Fundamental Journal of Mathematics and Applications, 4 (2) (2021) 77-82
Research Article

FUJMA

Fundamental Journal of Mathematics and Applications

Journal Homepage: www.dergipark.org.tr/en/pub/fujma
ISSN: 2645-8845
doi: https://dx.doi.org/10.33401/fujma.812274

Controllability and Accumulation of Errors Arising in a General Iteration Method

Faik Gürsoy[1*], Abdul Rahim Khan[2] and Kadri Doğan[3]

[1]*Department of Mathematics, Adiyaman University, Adiyaman 02040, Turkey*
[2]*Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia*
[3]*Department of Computer Engineering, Artvin Coruh University, 08000, Artvin, Turkey*
[*]*Corresponding author*

Article Info

Keywords: *Error analysis, Random errors, TS iteration method*
2010 AMS: *47H10, 97N20, 47J25, 54H25*
Received: *18 October 2020*
Accepted: *24 May 2021*
Available online: *27 May 2021*

Abstract

In this paper, we propose and analyze a three-step general iteration method which is a special case of an iteration method proposed in (S. Thianwan and S. Suantai, Convergence criteria of a new three-step iteration with errors for nonexpansive nonself-mappings, Comput. Math. Appl. 52 (2006), 1107-1118). Here we intend to study directly the accumulation, estimation and control of random errors in the newly proposed general iteration method. We give conditions under which the accumulated-error in our iteration method is bounded and controllable in a permissible range.

1. Introduction

The tools of fixed point theory are successfully applied to the solutions of a wide variety of problems arising in many disciplines of science. In particular, fixed point iteration methods have attracted the attention of researchers and in parallel with the extension of the application areas of fixed point theory, a great deal of effort has been devoted to the study of some important features of iteration methods (see, for instance, [1]-[9]).

Errors usually occur in the iterative calculations and so consideration of error estimates is of utmost importance in the study of iteration methods. A quick look at literature reveals that many paper have been devoted to the study of iteration methods with errors where the errors are calculated indirectly. There are only a few papers concerning direct estimation and control of errors of the iteration methods (see, e.g., [10]-[12]).

Throughout this exposition, we assume that $(B,\|\cdot\|)$ is an arbitrary real Banach space, S a nonempty closed and convex subset of B, $T:S\to S$ an operator, and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$,$\{\lambda_n\}_{n=0}^{\infty}$,$\{\mu_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$, $\{\alpha_n+\beta_n+\lambda_n\}_{n=0}^{\infty}$, $\{b_n+c_n+\mu_n\}_{n=0}^{\infty}$, $\{a_n+\gamma_n\}_{n=0}^{\infty}\subseteq[0,1]$ are parameter sequences satisfying certain control condition(s) and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$ are bounded sequences in S.

In 2006, Thianwan and Suantai [13] defined a three-step iteration method on S with error terms as:

$$\begin{cases} x_0\in S, \\ x_{n+1}=(1-\alpha_n-\beta_n-\lambda_n)x_n+\alpha_nTy_n+\beta_nTz_n+\lambda_nw_n \\ y_n=(1-b_n-c_n-\mu_n)x_n+b_nTz_n+c_nTx_n+\mu_nv_n \\ z_n=(1-a_n-\gamma_n)x_n+a_nTx_n+\gamma_nu_n, \text{ for all } n\in\mathbb{N}. \end{cases} \tag{1.1}$$

The iteration method (1.1) has been used for approximation of fixed points of various nonlinear mappings (see, for instance,

Email addresses and ORCID numbers: **faikgursoy02@hotmail.com, 0000-0002-7118-9088 (F. Gürsoy), arahim@kfupm.edu.sa, 0000-0001-6695-0939 (A. Khan), dogankadri@hotmail.com, 0000-0002-6622-3122 (K. Doğan)**

[14, 15]). If we put $\lambda_n = \mu_n = \gamma_n = 0$ for all $n \in \mathbb{N}$ in (1.1), then we obtain

$$\begin{cases} x_0 \in S, \\ x_{n+1} = (1-\alpha_n-\beta_n)x_n + \alpha_n T y_n + \beta_n T z_n \\ y_n = (1-b_n-c_n)x_n + b_n T z_n + c_n T x_n \\ z_n = (1-a_n)x_n + a_n T x_n, \text{ for all } n \in \mathbb{N}. \end{cases} \tag{1.2}$$

Remark 1.1. *The iteration method (1.1) reduces to:*
(i) Noor iteration method [16] if $c_n = \beta_n = \gamma_n = \lambda_n = \mu_n = 0$ for all $n \in \mathbb{N}$,
(ii) Ishikawa iteration method [17] if $a_n = c_n = \beta_n = \gamma_n = \lambda_n = \mu_n = 0$ for all $n \in \mathbb{N}$,
(iii) Mann iteration method [18] if $a_n = b_n = c_n = \beta_n = \gamma_n = \lambda_n = \mu_n = 0$ for all $n \in \mathbb{N}$.

2. Main results

Here we intend to study directly the accumulation, estimation and control of random errors in the iteration method (1.2). Define the errors of Tx_n, Ty_n and Tz_n by

$$u_n = Tx_n - \overline{Tx_n},\ v_n = Tz_n - \overline{Tz_n} \text{ and } w_n = Ty_n - \overline{Ty_n} \tag{2.1}$$

for all $n \in \mathbb{N}$, where $\overline{Tx_n}$, $\overline{Ty_n}$ and $\overline{Tz_n}$ are the exact values of Tx_n, Ty_n and Tz_n respectively, that is, Tx_n, Ty_n and Tz_n are approximate values of $\overline{Tx_n}$, $\overline{Ty_n}$ and $\overline{Tz_n}$, respectively. The theory of errors implies that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ and $\{w_n\}_{n=0}^{\infty}$ are bounded. Set

$$B = \max\{B_u, B_v, B_w\} \tag{2.2}$$

where $B_u = \sup_{n\in\mathbb{N}} \|u_n\|$, $B_v = \sup_{n\in\mathbb{N}} \|v_n\|$ and $B_w = \sup_{n\in\mathbb{N}} \|w_n\|$ are the bounds on the absolute errors of $\{Tx_n\}_{n=0}^{\infty}$, $\{Tz_n\}_{n=0}^{\infty}$ and $\{Ty_n\}_{n=0}^{\infty}$, respectively.
The main part of accumulation of errors from (1.2) comes essentially from u_n, v_n and w_n; hence we can set

$$\begin{cases} \overline{x_0} \in S, \\ \overline{x_{n+1}} = (1-\alpha_n-\beta_n)\overline{x_n} + \alpha_n \overline{Ty_n} + \beta_n \overline{Tz_n} \\ \overline{y_n} = (1-b_n-c_n)\overline{x_n} + b_n \overline{Tz_n} + c_n \overline{Tx_n} \\ \overline{z_n} = (1-a_n)\overline{x_n} + a_n \overline{Tx_n}, \text{ for all } n \in \mathbb{N}. \end{cases} \tag{2.3}$$

where $\overline{x_n}$, $\overline{y_n}$ and $\overline{z_n}$ are exact values of x_n, y_n and z_n, respectively. Clearly, the errors of last iteration will affect the next $(n+1)$ steps. So, utilizing (1.2), (2.1) and (2.3), we have

$$\begin{aligned}
x_0 &= \overline{x_0}; \\
z_0 &= (1-a_0)x_0 + a_0 T x_0 \\
&= (1-a_0)\overline{x_0} + a_0\overline{Tx_0} + a_0 u_0 = \overline{z_0} + a_0 u_0; \\
y_0 &= (1-b_0-c_0)x_0 + b_0 T z_0 + c_0 T x_0 \\
&= (1-b_0-c_0)\overline{x_0} + b_0\overline{Tz_0} + c_0\overline{Tx_0} + b_0 v_0 + c_0 u_0 \\
&= \overline{y_0} + b_0 v_0 + c_0 u_0; \\
x_1 &= (1-\alpha_0-\beta_0)x_0 + \alpha_0 T y_0 + \beta_0 T z_0 \\
&= (1-\alpha_0-\beta_0)\overline{x_0} + \alpha_0\overline{Ty_0} + \beta_0\overline{Tz_0} + \alpha_0 w_0 + \beta_0 v_0 \\
&= \overline{x_1} + \alpha_0 w_0 + \beta_0 v_0;
\end{aligned}$$

$$\begin{aligned}
z_1 &= \overline{z_1} + (1-a_1)(\alpha_0 w_0 + \beta_0 v_0) + a_1 u_1; \\
y_1 &= \overline{y_1} + (1-b_1-c_1)(\alpha_0 w_0 + \beta_0 v_0) + b_1 v_1 + c_1 u_1; \\
x_2 &= \overline{x_2} + (1-\alpha_1-\beta_1)(\alpha_0 w_0 + \beta_0 v_0) + \alpha_1 w_1 + \beta_1 v_1;
\end{aligned}$$

$$\begin{aligned}
z_2 &= \overline{z_2} + (1-a_2)(1-\alpha_1-\beta_1)(\alpha_0 w_0 + \beta_0 v_0) \\
&\quad + (1-a_2)(\alpha_1 w_1 + \beta_1 v_1) + a_2 u_2; \\
y_2 &= \overline{y_2} + (1-b_2-c_2)[(1-\alpha_1-\beta_1)(\alpha_0 w_0 + \beta_0 v_0) \\
&\quad + (\alpha_1 w_1 + \beta_1 v_1)] + b_2 v_2 + c_2 u_2; \\
x_3 &= \overline{x_3} + (1-\alpha_2-\beta_2)(1-\alpha_1-\beta_1)(\alpha_0 w_0 + \beta_0 v_0) \\
&\quad + (1-\alpha_2-\beta_2)(\alpha_1 w_1 + \beta_1 v_1) + \alpha_2 w_2 + \beta_2 v_2;
\end{aligned}$$

Repeating the above process, we obtain

$$x_{n+1}=\overline{x_{n+1}}+\sum_{k=0}^{n}(\alpha_k w_k+\beta_k v_k)\left[\prod_{i=k+1}^{n}(1-\alpha_i-\beta_i)\right],$$

$$\begin{aligned} y_n &= \overline{y_n}+b_n v_n+c_n u_n+(1-b_n-c_n)\sum_{k=0}^{n-1}(\alpha_k w_k+\beta_k v_k)\left[\prod_{i=k+1}^{n-1}(1-\alpha_i-\beta_i)\right] \\ &= \overline{y_n}+b_n v_n+c_n u_n+(1-b_n-c_n)(x_n-\overline{x_n}), \end{aligned}$$

and

$$\begin{aligned} z_n &= \overline{z_n}+a_n u_n+(1-a_n)\sum_{k=0}^{n-1}(\alpha_k w_k+\beta_k v_k)\left[\prod_{i=k+1}^{n-1}(1-\alpha_i-\beta_i)\right] \\ &= \overline{z_n}+a_n u_n+(1-a_n)(x_n-\overline{x_n}) \text{ for all } n\in\mathbb{N}. \end{aligned}$$

Define

$$Q_n^{(1)}:=x_{n+1}-\overline{x_{n+1}}=\sum_{k=0}^{n}(\alpha_k w_k+\beta_k v_k)\left[\prod_{i=k+1}^{n}(1-\alpha_i-\beta_i)\right], \tag{2.4}$$

$$Q_n^{(2)}:=y_n-\overline{y_n}=b_n v_n+c_n u_n+(1-b_n-c_n)\,Q_{n-1}^{(1)}, \tag{2.5}$$

and

$$Q_n^{(3)}:=z_n-\overline{z_n}=a_n u_n+(1-a_n)\,Q_{n-1}^{(1)} \text{ for all } n\in\mathbb{N}. \tag{2.6}$$

Obviously, the errors of iteration method, after $(n+1)$ times iterations, are added up to $Q_n^{(1)}$, $Q_n^{(2)}$ and $Q_n^{(3)}$.

Now, we are in a position to give the following result.

Theorem 2.1. *Let S, T, B, $Q_n^{(1)}$, $Q_n^{(2)}$ and $Q_n^{(3)}$ be as above.*

(i) If $\sum_{i=0}^{\infty}(\alpha_i+\beta_i)=+\infty$, then the accumulation of errors in (1.2) is bounded and does not exceed the number B;

(ii) If $\sum_{i=0}^{\infty}(\alpha_i+\beta_i)<+\infty$, $\lim_{n\to\infty}a_n=0$ and $\lim_{n\to\infty}(b_n+c_n)=0$, then random errors of (1.2) are controllable.

Proof. (i) It is well known that $\sum_{i=0}^{\infty}(\alpha_i+\beta_i)=+\infty$ implies $\prod_{i=0}^{\infty}(1-\alpha_i-\beta_i)=0$ (see, e.g., (Remark 2.1 of [19])). From (2.2), (2.4)-(2.6) we have

$$\begin{aligned}
\left\|Q_n^{(1)}\right\| &= \left\|(\alpha_0 w_0+\beta_0 v_0)\prod_{i=1}^{n}(1-\alpha_i-\beta_i)\right.\\
&\quad +(\alpha_1 w_1+\beta_1 v_1)\prod_{i=2}^{n}(1-\alpha_i-\beta_i)\\
&\quad \left.+\cdots+(\alpha_{n-1} w_{n-1}+\beta_{n-1} v_{n-1})\prod_{i=n}^{n}(1-\alpha_i-\beta_i)+\alpha_n w_n+\beta_n v_n\right\|\\
&\leq \left\|(\alpha_0 w_0+\beta_0 v_0)\prod_{i=1}^{n}(1-\alpha_i-\beta_i)\right\|\\
&\quad +\left\|(\alpha_1 w_1+\beta_1 v_1)\prod_{i=2}^{n}(1-\alpha_i-\beta_i)\right\|\\
&\quad +\cdots+\left\|(\alpha_{n-1} w_{n-1}+\beta_{n-1} v_{n-1})\prod_{i=n}^{n}(1-\alpha_i-\beta_i)\right\|\\
&\quad +\|\alpha_n w_n+\beta_n v_n\|\\
&\leq (\alpha_0\|w_0\|+\beta_0\|v_0\|)\prod_{i=1}^{n}(1-\alpha_i-\beta_i)\\
&\quad +(\alpha_1\|w_1\|+\beta_1\|v_1\|)\prod_{i=2}^{n}(1-\alpha_i-\beta_i)\\
&\quad +\cdots+(\alpha_{n-1}\|w_{n-1}\|+\beta_{n-1}\|v_{n-1}\|)\prod_{i=n}^{n}(1-\alpha_i-\beta_i)\\
&\quad +\alpha_n\|w_n\|+\beta_n\|v_n\|\\
&\leq B\left\{(\alpha_0+\beta_0)\prod_{i=1}^{n}(1-\alpha_i-\beta_i)+(\alpha_1+\beta_1)\prod_{i=2}^{n}(1-\alpha_i-\beta_i)\right.\\
&\quad \left.+\cdots+(\alpha_{n-1}+\beta_{n-1})\prod_{i=n}^{n}(1-\alpha_i-\beta_i)+\alpha_n+\beta_n\right\}\\
&= B\left\{\prod_{i=0}^{n}(1-\alpha_i-\beta_i)+(\alpha_0+\beta_0)\prod_{i=1}^{n}(1-\alpha_i-\beta_i)\right.\\
&\quad +(\alpha_1+\beta_1)\prod_{i=2}^{n}(1-\alpha_i-\beta_i)+\cdots+(\alpha_{n-1}+\beta_{n-1})\prod_{i=n}^{n}(1-\alpha_i-\beta_i)\\
&\quad \left.+\alpha_n+\beta_n-\prod_{i=0}^{n}(1-\alpha_i-\beta_i)\right\}\\
&= B\left[1-\prod_{i=0}^{n}(1-\alpha_i-\beta_i)\right]\leq B\left[1-\prod_{i=0}^{\infty}(1-\alpha_i-\beta_i)\right]=B, \qquad (2.7)
\end{aligned}$$

$$\begin{aligned}
\left\|Q_n^{(2)}\right\| &= \left\|b_n v_n+c_n u_n+(1-b_n-c_n)Q_{n-1}^{(1)}\right\|\\
&\leq b_n\|v_n\|+c_n\|u_n\|+(1-b_n-c_n)\left\|Q_{n-1}^{(1)}\right\|\\
&\leq B(b_n+c_n)+(1-b_n-c_n)B=B, \qquad (2.8)
\end{aligned}$$

and

$$\begin{aligned}
\left\|Q_n^{(3)}\right\| &= \left\|a_n u_n+(1-a_n)Q_{n-1}^{(1)}\right\|\\
&\leq a_n\|u_n\|+(1-a_n)\left\|Q_{n-1}^{(1)}\right\|\\
&\leq a_n B+(1-a_n)B=B \text{ for all } n\in\mathbb{N}. \qquad (2.9)
\end{aligned}$$

Hence, we have $\max_{n\in\mathbb{N}}\left\{\left\|Q_n^{(1)}\right\|,\left\|Q_n^{(2)}\right\|,\left\|Q_n^{(3)}\right\|\right\}\leq B$.

(ii) Indeed, $\sum_{i=0}^{\infty}(\alpha_i+\beta_i)<+\infty$ implies that $\prod_{i=0}^{\infty}(1-\alpha_i-\beta_i)\in(0,1)$. Let $1-\prod_{i=0}^{\infty}(1-\alpha_i-\beta_i)=\ell\in(0,1)$. Thus, from (2.7), we obtain

$$\left\|Q_n^{(1)}\right\|\leq B\left[1-\prod_{i=0}^{\infty}(1-\alpha_i-\beta_i)\right]\leq \ell B \text{ for all } n\in\mathbb{N}. \tag{2.10}$$

On the other hand, the condition $\lim_{n\to\infty}(b_n+c_n)=0$ implies the existence of an $n_0\in\mathbb{N}$ such that for all $n\geq n_0$ we have $b_n+c_n\leq \ell/(1-\ell)$. Using this fact together with (2.8) and (2.10), we get

$$\begin{aligned}\left\|Q_n^{(2)}\right\| &\leq (b_n+c_n)B+(1-b_n-c_n)\left\|Q_{n-1}^{(1)}\right\|\\ &\leq (b_n+c_n)B(1-\ell)+B\ell\\ &\leq \frac{\ell}{1-\ell}B(1-\ell)+B\ell=2B\ell \text{ for all } n\geq n_0.\end{aligned} \tag{2.11}$$

Similarly, the condition $\lim_{n\to\infty}a_n=0$ implies the existence of an $n_0\in\mathbb{N}$ such that for all $n\geq n_0$ we have $a_n\leq \ell/(1-\ell)$. Hence, from (2.9) and (2.10), we have

$$\begin{aligned}\left\|Q_n^{(3)}\right\| &\leq a_n\|u_n\|+(1-a_n)\left\|Q_{n-1}^{(1)}\right\|\\ &\leq a_nB(1-\ell)\ +B\ell\\ &\leq \frac{\ell}{1-\ell}B(1-\ell)\ +B\ell=2B\ell \text{ for all } n\geq n_0.\end{aligned} \tag{2.12}$$

Thus, we conclude that $\left\|Q_n^{(1)}\right\|$, $\left\|Q_n^{(2)}\right\|$ and $\left\|Q_n^{(3)}\right\|$ can be controlled for suitable choice of the parameter sequences $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ for all $n\geq n_0$. □

Example 2.2. *Let $\alpha_n+\beta_n=\frac{1}{(n^2+4n+3)^2}$ for all $n\in\mathbb{N}$. Then, we have by the Wolfram Mathematica 9 software package that $\sum_{i=0}^{\infty}(\alpha_i+\beta_i)=\frac{1}{48}(4\pi^2-33)<+\infty$ and $\ell=1-\prod_{i=0}^{\infty}(1-\alpha_i-\beta_i)=1+\frac{2\sqrt{2}\sin(\sqrt{2}\pi)}{\pi}\approx 0.132183\in(0,1)$ which implies together with (2.10)-(2.12) that $\left\|Q_n^{(1)}\right\|\leq\left(1+\frac{2\sqrt{2}\sin(\sqrt{2}\pi)}{\pi}\right)B$, $\left\|Q_n^{(2)}\right\|\leq 2\left(1+\frac{2\sqrt{2}\sin(\sqrt{2}\pi)}{\pi}\right)B$ and $\left\|Q_n^{(3)}\right\|\leq 2\left(1+\frac{2\sqrt{2}\sin(\sqrt{2}\pi)}{\pi}\right)B$ for all $n\in\mathbb{N}$.*
Especially, for any $\varepsilon\in(0,1)$, if $\alpha_n+\beta_n=\frac{5^{n+2}}{7^{n+3}}\varepsilon$ for all $n\in\mathbb{N}$, then

$$\prod_{i=0}^{\infty}(1-\alpha_i-\beta_i)\geq 1-\sum_{i=0}^{\infty}(\alpha_i+\beta_i)=1-\frac{25}{98}\varepsilon,$$

which yields $\ell<\frac{25}{98}\varepsilon$, so that

$$\left\|Q_n^{(1)}\right\|\leq\frac{25}{98}\varepsilon B \text{ for all } n\in\mathbb{N},$$

$$\left\|Q_n^{(2)}\right\|\leq\frac{25}{49}\varepsilon B \text{ for all } n\geq n_0,$$

and

$$\left\|Q_n^{(3)}\right\|\leq\frac{25}{49}\varepsilon B \text{ for all } n\geq n_0,$$

where n_0 belongs to $\mathbb{N}$ and the inequalities $a_n\leq\frac{\varepsilon}{3.92-\varepsilon}$ and $b_n+c_n\leq\frac{\varepsilon}{3.92-\varepsilon}$ hold. Hence, the random errors is controllable in a permissible range for suitable choice of the parameter sequences $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ for all $n\geq n_0$.

Acknowledgements

The authors would like to express their sincere thanks to the editor and the anonymous reviewers for their helpful comments and suggestions.

Funding

There is no funding for this work.

Availability of data and materials

Not applicable.

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

References

[1] C. Garodia, I. Uddin, *A new iterative method for solving split feasibility problem*, J. Appl. Anal. Comput., **10**(3) (2020), 986-1004.
[2] C. Garodia, I. Uddin, *A new fixed point algorithm for finding the solution of a delay differential equation*, AIMS Math., **5** (4) (2020), 3182-3200.
[3] E. Hacıoğlu, F. Gürsoy, S. Maldar, Y. Atalan, G. V. Milovanović, *Iterative approximation of fixed points and applications to two-point second-order boundary value problems and to machine learning*, Appl. Numer. Math., **167** (2021), 143–172.
[4] S. Maldar, F. Gürsoy, Y. Atalan, M. Abbas, *On a three-step iteration process for multivalued Reich-Suzuki type $\alpha-$nonexpansive and contractive mappings*, J. Appl. Math. Comput., (2021). https://doi.org/10.1007/s12190-021-01552-7.
[5] S. Maldar, Y. Atalan, K. Doğan, *Comparison rate of convergence and data dependence for a new iteration method*, Tbilisi Math. J., **13**(4) (2020), 65–79.
[6] S. Maldar, *An examination of data dependence for Jungck-type iteration method*, Erciyes Univ. J. Inst. Sci. Tech., 36 (3) (2020), 374–384.
[7] E. Hacıoğlu, V. Karakaya, *Existence and convergence for a new multivalued hybrid mapping in CAT(κ) spaces*, Carpathian J. Math., **33**(3) (2017), 319–326.
[8] E. Hacıoğlu, V. Karakaya, *Some fixed point results for a multivalued generalization of generalized hybrid mappings in CAT(κ)-spaces*, Konuralp J. Math., **6**(1) (2018), 26–34.
[9] E. Hacıoğlu, V. Karakaya, *A new contraction-like multivalued mapping on geodesic spaces*, Sci. Stud. Res. Ser. Math. Inform., **29**(1) (2019), 89–102.
[10] F. Gürsoy, K. Doğan, A. R. Khan, *Direct estimate of accumulated errors for a general iteration method*, Math. Adv. Pure Appl. Sci. (MAPAS), **2**(2019), 19–24.
[11] Y. Xu, Z. Liu, *On estimation and control of errors of the Mann iteration process*, J. Math. Anal. Appl., **286** (2003), 804-806.
[12] Y. Xu, Z. Liu, S. M. Kang, *Accumulation and control of random errors in the Ishikawa iterative process in arbitrary Banach space*, Comput. Math. Appl., **61** (2011), 2217-2220.
[13] S. Thianwan, S. Suantai, *Convergence criteria of a new three-step iteration with errors for nonexpansive nonself-mappings*, Comput. Math. Appl., **52** (2006), 1107-1118.
[14] K. Nammanee, S. Suantai, *The modified Noor iterations with errors for non-Lipschitzian mappings in Banach spaces*, Appl. Math. Comput., **187** (2007), 669-679.
[15] K. Nammanee, M. A. Noor, *S. Suantai, Convergence criteria of modified Noor iterations with errors for asymptotically nonexpansive mappings*, J. Math. Anal. Appl., 314 (2006), 320-334.
[16] M. A. Noor, *New approximation schemes for general variational inequalities*, J. Math. Anal. Appl., 251 (2000) 217–229.
[17] S. Ishikawa, *Fixed points by a new iteration method*, Proc. Amer. Math. Soc., **44** (1974), 147-150.
[18] W. R. Mann, *Mean value methods in iteration*, Proc. Amer. Math. Soc., **4** (1953), 506-510.
[19] S. M. Şoltuz, T. Grosan, *Data dependence for Ishikawa iteration when dealing with contractive-like operators*, Fixed Point Theory A., **2008** (2008), 1-7.