



Discrete-Time Gompertz Model for Adana Breed Pigeons

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Highlights

- In this article, the Gompertz growth model for Adana pigeon chicks is discussed.
- The estimation results are based on the data of 28 Adana pigeons.
- Estimation results show that using the discrete-time GM and AKF provides efficient analysis.

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Abstract

The mathematical animal growth models in the literature are not in the form of linear models. These growth models in the literature are not in linear form. There are different numerical analysis methods for the estimation of the parameters found in these functions and specific software have been produced to estimate the unknown parameters in these mathematical models and to apply these methods. In these nonlinear mathematical growth models, there may be more than one parameter. For these and other reasons, the number of mathematical numerical operations in estimating parameters is quite high. In this study, the discrete time stochastic Gompertz model (DTSGM) was considered to determine the growth of Adana pigeons. A model is used in which the parameter in the model is estimated by an adaptive Kalman filter (AKF). The aim of this research is to reveal the validity of both the model and the estimation method for Adana breed domestic pigeons. Daily weight measurements of 28 Adana pigeons were considered and estimated using DTSGM and AKF methods in this framework. DTSGM in conjunction with AKF has been shown to provide a convenient analysis tool for modeling daily weight estimates of Adana pigeons.

1. INTRODUCTION

Domestic Pigeons have taken place quite a lot in human life throughout human history. Pigeons were used as carriers in the transmission of mail, messages and notes in wars. Genetic studies on the descent of domestic pigeon species from a single ancestor are included in the studies in [1-2]. It is unclear exactly when wild pigeons were domesticated [3-4].

Although Adana pigeons are known by different names in the world, they are known as Adana natives in Turkey. Adana pigeons are generally bred in Çukurova Region. It is widely grown in Adana, Ceyhan, Mersin and Tarsus. Today, the same species of this pigeon is also produced in Syria and is even known as Adana in the Syrian geography. They are exactly famous as performance birds. Its flight characteristics are different from other pigeons. Adana pigeons do not fly long and in large groups. They often fly as a small group of two or three pigeons. Adana pigeons are housed in cabinets with small cells that can accommodate only two birds [5].

The word "growth" is used to express the growth of an organism or an increase in living species [6-8]. GM is widely used in many subfields of biological sciences [9-10]. Various number of parameterizations and reparameterizations of GM can be found in the literature [11]. The use of mathematical population growth models in various fields has become a very useful research area in recent years [12-28]. In these research published in the literature, nonlinear mathematical growth models were used, in which more than one parameter should be estimated. In order to make a realistic and accurate analysis in these mathematical models, the data in the models must be updated daily and all of them must be available. The basis of the methods is offline and all data must be available by a certain date in order to estimate the parameters in these mathematical models.

In the paper [29] continuous GM, Recursive Linear Model and Linear Regression Model were used for broiler growth model. The authors stated that the iterative linear model gives more accurate prediction results than the static linear growth model. Zeng et al. [30] proposed a class of complex population dynamic models incorporating novel time-varying parameters to describe univariate ecological time series data. They used second order stochastic Ricker and GM. In addition, they explained the advantages of KF in ecological time series analysis. The structural time series modeling method using KF has many advantages [31-32]. Jonas Knappe and Perry de Valpine [33] analyze 627 datasets from The Global Population Dynamics Database using GM and account for uncertainty through the KF.

This study emphasizes the growth model in order to estimate the growth of the Adana pigeons using DTSGM and AKF. The estimation results are based on the data of 28 Adana pigeons. There is only one parameter in the DTSGM, autoregressive time-series AR(1) model and that parameter is estimated online through AKF.

2. MATERIAL METHOD

The breeding season for pigeons is generally between February and August. During this period, they lay an average of 4 eggs and incubate. Two eggs are laid at a time. The incubation period is approximately 15-20 days. Both male and female pigeons lie in turns in the brood. Usually male pigeons incubate during the day and more female pigeons at night. For various reasons, one or both of the offspring die before they reach adulthood. For these reasons, it is very difficult to observe the weight of pigeon chicks during growth. In this study, daily weight measurements of 28 pigeon chicks were discussed. The study was carried out by taking the average daily weight of all 28 pigeons that were measured in the study. During the study, these 28 pigeons were weighed for 43 days and analyzed using these 43 day weight measurements. In order to estimate the parameter in the statistical model discussed in the study, a special code was written in the matlab programming language.

We use a DTSGM for the weight of the pigeon. Let's define the daily weight of the pigeon with n_t . The DTSGM is

$$n_t = n_{t-1} \exp(a + b \ln n_{t-1} + e_t) \quad (1)$$

here a and b are constants. e_t is defined by $e_t \sim N(0, \sigma^2)$. In the logarithmic case, DTGM can be considered as a linear, autoregressive time series AR(1) model.

$$y_t = y_{t-1} + a + by_{t-1} + y_t = a + cy_{t-1} + e_t \quad (2)$$

where $y_t = \ln n_t$ and $c = b + 1$. The statistical properties of the discrete-time stochastic GM are well-known [34].

There are many technological applications of Kalman filtering. Applications related to KF are encountered in almost all fields of science [35-42]. KF was used to recursive estimate the time-varying parameter in the DTSGM model discussed in this article.

If $a = 0$ in Equation (2)

$$y_t = cy_{t-1} + e_t \quad (3)$$

is obtained. In the case where the c parameter in Equation (3) is time-varying and assumes as

$$y_t = c_t y_{t-1} + e_t \quad (4)$$

$$c_t = c_{t-1} + w_t \quad (5)$$

is written. Here, c_t is the state variable. The Kalman filter is a recursive estimator (see Appendix).

If this parameter c_t is estimated using AKF, future growth forecasts can be made. In Table 1, MSE, MAPE and R^2 values are given for the estimation results obtained using daily weight measurements of pigeons. Actual logarithmic daily weight values and estimated values using the DTSGM and AKF is given in Figure 1. Actual daily weight values and estimated values are given in Figure 2.

Table 1. Values of MSE, MAPE, R^2

	<u>MSE</u>	<u>R²</u>	<u>MAPE</u>
Logarithmic weight	0.0066	0.9915	0.4875
Weight	270	0.9861	2.3045

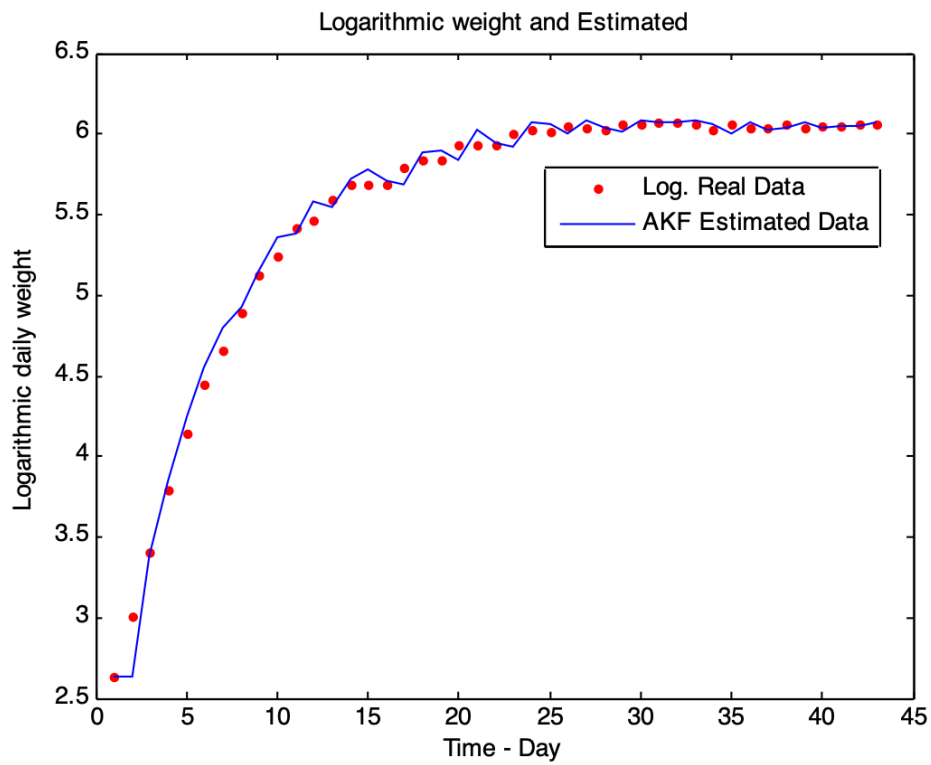


Figure 1. Logarithmic weight and estimated weight

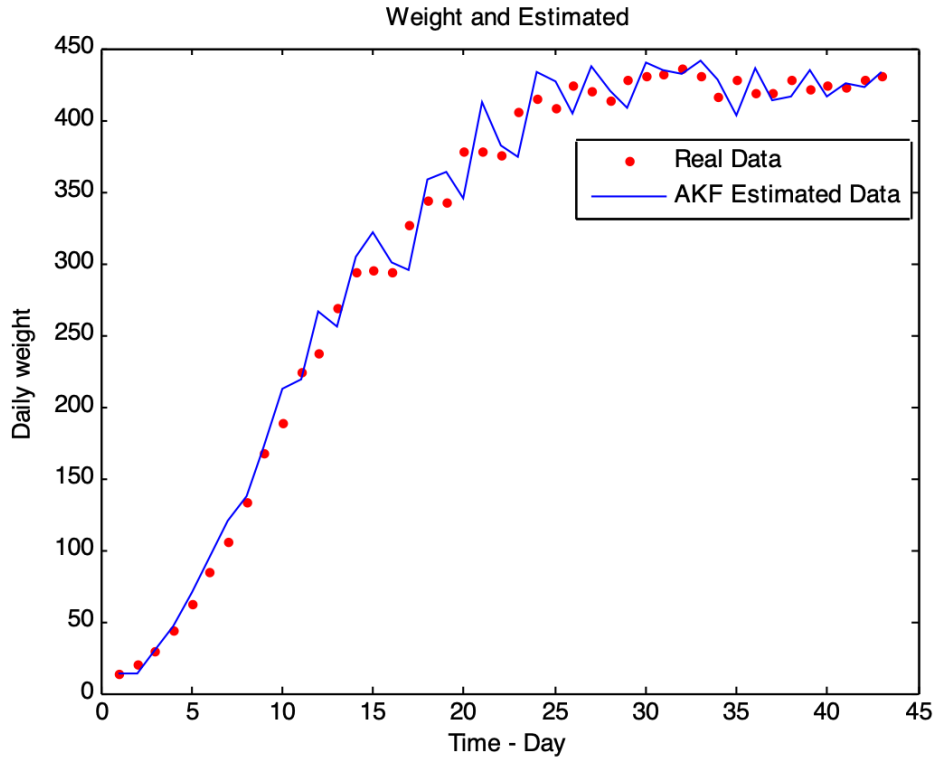


Figure 2. Weight and estimated weight

These values given in the table indicate that the model used has a very high compatibility with the real data and says that estimating the daily weight through DTSGM is a useful and simple technique. The model and estimation process is very simple as it requires no more assumptions. AKF is a very advantageous method as it uses only the observation at time t and the previous estimate.

3. RESULTS AND DISCUSSION

The most used nonlinear mathematical models in the literature can be found in [43]. As far as we know, there is only [43] article in the literature that analyzes the growth dynamics of Adana pigeons. There is no other published article comparing these models and/or the results of other models with these models for Adana pigeons.

In this study, we estimated the daily weights of 28 Adana pigeons using DTSGM and AKF with on-line basis. The daily weight measurements are modelled with DTSGM, and the time-varying parameters of the obtained AR(1) stochastic time series are estimated by on-line AKF. Estimation results show that using the DTSGM and AKF in terms of MSE, MAPE, and R^2 provides efficient analysis for modeling the weight estimations. It is proposed that the use of DTSGM and AKF is appropriate for such a modelling and AR(1) model is also an appropriate estimation.

Modeling the time series obtained from daily weight measurements of pigeon pups with AR(1) and estimating the time-variable parameter with AKF leads to an accurate estimation of daily weights without the need for any further calculations. It is quite easy to model this time series with time variable AR(1) and estimate the varying parameter with online AKF. Based on the estimates obtained using the data of 28 pigeon chicks, we suggest that the effective analysis for weight modeling is GM and ACF. The proposed method is a fairly simple method for estimating growth. It can be realized by modeling the daily weight time series using AR(1). We can conclude that our proposed method is suitable for estimating the weight of all kinds of living species.

Appendix.

The discrete time state space models are given in the form

$$x_{t+1} = F_t x_t + G_t w_t \quad (A1)$$

$$y_t = H_t x_t + v_t \quad (A2)$$

where x_t is a system, y_t is an observation vector. The covariance matrices w_t and v_t are

$$w_t \sim N(0, Q_t), \quad v_t \sim N(0, R_t).$$

The optimum KF equations are,

$$\hat{x}_{t|t-1} = F_{t-1} \hat{x}_{t-1} \quad (A3)$$

$$P_{t|t-1} = F_{t-1} P_{t-1|t-1} F_{t-1}' + G_{t-1} Q_{t-1} G_{t-1}' \quad (A4)$$

$$K_t = P_{t|t-1} H_t' (H_t P_{t|t-1} H_t' + R_t)^{-1} \quad (A5)$$

$$P_{t|t} = [I - K_t H_t] P_{t|t-1} \quad (\text{A6})$$

$$\hat{x}_t = \hat{x}_{t|t-1} + K_t (y_t - H_t \hat{x}_{t|t-1}) \quad (\text{A7})$$

[35-42]. In some cases, incorrect estimation divergence problems may occur in KF due to incorrect model setup. Various adaptive methods are used to circumvent this situation in KF [44-46]. The most commonly used one is the forgetting factor in KF equations. A new forgetting factor was proposed by Özbek and Aliev [44]

$$P_{t|t-1} = \alpha \left(F_{t-1} P_{t-1|t-1} F_{t-1}' + G_{t-1} Q_{t-1} G_{t-1}' \right) \quad (\text{A8})$$

CONFLICTS OF INTEREST

No conflict of interest was declared by the author.

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