*Journal of Mathematical Sciences and Modelling*, 4 (1) (2021) 1-6
**Research Article**

**Journal of Mathematical Sciences and Modelling**

*Journal Homepage:* www.dergipark.gov.tr/jmsm
*ISSN 2636-8692*
*DOI: http://dx.doi.org/10.33187/jmsm.815125*

# Kinematics of Supination and Pronation with Stewart Platform

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**Article Info**

***Keywords:*** *Artificial intelligence, Kinematics, Modelling, Pronation, Stewart platform, Supination.*
***2010 AMS:*** *53A04, 53A17, 57R25.*
***Received:*** *22 October 2020*
***Accepted:*** *12 March 2021*
***Available online:*** *30 April 2021*

**Abstract**

This paper presents kinematics form of pronation and supination movement. The algorithm of Stewart platform motion can be used to create a new motion of supination (or pronation) motion. Pronation motion can be taken as Stewart motion which has not any rotation on x-axis and y-axis. In this case, pronation motion has only one parameter. Supination movement creates a helix curve. Additionally, the correlation between rotation angle and extension is 1. This allows us to use artificial intelligence in pronation motion. In this article, the algorithm and Matlab applications of pronation motion are given in the concepts of artificial intelligence approach. This is a new and important approach.

## 1. Stewart platform

A Stewart platform is called a form of manipulator with six degrees of freedom, which allows one to provide a given position and orientation of the surface in the vicinity of any point of the platform on its three cartesian coordinates and projections of the unit normal vector [1]. A mathematical model of the mechanism of movement of an undeformed platform with six degrees of freedom is proposed [2].
The Stewart platform consists of two rigid frames connected by 6 variable length legs. The base is considered to be the reference frame work, with orthogonal axes x, y, z. The platform has 6 degrees of freedom with respect to the base. The origin of the platform coordinates can be defined by 3 translational displacements with respect to the base, one for each axis [3, 4].
Three angular displacements then define the orientation of the platform with respect to the base. A set of Euler angles is used in the following sequence:
1. Rotate an angle $\psi$ (yaw) around the $z$-axis,
2. Rotate an angle $\theta$(pitch) around the $y$-axis,
3. Rotate an angle $\varphi$ (roll) around the $x$-axis.

$$P = i'x' + j'y' + k'z' = ix + jy + kz$$

$$\begin{aligned} x &= OA - BC = x'\cos\psi - y'\sin\psi \\ y &= AB + PC = x'\sin\psi + y'\cos\psi \\ z &= z' \end{aligned}$$

(Figure 1.1). The rotation matrix of the platform relative to the base is given by

$$\begin{aligned} {}^{P}R_B &= R_z(\psi)R_y(\theta)R_x(\varphi) \\ &= \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta\cos\psi & -\cos\varphi\sin\psi + \sin\theta\cos\psi\sin\varphi & \sin\psi\sin\varphi + \sin\theta\cos\psi\cos\varphi \\ \cos\theta\sin\psi & \cos\psi\cos\varphi + \sin\theta\sin\psi\sin\varphi & -\cos\psi\sin\varphi + \sin\theta\cos\varphi\sin\psi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{bmatrix} \end{aligned}$$

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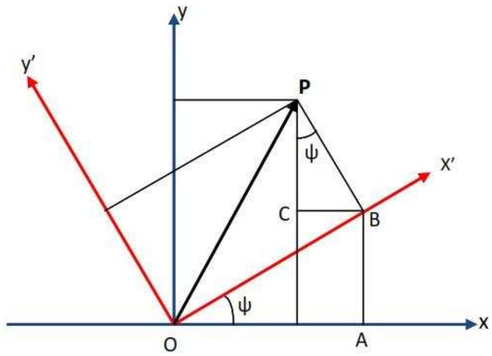


**Figure 1.1:** Rotation around z-axis

[5, 6]. Now consider a Stewart platform. For the $i-th$ leg (Figure 1.2):

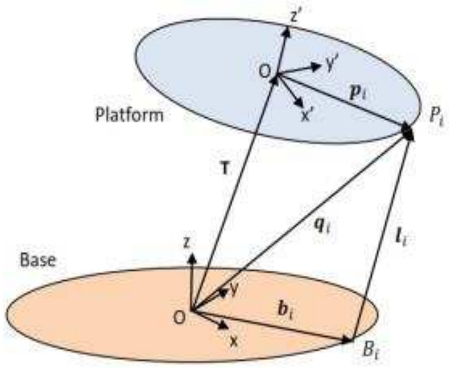


**Figure 1.2:** Stewart platform

The coordinates $q_i$ of the anchor point with respect to the base reference framework are given by the equation

$$q_i = T +^P R_B.p_i, i = 1,2,3$$

where $T$ is the translation vector, giving the positional linear displacement of the origin of the platform frame with respect to the base reference framework, and $p_i$ is the vector defining the coordinates of the anchor point $p_i$ with respect to the platform framework.
Similarly the length of the $i-th$ leg is given by

$$l_i = T +^P R_B.p_i - b_i, i = 1,2,3$$

where $b_i$ is the vector defining the coordinates of the lower anchor point $B_i$. These 6 equations give the lengths of the 6 legs to achieve the desired position and attitude of the platform.

## 2. Pronation motion in the concepts of artificial intelligence approach

In kinematics applications, axis, points, orbits are main and important. Especially orbits of points are important and informative. For example, if the orbit of a point under a displacement is on the sphere with radius $r$ and center $P$, then the displacement is a rotation with pole point $P$. If the orbits of every points under the displacement is on the perpendicular circular cylinder then the displacement is a rotation with translation. Stewart platform can make rotation, translation or rotation with translation. Let S be a cylinder in Figure 2.1.
Bottom cover is fixed platform, and top cover is moving platform. Every point moves during the movement, and movement is rotation with translation.
Supination and pronation motions can be considered inverse motion each other [7, 8, 9]. So we study only one of them in this study as modelling. Main structure of the model is as follows.
1. The forearm is considered as a cylinder or cone. We consider cylinder.
2. The planes at the elbow and wrist are considered as fixed and moving planes of Stewart platform. The elbow plane is fixed and wrist plane is moving plane.
3. Suppose that the forearm is the cylinder (Figure 2.1).
In (Figure 2.2), $L$ line part is

$$L : (r, 0, z)$$

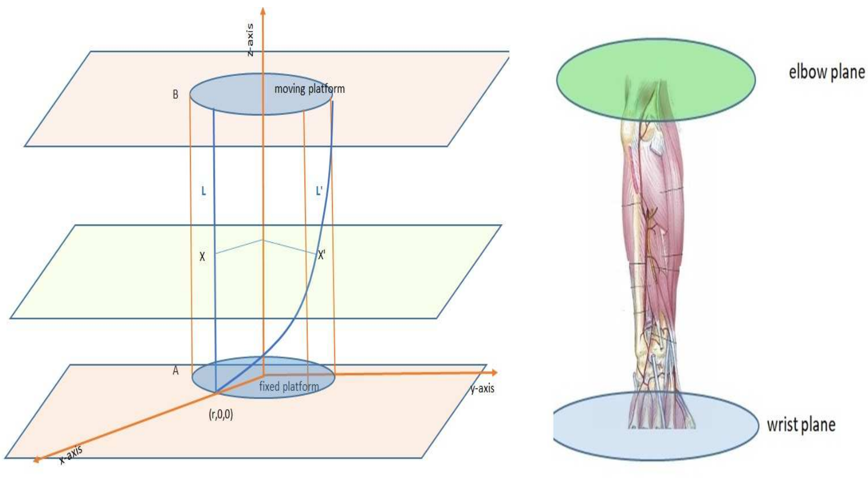


**Figure 2.1:** Elbow and wrist plane

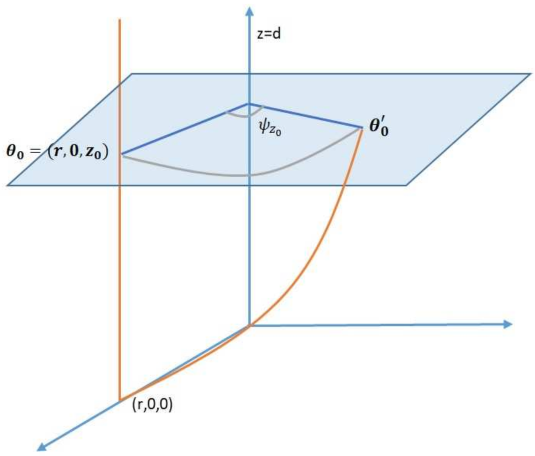


**Figure 2.2:** $\psi(z_0)$ rotating angle

$a < z < b,\ \ a, b \in \mathbb{R}$. As a Stewart platform, the matrix of displacement is

$$T + R_z(\psi)R_y(\theta)R_x(\varphi).$$

There are not rotation around $x-$axis and $y-$axis because fixed and moving platforms are parallel to each other. So, displacement matrix in this case is

$$T + R_z(\psi).$$

Translation part of displacement, in case of pronation motion, must be evaluated in pronation. Rotation angle is limited $\psi_0$ and $\psi_e$, $\psi_0 \leq \psi_z \leq \psi_e$ where $\psi_0 = 0$ and $\psi_e$ is final value.
At the $z = z_0$ rotation plane, rotation angle is

$$\psi_{z_0} = \frac{z_0}{L}\psi_e.$$

So, $P_m$ pronation rotation matrix is given as follows

$$P_m = \begin{bmatrix} \cos\psi_z & -\sin\psi_z & 0 \\ \sin\psi_z & \cos\psi_z & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2.1}$$

Let $X$ be a representative point of the line $d$, $d = \{(x_0, y_0, z) \mid 0 \leq z \leq z_e\}$, then we have

$$\alpha(z) = P_m X = \begin{bmatrix} \cos\psi_z & -\sin\psi_z & 0 \\ \sin\psi_z & \cos\psi_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z \end{bmatrix}, \quad X = (x_0, y_0, z), \quad x_0^2 + y_0^2 = r^2$$

and

$$\alpha(z) = (x_0\cos\psi_z - y_0\sin\psi_z, x_0\sin\psi_z + y_0\cos\psi_z, z). \tag{2.2}$$

## 3. Properties of the motion

We will give some theorems about the motion using the equations (2.1) and (2.2).

**Theorem 3.1.** *$\alpha(z) = P_m(d)$ is a helix.*

*Proof.*

$$\begin{aligned}
\alpha(z) &= (x_0\cos\psi_z - y_0\sin\psi_z, x_0\sin\psi_z + y_0\cos\psi_z, z)\\
\alpha'(z) &= (-\lambda x_0\sin\psi_z - \lambda y_0\cos\psi_z, \lambda x_0\cos\psi_z - \lambda y_0\sin\psi_z, 1)\\
\alpha''(z) &= (-\lambda^2 x_0\cos\psi_z + \lambda^2 y_0\sin\psi_z, -\lambda^2 x_0\sin\psi_z - \lambda^2 y_0\cos\psi_z, 0)
\end{aligned}$$

$$\begin{aligned}
\left\|\alpha'(z)\right\| &= \lambda^2 x_0^2\sin^2\psi_z + \lambda^2 y_0^2\cos^2\psi_z + 2\lambda^2 x_0 y_0\sin\psi_z\cos\psi_z\\
&\quad +\lambda^2 x_0^2\cos^2\psi_z + \lambda^2 y_0^2\sin^2\psi_z - 2\lambda^2 x_0 y_0\sin\psi_z\cos\psi_z + 1)\\
&= \sqrt{\lambda^2 x_0^2\sin^2\psi_z + \lambda^2 x_0^2\cos^2\psi_z + \lambda^2 y_0^2\cos^2\psi_z + \lambda^2 y_0^2\sin^2\psi_z + 1}\\
&= \sqrt{\lambda^2 x_0^2 + \lambda^2 y_0^2 + 1}\\
&= \sqrt{\lambda^2 r^2 + 1}.
\end{aligned}$$

Let $k = \sqrt{\lambda^2 r^2 + 1}$, $\lambda = \frac{\psi_e}{L}$, so we have Frenet vectors as follows.

$$\begin{aligned}
\overrightarrow{t}(z) &= \frac{1}{k}(-\lambda x_0\sin\psi_z - \lambda y_0\cos\psi_z, \lambda x_0\cos\psi_z - \lambda y_0\sin\psi_z, 1)\\
\overrightarrow{n}(z) &= \frac{1}{\lambda^2 r}(-\lambda^2 x_0\cos\psi_z + \lambda^2 y_0\sin\psi_z, -\lambda^2 x_0\sin\psi_z - \lambda^2 y_0\cos\psi_z, 0)\\
&= \frac{1}{r}(-x_0\cos\psi_z + y_0\sin\psi_z, -x_0\sin\psi_z - y_0\cos\psi_z, 0)\\
\overrightarrow{b}(z) &= (\frac{1}{k}(x_0\sin\psi_z + y_0\cos\psi_z), \frac{1}{k}(-x_0\cos\psi_z + y_0\sin\psi_z), \frac{\lambda r}{k})\\
&= (\frac{x_0}{k}\sin\psi_z + \frac{y_0}{k}\cos\psi_z, \frac{-x_0}{k}\cos\psi_z + \frac{y_0}{k}\sin\psi_z, \frac{\lambda r}{k}).
\end{aligned}$$

The first and second curvature of $\alpha(z)$ are

$$\begin{aligned}
\kappa &= \left\|\alpha''(z)\right\| = \lambda^2 r = (\frac{\psi_e}{L})^2 r,\\
\tau &= \left\|b'(z)\right\| = \frac{\lambda r}{k} = \frac{\psi_e r}{Lk}.
\end{aligned}$$

Therefore,

$$\frac{\kappa}{\tau} = \frac{\psi_e k}{L}$$

is fixed. Hence, $\alpha(z)$ is a helix. □

**Theorem 3.2.** *Let $X$ be series of $z$, and $Y$ be series of the length of the curve $\alpha(z)$. Then correlation between $X$ and $Y$ is equal to 1.*

*Proof.*

$$\alpha(z) = (x_0\cos\psi_z - y_0\sin\psi_z, x_0\sin\psi_z + y_0\cos\psi_z, z)$$

is the picture of $X$.

$$\alpha'(z) = (-\lambda x_0\sin\psi_z - \lambda y_0\cos\psi_z, \lambda x_0\cos\psi_z - \lambda y_0\sin\psi_z, 1)$$

and we calculate $\left\|\alpha'(z)\right\|$ as

$$\left\|\alpha'(z)\right\| = \sqrt{\lambda^2 r^2 + 1}, \quad \lambda = \frac{\psi_e}{L}.$$

Then every point at $z_0$, we have

$$L_{z_0} = \int\limits_0^{z_0} \sqrt{\lambda^2 r^2 + 1}dz = \sqrt{\lambda^2 r^2 + 1}z_0.$$

In this case, the series $X$ and $Y$ are written.

$$\begin{aligned}
X &= \{0, 0.1, ..., Z_e\},\\
Y &= \{0, ..., \varphi_e\},
\end{aligned}$$

where $Z_e$ is the last value of $z$ and $\varphi_e$ is the last rotation angle. For every $x_i$, we have $y_i = \sqrt{\lambda^2 r^2 + 1} x_i$, $\sqrt{\lambda^2 r^2 + 1} = c, c > 0$. We know that correlation of the series $X$ and $Y$ is given by

$$cor(X,Y) = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\sqrt{n\sum y_i^2 - (\sum y_i)^2}}.$$

For our series, we have

$$\begin{aligned} cor(X,Y) &= = \frac{n\sum x_i(cx_i) - (\sum x_i)(\sum cx_i)}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\sqrt{n\sum (cx_i)^2 - (\sum cx_i)^2}} \\ &= \frac{c(n\sum x_i^2 - (\sum x_i)^2)}{c\sqrt{n\sum x_i^2 - (\sum x_i)^2}\sqrt{n\sum x_i{}^2 - (\sum x_i)^2}} \\ &= 1. \end{aligned}$$

□

## 4. Application

**Example 4.1.** *Let $L = 20cm$, $r = 3cm$, $\psi_e = \frac{\pi}{3}$, step$= 0.5$. Then we have the series $X$ and $Y$.*
$X = \{$ 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, ..., 19, 19.5, 20 $\}$
$Y = \{$ 0.50615, 1.0123, 1.51845, ..., 18.2214, 18.72755, 19.2337, 19.73985, 20.246 $\}$
*Thus, we have $cor(X,Y) = 1$.*

Matlab programme m-file and (Figure 4.1) of this example are as follows.

```
cc=5
grid on
axis([-cc cc -cc cc 0 4*cc])
xlabel('x axis'); ylabel('y axis'); zlabel('z axis')
line([2 2],[0 0],[0 20],'LineWidth',4,'color',[.2 .2 .5]);
line([0 0 ],[2 2 ],[0 20],'LineWidth',4,'color',[.2 .3 .5]);
line([-2 -2],[0 0],[0 20],'LineWidth',4,'color',[.2 .4 .5])
L=20
for z=0:0.1:20
d=(pi/3)/20
r=3
c=((r^2*d^2+1))^(1/2)
u=((pi/3)/L)*z
A=[ cos(u) sin(u) 0
-sin(u) cos(u) 0
0 0 1]
P=[2 ; 0; z]
Q=[0 ;2; z]
D=[-2 ; 0; z]
E=A*Q
B=A*P
C=A*D
hold on
plot3(C(1),C(2),C(3),'.r')
plot3(B(1),B(2),B(3),'.r')
plot3(E(1),E(2),E(3),'.r')
pause(0.01)
end
hold on
text(2+0.5,0, 0, 'P')
text(2,0,21,'Q')
for t=0:1:360
r=2
plot3(r*cosd(t), r*sind(t), 0,'.r')
plot3(r*cosd(t), r*sind(t), 20,'.r')
end
```

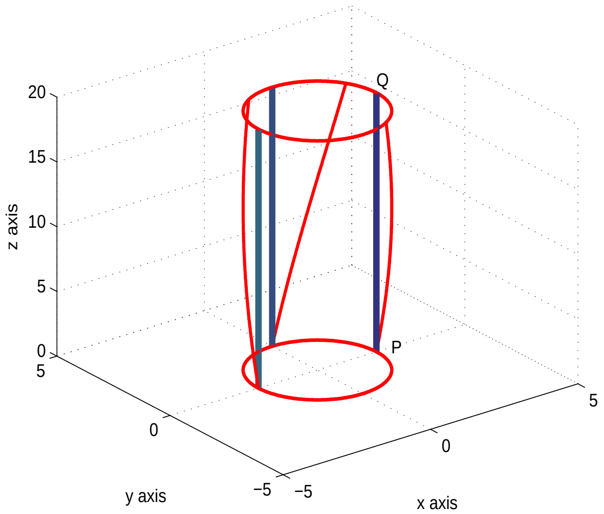


**Figure 4.1:** Orbits of points

## 5. Conclusion

We can use the algorithm of Stewart platform motion to create a new motion of supination(or pronation) motion. Supination and pronation motions are inverse motion to each other. So it is sufficient study only one of them. Pronation motion can be taken as Stewart motion which no rotation on x-axis and y-axis. In this case pronation motion has only one parameter. Translation part of pronation motion is uploaded the moving points. We give a relation between rotation angle and third component of moving platform which first and second components are fixed. The Frenet elements of the curve of the motion are calculated. We prove that the image curve is a helix. The correlation between rotation angle and extension of the image curve is exactly equal to 1.

## Acknowledgement

This work was supported by YYU BAP (Scientific Research Projects of Van Yuzuncu Yil University) with project number: FBA-2017-5435.

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