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A Comparison Between Analytical and Numerical Solutions for Time-Fractional Coupled Dispersive Long-Wave Equations

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Abstract: In this article, a technique namely Tanh method is applied to obtain some traveling wave solutions for coupled Dispersive Long-Wave equations, and by using LADM we obtain an approximate solution to TFDLW (time-fractional DLW equations). A comparison between the traveling wave solution and the approximate one of the DLW system indicates that Laplace Adomian Decomposition Method (LADM) is highly accurate and can be considered a very useful and valuable method. The availability of computer systems like ***Mathematica 11*** software facilitates the tedious algebraic calculations and plots of surfaces of solutions. The methods which we will propose in this paper are also standard, direct and computerizable methods, which allows us to do complicated and tedious algebraic calculation.

Keywords: Adomian polynomials, Caputo's fractional derivative, dispersive long-wave, LADM, Tanh method.

1. Introduction

In recent years, many authors have interested to the research of exact solutions to nonlinear evolution equations [17, 24], because it is the key to understanding the various physical phenomena in many fields such as plasma physics and fluid dynamics and nonlinear optics and other phenomena in life sciences like chemical and biological reactions. The exact traveling wave solution is one of the important solutions to partial differential equations in general [11, 15–17].

To investigate the traveling wave solutions, we propose in this work the Tanh method (or hyperbolic tangent method), because it is a powerful technique to search for traveling waves coming out from one-dimensional nonlinear wave and evolution equations [13, 20, 21, 34].

But, some evolution problems (differential equations or systems) do not admit the traveling wave solutions, due to that, we propose a semi-analytical method called Laplace Adomian De-

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composition Method (LADM), it is a combination of the Adomian Decomposition Method (ADM) and Laplace transforms. This method was successfully used for solving different problems as in [6, 12, 14, 29–31].

In this paper, we will study the coupled dispersive long-wave equations. the system appear in hydrodynamics, it describes the evolution of the horizontal velocity component of water waves propagating in both directions in an infinite narrow channel of constant depth [5, 8, 10, 15, 16, 27].

Our goal is to obtain the approximate solutions of the time-fractional DLW system, and compare this solution (in particular case) with the traveling wave solution of the system to show that the proposed algorithm (LADM) is suitable for such problems and is very efficient.

2. Preliminaries

Before the beginning of this research, we are trying in a hurry to get to know the supporting materials to accomplish this work.

2.1. The Tanh Method

To describe the basic idea of Tanh method we consider the general nonlinear partial differential equation for a unknown function $u(x,t)$:

$$F[u, u_t, , u_x, u_{xx}, u_{xt}, \dots] = 0. \tag{1}$$

We define the traveling wave transformation, $\zeta = c(x - \mu t)$. Here $c(>0)$ represents the wave number and μ is the (unknown) velocity of the traveling wave, and we investigate the traveling wave solutions of (1) of the form: $u(x,t) = U(\zeta)$, then (1) reduces to a nonlinear ordinary differential equation of the form:

$$G[U, U_\zeta, , U_{\zeta\zeta}, U_{\zeta\zeta\zeta}, \dots] = 0. \tag{2}$$

Our main goal is to derive exact solutions for this ODE. So we introduce a new variable $\Phi = \tanh\zeta$ in the ODE. The latter equation then solely depends on Φ, because all derivatives $\frac{d}{d\zeta}$ in (2) are now replaced by $(1 - \Phi^2)\frac{d}{d\zeta}$. The Tanh method admits the use of the finite expansion

$$U(\zeta) = W(\Phi) = \sum_{n=0}^{N} a_n \Phi^n. \tag{3}$$

To obtain the positive integer N (highest order of Φ), we balance the linear term of the highest order in the equation (2) with the highest order nonlinear term. Supposing $U^{(s)}$ and $U^p(U^{(\ell)})^r$ are linear and nonlinear terms respectively, we have $N + s = pN + r(N + \ell)$ or equivalent to $N = \frac{s-r\ell}{p+r-1}$. Substituting (3) into (2), results a polynomial of $\Phi^n, k = 0, 1, 2, \dots$. Equating the coefficients of the same powers of Φ to zero. Solving the obtained algebraic system, yields the values of a_n, c, μ, where $0 \leq n \leq N$.

2.2. Laplace Transform

Given a suitable function $F(t)$ the Laplace transform [26, 28], written $f(s)$ is defined by

$$\mathcal{L}[F(t)] = f(s) = \int_0^{\infty} F(t)e^{-st}dt, \tag{4}$$

the inverse Laplace transform is defined by

$$\mathcal{L}^{-1}[f(s)] = F(t). \tag{5}$$

The important properties of Laplace transform and its inverse that will be used in this paper are

- If $F_1(t)$ and $F_2(t)$ are two functions whose Laplace transform exists, then
- $\mathcal{L}[aF_1(t) + bF_2(t)] = a\mathcal{L}[F_1(t)] + b\mathcal{L}[F_2(t)]$,
- $\mathcal{L}(t^{\alpha}) = \Gamma(\alpha+1)s^{-\alpha-1}, \quad \alpha > 0$,
- $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}, \quad n$ a positive integer.
- The inverse Laplace transform is linear,i.e $\mathcal{L}^{-1}[af_1(s)+bf_2(s)] = a\mathcal{L}^{-1}[f_1(s)]+b\mathcal{L}^{-1}[f_2(s)]$,
- $\mathcal{L}^{-1}(\frac{1}{s^{\alpha}}) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}, \quad \alpha > 0.$

2.3. Caputo Derivative

The Caputo derivative of order α is defined by the formula [18, 23]:

$$D_*^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f^{(m)}(\tau)d\tau, & \text{if } m-1 < \alpha < m \\ \frac{d^m}{dt^m} f(t), & \text{if } \alpha = m \end{cases} \tag{6}$$

where $m \in \mathbb{N}^*$ and $\Gamma(.)$ denotes the Gamma function defined by $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt, \quad x > 0$.

The important properties of the Caputo derivative that will be used in this paper are [22, 25, 33]:

$$D^{\alpha}t^{\beta} = \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)}t^{\beta-\alpha} \tag{7}$$

$$D^{\alpha}c = 0. \tag{8}$$

The Laplace transform of the Caputo derivative is:

$$\mathcal{L}[D_t^{\alpha}u(x,t)] = s^{\alpha}u(x,s) - \sum_{i=0}^{n-1} u^{(i)}(x,0^{+})s^{\alpha-1-i}, \quad n-1 \le \alpha \le n. \tag{9}$$

2.4. The Laplace Transform Adomian Decomposition Method

The ADM was introduced by Adomian [1–4, 9, 10, 19], the underlying idea of the technique is to assume an infinite solution of the form $u = \sum_{n=0}^{\infty} u_n$, then apply Laplace transformation to the differential equation. The method identifies and separates the linear and nonlinear parts of a differential equation, the nonlinear terms are then decomposed in terms of Adomian polynomials [7, 32] and an iterative algorithm is constructed for the determination of the u_n in a recursive manner.

Given a partial differential equation

$$Fu(x,t) = h(x,t) \ \text{ with initial condition } \ u(x,0) = f(x) \tag{10}$$

where F is a differential operator that could, in general, be nonlinear and therefore includes some linear and nonlinear terms. In general, Eq. (10) could be written as

$$L_t u(x,t) = Ru(x,t) + Nu(x,t) + h(x,t) \tag{11}$$

where $L_t = \frac{\partial^{\alpha}}{\partial t^{\alpha}}, 0 < \alpha \le 1$ (in this paper) , R is a linear operator that includes partial derivatives with respect to x , N is a nonlinear operator and h is a non-homogeneous term that is u -independent.

The LADM consists of applying Laplace transform first on both sides of Eq. (11), obtaining

$$\mathcal{L}\{L_t u(x,t)\} = \mathcal{L}\{Ru(x,t) + Nu(x,t) + h(x,t)\}. \tag{12}$$

An equivalent expression to (12) is

$$s^{\alpha}u(x,s) - u(x,0)s^{\alpha-1} = \mathcal{L}\{Ru(x,t) + Nu(x,t) + h(x,t)\}. \tag{13}$$

In the homogeneous case, $h(x,t) = 0$, we have

$$u(x,s) = \frac{f(x)}{s} + \frac{1}{s^{\alpha}}\mathcal{L}\{Ru(x,t) + Nu(x,t)\}. \tag{14}$$

Now, applying the inverse Laplace transform to Eq. (14)

$$u(x,s) = f(x) + \mathcal{L}^{-1}[\frac{1}{s^{\alpha}}\mathcal{L}\{Ru(x,t) + Nu(x,t)\}]. \tag{15}$$

The ADM method proposes a series solution $u(x,t)$ given by,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{16}$$

The nonlinear term $Nu(x,t)$ is given by

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n(u_0, u_1, u_2, \ldots, u_n) \tag{17}$$

where $\{A_n\}_{n=0}^{\infty}$ is the so-called Adomian polynomials sequence established in [32] , in general, give us term to term:

$$\begin{aligned}
&A_0 = N(u_0)\\
&A_1 = u_1 N'(u_0)\\
&A_2 = u_2 N'(u_0) + \frac{1}{2} u_1^2 N''(u_0)\\
&A_3 = u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{3!} u_1^3 N^{(3)}(u_0)\\
&A_4 = u_4 N'(u_0) + (\frac{1}{2} u_2^2 + u_1 u_3) N''(u_0) + \frac{1}{2!} u_1^2 u_2 N^{(3)}(u_0) + \frac{1}{4!} u_1^4 N^{(4)}(u_0)\\
&\vdots
\end{aligned}$$

Some other approaches to obtain Adomian's polynomials can be found in [32].

Using (16) and (17) into Eq. (15), we obtain,

$$\sum_{n=0}^{\infty} u_n(x,t) = f(x) + \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_n(x,t) + \sum_{n=0}^{\infty} A_n(u_0, u_1, u_2, \ldots, u_n)\right\}\right]. \tag{18}$$

We deduce the following recurrence formulas

$$\begin{cases} u_0(x,t) = f(x) \\ u_{n+1}(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left\{Ru_n(x,t) + A_n(u_0, u_1, u_2, \ldots, u_n)\right\}\right], \; n = 0, 1, 2, \ldots \end{cases} \tag{19}$$

Using (19) we can obtain an approximate solution of (10), using

$$u(x,t) \approx \sum_{n=0}^{k} u_n(x,t), \quad \text{where} \lim_{t\to\infty} \sum_{n=0}^{k} u_n(x,t) = u(x,t) \tag{20}$$

Remark 2.1 *All results and 2D and 3D plots below are obtained by using* ***Mathematica 11*** *software.*

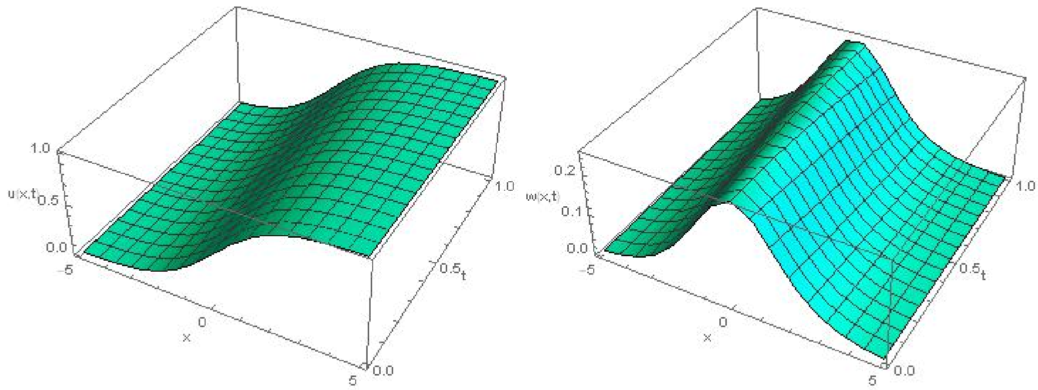
3. Main Results

3.1. Dispersive Long-Wave Equations Solutions by Using Tanh Method

In this section, we will apply the Tanh method to find the axact solutions to the system of *DLW* equations in the form,

$$\begin{cases} u_t - 2uu_x + u_{xx} - 2w_x = 0, \\ w_t - 2u_x w - 2uw_x - w_{xx} = 0, \end{cases} \tag{21}$$

which describe an interaction of two long waves with different dispersion.



(a) $u(x,t)$ (b) $w(x,t)$

Figure 1: Plot of the exact solution given by Eq. (21) for $(x,t) \in [-5,5] \times [0,1]$

Table 1: A comparison between approximate solution and exact solution of (21) for $t = 1$

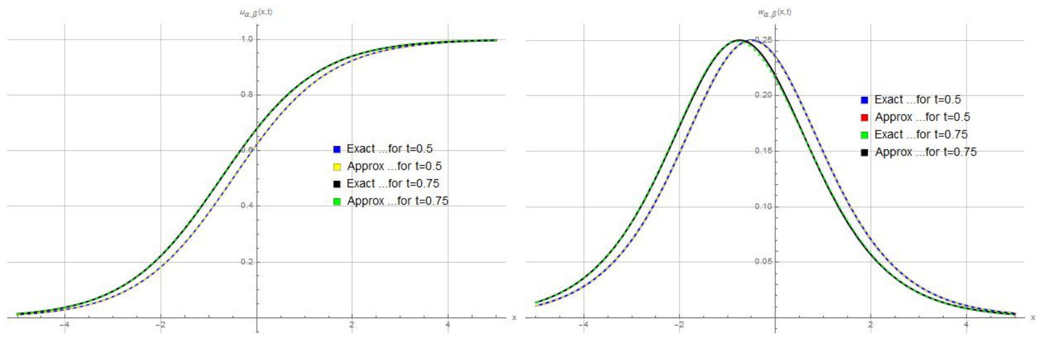
$t=1$						
x	U_{LADM}	U_{Exact}	$Error$	W_{LADM}	W_{Exact}	$Error$
-5	0.0176843	0.0179862	0.000301956	0.0174045	0.0176627	0.000258176
-4	0.0467944	0.0474259	0.000631476	0.0448219	0.0450108	0.000354786
-3	0.118537	0.119203	0.000666089	0.105654	0.104994	0.00066076
-2	0.270653	0.268941	0.00171161	0.200918	0.196612	0.00430597
-1	0.505095	0.5	0.00509463	0.249222	0.25	0.000777586
0	0.729167	0.731059	0.00189191	0.1875	0.196612	0.00911193
1	0.876354	0.880797	0.00444304	0.108676	0.104994	0.00368229
2	0.952285	0.952574	0.000289436	0.0479209	0.0451767	0.00274427
3	0.982794	0.982014	0.000779722	0.0176301	0.0176627	0.000032656
4	0.993795	0.993307	0.000487492	0.00629442	0.00664806	0.000353639
5	0.997739	0.997527	0.000212108	0.00227446	0.00246651	0.00019205

Table 2: A comparison between approximate solution and exact solution of (21) for $t = 0.5$

$t = 0.5$						
x	U_{LADM}	U_{Exact}	$Error$	W_{LADM}	W_{Exact}	$Error$
-5	0.0109697	0.0109869	0.0000172033	0.0108512	0.0108662	0.0000150336
-4	0.029275	0.0293122	0.000037274	0.0284298	0.028453	0.0000232409
-3	0.0758117	0.0758582	0.0000464661	0.0701289	0.0701037	0.0000251867
-2	0.182504	0.182426	0.0000789133	0.149398	0.149146	0.00025184
-1	0.377869	0.377541	0.000327988	0.235052	0.235004	0.0000483377
0	0.622396	0.622459	0.0000634979	0.234375	0.235004	0.000628712
1	0.817271	0.817574	0.000303099	0.14934	0.149146	0.000193969
2	0.924108	0.924142	0.0000338522	0.0703018	0.0701037	0.000198079
3	0.970737	0.970688	0.0000493047	0.0284572	0.028453	$4.16143 * 10^{-6}$
4	0.989046	0.989013	0.0000326454	0.0108433	0.0108662	0.0000228805
5	0.995944	0.99593	0.0000144083	0.00404064	0.00405357	0.0000129353

Table 3: A comparison between approximate solution and exact solution of (21) for $t = 0.75$

$t = 0.75$						
x	U_{LADM}	U_{Exact}	$Error$	W_{LADM}	W_{Exact}	$Error$
-5	0.0139724	0.0140636	0.0000911998	0.0137869	0.0138658	0.0000789072
-4	0.0371325	0.0373269	0.000194413	0.0358179	0.0359336	0.00011572
-3	0.0951246	0.0953495	0.000224875	0.0864237	0.0862579	0.000165753
-2	0.223169	0.2227	0.000469184	0.174431	0.173105	0.00132632
-1	0.43947	0.437823	0.00164678	0.246136	0.246134	$1.76555 * 10^{-6}$
0	0.678711	0.679179	0.000467762	0.214844	0.217895	0.00305124
1	0.85048	0.851953	0.00147284	0.127217	0.126129	0.00108815
2	0.939785	0.939913	0.000128778	0.0574101	0.0564762	0.000933878
3	0.977271	0.977023	0.000248702	0.0224534	0.0224494	$3.94183 * 10^{-6}$
4	0.991582	0.991423	0.000159663	0.00838991	0.00850391	0.000114001
5	0.996897	0.996827	0.0000699326	0.00309955	0.00316262	0.0000630697



(a) $u(x,t)$ (b) $w(x,t)$

Figure 2: Plot of the exact and approximate solution of Eq. (21) for $t = 0.5, 0.75$

We consider the traveling wave transformation defined by,

$$U(\zeta) = u(x,t),\ W(\zeta) = w(x,t),\ \zeta = c(x - \mu t). \tag{22}$$

Using traveling wave Eqs. (22), then (21) transform into the following ordinary differential equations

$$\mu U' + 2UU' - cU'' + 2W' = 0, \tag{23}$$

$$\mu W' + 2U'W + 2UW' + cW'' = 0. \tag{24}$$

Integrating Eqs. (23) and (24) with respect to ζ, choosing the constant of integration as zero, we obtain the following ordinary differential equations respectively:

$$\mu U + U^2 - cU' + 2W = 0. \tag{25}$$

$$\mu W + 2UW + cW' = 0, \tag{26}$$

From Eq. (25), we get

$$W = -\frac{\mu}{2}U - \frac{1}{2}U^2 + \frac{c}{2}U', \tag{27}$$

derivative (27) with respect to ζ, yields

$$W' = -\frac{\mu}{2}U' - UU' + \frac{c}{2}U'', \tag{28}$$

substituting Eqs. (27) and (28) into Eq. (26), we obtain

$$\frac{\mu^2}{2}U + \frac{3\mu}{2}U^2 + U^3 - \frac{c^2}{2}U'' = 0. \tag{29}$$

Now balancing the highest order derivative U'' and nonlinear term U^3, we get $3N = N + 2$ or equivalent to $N = 1$. Therefore, Eq. (3) reduces to

$$U(\zeta) = a_0 + a_1 \tanh(\zeta), \tag{30}$$

substituting Eq. (30) into Eq. (29) and using ***Mathematica 11*** software, we get a polynomial of $\tanh(\zeta)^k$, (k = 0, 1, 2, ...). Equating the coefficients of this polynomial of the same powers of $\tanh(\zeta)$ to zero, we obtain a system of algebraic equations for a_0, a_1, c, μ.

$$\frac{a_0\mu^2}{2} + \frac{3a_0^2\mu}{2} + a_0^3 = 0$$

$$a_1c^2 + \frac{a_1\mu^2}{2} + 3a_0a_1\mu + 3a_0^2a_1 = 0$$

$$\frac{3a_1^2\mu}{2} + 3a_0a_1^2 = 0$$

$$a_1^3 - a_1c^2 = 0$$

where $a_1 \neq 0$.

Solving them by means of ***Mathematica 11*** software gives:

$\{a_0 = c, a_1 = c, \mu = -2c\}$, $\{a_0 = -c, a_1 = c, \mu = 2c\}$, substituting into Eq. (30) it follows

$$u_1(x,t) = c + c\tanh(cx + 2c^2 t) \tag{31}$$

$$u_2(x,t) = -c + c\tanh(cx - 2c^2 t), \tag{32}$$

substituting Eqs. (31) and (32) into Eq. (27) yields,

$$w_1(x,t) = c^2 - c^2\tanh^2(cx + 2c^2 t), \tag{33}$$

$$w_2(x,t) = c^2 - c^2\tanh^2(cx - 2c^2 t). \tag{34}$$

In particular, take $c = \frac{1}{2}$, we get

$$u_1(x,t) = \frac{1}{2} + \frac{1}{2}\tanh(\frac{1}{2}x + \frac{1}{2}t), \quad w_1(x,t) = \frac{1}{4} - \frac{1}{4}\tanh^2(\frac{1}{2}x + \frac{1}{2}t), \tag{35}$$

3.2. The Approximate Solution of Time-Fractional DLW System by LADM

Consider the system of DLW equations

$$\begin{cases} D_t^{\alpha} u = -u_{xx} + 2w_x + 2uu_x, \\ D_t^{\beta} w = w_{xx} + 2wu_x + 2uw_x \end{cases} \tag{36}$$

subject to the initial conditions

$$u(x,0) = f(x) = \frac{e^x}{1+e^x}, \; w(x,0) = g(x) = \frac{e^x}{(1+e^x)^2} \tag{37}$$

where $0 < \alpha, \beta \leq 1$ and $D_t^{\alpha} = \frac{\partial^{\alpha}}{\partial t^{\alpha}}, D^{\beta} = \frac{\partial^{\beta}}{\partial t^{\beta}}$ the derivatives in the sense of Caputo.

Comparing (36) with Eq. (11) we have that $h(x,t) = 0$, L_t and R becomes:

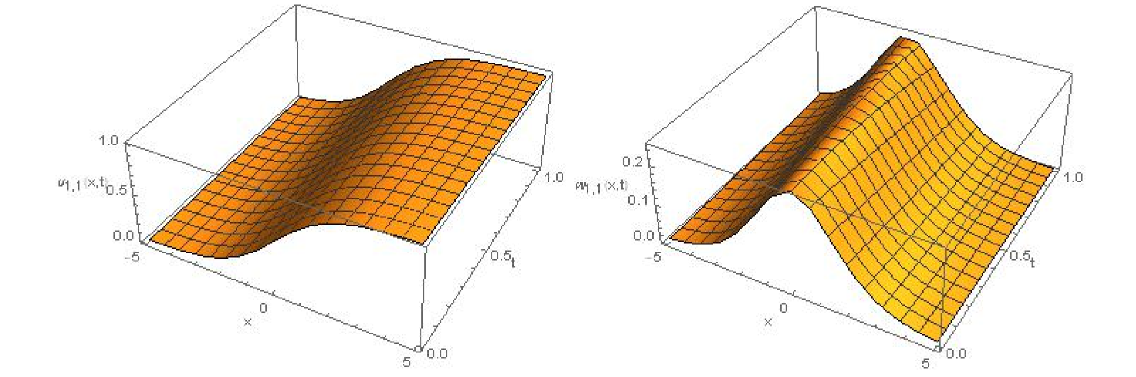
$$L_t u = D_t^{\alpha} u = \frac{\partial^{\alpha}}{\partial t^{\alpha}} u, \quad R_1(u,w) = -u_{xx} + 2w_x, \tag{38}$$

similarly, we have

$$L_t w = D_t^{\beta} w = \frac{\partial^{\beta}}{\partial t^{\beta}} u, \quad R_2(u,w) = w_{xx} \tag{39}$$

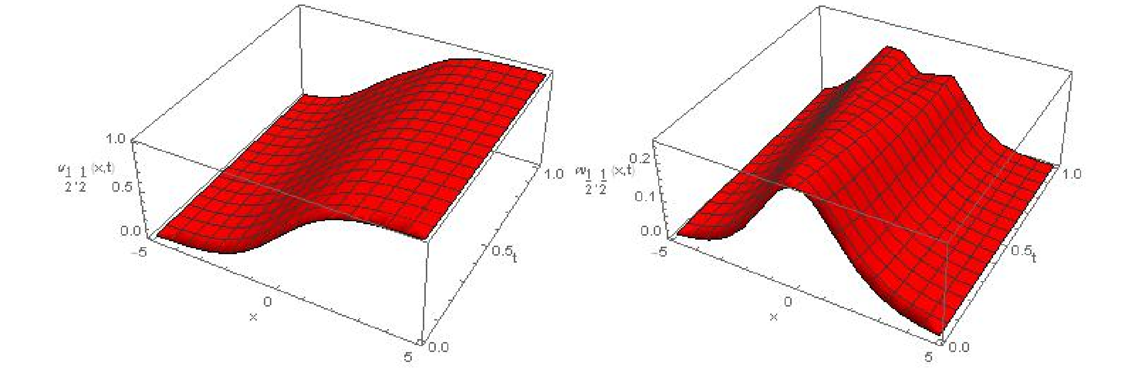
while the nonlinear terms are given by

$$N_1 u = 2uu_x, \quad N_2(u,w) = 2wu_x + 2uw_x. \tag{40}$$



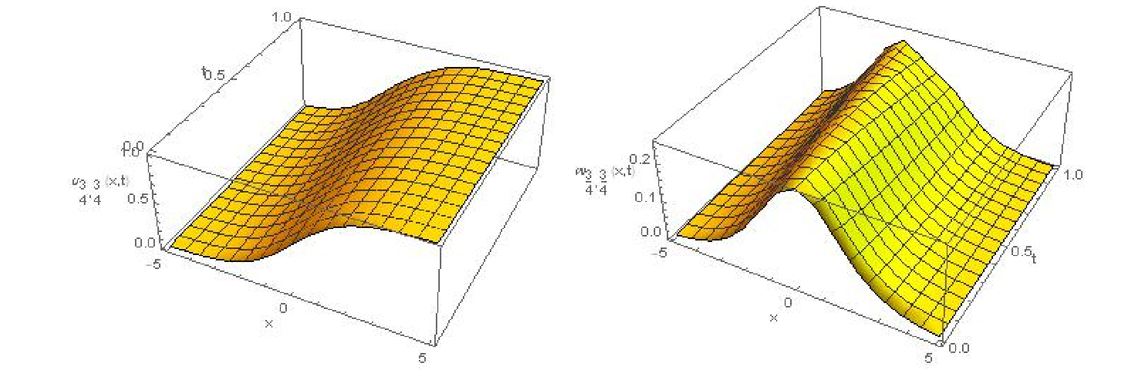
(a) $u_{1,1}(x,t)$ (b) $w_{1,1}(x,t)$

Figure 3: Plot of the approximate solution $(u,w)_{LADM}$ given by Eq. (36) when $\alpha = \beta = 1$ for $(x,t) \in [-5,5] \times [0,1]$.



(a) $u_{\frac{1}{2},\frac{1}{2}}(x,t)$ (b) $w_{\frac{1}{2},\frac{1}{2}}(x,t)$

Figure 4: Plot of the approximate solution $(u,w)_{LADM}$ given by Eq. (36) when $\alpha = \beta = \frac{1}{2}$ for $(x,t) \in [-5,5] \times [0,1]$

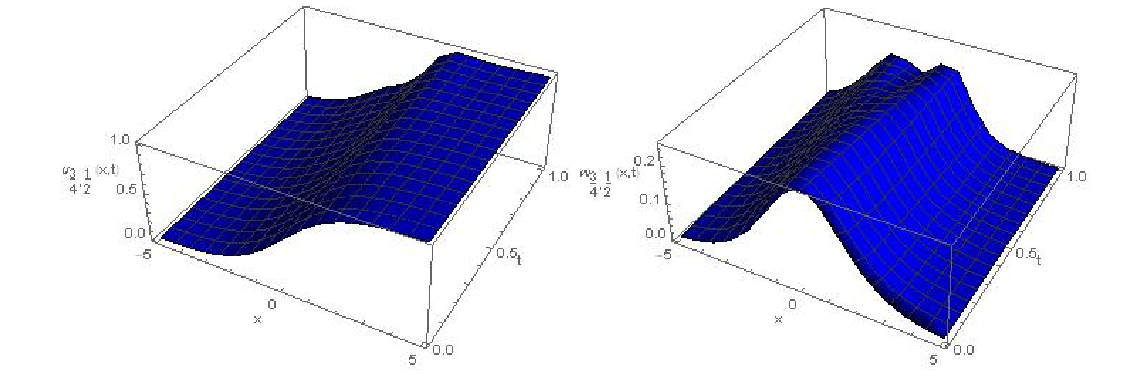


(a) $u_{\frac{3}{4},\frac{3}{4}}(x,t)$ (b) $w_{\frac{3}{4},\frac{3}{4}}(x,t)$

Figure 5: Plot of the approximate solution $(u,w)_{LADM}$ given by Eq. (36) when $\alpha = \beta = \frac{3}{4}$ for $(x,t) \in [-5,5] \times [0,1]$

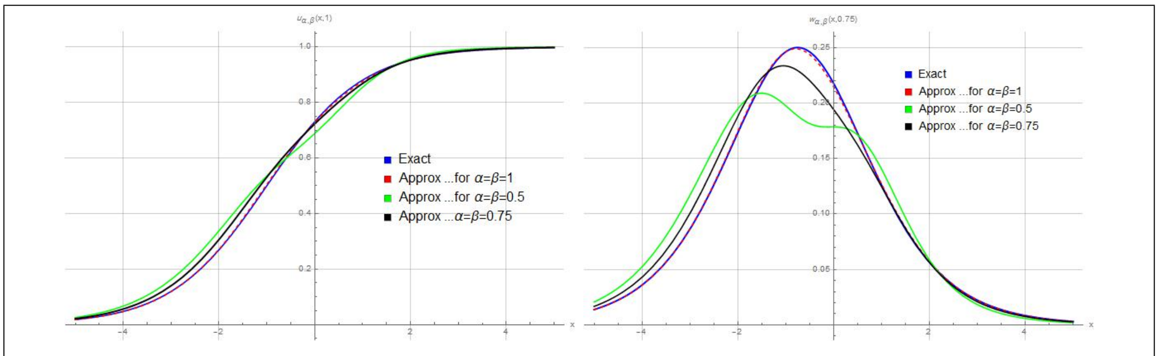
By using now Eq. (19) through the LADM method we obtain recursively

$$\begin{cases} u_0(x,t) = f(x) \\ u_{n+1}(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^\alpha}\mathcal{L}\left\{R_1(u_n,w_n) + A_n(u_0,u_1,u_2,\ldots,u_n)\right\}\right], \; n = 0,1,2,\ldots \end{cases} \tag{41}$$



(a) $u_{\frac{3}{4},\frac{1}{2}}(x,t)$ (b) $w_{\frac{3}{4},\frac{1}{2}}(x,t)$

Figure 6: Plot of the approximate solution $(u,w)_{LADM}$ given by Eq. (36) when $\alpha = \frac{3}{4}, \beta = \frac{1}{2}$ for $(x,t) \in [-5,5] \times [0,1]$



(a) $u_{\alpha,\beta}(x,t)$ (b) $w_{\alpha,\beta}(x,t)$

Figure 7: Series approximation solution of Eq. (36), when $\alpha = \beta = 0.5$, $\alpha = \beta = 0.75$, $\alpha = \beta = 1$ and the exact solution when $\alpha = \beta = 1$ at time $t = 1$ for $(u_{\alpha,\beta}(x,t))$, $t = 0.75$ for $(w_{\alpha,\beta}(x,t))$ and $x \in [-5,5]$ with the first four terms

and

$$\begin{cases} w_0(x,t) = g(x) \\ w_{n+1}(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^\beta}\mathcal{L}\left\{R_2(u_n,w_n) + B_n(u_0,u_1,u_2,\ldots,u_n,w_0,w_1,w_2,\ldots,w_n)\right\}\right], \; n = 0,1,2,\ldots \end{cases} \tag{42}$$

Note that, the nonlinear term $N_2(u,w) = 2wu_x + 2uw_x$ can be split into two terms to facilitate calculations

$$N_{2_1}(u,w) = 2uw_x, \; N_{2_2}(u,w) = 2wu_x,$$

from this, we will consider the decomposition of the nonlinear terms into Adomian polynomials as

$$N_1 u = 2uu_x = \sum_{n=0}^{\infty} A_n(u_0,u_1,u_2,\ldots,u_n), \tag{43}$$

$$N_{2_1}(u,w) = 2uw_x = \sum_{n=0}^{\infty} P_n(u_0,u_1,u_2,\ldots,u_n,w_0,w_1,w_2,\ldots,w_n), \tag{44}$$

$$N_{2_2}(u,w) = 2wu_x = \sum_{n=0}^{\infty} Q_n(u_0,u_1,u_2,\dots,u_n,w_0,w_1,w_2,\dots,w_n) \tag{45}$$

where $B_n = P_n + Q_n$.

Using ADM, Eq. (16) gives

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \quad \text{and} \quad w(x,t) = \sum_{n=0}^{\infty} w_n(x,t). \tag{46}$$

The Adomian polynomials A_n, P_n, Q_n are in the forms [32]

$$A_n = \sum_{i=0}^{n} u_i u_{n-i_x}, \quad P_n = \sum_{i=0}^{n} u_i w_{n-i_x}, \quad Q_n = \sum_{i=0}^{n} w_i u_{n-i_x}. \tag{47}$$

We have

$$\begin{aligned}
A_0 &= 2u_0 u_{0_x} \\
A_1 &= 2u_0 u_{1_x} + 2u_1 u_{0_x} \\
A_2 &= 2u_0 u_{2_x} + 2u_1 u_{1_x} + 2u_2 u_{0_x} \\
A_3 &= 2u_0 u_{3_x} + 2u_1 u_{2_x} + 2u_2 u_{1_x} + 2u_3 u_{0_x} \\
A_4 &= 2u_0 u_{4_x} + 2u_1 u_{3_x} + 2u_2 u_{2_x} + 2u_3 u_{1_x} + 2u_4 u_{0_x} \\
&\vdots
\end{aligned} \tag{48}$$

and

$$\begin{aligned}
P_0 &= 2u_0 w_{0_x} \\
P_1 &= 2u_0 w_{1_x} + 2u_1 w_{0_x} \\
P_2 &= 2u_0 w_{2_x} + 2u_1 w_{1_x} + 2u_2 w_{0_x} \\
P_3 &= 2u_0 w_{3_x} + 2u_1 w_{2_x} + 2u_2 w_{1_x} + 2u_3 w_{0_x} \\
P_4 &= 2u_0 w_{4_x} + 2u_1 w_{3_x} + 2u_2 w_{2_x} + 2u_3 w_{1_x} + 2w_4 u_{0_x} \\
&\vdots
\end{aligned} \tag{49}$$

$$\begin{aligned}
Q_0 &= 2w_0 u_{0_x} \\
Q_1 &= 2w_0 u_{1_x} + 2w_1 u_{0_x} \\
Q_2 &= 2w_0 u_{2_x} + 2w_1 u_{1_x} + 2w_2 u_{0_x} \\
Q_3 &= 2w_0 u_{3_x} + 2w_1 u_{2_x} + 2w_2 u_{1_x} + 2w_3 u_{0_x} \\
Q_4 &= 2w_0 u_{4_x} + 2w_1 u_{3_x} + 2w_2 u_{2_x} + 2w_3 u_{1_x} + 2w_4 u_{0_x} \\
&\vdots
\end{aligned} \tag{50}$$

Through the LADM we obtain recursively

$$
\begin{aligned}
&u_0(x,t) = f(x),\\
&u_1(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left\{-u_{0_{xx}} + 2w_{0_{xx}} + A_0\right\}\right],\\
&u_2(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left\{-u_{1_{xx}} + 2w_{1_{xx}} + A_1\right\}\right],\\
&u_3(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left\{-u_{2_{xx}} + 2w_{2_{xx}} + A_2\right\}\right],\\
&\vdots \qquad \vdots\\
&u_{n+1}(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\left\{-u_{n_{xx}} + 2w_{n_{xx}} + A_n\right\}\right],
\end{aligned}
\tag{51}
$$

and

$$
\begin{aligned}
&w_0(x,t) = g(x),\\
&w_1(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{w_{0_{xx}} + B_0\right\}\right],\\
&w_2(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{w_{1_{xx}} + B_1\right\}\right],\\
&w_3(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{w_{2_{xx}} + B_2\right\}\right],\\
&\vdots \qquad \vdots\\
&w_{n+1}(x,t) = \mathcal{L}^{-1}\left[\frac{1}{s^{\beta}}\mathcal{L}\left\{w_{n_{xx}} + B_n\right\}\right].
\end{aligned}
\tag{52}
$$

Besides

$$
\begin{aligned}
&A_0 = \frac{2e^{2x}}{(e^x+1)^2} - \frac{2e^{3x}}{(e^x+1)^3}\\
&B_0 = P_0 + Q_0 = \frac{4e^{2x}}{(e^x+1)^3} - \frac{6e^{3x}}{(e^x+1)^4},
\end{aligned}
\tag{53}
$$

With the above, we have

$$u_0(x,t) = \frac{e^x}{e^x+1}$$

$$u_1(x,t) = -\frac{e^x t^\alpha}{(e^x+1)\Gamma(\alpha+1)} + \frac{2e^x t^\alpha}{(e^x+1)^2\Gamma(\alpha+1)} + \frac{5e^{2x} t^\alpha}{(e^x+1)^2\Gamma(\alpha+1)} - \frac{4e^{2x} t^\alpha}{(e^x+1)^3\Gamma(\alpha+1)} - \frac{4e^{3x} t^\alpha}{(e^x+1)^3\Gamma(\alpha+1)}, \tag{54}$$

and

$$w_0(x,t) = \frac{e^x}{(e^x+1)^2}$$

$$w_1(x,t) = \frac{e^x t^\beta}{(e^x+1)^2\Gamma(\beta+1)} - \frac{2e^{2x} t^\beta}{(e^x+1)^3\Gamma(\beta+1)}, \tag{55}$$

and proceeding in a similar way we get

$$A_1 = -\frac{4e^{2x} t^\alpha}{(e^x+1)^2\Gamma(\alpha+1)} + \frac{8e^{2x} t^\alpha}{(e^x+1)^3\Gamma(\alpha+1)} + \frac{34e^{3x} t^\alpha}{(e^x+1)^3\Gamma(\alpha+1)} - \frac{36e^{3x} t^\alpha}{(e^x+1)^4\Gamma(\alpha+1)} - \frac{62e^{4x} t^\alpha}{(e^x+1)^4\Gamma(\alpha+1)} + \frac{32e^{4x} t^\alpha}{(e^x+1)^5\Gamma(\alpha+1)} + \frac{32e^{5x} t^\alpha}{(e^x+1)^5\Gamma(\alpha+1)}, \tag{56}$$

$$B_1 = P_1 + Q_1 = \frac{4e^{2x} t^\beta}{(e^x+1)^3\Gamma(\beta+1)} - \frac{18e^{3x} t^\beta}{(e^x+1)^4\Gamma(\beta+1)} + \frac{16e^{4x} t^\beta}{(e^x+1)^5\Gamma(\beta+1)} - \frac{4e^{2x} t^\alpha}{(e^x+1)^3\Gamma(\alpha+1)} + \frac{8e^{2x} t^\alpha}{(e^x+1)^4\Gamma(\alpha+1)} + \frac{36e^{3x} t^\alpha}{(e^x+1)^4\Gamma(\alpha+1)} - \frac{40e^{3x} t^\alpha}{(e^x+1)^5\Gamma(\alpha+1)} - \frac{72e^{4x} t^\alpha}{(e^x+1)^5\Gamma(\alpha+1)} + \frac{40e^{4x} t^\alpha}{(e^x+1)^6\Gamma(\alpha+1)} + \frac{40e^{5x} t^\alpha}{(e^x+1)^6\Gamma(\alpha+1)}, \tag{57}$$

thus,

$$u_2 = \frac{2e^x t^{\alpha+\beta}}{(e^x+1)^4\Gamma(\alpha+\beta+1)} - \frac{8e^{2x} t^{\alpha+\beta}}{(e^x+1)^4\Gamma(\alpha+\beta+1)} + \frac{2e^{3x} t^{\alpha+\beta}}{(e^x+1)^4\Gamma(\alpha+\beta+1)} - \frac{e^x t^{2\alpha}}{(e^x+1)^4\Gamma(2\alpha+1)} + \frac{8e^{2x} t^{2\alpha}}{(e^x+1)^4\Gamma(2\alpha+1)} - \frac{3e^{3x} t^{2\alpha}}{(e^x+1)^4\Gamma(2\alpha+1)}, \tag{58}$$

$$w_2 = \frac{4e^{2x} t^{\alpha+\beta}}{(e^x+1)^5\Gamma(\alpha+\beta+1)} - \frac{4e^{3x} t^{\alpha+\beta}}{(e^x+1)^5\Gamma(\alpha+\beta+1)} + \frac{e^x t^{2\beta}}{(e^x+1)^5\Gamma(2\beta+1)} - \frac{7e^{2x} t^{2\beta}}{(e^x+1)^5\Gamma(2\beta+1)} + \frac{e^{3x} t^{2\beta}}{(e^x+1)^5\Gamma(2\beta+1)} + \frac{e^{4x} t^{2\beta}}{(e^x+1)^5\Gamma(2\beta+1)}, \tag{59}$$

$$
\begin{aligned}
A_2 =& \frac{8e^{2x}t^{\alpha+\beta}}{(e^x+1)^5\,\Gamma(\alpha+\beta+1)} - \frac{48e^{3x}t^{\alpha+\beta}}{(e^x+1)^5\,\Gamma(\alpha+\beta+1)} + \frac{16e^{4x}t^{\alpha+\beta}}{(e^x+1)^5\,\Gamma(\alpha+\beta+1)} - \frac{20e^{3x}t^{\alpha+\beta}}{(e^x+1)^6\,\Gamma(\alpha+\beta+1)} \\
&+ \frac{80e^{4x}t^{\alpha+\beta}}{(e^x+1)^6\,\Gamma(\alpha+\beta+1)} - \frac{20e^{5x}t^{\alpha+\beta}}{(e^x+1)^6\,\Gamma(\alpha+\beta+1)} + \frac{2e^{2x}t^{2\alpha}}{(e^x+1)^2\,\Gamma(\alpha+1)^2} - \frac{8e^{2x}t^{2\alpha}}{(e^x+1)^3\,\Gamma(\alpha+1)^2} \\
&- \frac{32e^{3x}t^{2\alpha}}{(e^x+1)^3\,\Gamma(\alpha+1)^2} + \frac{8e^{2x}t^{2\alpha}}{(e^x+1)^4\,\Gamma(\alpha+1)^2} + \frac{96e^{3x}t^{2\alpha}}{(e^x+1)^4\,\Gamma(\alpha+1)^2} + \frac{162e^{4x}t^{2\alpha}}{(e^x+1)^4\,\Gamma(\alpha+1)^2} \\
&- \frac{64e^{3x}t^{2\alpha}}{(e^x+1)^5\,\Gamma(\alpha+1)^2} - \frac{336e^{4x}t^{2\alpha}}{(e^x+1)^5\,\Gamma(\alpha+1)^2} - \frac{332e^{5x}t^{2\alpha}}{(e^x+1)^5\,\Gamma(\alpha+1)^2} + \frac{144e^{4x}t^{2\alpha}}{(e^x+1)^6\,\Gamma(\alpha+1)^2} \\
&+ \frac{440e^{5x}t^{2\alpha}}{(e^x+1)^6\,\Gamma(\alpha+1)^2} + \frac{296e^{6x}t^{2\alpha}}{(e^x+1)^6\,\Gamma(\alpha+1)^2} - \frac{96e^{5x}t^{2\alpha}}{(e^x+1)^7\,\Gamma(\alpha+1)^2} - \frac{192e^{6x}t^{2\alpha}}{(e^x+1)^7\,\Gamma(\alpha+1)^2} \\
&- \frac{96e^{7x}t^{2\alpha}}{(e^x+1)^7\,\Gamma(\alpha+1)^2} - \frac{4e^{2x}t^{2\alpha}}{(e^x+1)^5\,\Gamma(2\alpha+1)} + \frac{48e^{3x}t^{2\alpha}}{(e^x+1)^5\,\Gamma(2\alpha+1)} - \frac{24e^{4x}t^{2\alpha}}{(e^x+1)^5\,\Gamma(2\alpha+1)} \\
&+ \frac{10e^{3x}t^{2\alpha}}{(e^x+1)^6\,\Gamma(2\alpha+1)} - \frac{80e^{4x}t^{2\alpha}}{(e^x+1)^6\,\Gamma(2\alpha+1)} + \frac{30e^{5x}t^{2\alpha}}{(e^x+1)^6\,\Gamma(2\alpha+1)},
\end{aligned} \tag{60}
$$

$$
\begin{aligned}
B_2 =& -\frac{4e^{2x}t^{2\alpha}}{(1+e^x)^6\,\Gamma(2\alpha+1)} + \frac{48e^{3x}t^{2\alpha}}{(1+e^x)^6\,\Gamma(2\alpha+1)} - \frac{24e^{4x}t^{2\alpha}}{(1+e^x)^6\,\Gamma(2\alpha+1)} + \frac{12e^{3x}t^{2\alpha}}{(1+e^x)^7\,\Gamma(2\alpha+1)} \\
&- \frac{96e^{4x}t^{2\alpha}}{(1+e^x)^7\,\Gamma(2\alpha+1)} + \frac{36e^{5x}t^{2\alpha}}{(1+e^x)^7\,\Gamma(2\alpha+1)} + \frac{4e^{2x}t^{2\beta}}{(1+e^x)^6\,\Gamma(2\beta+1)} - \frac{42e^{3x}t^{2\beta}}{(1+e^x)^6\,\Gamma(2\beta+1)} \\
&+ \frac{8e^{4x}t^{2\beta}}{(1+e^x)^6\,\Gamma(2\beta+1)} + \frac{10e^{5x}t^{2\beta}}{(1+e^x)^6\,\Gamma(2\beta+1)} - \frac{12e^{3x}t^{2\beta}}{(1+e^x)^7\,\Gamma(2\beta+1)} + \frac{84e^{4x}t^{2\beta}}{(1+e^x)^7\,\Gamma(2\beta+1)} \\
&- \frac{12e^{5x}t^{2\beta}}{(1+e^x)^7\,\Gamma(2\beta+1)} - \frac{12e^{6x}t^{2\beta}}{(1+e^x)^7\,\Gamma(2\beta+1)} - \frac{4e^{2x}t^{\alpha+\beta}}{(1+e^x)^3\,\Gamma(\alpha+1)\Gamma(\beta+1)} + \frac{8e^{2x}t^{\alpha+\beta}}{(1+e^x)^4\,\Gamma(\alpha+1)\Gamma(\beta+1)} \\
&+ \frac{48e^{3x}t^{\alpha+\beta}}{(1+e^x)^4\,\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{64e^{3x}t^{\alpha+\beta}}{(1+e^x)^5\,\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{168e^{4x}t^{\alpha+\beta}}{(1+e^x)^5\,\Gamma(\alpha+1)\Gamma(\beta+1)} \\
&+ \frac{144e^{4x}t^{\alpha+\beta}}{(1+e^x)^6\,\Gamma(\alpha+1)\Gamma(\beta+1)} + \frac{220e^{5x}t^{\alpha+\beta}}{(1+e^x)^6\,\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{96e^{5x}t^{\alpha+\beta}}{(1+e^x)^7\,\Gamma(\alpha+1)\Gamma(\beta+1)} \\
&- \frac{96e^{6x}t^{\alpha+\beta}}{(1+e^x)^7\,\Gamma(\alpha+1)\Gamma(\beta+1)} + \frac{8e^{2x}t^{\alpha+\beta}}{(1+e^x)^6\,\Gamma(\alpha+\beta+1)} - \frac{24e^{3x}t^{\alpha+\beta}}{(1+e^x)^6\,\Gamma(\alpha+\beta+1)} - \frac{16e^{4x}t^{\alpha+\beta}}{(1+e^x)^6\,\Gamma(\alpha+\beta+1)} \\
&- \frac{24e^{3x}t^{\alpha+\beta}}{(1+e^x)^7\,\Gamma(\alpha+\beta+1)} + \frac{48e^{4x}t^{\alpha+\beta}}{(1+e^x)^7\,\Gamma(\alpha+\beta+1)} + \frac{24e^{5x}t^{\alpha+\beta}}{(1+e^x)^7\,\Gamma(\alpha+\beta+1)},
\end{aligned} \tag{61}
$$

$$
\begin{aligned}
u_3 = &- \frac{2e^{x}t^{2\alpha+\beta}}{(e^x+1)^6\,\Gamma(2\alpha+\beta+1)} + \frac{76e^{2x}t^{2\alpha+\beta}}{(e^x+1)^6\,\Gamma(2\alpha+\beta+1)} - \frac{240e^{3x}t^{2\alpha+\beta}}{(e^x+1)^6\,\Gamma(2\alpha+\beta+1)} + \frac{116e^{4x}t^{2\alpha+\beta}}{(e^x+1)^6\,\Gamma(2\alpha+\beta+1)} \\
&- \frac{6e^{5x}t^{2\alpha+\beta}}{(e^x+1)^6\,\Gamma(2\alpha+\beta+1)} + \frac{2e^{x}t^{\alpha+2\beta}}{(e^x+1)^6\,\Gamma(\alpha+2\beta+1)} - \frac{36e^{2x}t^{\alpha+2\beta}}{(e^x+1)^6\,\Gamma(\alpha+2\beta+1)} + \frac{48e^{3x}t^{\alpha+2\beta}}{(e^x+1)^6\,\Gamma(\alpha+2\beta+1)} \\
&+ \frac{4e^{4x}t^{\alpha+2\beta}}{(e^x+1)^6\,\Gamma(\alpha+2\beta+1)} - \frac{2e^{5x}t^{\alpha+2\beta}}{(e^x+1)^6\,\Gamma(\alpha+2\beta+1)} + \frac{e^{x}t^{3\alpha}}{(e^x+1)^6\,\Gamma(3\alpha+1)} - \frac{46e^{2x}t^{3\alpha}}{(e^x+1)^6\,\Gamma(3\alpha+1)} \\
&+ \frac{186e^{3x}t^{3\alpha}}{(e^x+1)^6\,\Gamma(3\alpha+1)} - \frac{118e^{4x}t^{3\alpha}}{(e^x+1)^6\,\Gamma(3\alpha+1)} + \frac{9e^{5x}t^{3\alpha}}{(e^x+1)^6\,\Gamma(3\alpha+1)} + \frac{2e^{2x}\Gamma(2\alpha+1)t^{3\alpha}}{(e^x+1)^6\,\Gamma(\alpha+1)^2\Gamma(3\alpha+1)} \\
&- \frac{2e^{4x}\Gamma(2\alpha+1)t^{3\alpha}}{(e^x+1)^6\,\Gamma(\alpha+1)^2\Gamma(3\alpha+1)},
\end{aligned} \tag{62}
$$

$$
\begin{aligned}
w_3 = &- \frac{4e^{2x}t^{2\alpha+\beta}}{(e^x+1)^7\,\Gamma(2\alpha+\beta+1)} + \frac{56e^{3x}t^{2\alpha+\beta}}{(e^x+1)^7\,\Gamma(2\alpha+\beta+1)} - \frac{72e^{4x}t^{2\alpha+\beta}}{(e^x+1)^7\,\Gamma(2\alpha+\beta+1)} + \frac{12e^{5x}t^{2\alpha+\beta}}{(e^x+1)^7\,\Gamma(2\alpha+\beta+1)} \\
&+ \frac{24e^{2x}t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+2\beta+1)} - \frac{144e^{3x}t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+2\beta+1)} + \frac{112e^{4x}t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+2\beta+1)} - \frac{8e^{5x}t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+2\beta+1)} \\
&+ \frac{4e^{2x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} - \frac{8e^{3x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} \\
&- \frac{8e^{4x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} + \frac{4e^{5x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(e^x+1)^7\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} \\
&+ \frac{e^{x}t^{3\beta}}{(e^x+1)^7\,\Gamma(3\beta+1)} - \frac{37e^{2x}t^{3\beta}}{(e^x+1)^7\,\Gamma(3\beta+1)} + \frac{94e^{3x}t^{3\beta}}{(e^x+1)^7\,\Gamma(3\beta+1)} - \frac{14e^{4x}t^{3\beta}}{(e^x+1)^7\,\Gamma(3\beta+1)} \\
&- \frac{3e^{5x}t^{3\beta}}{(e^x+1)^7\,\Gamma(3\beta+1)} - \frac{e^{6x}t^{3\beta}}{(e^x+1)^7\,\Gamma(3\beta+1)}.
\end{aligned} \tag{63}
$$

Thus, the solution approximate of Time-fractional DLW system (36) with the first four terms is:

$$\begin{aligned}
u(x,t) = &-\frac{e^{x}t^{\alpha}}{(1+e^{x})\,\Gamma(\alpha+1)} + \frac{2e^{x}t^{\alpha}}{(1+e^{x})^{2}\,\Gamma(\alpha+1)} + \frac{5e^{2x}t^{\alpha}}{(1+e^{x})^{2}\,\Gamma(\alpha+1)} - \frac{4e^{2x}t^{\alpha}}{(1+e^{x})^{3}\,\Gamma(\alpha+1)} \\
&- \frac{4e^{3x}t^{\alpha}}{(1+e^{x})^{3}\,\Gamma(\alpha+1)} - \frac{e^{x}t^{2\alpha}}{(1+e^{x})^{4}\,\Gamma(2\alpha+1)} + \frac{8e^{2x}t^{2\alpha}}{(1+e^{x})^{4}\,\Gamma(2\alpha+1)} - \frac{3e^{3x}t^{2\alpha}}{(1+e^{x})^{4}\,\Gamma(2\alpha+1)} \\
&+ \frac{e^{x}t^{3\alpha}}{(1+e^{x})^{6}\,\Gamma(3\alpha+1)} - \frac{46e^{2x}t^{3\alpha}}{(1+e^{x})^{6}\,\Gamma(3\alpha+1)} + \frac{186e^{3x}t^{3\alpha}}{(1+e^{x})^{6}\,\Gamma(3\alpha+1)} - \frac{118e^{4x}t^{3\alpha}}{(1+e^{x})^{6}\,\Gamma(3\alpha+1)} \\
&+ \frac{9e^{5x}t^{3\alpha}}{(1+e^{x})^{6}\,\Gamma(3\alpha+1)} + \frac{2e^{2x}\Gamma(2\alpha+1)t^{3\alpha}}{(1+e^{x})^{6}\,\Gamma(\alpha+1)^{2}\Gamma(3\alpha+1)} - \frac{2e^{4x}\Gamma(2\alpha+1)t^{3\alpha}}{(1+e^{x})^{6}\,\Gamma(\alpha+1)^{2}\Gamma(3\alpha+1)} \\
&+ \frac{2e^{x}t^{\alpha+\beta}}{(1+e^{x})^{4}\,\Gamma(\alpha+\beta+1)} - \frac{8e^{2x}t^{\alpha+\beta}}{(1+e^{x})^{4}\,\Gamma(\alpha+\beta+1)} + \frac{2e^{3x}t^{\alpha+\beta}}{(1+e^{x})^{4}\,\Gamma(\alpha+\beta+1)} \\
&- \frac{2e^{x}t^{2\alpha+\beta}}{(1+e^{x})^{6}\,\Gamma(2\alpha+\beta+1)} + \frac{76e^{2x}t^{2\alpha+\beta}}{(1+e^{x})^{6}\,\Gamma(2\alpha+\beta+1)} - \frac{240e^{3x}t^{2\alpha+\beta}}{(1+e^{x})^{6}\,\Gamma(2\alpha+\beta+1)} \\
&+ \frac{116e^{4x}t^{2\alpha+\beta}}{(1+e^{x})^{6}\,\Gamma(2\alpha+\beta+1)} - \frac{6e^{5x}t^{2\alpha+\beta}}{(1+e^{x})^{6}\,\Gamma(2\alpha+\beta+1)} + \frac{2e^{x}t^{\alpha+2\beta}}{(1+e^{x})^{6}\,\Gamma(\alpha+2\beta+1)} \\
&- \frac{36e^{2x}t^{\alpha+2\beta}}{(1+e^{x})^{6}\,\Gamma(\alpha+2\beta+1)} + \frac{48e^{3x}t^{\alpha+2\beta}}{(1+e^{x})^{6}\,\Gamma(\alpha+2\beta+1)} + \frac{4e^{4x}t^{\alpha+2\beta}}{(1+e^{x})^{6}\,\Gamma(\alpha+2\beta+1)} \\
&- \frac{2e^{5x}t^{\alpha+2\beta}}{(1+e^{x})^{6}\,\Gamma(\alpha+2\beta+1)} + \frac{e^{x}}{1+e^{x}},
\end{aligned} \tag{64}$$

and

$$\begin{aligned}
w(x,t) = &\frac{e^{x}t^{\beta}}{(1+e^{x})^{2}\,\Gamma(\beta+1)} - \frac{2e^{2x}t^{\beta}}{(1+e^{x})^{3}\,\Gamma(\beta+1)} + \frac{e^{x}t^{2\beta}}{(1+e^{x})^{5}\,\Gamma(2\beta+1)} - \frac{7e^{2x}t^{2\beta}}{(1+e^{x})^{5}\,\Gamma(2\beta+1)} \\
&+ \frac{e^{3x}t^{2\beta}}{(1+e^{x})^{5}\,\Gamma(2\beta+1)} + \frac{e^{4x}t^{2\beta}}{(1+e^{x})^{5}\,\Gamma(2\beta+1)} + \frac{e^{x}t^{3\beta}}{(1+e^{x})^{7}\,\Gamma(3\beta+1)} - \frac{37e^{2x}t^{3\beta}}{(1+e^{x})^{7}\,\Gamma(3\beta+1)} \\
&+ \frac{94e^{3x}t^{3\beta}}{(1+e^{x})^{7}\,\Gamma(3\beta+1)} - \frac{14e^{4x}t^{3\beta}}{(1+e^{x})^{7}\,\Gamma(3\beta+1)} - \frac{3e^{5x}t^{3\beta}}{(1+e^{x})^{7}\,\Gamma(3\beta+1)} - \frac{e^{6x}t^{3\beta}}{(1+e^{x})^{7}\,\Gamma(3\beta+1)} \\
&+ \frac{4e^{2x}t^{\alpha+\beta}}{(1+e^{x})^{5}\,\Gamma(\alpha+\beta+1)} - \frac{4e^{3x}t^{\alpha+\beta}}{(1+e^{x})^{5}\,\Gamma(\alpha+\beta+1)} - \frac{4e^{2x}t^{2\alpha+\beta}}{(1+e^{x})^{7}\,\Gamma(2\alpha+\beta+1)} \\
&+ \frac{56e^{3x}t^{2\alpha+\beta}}{(1+e^{x})^{7}\,\Gamma(2\alpha+\beta+1)} - \frac{72e^{4x}t^{2\alpha+\beta}}{(1+e^{x})^{7}\,\Gamma(2\alpha+\beta+1)} + \frac{12e^{5x}t^{2\alpha+\beta}}{(1+e^{x})^{7}\,\Gamma(2\alpha+\beta+1)} \\
&+ \frac{24e^{2x}t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+2\beta+1)} - \frac{144e^{3x}t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+2\beta+1)} + \frac{112e^{4x}t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+2\beta+1)} \\
&- \frac{8e^{5x}t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+2\beta+1)} + \frac{4e^{2x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} \\
&- \frac{8e^{3x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} - \frac{8e^{4x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} \\
&+ \frac{4e^{5x}\Gamma(\alpha+\beta+1)t^{\alpha+2\beta}}{(1+e^{x})^{7}\,\Gamma(\alpha+1)\Gamma(\beta+1)\Gamma(\alpha+2\beta+1)} + \frac{e^{x}}{(1+e^{x})^{2}}.
\end{aligned} \tag{65}$$

Setting $u(x,t) = u_{\alpha,\beta}(x,t),\ w(x,t) = w_{\alpha,\beta}(x,t)$, we have in particular:

$$\begin{aligned} u_{1,1}(x,t) =& -\frac{t^3e^{3x}}{(e^x+1)^6} - \frac{t^3e^{2x}}{3(e^x+1)^6} - \frac{t^3e^{4x}}{3(e^x+1)^6} + \frac{t^3e^{x}}{6(e^x+1)^6} + \frac{t^3e^{5x}}{6(e^x+1)^6} + \frac{t^2e^{x}}{2(e^x+1)^4} \\ & - \frac{t^2e^{3x}}{2(e^x+1)^4} - \frac{te^x}{e^x+1} + \frac{2te^x}{(e^x+1)^2} + \frac{5te^{2x}}{(e^x+1)^2} - \frac{4te^{2x}}{(e^x+1)^3} - \frac{4te^{3x}}{(e^x+1)^3} + \frac{e^x}{e^x+1}, \end{aligned} \tag{66}$$

$$\begin{aligned} u_{\frac{1}{2},\frac{1}{2}}(x,t) =& \frac{32t^{3/2}e^{2x}}{3\pi^{3/2}(e^x+1)^6} - \frac{32t^{3/2}e^{4x}}{3\pi^{3/2}(e^x+1)^6} - \frac{8t^{3/2}e^{2x}}{\sqrt{\pi}(e^x+1)^6} - \frac{8t^{3/2}e^{3x}}{\sqrt{\pi}(e^x+1)^6} + \frac{4t^{3/2}e^{x}}{3\sqrt{\pi}(e^x+1)^6} \\ & + \frac{8t^{3/2}e^{4x}}{3\sqrt{\pi}(e^x+1)^6} + \frac{4t^{3/2}e^{5x}}{3\sqrt{\pi}(e^x+1)^6} + \frac{te^{x}}{(e^x+1)^4} - \frac{te^{3x}}{(e^x+1)^4} + \frac{4\sqrt{t}e^{x}}{\sqrt{\pi}(e^x+1)^2} \\ & + \frac{10\sqrt{t}e^{2x}}{\sqrt{\pi}(e^x+1)^2} - \frac{2\sqrt{t}e^{x}}{\sqrt{\pi}(e^x+1)} - \frac{8\sqrt{t}e^{2x}}{\sqrt{\pi}(e^x+1)^3} - \frac{8\sqrt{t}e^{3x}}{\sqrt{\pi}(e^x+1)^3} + \frac{e^x}{e^x+1}, \end{aligned} \tag{67}$$

$$\begin{aligned} u_{\frac{3}{4},\frac{3}{4}}(x,t) =& \frac{4t^{3/2}e^{x}}{3\sqrt{\pi}(e^x+1)^4} - \frac{4t^{3/2}e^{3x}}{3\sqrt{\pi}(e^x+1)^4} - \frac{t^{3/4}e^{x}}{(e^x+1)\Gamma\left(\frac{7}{4}\right)} + \frac{2t^{3/4}e^{x}}{(e^x+1)^2\Gamma\left(\frac{7}{4}\right)} + \frac{5t^{3/4}e^{2x}}{(e^x+1)^2\Gamma\left(\frac{7}{4}\right)} \\ & - \frac{4t^{3/4}e^{2x}}{(e^x+1)^3\Gamma\left(\frac{7}{4}\right)} - \frac{4t^{3/4}e^{3x}}{(e^x+1)^3\Gamma\left(\frac{7}{4}\right)} + \frac{3\sqrt{\pi}t^{9/4}e^{2x}}{2(e^x+1)^6\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} + \frac{t^{9/4}e^{x}}{(e^x+1)^6\Gamma\left(\frac{13}{4}\right)} - \frac{6t^{9/4}e^{2x}}{(e^x+1)^6\Gamma\left(\frac{13}{4}\right)} \\ & - \frac{6t^{9/4}e^{3x}}{(e^x+1)^6\Gamma\left(\frac{13}{4}\right)} + \frac{2t^{9/4}e^{4x}}{(e^x+1)^6\Gamma\left(\frac{13}{4}\right)} + \frac{t^{9/4}e^{5x}}{(e^x+1)^6\Gamma\left(\frac{13}{4}\right)} - \frac{3\sqrt{\pi}t^{9/4}e^{4x}}{2(e^x+1)^6\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} + \frac{e^x}{e^x+1}, \end{aligned} \tag{68}$$

$$\begin{aligned} u_{\frac{3}{4},\frac{1}{2}}(x,t) =& -\frac{4e^{3x}t^{3/2}}{(1+e^x)^4\sqrt{\pi}} - \frac{4e^{x}t^{3/2}}{3(1+e^x)^4\sqrt{\pi}} + \frac{32e^{2x}t^{3/2}}{3(1+e^x)^4\sqrt{\pi}} - \frac{e^{x}t^{3/4}}{(1+e^x)\Gamma\left(\frac{7}{4}\right)} + \frac{2e^{x}t^{3/4}}{(1+e^x)^2\Gamma\left(\frac{7}{4}\right)} \\ & + \frac{5e^{2x}t^{3/4}}{(1+e^x)^2\Gamma\left(\frac{7}{4}\right)} - \frac{4e^{2x}t^{3/4}}{(1+e^x)^3\Gamma\left(\frac{7}{4}\right)} - \frac{4e^{3x}t^{3/4}}{(1+e^x)^3\Gamma\left(\frac{7}{4}\right)} + \frac{2e^{x}t^{5/4}}{(1+e^x)^4\Gamma\left(\frac{9}{4}\right)} - \frac{8e^{2x}t^{5/4}}{(1+e^x)^4\Gamma\left(\frac{9}{4}\right)} \\ & + \frac{2e^{3x}t^{5/4}}{(1+e^x)^4\Gamma\left(\frac{9}{4}\right)} + \frac{2e^{x}t^{7/4}}{(1+e^x)^6\Gamma\left(\frac{11}{4}\right)} - \frac{36e^{2x}t^{7/4}}{(1+e^x)^6\Gamma\left(\frac{11}{4}\right)} + \frac{48e^{3x}t^{7/4}}{(1+e^x)^6\Gamma\left(\frac{11}{4}\right)} \\ & + \frac{4e^{4x}t^{7/4}}{(1+e^x)^6\Gamma\left(\frac{11}{4}\right)} - \frac{2e^{5x}t^{7/4}}{(1+e^x)^6\Gamma\left(\frac{11}{4}\right)} + \frac{3e^{2x}\sqrt{\pi}t^{9/4}}{2(1+e^x)^6\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} + \frac{e^{x}t^{9/4}}{(1+e^x)^6\Gamma\left(\frac{13}{4}\right)} \\ & - \frac{46e^{2x}t^{9/4}}{(1+e^x)^6\Gamma\left(\frac{13}{4}\right)} + \frac{186e^{3x}t^{9/4}}{(1+e^x)^6\Gamma\left(\frac{13}{4}\right)} - \frac{118e^{4x}t^{9/4}}{(1+e^x)^6\Gamma\left(\frac{13}{4}\right)} + \frac{9e^{5x}t^{9/4}}{(1+e^x)^6\Gamma\left(\frac{13}{4}\right)} \\ & - \frac{3e^{4x}\sqrt{\pi}t^{9/4}}{2(1+e^x)^6\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} - \frac{e^{x}t^{2}}{(1+e^x)^6} + \frac{38e^{2x}t^{2}}{(1+e^x)^6} - \frac{120e^{3x}t^{2}}{(1+e^x)^6} + \frac{58e^{4x}t^{2}}{(1+e^x)^6} - \frac{3e^{5x}t^{2}}{(1+e^x)^6} + \frac{e^x}{1+e^x}, \end{aligned} \tag{69}$$

and similarly, we have

$$\begin{aligned} w_{1,1}(x,t) =& -\frac{3t^3e^{2x}}{2(e^x+1)^7} + \frac{3t^3e^{5x}}{2(e^x+1)^7} - \frac{5t^3e^{3x}}{3(e^x+1)^7} + \frac{5t^3e^{4x}}{3(e^x+1)^7} + \frac{t^3e^{x}}{6(e^x+1)^7} - \frac{t^3e^{6x}}{6(e^x+1)^7} \\ & + \frac{t^2e^{x}}{2(e^x+1)^5} - \frac{3t^2e^{2x}}{2(e^x+1)^5} - \frac{3t^2e^{3x}}{2(e^x+1)^5} + \frac{t^2e^{4x}}{2(e^x+1)^5} + \frac{te^{x}}{(e^x+1)^2} - \frac{2te^{2x}}{(e^x+1)^3} + \frac{e^x}{(e^x+1)^2}, \end{aligned} \tag{70}$$

$$\begin{aligned} w_{\frac{1}{2},\frac{1}{2}}(x,t) =& \frac{64t^{3/2}e^{2x}}{3\pi^{3/2}(e^x+1)^7} - \frac{128t^{3/2}e^{3x}}{3\pi^{3/2}(e^x+1)^7} - \frac{128t^{3/2}e^{4x}}{3\pi^{3/2}(e^x+1)^7} + \frac{64t^{3/2}e^{5x}}{3\pi^{3/2}(e^x+1)^7} + \frac{8t^{3/2}e^{3x}}{\sqrt{\pi}(e^x+1)^7} \\ &+ \frac{4t^{3/2}e^{x}}{3\sqrt{\pi}(e^x+1)^7} - \frac{68t^{3/2}e^{2x}}{3\sqrt{\pi}(e^x+1)^7} + \frac{104t^{3/2}e^{4x}}{3\sqrt{\pi}(e^x+1)^7} + \frac{4t^{3/2}e^{5x}}{3\sqrt{\pi}(e^x+1)^7} - \frac{4t^{3/2}e^{6x}}{3\sqrt{\pi}(e^x+1)^7} \\ &+ \frac{te^x}{(e^x+1)^5} - \frac{3te^{2x}}{(e^x+1)^5} - \frac{3te^{3x}}{(e^x+1)^5} + \frac{te^{4x}}{(e^x+1)^5} + \frac{2\sqrt{t}e^x}{\sqrt{\pi}(e^x+1)^2} - \frac{4\sqrt{t}e^{2x}}{\sqrt{\pi}(e^x+1)^3} + \frac{e^x}{(e^x+1)^2}, \end{aligned} \tag{71}$$

$$\begin{aligned} w_{\frac{3}{4},\frac{3}{4}}(x,t) =& -\frac{4t^{3/2}e^{2x}}{\sqrt{\pi}(e^x+1)^5} - \frac{4t^{3/2}e^{3x}}{\sqrt{\pi}(e^x+1)^5} + \frac{4t^{3/2}e^{x}}{3\sqrt{\pi}(e^x+1)^5} + \frac{4t^{3/2}e^{4x}}{3\sqrt{\pi}(e^x+1)^5} + \frac{t^{3/4}e^x}{(e^x+1)^2\Gamma\left(\frac{7}{4}\right)} \\ &- \frac{2t^{3/4}e^{2x}}{(e^x+1)^3\Gamma\left(\frac{7}{4}\right)} + \frac{3\sqrt{\pi}t^{9/4}e^{2x}}{(e^x+1)^7\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} + \frac{3\sqrt{\pi}t^{9/4}e^{5x}}{(e^x+1)^7\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} + \frac{t^{9/4}e^x}{(e^x+1)^7\Gamma\left(\frac{13}{4}\right)} \\ &- \frac{17t^{9/4}e^{2x}}{(e^x+1)^7\Gamma\left(\frac{13}{4}\right)} + \frac{6t^{9/4}e^{3x}}{(e^x+1)^7\Gamma\left(\frac{13}{4}\right)} + \frac{26t^{9/4}e^{4x}}{(e^x+1)^7\Gamma\left(\frac{13}{4}\right)} + \frac{t^{9/4}e^{5x}}{(e^x+1)^7\Gamma\left(\frac{13}{4}\right)} - \frac{t^{9/4}e^{6x}}{(e^x+1)^7\Gamma\left(\frac{13}{4}\right)} \\ &- \frac{6\sqrt{\pi}t^{9/4}e^{3x}}{(e^x+1)^7\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} - \frac{6\sqrt{\pi}t^{9/4}e^{4x}}{(e^x+1)^7\Gamma\left(\frac{7}{4}\right)^2\Gamma\left(\frac{13}{4}\right)} + \frac{e^x}{(e^x+1)^2}, \end{aligned} \tag{72}$$

$$\begin{aligned} w_{\frac{3}{4},\frac{1}{2}}(x,t) =& -\frac{4t^{3/2}e^{5x}}{\sqrt{\pi}(e^x+1)^7} + \frac{4t^{3/2}e^{x}}{3\sqrt{\pi}(e^x+1)^7} - \frac{148t^{3/2}e^{2x}}{3\sqrt{\pi}(e^x+1)^7} + \frac{376t^{3/2}e^{3x}}{3\sqrt{\pi}(e^x+1)^7} - \frac{56t^{3/2}e^{4x}}{3\sqrt{\pi}(e^x+1)^7} - \frac{4t^{3/2}e^{6x}}{3\sqrt{\pi}(e^x+1)^7} \\ &- \frac{4t^{5/4}e^{3x}}{(e^x+1)^5\Gamma\left(\frac{9}{4}\right)} + \frac{24t^{7/4}e^{2x}}{(e^x+1)^7\Gamma\left(\frac{11}{4}\right)} - \frac{144t^{7/4}e^{3x}}{(e^x+1)^7\Gamma\left(\frac{11}{4}\right)} + \frac{112t^{7/4}e^{4x}}{(e^x+1)^7\Gamma\left(\frac{11}{4}\right)} - \frac{8t^{7/4}e^{5x}}{(e^x+1)^7\Gamma\left(\frac{11}{4}\right)} \\ &+ \frac{8t^{7/4}e^{2x}\Gamma\left(\frac{9}{4}\right)}{\sqrt{\pi}(e^x+1)^7\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{11}{4}\right)} - \frac{16t^{7/4}e^{3x}\Gamma\left(\frac{9}{4}\right)}{\sqrt{\pi}(e^x+1)^7\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{11}{4}\right)} - \frac{16t^{7/4}e^{4x}\Gamma\left(\frac{9}{4}\right)}{\sqrt{\pi}(e^x+1)^7\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{11}{4}\right)} \\ &+ \frac{8t^{7/4}e^{5x}\Gamma\left(\frac{9}{4}\right)}{\sqrt{\pi}(e^x+1)^7\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{11}{4}\right)} + \frac{4t^{5/4}e^{2x}}{(e^x+1)^5\Gamma\left(\frac{9}{4}\right)} - \frac{2t^2e^{2x}}{(e^x+1)^7} + \frac{28t^2e^{3x}}{(e^x+1)^7} - \frac{36t^2e^{4x}}{(e^x+1)^7} + \frac{6t^2e^{5x}}{(e^x+1)^7} \\ &+ \frac{te^x}{(e^x+1)^5} - \frac{7te^{2x}}{(e^x+1)^5} + \frac{te^{3x}}{(e^x+1)^5} + \frac{te^{4x}}{(e^x+1)^5} + \frac{2\sqrt{t}e^x}{\sqrt{\pi}(e^x+1)^2} - \frac{4\sqrt{t}e^{2x}}{\sqrt{\pi}(e^x+1)^3} + \frac{e^x}{(e^x+1)^2}. \end{aligned} \tag{73}$$

Remark 3.1 *(i) Under the initial conditions (37), if $\alpha = \beta = 1$, then the (36) becomes (21), and an exact solution is (35).*

(ii) Through our comparison of a 3D and 2D graphs of the solutions (Figures 1, 2, 3, 4, 5, 6 and 7) and for the values shown in the Tables 1, 2 and 3, we can see that the approximate solution converges towards the exact solution ***in few time****.*

4. Conclusion

In this paper, we discussed three stages related to the study of system of two non-linear partial differential equations (DLW). First we used the Tanh method to get the exact solution to the system. In the second stage of the study, thanks to the LADM method (ADM in combination with the Laplace transform), we get the approximate solutions to the TFDLW (time-fractional DLW) system. Finally, in order to show the accuracy and efficiency of our method, compare our results with the exact solution of the system obtained by the Tanh method.

Therefore, we concluded that the LADM method gives a better approximation value of the exact solution **in few time**, by calculating the first terms.

In general, LADM is a very effective and powerful mathematical tool, can be further applied to solve various types of nonlinear fractional partial differential equations and also can be extended to physical mathematics, engineering and other nonlinear sciences.

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