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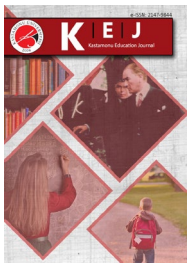

|Research Article / Araştırma Makalesi|

Analysis on the Reflective Thinking Skills of the 7th and 8th Grade Students toward Problem Solving

7. ve 8. Sınıf Öğrencilerinin Problem Çözmeye Yönelik Yansıtıcı Düşünme Becerilerinin İncelenmesi

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Keywords

1. Mathematics education
2. Middle school students
3. Problem solving
4. Reflective thinking skill

Anahtar Kelimeler

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Abstract

This study aims to analyze the reflective thinking skills of the 7th and 8th grade students toward problem solving. Using both qualitative and quantitative data collection tools, this study draws on the mixed-methods design. The sample consists of a total of 167 students, including 78 female and 89 male students studying in a middle school in the district of Fatih, province of Istanbul, Turkey in the 2019-2020 academic year. To that end, this study benefits from "The Reflective Thinking Ability Scale towards Problem Solving" and a personal information form to gather quantitative data. Qualitative data are obtained through reflective dialogue forms that present the semi-structured interview questions. The analysis of the quantitative data is performed through descriptive statistics methods, independent samples t-test, correlation analysis whereas the analysis of the qualitative data is conducted through descriptive analysis. The analyses performed in this study ascertain that the reflective thinking ability of the students toward problem solving is high. This study also concludes that there is a significant correlation between this ability and mathematics achievement whilst there is no such significant correlation across genders and grade levels. Based on the analysis of the qualitative data, it is notable that the reflections of the students are lacking and inadequate in each sub-dimension of reflective thinking, namely, questioning, reasoning and evaluation sub-dimensions, as well as in this ability in general.

Öz

Bu çalışmanın amacı 7. ve 8. sınıf öğrencilerinin problem çözmeye yönelik yansıtıcı düşünme becerilerini incelemektir. Bu amaçla nicel ve nitel veri toplama araçlarının birlikte kullanıldığı karma desen benimsenmiştir. Çalışmanın örneklemini 2019-2020 eğitim öğretim yılında İstanbul ilinin Fatih ilçesinde yer alan bir ortaokulda öğrenim görmekte olan 78 kız 89 erkek olmak üzere 167 öğrenci oluşturmaktadır. Araştırmada, nicel verileri toplamak için "Problem Çözmeye Yönelik Yansıtıcı Düşünme Becerisi Ölçeği" ve kişisel bilgi formu kullanılmıştır. Nitel veriler, yarı yapılandırılmış görüşme sorularından oluşan yansıtıcı diyalog formu ile elde edilmiştir. Verilerin analiz sürecinde nicel veriler için betimsel istatistiki yöntemler, bağımsız örneklemler t-testi, korelasyon analizi, ve nitel veriler için betimsel analiz kullanılmıştır. Analizler sonucunda öğrencilerin problem çözmeye yönelik yansıtıcı düşünme becerilerinin yüksek olduğu bulunmuştur. Bu beceri ile matematik başarısı arasında anlamlı bir ilişki olduğu bulunurken, cinsiyet ve sınıf düzeyine göre problem çözmeye yönelik yansıtıcı düşünme becerisinin anlamlı farklılık göstermediği sonucuna ulaşılmıştır. Nitel verilerin analizleri sonucunda, yansıtıcı düşünmenin sorgulama, nedenleme ve değerlendirme alt boyutlarının her birinde ve bu becerinin genelinde, öğrencilerin problem çözümlerine yönelik yansıtıcı diyalog bakımından eksik ve yetersiz yansıtmalar yaptığı görülmüştür.

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INTRODUCTION

The concepts of problem solving, critical thinking, metacognitive thinking, creative thinking and reflective thinking are among higher-order thinking skills and are closely related to each other (Erdoğan, 2019). Reflective thinking as one of higher-order thinking skills has been widely addressed in the literature (Erdoğan and Şengül, 2018; Griffin, 2003; Gürol, 2011; Jansen and Spitzer, 2009). It is notable that researchers define reflective thinking in different ways. Indeed, Mezirow (1991) argues that reflective thinking is "an important part of the learning process that combines past interpretations and new experiences to construct conceptualizations of the future by performing different functions to meet the needs of the individual." (p.101). Kember et al. (2000) defines reflective thinking skill as one's thinking about what s/he can do to overcome a problem, questioning about, evaluating and comparing his/her thoughts. Ersözlü (2008) describes this skill as reflection on and questioning past, present and future experiences, and then evaluating the steps to be taken to solve the problems arising from such reflection and questioning. Reflective thinking is a reflection process through which an individual thoroughly questions any subject s/he deals with, examines reasons while developing assumptions on the subject, and lastly identifies his/her misconceptions about the subject and explores new ways.

In the relevant literature, Dewey, Mezirow and Schön are the scholars who led the main discussion on reflective thinking. Dewey (1991) approached this concept based on problem-solving stages; Mezirow (1991) highlighted emotions in reflective thinking and Schön (1987) distinguished between two types of reflective practice: reflection in action and reflection on action. While reflection in action focuses on solving problems that emerge during the action, reflection on action is about an evaluation, respective analysis of the action, after the action is taken, and a systematic and deliberate reflection on it in every aspect (Dewey, 1991; Mezirow; 1991; Schön, 1987; cited from Bayrak and Usluel, 2011). Dewey (1933) reports that reflective thinking process consists of five stages: suggestions, problems, hypotheses, reasoning and testing. Suggestions: the ideas that appear in the mind of an individual when faced with a complex situation. Problems: the effort to see the bigger picture, rather than facing the details. Hypothesis: is what can be done with due consideration of the suggestions. It requires further thinking over the information. Reasoning: new developments created with knowledge, ideas and past experiences. Testing: process of testing problems to produce a solution. In the last step, a new problem may emerge or the existing problem may be solved. Each stage of reflective thinking process does not follow a certain order and is in harmony with each other (Kızılkaya and Aşkar, 2009). Different tools are used for reflective thinking. For example, activities such as reflective diaries, reflective dialogue, reflective writing, videotapes, thinking aloud, and group discussions are the tools that allow a reflective thinking process (Kızılkaya and Aşkar, 2009). Previous studies have investigated the effects of these reflective thinking tools on success (Çontay, 2012), problem-solving skills, attitude and anxiety (Küçük, 2019), as well as reflective thinking skills (Bayrak and Usluel, 2011; Can and Altuntaş, 2016).

Reflective thinking skills in mathematics education have gained more importance since they were associated with problem solving. The basic skill of mathematics is problem solving (Erdoğan, 2019). Problem solving process is a process whereby higher-order thinking skills are acquired. The PISA 2003 technical report prepared by OECD explains the steps to be followed in problem solving process as follows: defining the problem in real terms, determining appropriate information or limitations, presenting possible options or solutions, solving the problem, controlling the solution and sharing the results (Cited from Kızılkaya and Aşkar, 2009). The use of reflective thinking in problem solving process indicates the reflective problem-solving skill. In other words, if reflective thinking skills are dominantly present in the problem-solving steps, problem solving skills are involved too (Saygılı and Atahan, 2014). Numerous studies are available on reflective thinking skills toward problem solving (Baki, Güç and Özmen, 2012; Baş and Kıvılcım, 2013; Bjuland, 2004; Can, 2015; Demirel, Derman and Karagedik, 2015; Erdoğan, 2019; Güneş, 2015; Hong and Choi, 2015; Saygılı and Atahan, 2014; Şen, 2013; Tat, 2015; Tuncer and Özeren, 2012; Ülker, 2019). Among them, the study by Hong and Choi (2015) developed a reliable and valid questionnaire to measure reflective thinking toward problem solving. Baş and Kıvılcım (2013) examined the relationship between high-school students' achievement in geometry and their reflective thinking skills toward problem solving and found a high positive correlation between these two variables. Saygılı and Atahan (2014) performed a study to determine the reflective thinking skill levels of gifted children toward problem solving and concluded that the reflective thinking skills of these children are high. Also, through an experimental study, Sarıtepeci (2017) studied the effect of the digital storytelling method on the reflective thinking skills of the participants at the middle school level and ascertained that it had a positive effect. The study also found out that the number of activities/homework performed by the participants in an online learning environment affected the development of reflective thinking skills.

The actions involved in reflective thinking process toward problem solving are questioning, reasoning and evaluation (Kızılkaya and Aşkar, 2009). Questioning is the process where one seeks answers to herself/himself or to questions from others. The process of reasoning urges one to investigate the cause of his/her actions by establishing cause-effect relationships. Evaluation allows one to look back at her/his previous actions and distinguish between what is wrong and right. Individuals with reflective thinking skills question, think, produce, synthesize previous learning with new information, adapt and apply the information they produce to their daily life (Yıldırım and Yıldız, 2019). Moreover, reflective thinking leads students to learn through their experiences, to become aware of what they are doing, to reflect on them, to take responsibility for their own learning, to identify and correct their own mistakes, to think critically, to develop problem solving and research skills (Tok, 2008). Albayrak and Simşek (2018) state that offering an effective learning-teaching setting, reflective thinking activities provide students and teachers with a solution-oriented approach. Students are guides who observe and shape their own work in the reflective thinking process (Yıldırım, 2013). Thus, students who are responsible for their own learning will be hopefully involved in an effective learning process. It appears

that reflective thinking skills are important skills for achieving the student profile targeted in new curricula Reflective thinking skills help individuals in the problem-solving process and can be best revealed in the problem-solving process (Kızılkaya and Aşkar, 2009). Reflective dialogue, which involves reflective thinking, offers students the opportunity to discuss a topic (Kramarski and Kohen, 2017). Students can reflect on the topic during the discussion and review, question and evaluate what they have learned in this reflection process. These being said, addressing problem solving and reflective thinking together, this study explores the reflective thinking skills of students toward problem solving through reflective dialogues.

This study seeks to analyze the reflective thinking skills of the 7th and 8th grade students toward problem solving and to reveal the reflective thinking process whereby the students reflect on certain problems. Accordingly, it aims to answer the following research questions:

1) At what level are the reflective thinking skills of the 7th and 8th grade students toward problem solving?

1.1) Do the reflective thinking skills of the students toward problem solving significantly differ by grade level?

1.2) Do the reflective thinking skills of the students toward problem solving significantly differ by gender?

1.3) Are reflective thinking skills toward problem solving significantly correlated with mathematics achievement?

2) What reflective thinking skills do the 7th and 8th grade students use toward problem solving?

METHOD

Research Model

This study draws on the mixed-methods sequential explanatory design, which is one of the mixed-methods research designs. The mixed-methods sequential explanatory design is intended to use qualitative data to explain quantitative findings in more detail (Creswell, 2017). The first part of this study includes the process of collecting and analyzing qualitative data through various measurement tools. Then, the qualitative data are explained through interviews made in the quantitative part of this study.

Study Group

The study population consists of 7th and 8th grade students studying in middle schools in the district of Fatih, province of Istanbul, Turkey. A total of 167 students, including 78 female and 89 male students studying in a middle school in the district of Fatih, province of Istanbul, participated in this study. 118 of them are 7th graders whereas 49 are 8th graders. The participants who are easily accessible were selected for the sample. Within the qualitative part of this study, 12 students (10 female and 2 male), including six 7th graders and six 8th graders, were interviewed through semi-structured interviews. Two students in each mathematics achievement group (low, moderate and high) for each grade are included in this study to achieve maximum variation sampling. The success of students in the mathematics course was considered as a criterion in this process. The eligibility of the students selected was also confirmed by their mathematics teachers.

Table 1. The demographic characteristics of the students

Student	Grade Level	Gender	Mathematics achievement
S1	7	Male	Low
S2	7	Female	Low
S3	7	Female	Moderate
S4	7	Female	Moderate
S5	7	Female	High
S6	7	Male	High
S7	8	Female	Low
S8	8	Female	Low
S9	8	Female	Moderate
S10	8	Female	Moderate
S11	8	Female	High
S12	8	Female	High

Data Collection Tools and Process

The first part of this study includes the process of collecting qualitative data. The data collection in the qualitative part of this study was performed through the personal information form designed by the researcher and "The Reflective Thinking Ability Scale towards Problem Solving" developed by Kızılkaya and Aşkar (2009). This scale consists of 14 items and three sub-dimensions. The questioning dimension includes five items (1st, 3rd, 7th, 9th and 13th items); the evaluation dimension includes five items (2nd, 4th, 6th, 10th and 14th items) and the reasoning dimension includes four items (5th, 8th, 11th and 12th items). Examples of the items in this scale are: the 13th item in the scale reads "When I read a problem, I ask myself questions to understand what the problem says and asks." In the questioning sub-dimension, the 4th item reads "I evaluate the possible solutions one by one to find a better

solution to the next problem" in the evaluation sub-dimension and the 8th item reads "While solving a problem, I think about the reasons for the operations and try to establish a connection with the results I have found" in the reasoning sub-dimension. This scale has no reverse item. Kızılkaya and Aşkar (2009) conducted a confirmatory factor analysis for validating the reflective thinking ability scale towards problem solving with 7th grade students and concluded that the fit indices were acceptable. They found the Cronbach's Alpha reliability coefficient of this scale as 0.872. This study measured its Cronbach's Alpha reliability coefficient as 0.855. The items in this 5-point Likert scale represent five frequency responses as "never, rarely, sometimes, often and always." These responses are scored as Always=5, Often=4, Sometimes=3, Rarely=2, Never=1. The students were asked to consider the frequency of the action stated while filling the scale. The total scale score was calculated by summing the scores that represent the answers given to 14 items in the scale. Thus, the highest possible score for this score is 70 whilst the lowest score is 14. The range of the total score symbolizes the degree to which one has reflective thinking skills. To interpret the data more precisely, the range coefficients were determined using the [(Number of options - 1) / formula for number of options]. Accordingly, the values with a range of 1.00-1.80 are very low; of 1.81-2.60 are low, of 2.61-3.40 are moderate, of 3.41-4.20 are high and of 4.21-5.00 are very high.

The researchers, based on the analyses and results of the data obtained, supported the quantitative findings with qualitative data and provided more in-depth information. To collect data for the qualitative part of this study, the researcher designed and used "the reflective dialogue form" with 10 open-ended questions in the form of semi-structured interview. The questions in the form of reflective dialogue were prepared using the three sub-dimensions of reflective thinking reported by Kızılkaya and Aşkar (2009). The 1st, 4th, 6th and 7th questions in the form are related to the reasoning sub-dimension; the 2nd, 3rd and 5th questions are linked to the questioning sub-dimension and the 8th, 9th and 10th questions are associated with the evaluation sub-dimension. The questions in the form were reviewed by two field experts on mathematics education and finalized based on their feedback. Examples of the questions in the reflective dialogue form are presented below:

Questioning: *"Which method did you use in solving this problem? How did you decide to apply this method?"*

Reasoning: "*How did you utilize the mathematical tools you used in problem solving? Please explain."*

Evaluation: "*After you solved the problem, what did you do to be sure of the result you calculated? Please explain."*

In the qualitative part, a total of 12 students, two students in each mathematics achievement group (low, moderate and high) for each grade, were interviewed. Prior to the interviews, each student was asked two problems for the problem-solving process. Then, the students were asked the questions in the reflective dialogue form to reveal their reflecting thinking ability toward problem solving. Once the students completed the problem-solving process, the reflective dialogues between them and the researchers on problem solutions were voice-recorded. The interviews lasted about 20-30 minutes. The problems used in the interviews were obtained from the skill-based tests and sample questions prepared by the Turkish Ministry of National Education, provided that the answers were not shown. An expert on mathematics education and the teacher teaching mathematics to these students were consulted for the suitability of these problems to the level of the students and their effectiveness in revealing the reflective thinking ability toward problem solving. Appendix 1 presents the screenshots of the problems asked to the students.

Data Analysis

In the quantitative part of this study, the data were analyzed using the SPSS 22.0 statistical package program. Independent samples t-test was performed to determine whether the reflective thinking skills of the students toward problem solving significantly differed by grade level and gender, and correlation analysis was applied to evaluate the correlation of these skills with mathematics achievement. Also, descriptive statistical methods (mean, standard deviation, minimum and maximum values) were used to determine the levels of reflective thinking skills of the students toward problem solving.

In the qualitative part of this study, a descriptive analysis was conducted. The data from the interviews were analyzed according to the sub-dimensions of questioning, reasoning and evaluation of the scale toward problem solving. The levels of the reflective thinking ability of the students toward problem solving were determined based on the two problems asked to the students; their levels are presented as adequate, partially adequate or inadequate in tables. If the student gave appropriate answers to each question in a clear and understandable way during the reflective dialogues, it was considered that s/he had adequate reflective thinking skills. If the student gave appropriate answers to 1 out of 3 questions or 2 out of 4 questions in a clear and understandable way during the reflective dialogues, it was considered that s/he had partially adequate reflective thinking skills. And, if the student failed to give appropriate answers to any question in a clear and understandable way and to express himself/herself during the reflective dialogues, it was considered that s/he had inadequate reflective thinking skills. The data from the interviews with a total of 12 students, including two 7th graders and two 8th graders in each of the three sub-dimensions (questioning, reasoning and evaluation) and the analyses of these data were presented in this study. The analyses were separately coded by the researchers and the agreement percentage was 86%. The researchers discussed the discrepancies between the codes to reconcile any disagreements on the codes and to reach a conclusion.

FINDINGS

Findings from the Quantitative Data

This study, which seeks to analyze the reflective thinking ability of the 7th and 8th grade students toward problem solving, demonstrates how their ability differed by grade level, gender and mathematics achievement. The descriptive statistics of the answers of the students to the "Questioning", "Evaluation" and "Reasoning" dimensions in the scale were calculated and to reveal the levels of the reflective thinking skills of the 7th and 8th graders toward problem solving, and these statistics are presented below.

Table 2. Descriptive Results of the Scores on the Reflective Thinking Ability Scale towards Problem Solving

Variables	N	Minimum	Maximum	$\overline{X}$	ss	Level
Questioning	167	1	5	3,52	0,79	High
Evaluation	167	1	5	3,67	0,82	High
Reasoning	167	1	5	3,46	0,84	High

Table 2 shows that the average scores of the participants on the sub-dimensions of questioning, evaluation and reasoning in the scale are 3,52± 0,79; 3,67±0,82 and 3,46±0,84, respectively. These values imply that the perceptions of the participants on their questioning, evaluation and reasoning skills are high. The average of the answers of the participants to the overall scale is 3,59±0,69. This value means that the reflective thinking ability of the participants toward problem solving is high.

Table 3 demonstrates the answers of the participants to the "Questioning", "Evaluation" and "Reasoning" dimensions as well as their descriptive statistics and results of independent samples t-test on grade level to determine whether the reflective thinking ability of the students toward problem solving significantly differed by grade level or not.

Table 3. Comparison of the reflective thinking ability toward problem solving across grade levels

Variable	Grade	N	$\overline{X}$	Ss	F	p	t
Questioning	7. grade	118	3,42	0,76	0,56	0,45	-2,520
	8. grade	49	3,76	0,83			
Evaluation	7. grade	118	3,64	0,83	0,19	0,66	-0,712
	8. grade	49	3,74	0,80			
Reasoning	7. grade	118	3,46	0,82	0,03	0,85	-2,36
	8. grade	49	3,80	0,84			
Total	**7. grade**	**118**	**3,52**	**0,68**	**0,02**	**0,86**	**-1,85**
	8. grade	**49**	**3,75**	**0,69**			

As seen in Table 3, the mean scores on the questioning, evaluation and reasoning sub-dimensions in the scale did not significantly differ across grade levels (p>0,05). The mean scores in the table imply that there was no significant difference between grade level and reflective thinking ability toward problem solving for the overall scale (p>0,05). In other words, as grade level changes, the reflective thinking skills of the students do not change at all.

Table 4 demonstrates the answers of the participants to the "Questioning", "Evaluation" and "Reasoning" dimensions as well as their descriptive statistics and results of independent samples t-test on gender to determine whether the reflective thinking ability of the students toward problem solving significantly differed by gender or not.

Table 4. Comparison of the reflective thinking ability toward problem solving across genders

Variable	Grade	N	$\overline{X}$	Ss	F	p	t
Questioning	Female	78	3,66	0,66	5,19	0,024	2,06
	Male	89	3,40	0,88			
Evaluation	Female	78	3,78	0,73	2,67	0,10	1,61
	Male	89	3,57	0,88			
Reasoning	Female	78	3,64	0,78	1,62	0,31	1,11
	Male	89	3,49	0,88			
Total	**Female**	**78**	**3,71**	**0,58**	**2,56**	**0,11**	**1,98**
	Male	**89**	**3,75**	**0,75**			

As observed in Table 4, the mean score on the questioning sub-dimension in the scale significantly differed by the independent variable of gender (p>0,05) for the male students. On the other hand, the mean scores on the evaluation and reasoning sub-dimensions in the scale did not significantly differ across genders (p>0,05). For the overall scale, there was no significant difference between gender and reflective thinking ability toward problem solving (p>0,05). To determine any significant difference between the reflective thinking ability of the students toward problem solving and mathematics achievement, this study also calculated correlation coefficient. The descriptive statistics on the data obtained are presented in the following table.

Table 5. Correlation between reflective thinking ability toward problem solving and mathematics achievement

Variable	N	$\overline{X}$	Ss	r	p
Mathematics achievement score	167	67,36	23,88	0,442	.000

The Pearson's correlation analysis, which was performed to measure the relationship between reflective thinking ability and mathematics achievement, found r = 0,442 and p=.000. Table 5 shows that there was a moderate, positive and significant correlation between reflective thinking ability toward problem solving and mathematics achievement.

Findings from the Qualitative Data

This section presents the qualitative findings on the reflective dialogues with the students to enrich the findings from the quantitative findings. Table 6 offers information on the levels of the reflective thinking skills of 12 students selected from the sample toward problem solving based on the reflective dialogues with them on the problems they solved. The 7th graders are abbreviated as S1, S2, S3, S4, S5, S6 whilst the 8th graders are abbreviated as S7, S8, S9, S10, S11 and S12.

Table 6. The levels of the reflective thinking skills of the students toward problem solving based on the reflective dialogues

Grade Level	Mathematics achievement	Student	Problem	Questioning			Reasoning			Evaluation		
				Adequate	Partially Adequate	Inadequate	Adequate	Partially Adequate	Inadequate	Adequate	Partially Adequate	Inadequate
7.	Low	S1	1.Problem			x			x			x
			2.Problem			x			x			x
		S2	1.Problem			x		x				x
			2.Problem			x			x		x	
	Moderate	S3	1.Problem			x		x			x	
			2.Problem			x			x		x	
		S4	1.Problem		x				x			x
			2.Problem		x				x			x
	High	S5	1.Problem			x			x			x
			2.Problem			x			x			x
		S6	1.Problem		x				x			x
			2.Problem			x			x			x
8.	Low	S7	1.Problem			x	x					x
			2.Problem			x			x			x
		S8	1.Problem		x		x				x	
			2.Problem			x			x		x	
	Moderate	S9	1.Problem			x			x			x
			2.Problem			x	x				x	
		S10	1.Problem		x			x				x
			2.Problem			x			x			x
	High	S11	1.Problem			x	x				x	
			2.Problem			x			x			x
		S12	1.Problem			x	x					x
			2.Problem			x		x				x
			n=24	**0**	**5(20,8%)**	**19(79,2%)**	**5(20,8%)**	**4(16,7%)**	**15(62,5%)**		**7(29,2%)**	**17(70,8%)**

The skills of the students were classified based on the reflective dialogues with the selected students with low, moderate and high mathematics achievement on 2 different problems. Their reflective ability is mostly inadequate. In the questioning sub-dimension, no student had any adequate reflective thinking skills whereas the students demonstrated partially adequate reflective thinking skills in solving 5 problems and showed inadequate reflective thinking skills in solving 19 problems. In the reasoning sub-dimension, the students demonstrated adequate reflective thinking skills in solving 5 problems, partially adequate skills in 4 problems and inadequate skills in 15 problems. The 7th graders failed to show adequate reflective thinking skills in the reasoning sub-dimension. However, the 8th graders, except from S10, demonstrated adequate reflective thinking skills in a problem. Also, in the evaluation sub-dimension, similar to the questioning sub-dimension, no student had any adequate reflective thinking skills whilst the students demonstrated partially adequate reflective thinking skills in solving 7 problems and showed inadequate reflective thinking skills in solving 17 problems. It is remarkable that the students showed largely inadequate reflective thinking skills in each sub-dimension. That is, the 7th graders and 8th graders had poor reflective thinking ability toward problem solving based on the reflective dialogues on the solutions they provided. The self-perceptions of the students about this ability were notably high, but their levels of displaying this ability were low.

Reflective thinking skills in the questioning sub-dimension

In the questioning sub-dimension, the students were asked to re-express the problems posed to them in their own words, to explain what is given and required in the problem, and to indicate the different solutions to the problem. No student showed any adequate reflective thinking skills in the questioning sub-dimension. Out of the 24 problem solutions, which were provided by 12 students (each solving 2 problems), the students demonstrated partially adequate reflective thinking ability only in 5 of them and inadequate reflective thinking ability in the remaining 19 solutions. This section presents some of the problem solutions provided by the students with different math achievement, grade levels and reflective thinking skills as well as some of the remarkable student statements.

S1 is a 7th grader with low mathematics achievement. This student incorrectly calculated the result of the first problem to be 64. He did not want to share how he solved the problem. In the questioning sub-dimension, the student was asked to re-express the problem in his own words and gave a short feedback, saying that "*I would express the numbers more clearly*" which is not related to the expression of the problem. Further, the student failed to correctly explain what the problem presents and asks and consider the different ways to solve the problem. Thus, the student showed inadequate reflective thinking skills in the questioning sub-dimension.

S4 is a 7th grader with moderate mathematics achievement. Figure 1 presents the solution provided by S4 to the 2nd problem.

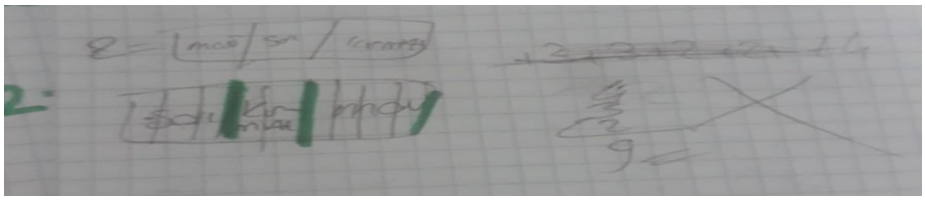

Figure 1. The solution provided by the 7th grader S4 for the 2nd problem

S4 drew the same shape given in the problem on the paper. Dissecting the colored areas in the same way, the student added the number of these areas and found the answer as 9.

In the questioning sub-dimension, S4 told that "I would express it as it is written here", rather than expressing the problem in her own words. However, the student correctly identified what the problem presents and asks. It is notable that she can actually comprehend what information the problem presents and what it asks. Still, she found the incorrect solution. She believed that there could not be different solutions to the problem and stated that she thought this while solving the problem. Thus, she failed to re-express the problem and present different solutions to the problem. Yet, the fact that she correctly identified what the problem presents and asks and that she thought if there could be different solutions to the problem means that the student showed partially adequate reflective thinking skills.

S10 is an 8th grader with moderate mathematics achievement. Figure 2 presents the solution provided by this student to the 1st problem.

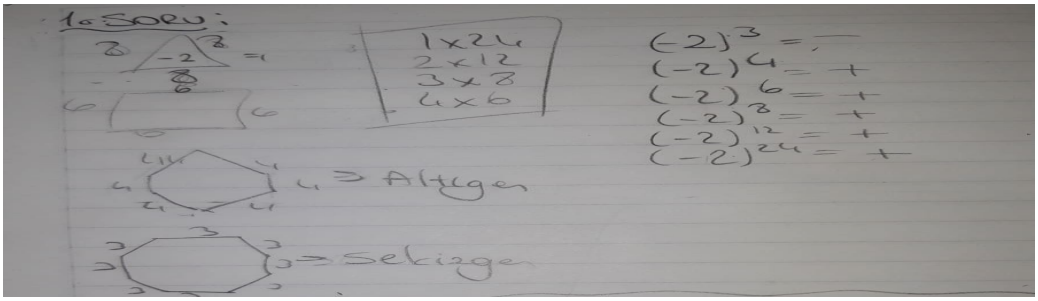

Figure 2. The solution provided by the 8th grader S10 for the 1st problem

S10 found the correct answer, calculating it as $\frac{1}{6}$. While solving the problem, S10 drew a triangle with a side length of 8 cm, a rectangle with a side length of 6 cm, a hexagon with a side length of 4 cm and an octagon with a side length of 3 cm. On the side, she found the positive divisors of 24. Then, she examined which ones are negative or positive in the positive multipliers, which she considered as the powers of -2. The student found the answer as $\frac{1}{6}$ because she determined that 1 out of 6 situations meets the condition.

S10 showed partially adequate reflective thinking ability in the questioning sub-dimension during the reflective dialogues. As S10 found the correct answer, she did not have any difficulty in understanding the problem and achieved to re-express it. Yet, she was confused about what the problem presents and asks, saying that: "*What the problem presents is different regular polygons with a circumference of 24 cm and a number to be written inside. What the problem asks is the probability of getting a negative integer on the card randomly selected.*"

S11 is an 8th grader with high mathematics achievement. She first said that the answer to the second problem is 167 and then stated that she wanted to recalculate it and said 28. Both answers are incorrect. Figure 3 presents the solution provided by her to for the second problem.

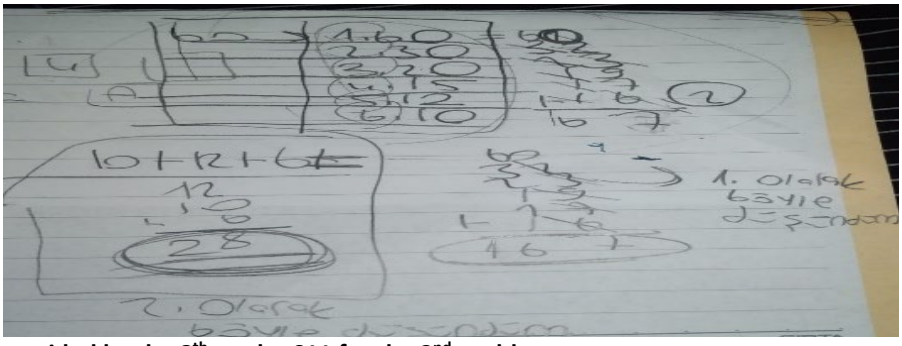

Figure 3. The solution provided by the 8th grader S11 for the 2nd problem

S11 first noted all factor pairs of 60 and added them on the side. She then added these numbers and calculated the answer as 167. Realizing that this is not correct, the student then wrote 28 by adding the numbers 12, 10 and 6. However, how she ended up with these numbers is not clearly stated.

The answers of S11 during the reflective dialogues were not at the expected level. When asked to re-express the problem, S11 instead explained how she solved it at length. Yet, her explanation was unclear. About what the problem presents and asks, S11 stated that: "*What it presents is 60 and what it asks to sum the factor pairs of 60*." This means that S11 could not successfully understand the problem. Although the problem only includes addition operations, the student mentioned about multipliers, which implies that the student had difficulty in answering this problem. As a result, the student showed poor reflective thinking ability in the questioning sub-dimension during the reflective dialogues.

Reflective thinking abilities in the reasoning sub-dimension

In the reasoning sub-dimension, the students were asked to compare the given problem with other problems, explain the method they followed in problem solving and how they decided to apply this method, and express the mathematical ways they used in problem solving. Out of the 24 problem solutions, which were provided by 12 students (each solving 2 problems), the students demonstrated adequate reflective thinking ability only in 5 of them and partially adequate reflective thinking ability in the 4 solutions and inadequate reflective thinking ability in the remaining 15 solutions. This section presents some of the problem solutions provided by the students with different math achievement, grade levels and reflective thinking skills as well as some of the remarkable student statements.

S2 is a 7th grader with low mathematics achievement. This student incorrectly calculated the answer to the cricket question as 21. Figure 4 shows the solution provided by her for the 1st problem.

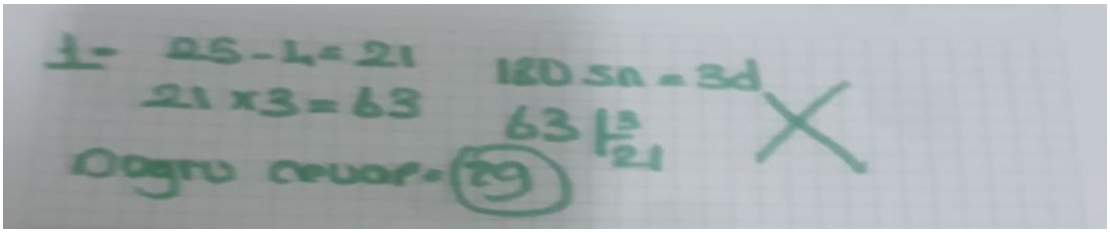

Figure 4. The solution provided by the 7th grader S1 for the 1st problem

S2 first subtracted 4 out of 25, presented in the problem, and then multiplied it by 3. Knowing that 180 seconds is equal to 3 minutes, she then divided 63 by 3 and found 21. First of all, the expression used instead of the unknown in the equation is false; the student should establish a ratio. She should calculate how many times it would chirp in 25 seconds if it chirps 180 times in 1 minute. However, she mistakenly used the number 25 as it is and directly subtracted 4 out of it and multiplied it by 3. The reason why she did such a mistake is that she failed to reflect what she read into her solution. She also made the time conversion with the wrong time unit.

In the reasoning sub-dimension, S2 clearly explained how she solved the problem: "*I used an equation first. That is, I calculated the second or minute as x. Then I performed 25-4 and multiplied it by 3. As a result, I found x to be 21*." She also reported that she previously solved similar problems. Though the solution of the student was incorrect, she did not have difficulty in justifying her solution. Nevertheless, the fact that S2 used concise statements means that she had partially adequate reflective thinking skills in the reasoning sub-dimension.

S5 is a 7th grader with high mathematics achievement. She found the answer of the 2nd problem as 9. But she also stated that she was not sure of her result. Figure 5 presents the solution provided by this student for the 2nd problem.

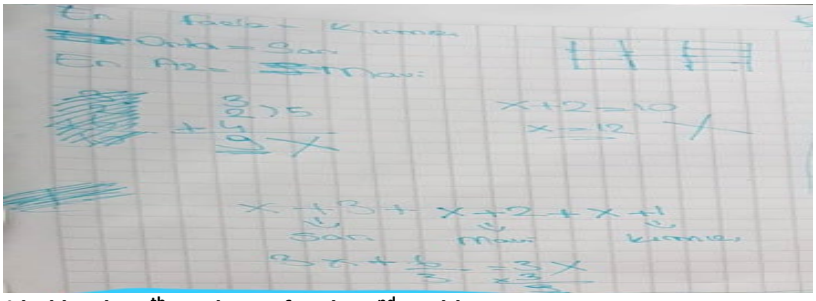

Figure 5. The solution provided by the 7th grader S5 for the 2nd problem

To fully understand the problem, S5 converted some mathematics expressions into text. She also calculated how many parts the shape is divided into. Then, considering the expression in the problem that reads it is 2 cm more than, she established an equation, but this equation was quite different from the correct solution to the problem. Further, she added the algebraic expressions called yellow, blue and red and found that the sum was equal to 3x. She still failed to find the correct result.

In the questioning sub-dimension, S5 stated that she previously solved similar problems, but could not identify in what way they were similar. When asked to describe the mathematical tools she used in problem solving, she said that: *"I listed it from the highest to the lowest. Then I established the equation to find out the difference between them. I did not draw a shape."* But she drew a shape, albeit small, on the paper. S5, who could not thoroughly explain how she used the equation tool and the mathematical operations she performed and failed to justify them, showed poor reflective thinking ability in the reasoning sub-dimension.

S8 is an 8th grader with low mathematics achievement. The answer of this student to the 1st problem was 0, so it is was incorrect. Figure 6 presents the solution provided by her for the 1st problem.

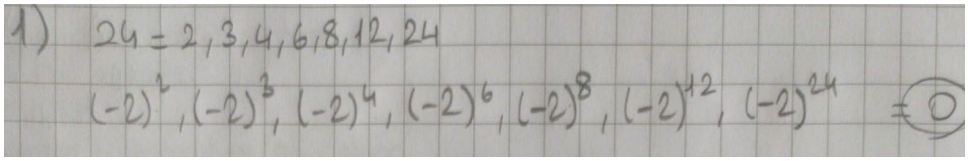

Figure 6. The solution provided by the 8th grader S8 for the 1st problem

S8 first wrote the positive integer divisors of 24. She did not include the number 1, but she included the number 2 to her list. She overlooked the fact that there cannot be a 2-sided polygon. Then, believing that any of these numbers could be a negative, she found the answer as 0.

In the reasoning sub-dimension, S8 stated that she solved problems similar to this one before and explained their similarity as follows: "*It is like finding its exponent based on the edge, well, it is a question type that I encounter*." S8 described how she solved the problem: "*I used the following method: I first found the divisors of 24, except from 1. Then, I positioned these divisors as the exponents of -2. So, I realized that since the exponents are all even, the results would not be negative*." Although her result was incorrect, she clearly expressed what she did and achieved to show her reflective thinking skills. The student acted carelessly in solving the problem. It appears that although she could not successfully solve the problem, she showed a high level of reflective thinking ability in the reasoning sub-dimension.

S10 is an 8th grader with moderate mathematics achievement. S10 failed to find the answer to the 2nd problem. Figure 7 presents the solution provided by her for the 2nd problem.

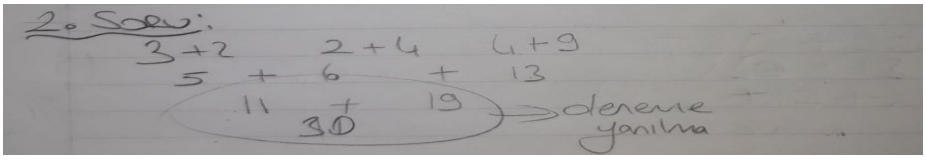

Figure 7. The solution provided by the 8th grader S10 for the 2nd problem

Thus, S10 showed a low level of reflective thinking ability in the reasoning sub-dimension. Because she failed to provide a detail description of how she tried to solve the problem and stated that she did not use any mathematical tools to solve it. She also reported that she did not solve any problem similar to this one.

Reflective thinking abilities in the evaluation dimension

In the evaluation sub-dimension, the students were asked to explain what they did to be sure of their result, if they could not solve the problem, to tell what questions they asked themselves about why they could not solve the problem, and to create a new problem by using the given problem. Out of the 24 problem solutions, which were provided by 12 students (each solving 2 problems), the students demonstrated partially adequate reflective thinking ability in 7 of them and inadequate reflective thinking ability in the remaining 18 solutions. This section presents some of the problem solutions provided by the students with different math achievement, grade levels and reflective thinking skills as well as some of the remarkable student statements.

S3 is a 7th grader with moderate mathematics achievement. This student incorrectly calculated the answer to the 1st problem as 150. Figure 8 presents the solution provided by S3.

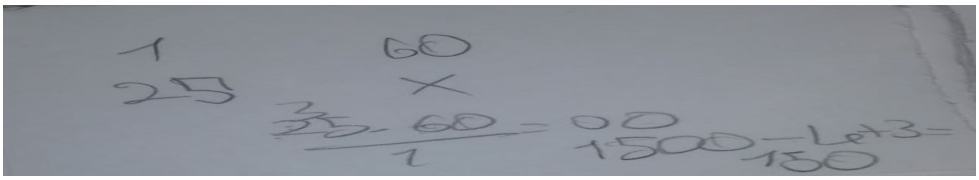

Figure 8. The solution provided by the 7th grader S3 for the 1st problem

S3 tried to establish a ratio between a minute being 60 seconds and the number of chirping in 25 seconds. She wanted to subtract 4 out of the resulting number and then multiply it by 3, but she made an error in the operations. That is, the student used an incorrect ratio in problem solving and the operations she performed did not match the expressions given in the problem.

In the evaluation sub-dimension, S3 did not perform any operation to make sure of the result she found, saying that "*I did not redo the equation to be sure, so I just checked the question again and thought that made sense.*" The reason for this is that S3 did not consider the problem-solving steps during the problem-solving process. Also, when asked to pose a similar problem, the student stated that: "*I would create a problem like this: if s/he runs 100 m in a minute, how far will s/he go in 5 minutes?"* and thus came up with a commonly used problem. S3 did not do anything to make sure of her result, but she gave positive feedback that she could pose a new problem. It therefore appears that S3 showed partially adequate reflective thinking skills.

S6 is a 7th grader with high mathematics achievement. He could not find the correct answer for the 2nd problem. Figure 9 shows his solution.

Figure 9. The solution provided by the 7th grader S6 for the 2nd problem

S6 first drew the shape given in the problem and wrote some algebraic expressions on it. Yet, these algebraic expressions were not clear and the student failed to solve the problem.

In the evaluation sub-dimension, the student stated that he did not reflect on why he could not solve the problem and did not check the way he tried to solve it, and told that "*I did not do much. I just put 6 or so. I already realized that this was not the solution*." When asked to pose a new problem, he stated that: "*Instead of dividing it in the same way and saying that one is longer than the other, I would say here is a piece of red in two parts, each one is 12 cm. Then, I ask about the blue box*." In other words, the student offered a less complex problem by reducing the unknowns of the problem, rather than creating an original problem. It follows that S6 showed inadequate reflective thinking ability in the evaluation sub-dimension.

S9 is an 8th grader with moderate mathematics achievement. Figure 10 presents the solution provided by her for the 1st problem.

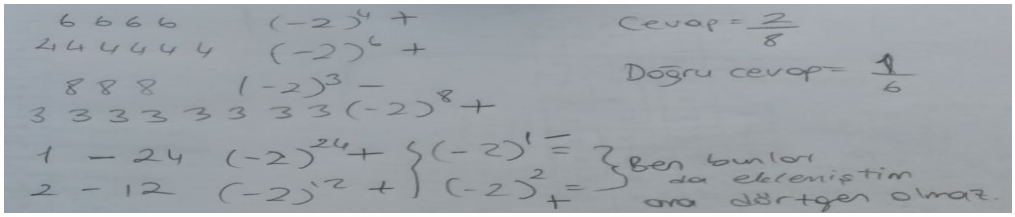

Figure 10. The solution provided by the 8th grader S9 for the 1st problem

Thinking that polygons with side lengths of 6 cm, 4 cm, 8 cm, 3 cm, 2 cm and 1 cm are possible, S9 listed these polygons one under the other. She then wrote them as the powers of -2, noting whether they are negative or positive next to them. The student, who much later realized that 1 and 2 are not possible, wrote that she took into consideration these possibilities as well while solving the problem. As a result, she incorrectly calculated the answer as $\frac{1}{4}$.

In the evaluation sub-dimension, it is notable that though the student stated that she re-checked the operations she performed, she failed to realize her mistakes. The student also could not pose a new problem, despite her statement that reads "*Yes, I can. I would limit the circumference in numbers and pose a new problem.*" She further could justify the mathematical operations she performed and the numbers she used. Although some of her answers were reasonable in the evaluation sub-dimension, S9 offered very short answers during the reflective dialogues, which implies that she showed partially adequate reflective thinking ability.

S12 is an 8th grader with high mathematics achievement. This student successfully found the correct answer for the 2nd problem.

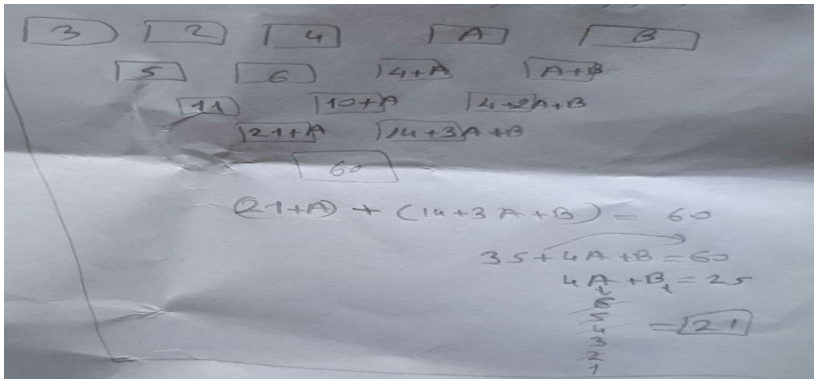

Figure 11. The solution provided by the 8th grader S12 for the 2nd problem

S12 drew the same shape in the problem on the paper and filled in all the numbers and algebraic expressions into the relevant boxes. As the last box read 60, she thought that 35+4A+B was equal to 60. Then, she thought that 4A+B was equal to 25. Lastly, summing all possible values for A, she found the result as 21.

In the evaluation sub-dimension, it is notable that S12 did not do anything to make sure of her solution. When asked to create a new problem, she stated that "*I would change the numbers and ask for their multiplication rather than addition*." The student did not change the structure of the problem; instead, she replaced addition with multiplication. It appears that although this student provided the correct answer in the evaluation sub-dimension, her justification for her mathematical thinking and the operations she performed was poor. Therefore, S12 showed inadequate reflective thinking ability in the evaluation sub-dimension.

DISCUSSION

The analysis of the qualitative data in this study reveals that the participants overall have a high level of reflective thinking ability toward problem solving. In other words, the students consider themselves as good in this skill. This study thus concludes that the reflective thinking ability of the students toward problem solving is high in all sub-dimensions of questioning, evaluation and reasoning. Erdoğan (2019) found that the reflective thinking ability of the middle school students toward problem solving is poor. The difference in the findings perhaps results from the samples that include students at a different education level.

The qualitative part of this study also investigates whether their reflective thinking ability significantly varies across grade levels and genders. No significant difference has been observed between grade level and reflective thinking ability toward problem solving. Similarly, Demirbaş (2012) reports that the reflective thinking ability levels of 4th and 5th graders do not differ across grade levels; on the other hand, Erdoğan (2019) argues that as the grade level among middle school students increases, their reflective thinking skills for problem solving improve too. It seems that the finding that reflective thinking ability significantly changes across grade levels depends on the age group of participants. Another finding of this study is that reflective thinking ability toward

problem solving significantly varies for male students in the questioning sub-dimension. However, there is no significant difference across genders in the other sub-dimensions. The overall score of the participants on the scale shows that the variable of gender does not have a significant impact on reflective thinking ability toward problem solving. This finding is congruent with the findings of Erdoğan (2019), Saygılı and Atahan (2014) and Yıldırım (2013). Yet, some studies in the literature claim that reflective thinking skills toward problem solving are higher among female students (Hoare, 2006; Aydın and Çelik, 2013) or male students (Gohindo, 2004).

In conclusion, this study ascertains that the mathematics achievement of the participants and their reflective thinking ability toward problem solving are correlated. This correlation is positive, significant and at a moderate level. There are certain studies in the literature that report the positive correlation between academic achievement and reflective thinking (Baş and Kıvılcım, 2013; Cengiz, 2014; Dursun, 2015; Karaoğlan Yılmaz, 2014; Kızılkaya, 2009; Mohamad et al., 2013). Experimental research that explore the impact of reflective thinking on mathematical performance show that this effect is positive and contributes to post-test achievement scores (Bölükbaş, 2004; Dursun, 2015; Kırnık, 2010; Tok, 2008). Baş and Kıvılcım (2013) and Şen (2011) determine a significant correlation between reflective thinking skills toward problem solving and academic achievement in mathematics course. The reason for this is perhaps that reflective thinking ability is one of the higher-order thinking skills and supports individuals in many ways. The reflective dialogues with the students reveal the reflective thinking skills of the 7th and 8th graders. The students first solved the problems and then reflected on these problems through reflective dialogues. The 7th graders had more difficulty in solving the problems compared to the 8th graders. The 8th graders are more familiar with exam questions as they are preparing for the High School Entrance Exam (LGS). This perhaps helps them solving the problems more easily. All of the students had difficulty in the reflective dialogues and their reflections were incomplete and inadequate. It is notable that the students had difficulty particularly in re-expressing the problem in their own words in the questioning sub-dimension. Instead of re-expressing the problem, most of the students offered explanations by repeating the same expressions in the problem, and some of them articulated their solutions for the problem. As the students had difficulty in solving the problems, they also failed to express alternative solutions to the problem. Although the students, including those with high mathematics achievement, generally stated that there may be alternative solutions, they failed to explain these alternative solutions.

The findings of this study clearly indicate that most of the students, those with low and moderate mathematics achievement in particular, had difficulty in explaining the problems and posing new problems. Though those with high mathematics achievement were more successful in understanding the problems than other students, they experienced similar difficulties in posing new problems. The majority of the students tended to pose same problems without making dramatic changes on the given problem. Some students even stated that they could not pose a new problem. Pusmaz and Tavşan (2019) investigates the reflective thinking skills of the students successful in solving mathematics problems toward problem solving and similarly finds that the students expressed similar problems instead of posing new problems with their own sentences. Tertemiz and Sulak (2013) posit that the technique used by the students while posing a new problem involves changing the values of the given data, without making any change in the conditions and the subject. The students are not familiar with problem posing activities, which may be the reason for that they had difficulty in posing new problems during the reflective dialogues. These dialogues demonstrate that students do not like lengthy problems, that the length of problems affects their willingness to solve the problem and their success, and that when there are unfamiliar terms in the problem, these terms, although they are not necessary to solve the problem, create confusion and make it difficult to find the correct result. Considering the qualitative and quantitative data together, one may understand that the average scores of the students on the scale are high, but their reflective thinking ability during the reflective dialogues is low. That is to say, the self-perceptions of the students on this skill are high, yet their ability to exhibit this skill is low. The fact that the qualitative and quantitative data are not parallel may result from the lack of self-understanding of the students on their reflective thinking skills. The concept of reflective thinking is new to the students, and they are not familiar with having dialogue on the problems they solved. For that reason, the students considered themselves good in the expressions in the scale, but they could not put it into practice. The acquisition of reflective thinking skills in the school environment is considered key (Wilson and Jan, 1993). Because reflective thinking is generally achieved through teaching (Meissner, 1999). In this regard, it is important to perform systematic and regular reflective thinking activities in schools. This will perhaps help allow students to gain reflective thinking skills.

CONCLUSION AND SUGGESTIONS

This study concludes that the reflective thinking skills of the 7th and 8th graders are high and that these skills do not significantly differ across genders and grade levels but have a significant and moderate correlation with mathematics achievement. Also, unlike the quantitative data, the qualitative data show that the students' reflections on their own problem solutions were insufficient. The students had the most difficulty in re-expressing the problems in their own words, explaining different ways to solve the problems and pose new problems. Based on the findings, future studies may enlarge the size of the sample, which is one of the limitations of this study. Qualitative studies may be performed to analyze the reflective thinking ability of elementary school students toward problem solving. This would help identifying the factors that affect the acquisition of this ability from an early age and take the necessary measures. Studies that investigate the reflective thinking ability of elementary school students as well as high school students would offer new insights into the literature. Students may participate in reflective dialogues after problem solving to improve their reflective thinking ability toward problem solving. Future studies may focus on other variables related to

the reflective thinking ability of students toward problem solving and their effects on mathematics achievement. The reflective thinking ability of both students and teachers may be further explored, and the effect of teachers on students in relation to this ability may be analyzed.

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Researchers' contribution rate

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APPENDIX-1 (Problems)

1 dakika = 60 saniye

Cırcır böceklerinin ötme sıklığı hava sıcaklığı hakkında bilgi verir.

Bir cırcır böceği 25 saniyede, derece selsiyus (°C) cinsinden hava sıcaklığı değerinin 4 eksiğinin 3 katı kadar öter.

Yukarıdaki bilgiyi kullanarak bulunduğu kamp alanındaki hava sıcaklığını ölçmek isteyen İdil, bir cırcır böceğinin dakikada 180 kere öttüğünü sayar.

Buna göre İdil kamp alanındaki hava sıcaklığını kaç derece selsiyus (°C) olarak hesaplar?

1 minute = 60 seconds

The chirping of crickets gives information about the air temperature.

A cricket chirps 4 less than 3 times the air temperature value in Celsius (°C) in 25 seconds. Using the information above, İdil wants to measure the air temperature in the camp area she stays and finds that a cricket chirps 180 times in 1 minute.

So, how many degree in Celsius (°C) does İdil calculate the air temperature in the camp area?

Figure 12. The 1st problem for the 7th graders

Düzgün çokgenlerin iç bölgelerine tam sayılar yazılarak aşağıda örnekleri verilen kurala göre üslü ifadeler elde edilecektir.

- Yukarıdaki kurala göre kenar uzunlukları santimetre cinsinden birer tam sayı ve çevre uzunluğu 24 cm olan farklı düzgün çokgenlerin tamamı oluşturuluyor.
- Bu düzgün çokgenlerin her birinin içerisine -2 yazılıp üslü ifadeler elde ediliyor.
- Oluşturulan üslü ifadeler eş kartlara yazılıp bir torbaya atılıyor.

Bu torbadan rastgele seçilen bir kartın üzerinde negatif bir tam sayının değerine eşit olan bir üslü ifade yazma olasılığı nedir?

According to the rule below, integers are written in the inner regions of regular polygons to obtain exponential expressions.

- According to the rule above, all possible regular polygons with side lengths integer in centimeters and a circumference of 24 cm are created.
- Then, -2 is added to the inner regions of each polygon to obtain exponential expressions.
- The resulting exponential expressions are written on papers and put in a bag.

What is the probability of randomly selecting a card on which an exponential expression equal to the value of a negative integer is written?

Figure 13. The 2nd problem for the 7th graders

Bir tahtanın üst yüzeyi çizgiler ile aşağıdaki gibi üç eş parçaya ayrılıp her bölge farklı bir renge boyanıyor.

Daha sonra tahtanın üst yüzeyindeki çizgilere paralel olacak şekilde sarı bölgeyi 3, kırmızı bölgeyi 2 ve mavi bölgeyi 4 eş parçaya ayıran çizgiler çiziliyor.

Sarı bölgedeki ardışık çizgiler arasındaki mesafe mavi bölgedeki ardışık çizgiler arasındaki mesafeden 2 cm daha fazladır.

Buna göre kırmızı bölgedeki ardışık çizgiler arasındaki mesafe kaç santimetredir?

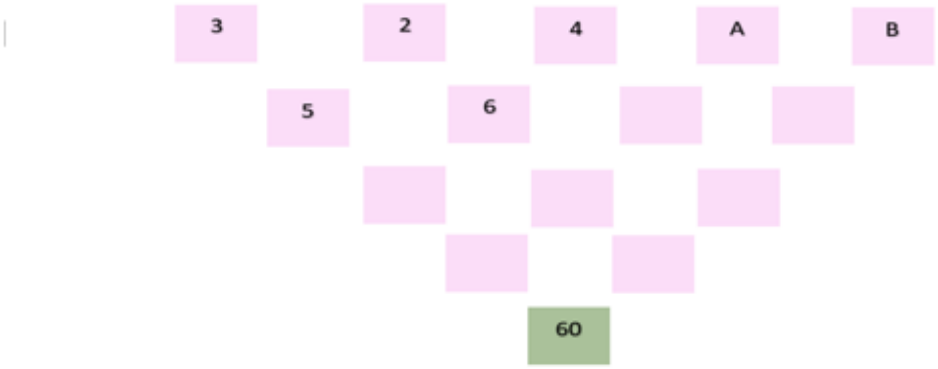
-The top surface of a board is divided into three equal parts as below, and each area is painted in a different color.

-Then, the yellow area is divided into 3 equal parts; the red area into 2 and the blue area into 4, parallel to the lines on the surface of the board.

-The distance between the consecutive lines in the yellow area is 2 cm greater than the distance between the consecutive lines in the blue area.

So, how many centimeters is the distance between the consecutive lines in the red area?

Figure 14. The 1st problem for the 8th graders



Yukarıdaki şekilde pembe olan hücrelere bazı kurallara göre sayılar yerleştirilmiştir. Bu kurallar

- A ve B pozitif tam sayı olmalıdır.
- Tabloda yukarıdan merkeze doğru (yeşil hücre) hücrelere sayılar yazılırken, bir üst satırdaki kesişen pembe hücrede bulunan sayılar toplanarak yazılır.

Buna göre A sayısının alabileceği değerlerin toplamı kaçtır?

In the above figure, the numbers are placed in the pink squares according to some rules. These rules are as follows:

- A and B must be a positive integer.
- While writing numbers from the top to the center (to the green square) in the table, the numbers in the pink squares that intersect in the upper row are added together.

So, what are the sum of all possible values for A?

Figure 15. The 2nd problem for the 8th graders