.00	$-.459^{b}$	.646
	Positive sequence	10	9.60	96.00		
	Equal	2				
Linear equation graph- Coordinate system	Negative sequence	2	10.25	20.50	-3.160^{b}	.002
	Positive sequence	18	10.53	189.50		
	Equal	-				
Linear equation graph– Linear correlation	Negative sequence	2	7.50	15.00	-3.363^{b}	.001
	Positive sequence	18	10.83	195.00		
	Equal	-				

b was arranged based on the negative sequence

Table 7 shows that there is no significant difference between the mean scores of the complexity level for the objectives of the linear relationship and coordinate system $(z = -0.459,\ p(0.646) > 0.05)$. However, between the mean scores for the objectives of equation graphs and coordinate system, there is a significant difference in favour of the objective of equation graphs $(z = -3.160,\ p < 0.05)$. The mean scores of the complexity level quality for the objectives of equation graphs and linear relationship, there is a significant difference in favour of the objective of equation graphs $(z = -3.363\ \ p < 0.05)$. As a result, it can be said that the problems posed in the subject of equation graphs are more complex than the problems posed in the subjects of the linear relationship and coordinate system. Although no significant difference was found between the mean scores obtained for the objectives of the linear relationship and coordinate system, the mean score obtained for the objective of linear relationship was found to be slightly higher than that of the objective of the coordinate system. In this regard, it can be concluded that the students were able to pose increasingly more complicated problems throughout the process. Thus, it can be argued that the problem-enhanced instruction had positive effects on the complexity level of the problem. According to the results of the Friedman test, the mean scores gotten from the pertinence to situation quality varied significantly depending on the objectives $(\chi^2 = 16.545,\ p < 0.05)$. In order to determine the source of this difference, the non-parametric Wilcoxon signed ranks test was administered. The results of this test are presented in Table 8.

Table 8. The results for the compliance with the quality of the conditions

Pertinence to situation qualifications		N	Mean Rank	Sum of Ranks	z	p
Linear correlation-Coordinate system	Negative sequence	15	10.67	160	-3.250^{b}	.001
	Positive sequence	3	3.67	11.00		
	Equal	2				
Linear equation graph-Coordinate system	Negative sequence	18	10.36	189.50	-3.685^{b}	.000
	Positive sequence	1	3.50	3.50		
	Equal	1				
Linear equation graph–Linear correlation	Negative sequence	10	10.65	106.50	$-.056^{b}$	.995
	Positive sequence	10	10.35	103.50		
	Equal	0				

b was arranged based on the negative sequence

According to the results in table 8, there is a significant difference between the mean scores acquired from the pertinence to situation qualifications for the objectives of the linear relationship and coordinate system $(z = -3.250,\ p < 0.05)$. Also, there is a significant difference between the mean scores for the objectives of equation graphs and coordinate system, in favour of the objective of the coordinate system ($(z = -3.685,\ p < 0.05)$. And, there is no significant difference between the mean scores for the objectives of equation graphs and linear relationship $(z = -0.056\ p(0.995) > 0.05)$. Thus, it can be argued that the problems posed by the students within the objective of the coordinate system are in greater pertinence to situation qualifications. When the problem-posing tasks given in the subject of the coordination system were examined, it was seen that there were fewer conditions when compared to those found in the subjects of the linear relationship and equation graphs. In the subjects of the linear relationship and equation graphs, there were more semi-structured problem-posing tasks, showing that there were certain conditions required to be met in the problem-posing task. It is argued that the types of problem-posing tasks rather than the content of the objective affected the scores gotten from the pertinence to situation quality.

Results, Discussion and Suggestions

When the scores obtained from the problem qualities are examined, it is seen that the thoroughly clear problems constitute 21% of all the problems posed. The difficulty experienced by students in posing problems has also been reported in other studies (e.g. Cetinkaya, 2017; Kwek & Lye, 2008). As for the current study, it should be noted that the students encountered problem posing for the first time and that a total of 12 class periods were allocated to the teaching of the subject of the linear equation. No significant difference was observed between the scores acquired from the clarity of the problem for the objectives. However, it is contended that when students pose more problems in classes, they will be able to improve the clarity of the problem, one of the qualities the students were found to be bad at.

In the analysis schemes used in some of the problem-posing activities, it was observed that the problems including mathematically missing/incorrect data were not included in the evaluation (Bonotto & Santo, 2015; Silver & Cai, 1996; Silver & Cai, 2005; Kwek & Lye, 2008). In a study conducted by Kwek and Lye (2008) with 120 gifted middle school students, they first separated the problems that could not be solved by using the scheme developed by Silverand Cai (1996) and then performed various analyses related to the level of complexity of the solvable problems. However, in these studies, especially when the problems that require calculations with four operations are examined, it is seen that it is not possible to reach a solution when there is missing data. In the current study, on the other hand, when the student's drawings, graphs and solutions are examined holistically in some posed problems, it appears that a general idea about what was asked could be gained. For example, a student used the expression "the sum of the origin's points" in his/her problem. Here, it is understood that the student wanted to refer to the numerical values of the apsis and ordinate components of the origin. This problem was subjected to evaluation and its errors were evaluated within the mathematical correctness quality.

When the mathematical accuracy of the problems was examined, only 38.7% of the problems were evaluated as completely and mathematically correct and received 3 points. As a result, given that fewer than half of the problems posed by the students were completely correct in mathematical terms, it can be said that the math problems posed were inadequate in terms of mathematical accuracy quality. The mean scores of the students in the mathematical accuracy quality were found to be not varying significantly depending on the objectives. Here, it should be noted that different math knowledge was used for each objective about the mathematical correctness quality. However, the determination of whether different results can be obtained when the period of the study directed to the same subject or same objective is extended can make some contribution to the field.

For the mathematical accuracy quality, it may not always be possible to understand whether students have posed their problems with the awareness of mathematical requirements involved in their problems based on the written documents of the posed problems. It has been revealed in previous studies that some students have various misconceptions especially about the subject of linearity (De Bock et al., 2002; Hadjidemetriou & Williams, 2002; Leinhardt et al., 1990). In the current study, interviews were not conducted with the students about all the problems they posed. However, it is believed that for the problems posed in subjects, such as linearity in which students are expected to experience some difficulties, conducting interviews or asking students to explain their problems through the courses will allow for conducting a better evaluation of the issue under investigation.

Another remarkable finding is that the mean scores acquired by the students from the contextual originality and originality in mathematical relationships qualities are lower than the mean scores from the other qualities addressed in the current study. It was found that 21.6% of the problems received 3 points for their originality; namely, they were evaluated as being original. Only 1.8% of the problems were found to be original in terms of mathematical relationships. Thus, it seems clear that the students were not successful in posing mathematically original problems. The high majority of the problems received 1 point for their mathematical originality; namely, it was seen that majority of the students posed problems including mathematical relationships that can be seen in textbooks. Similar results have been reported by the studies investigating problem-posing skills in terms of originality in the literature. To illustrate, Bonotto (2013) conducted a study on children aged 10-11 and found that the majority of the problems posed are similar to the problems found in textbooks. Korkmaz and Gür (2006) stated that while posing problems, the pre-service teachers adhered to textbooks. Cetinkaya (2017) also reported that while the 5th graders were posing problems, they used the same contexts and posed problems similar to the problems they had seen in their academic life. Therefore, it can be inferred that students tend to pose problems in the ways they have encountered before and that they experience difficulty in posing problems that can be considered original. On the other hand, towards the end of the research process, the scores obtained from the contextual originality and originality in mathematical relationships qualities can be said to have increased. Thus, it is suggested that the problems posed by the students have made positive contributions to the development of these qualities.

When the problems were examined in relation to the complexity of the problem, 42.8% of the problems were found to be at the low complexity level while 49.5% of the problems were found to be at the moderate complexity level. Only 2.7% of the problems were found to be highly complex and to include more reasoning and decision-making steps. Similar results were also found in the study conducted with the gifted middle school students in that more than half of whose problems were found to have a low level of complexity (Kwek & Lye, 2008). Investigating the mathematical complexity of problems, Silver and Cai (1996) gave a problem-posing task and asked the students to pose problems. As a result, problems like the ones requiring the distance taken by two cars driven by two different people were posed and the complexity of these problems was evaluated on the basis of the relationships they included. Apparently, there is no single way of evaluating the complexity; there are severaş different ways of doing this on the basis of the responses given depending on the structure of the problem-posing tasks. In the current study, on the basis of the complexity level rubric used by Kwek and Lye (2008), the complexity of the problem was evaluated in a more general structure. The complexity level quality of the rubric developed in the current study is thought to be suitable for the problems posed in all subjects.

When the mean scores obtained from the complexity level quality were examined, no significant difference was found between the objectives of the linear relationship and coordinate system. Though there is no significant difference between the mean complexity scores, the mean complexity score obtained for the objective of the linear relationship is slightly higher than the coordinate system. On the other hand, a significant difference was found between the mean complexity score in the subject of equation graphs and the linear relationship and coordinate system subjects in favour of the subject of equation graphs. In the course of the research process, the students were found to be able to pose more complex problems.

In relation to the pertinence to situation quality, 43.2% of the problems were found to have completely satisfied the conditions while 10.8% of them were found to have satisfied none of them. When the mean scores obtained from the pertinence to the situation were evaluated in terms of the objectives, a significant difference was found in favour of the mean score in the subject of the coordinate system. When the problem-posing tasks were examined, it was seen that there were fewer conditions involved in the problem-posing tasks given within the subject of the coordinate system; thus, it is thought that students were able to pose problems more pertinence to the situation in the subject of coordinate system.

A quality related to the compliance of the problems with the objectives, which is similar to pertinence to situation quality of the current study was addressed in a study conducted by Özgen et al. (2017). However, the structure of the problem-posing activities was analysed in relation to gender, achievement etc. yet no analysis was conducted in relation to qualities. Cai et al. (2013) reported that a small percentage of the students were able to write problems complying with the required conditions and that the percentage of the problems satisfying at least one condition is 14.3% for the subject of equation systems and 18.3% for the task of graphs. Similarly, in the study of Canadas et al. (2018), students had some difficulties in posing a suitable problem to the given situation. It was stated that posed problems syntactic structures were different from the given symbolic expressions. They stated that students posed problems by changing the relationships between variables, adding new variables or adding new relationships (Canadas et al., 2018).

In the current study, problem-posing was integrated into classes and the qualities of the problems posed by the students throughout the process were identified. However, as stated by Cai et al. (2013), it is thought that a measurement and evaluation approach to be applied after students have been engaged in problem posing for a longer period of time and gained more experience about it will yield more efficient outcomes. In the current study, the classes to which problem posing was integrated were limited to 12 class hours. Throughout the study, the scores obtained from the originality in mathematical relationships, contextual originality and complexity level qualities were observed to increase. As a result of the integration of problem-posing into classes for a longer period, it is believed that the subject will be understood better and more improvement will be observed in problem-posing skills.

When the literature was reviewed, it was seen that different rubrics were developed for different subjects. On the basis of the rubrics used in other studies for different subject areas, the qualities that could be found in the subject of linear relationships, which is the subject of the current study, and in other math subjects were detected (Clarity of the problem, mathematical accuracy, contextual originality, originality in terms of mathematical relations, complexity level, pertinence to situation qualifications). The rubric developed in the current study can also be used to evaluate the problems posed in different subjects and the results obtained in this way can be compared with the results reported in the literature.

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