Acta Materialia Turcica

Volume 5, September 2021, Pages 18-23

Optimization Method for Electron Energy Distribution Function in Non-Equilibrium Plasma

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Abstract:

Electron energy distribution function (EEDF) in non-equilibrium plasmas often differs from Maxwellian distribution. Therefore, it is essential to determine the EEDF experimentally for accurately deducing the plasma parameters and rates of plasma-chemical processes. Langmuir Probe is a robust diagnostic tool allowing the estimation of EEDF by the differentiation of I-V curve. However, possible distortions in differentiation process should be handled cautiously. This paper focuses on Maxwellian optimization method for the determination of EEDF from Langmuir probe I-V curve where EEDF is modeled as perturbations around Maxwellian distribution. The proposed optimization method is implemented on the Argon glow discharge plasma experimental data. The results obtained by the proposed Maxwellian optimization are compared with the ones obtained by commonly used Savitzky-Golay filtering method and the polynomial optimization method. The results indicate that filtering data may be disadvantageous as it may cause the useful information encoded in the experimental data to be lost. In addition, based on the results one can conclude that it is vital to choose the appropriate model function to begin with for the optimization procedure to be satisfactory. It has been shown that the second order polynomial fit yields the best fitting curve to the Maxwellian optimization.

Keywords: Thermodynamics of plasmas; Plasma properties; Plasma diagnostic techniques and instrumentation; Discharge in vacuum; Glow.

DOI:

1. INTRODUCTION

The Langmuir probe is essentially a conducting object in the form of a sphere or a circular cylinder used for plasma diagnostics. The electric potential of the probe is varied by an external circuit to obtain the curve of probe potential versus the current flowing between the probe and the plasma, the so called I-V diagram. Theoretical interest in the Langmuir probe I-V diagram is partly motivated by the versatility of the probe in plasma diagnostics [1].

Druyvestein established a remarkable relation between the electron energy distribution function (EEDF) and the I-V diagram [2] which is valid if the following two conditions are met:

- The mean free path of electrons is much greater than the Debye length
- The mean free path of electrons is much greater than the probe radius.

The Debye length is the fundamental property of the plasma representing the physical scale of the transition from plasma collectivity to individual particle behavior. Langmuir probe is screened from the plasma within the Debye region bounded by the Debye length. In case the first condition is satisfied, variations of EEDF due to the

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Acta Materialia Turcica

Volume 5, September 2021, Pages 18-23

collisions of electrons with charged particles within the Debye region will be negligibly small. Accordingly, one can establish a relationship between the EEDF near the probe with the EEDF in the plasma. Provided that the second condition is met, one may neglect the impact of the probe dimensions on the plasma.

In Druyvestein formula EEDF is given by the second derivative of the I-V curve. However, noise-amplification effect in differentiation may lead enormous distortions in EEDF even for the small fluctuations in I-V curve. Hence, it is essential to develop methods smoothing the I-V curve. Several commonly used digital filtering methods are the Savitzky–Golay filter (used for noise suppression also in the first and second derivatives of the filtered curve) [3, 4], finite impulse response filter (gives the second derivative via convolution) [5], Gaussian function [6] and Blackman window [7].

Although the filtering avails the suppression of noises, it may also lead to the loss of useful information encoded in the experimental data. This is why, it may be advantageous to approximate the experimental curves in terms of analytic functions by quadratic optimization. Once the analytic expression is obtained for the I-V curve, differentiations are carried on without causing additional distortions. However, the second derivative of the analytically defined I-V curve does not necessarily agree with the second derivative obtained from the experimental data. Therefore, there appear additional requirements on the choice of basic functions used in the quadratic optimization. The novelty of our work is that we propose Maxwellian optimization method for choosing a physical set of basic functions so that the second derivatives of both analytically defined I-V curve and the experimental data are consistent.

2. ELECTRON ENERGY DISTRIBUTION FUNCTION

This section is divided into two subsections where in the first one theoretical background for obtaining EEDF from I-V curve is described and in the second one the experimental set up and the results are presented.

2.1. Theoretical Background for EEDF

The typical Langmuir probe I-V diagram is given in Fig. 1. The potential held by an isolated conductor in the plasma is called as the floating potential, since the potential floats to a value sufficient to maintain equal fluxes of electrons and ions. Except around the probe, the remainder of the plasma is at equipotential, known as the plasma potential. The floating potential and the plasma potential divide the I-V curve into three regions: electron saturation region (Region I in Fig. 1), electron retardation region (Region II in Fig. 1) and ion saturation region (Region III in Fig. 1). The EEDF is determined by using the data on the electron retardation and ion saturation regions. The plasma potential V_p, separating the electron saturation region from the electron retardation one, can be obtained either by the study of the I-V curve in the logarithmic scale or by the differentiation of the I-V curve [9].

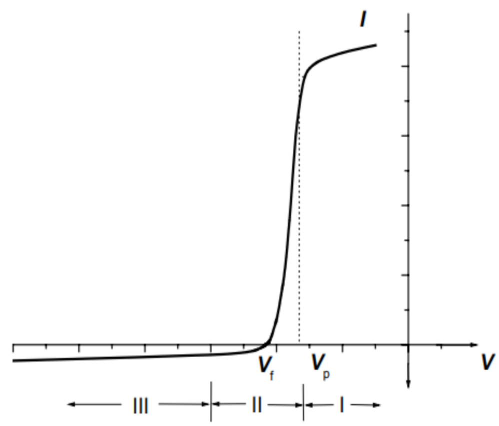

FIGURE 1. Langmuir probe I-V diagram.

Once the plasma potential is known, one can construct the current voltage characteristics I(E) of the plasma by the change of the variable $E=e(V-V_p)$, where e is the charge of electron and E is the energy in the electron volts unit. It is important to emphasize that both ion and electron fluxes from plasma to the probe contribute to I(E). In order to define the electron current as a function of energy $I_e(E)$, one should be able to model the behavior of the ion current. The theory of Langmuir probe is quite complicated unless some assumptions are made. In case the mean free path of ions is much larger than the Debye length, the ion current can be described by the Laframboise theory [10] so that the ion current dependence on the energy is given by $A\sqrt{E}$ where A is a constant which can be determined by interpolation of the I(E) curve at high energies. The EEDF is obtained from the electron current as a function of energy by the Druyvestein Formula

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Acta Materialia Turcica

Volume 5, September 2021, Pages 18-23

$$g(E) = -\frac{\sqrt{8mE}}{eS}\frac{d^2 I_e(E)}{dE^2}. \quad (2.1)$$

Here S is the area of the probe, m is the mass of electron. Direct differentiation of the electron current twice with respect to energy yields the EEDF given in Fig. 3.

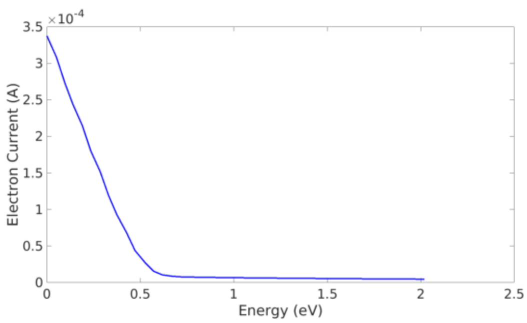

FIGURE 2. The energy dependence of electron current.

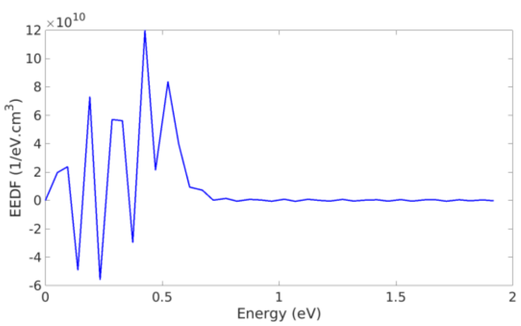

FIGURE 3. EEDF obtained by using Druyvestein formula.

It is apparent that the direct differentiation of electron current data gives rise to noise amplification. Hence, special methods are required for determining EEDF by using Druyvestein formula in (1). Savitzky-Golay filter is among the most commonly used smoothing method. In Figure 4, the result of using Savitzky-Golay filter on both electron current data and its first and second derivatives is given for illustration purposes.

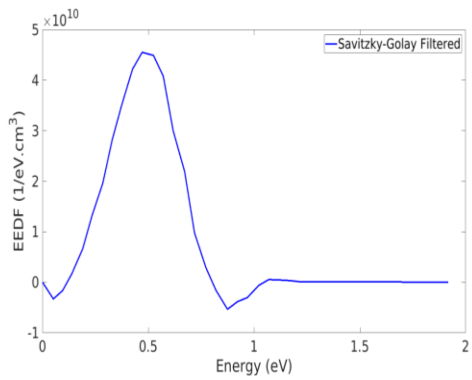

FIGURE 4. EEDF obtained by using Savitzky-Golay filter on both electron current data and its first and second derivatives.

As mentioned in the Introduction, filtering of the data may have its own drawback of loosing useful information encoded in experimental data. As in Druyvestein formula, one needs to apply this filtering consecutively to $I_e(E)$ and to its first and second derivatives, it may lead incoherent results. The unphysical negative regions in Fig. 4 exemplify this result. In the next Section, we develop an alternative approach based on the least square approximation of the electron current experimental curve by means of an analytic one.

3. OPTIMIZATION PROCEDURE

Apart from the filtering methods, polynomial fitting is also widely used for this purpose [8] where the experimental curve $I_e(E)$ is approximated by the polynomial model curve $I_m(E)$. The polynomial coefficients are determined by minimizing the quadratic form:

$$F = \sum_{n=1}^{N} [I_e(E_n) - I_m(E_n)]^2 \quad (3.1)$$

where n is the number index in a row of experimental data and E_n is the corresponding energy of the n^{th} electron current data. The experimental electron current and its fourth, fifth and sixth order polynomial fits are given in Fig. 5.

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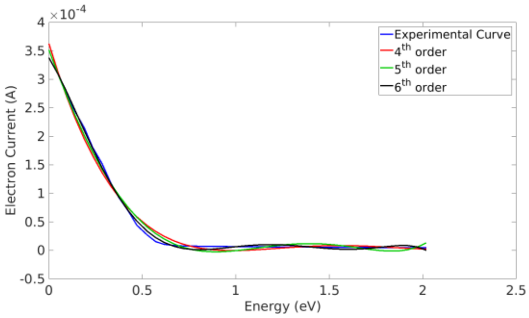

FIGURE 5. The experimental electron current and its fourth, fifth and sixth order polynomial fits.

In Fig. 6, EEDF obtained by analytic differentiation of these model curves are depicted.

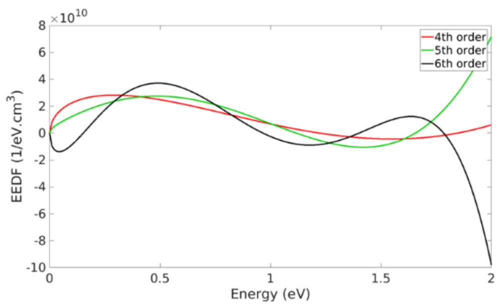

FIGURE 6. EEDF obtained by analytic differentiation of these model curves.

It is obvious from Fig. 5 that, the modeled electron current coincides better with experimental one for higher order polynomials. However, in Fig. 6 one can easily infer that, EEDF yields unphysical results at high energies for higher orders. In 5^{th} order, EEDF increases with increasing energies while in the 6th order it becomes negative. Therefore, the polynomial fit can be expanded up to the fourth order. In conclusion, even though the polynomial model curve fits well with the experimental one, its second derivatives may be quite dissimilar. This situation arises if the experimental curve does not feature a polynomial behavior. Therefore, it is vital to choose the appropriate model function to begin with for the optimization procedure to be satisfactory.

For distributions slightly different from the Maxwellian one, it is reasonable to assume the following model function for the electron current which is referred as Maxwellian optimization:

$$I_m(E) = e^{-\frac{E}{E_0}}\left(a_0 + a_1\frac{E}{E_0} + a_2\left(\frac{E}{E_0}\right)^2 + \cdots\right). \qquad (3.2)$$

In the zeroth order approximation, all coefficients are set to zero except a_0 and a pure Maxwellian distribution is obtained. Higher order coefficients describe the perturbations from the Maxwellian distribution. In the zeroth order, the energy scale E_0 is the mean energy of electron gas in plasma. This optimization parameter is determined by the the zeroth order optimization as shown in the Fig. 7. The best fitting is curve is obtained at E_0=0.3 eV.

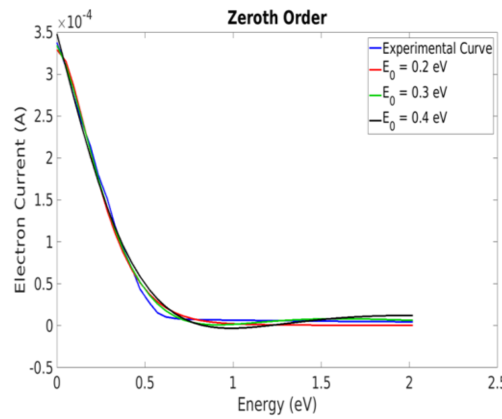

FIGURE 7. Estimation of energy scale E_0.

In Fig. 8 the first, the second, the third and the fourth order approximations for the electron current in are given in Maxwellian optimization. As it is expected, the modeled electron current versus energy curve fits better with the experimental one for higher orders in optimization.

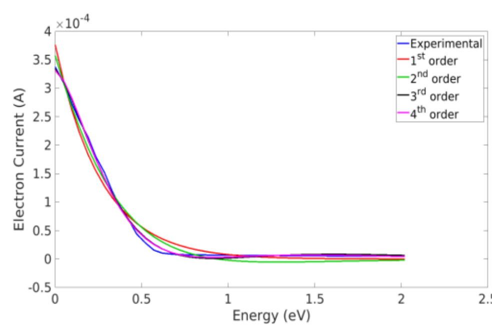

FIGURE 8. Experimental electron current and Maxwellian optimization of the electron current at 1st-4th orders

In EEDF, on the other hand, unphysical distributions are observed at higher orders (see Fig. 9). It is clear from Fig. 9 that, at the third order, there is a negative area at

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Acta Materialia Turcica

Volume 5, September 2021, Pages 18-23

small energies. Therefore, the second order approximation is the best fitting one in Maxwellian optimization.

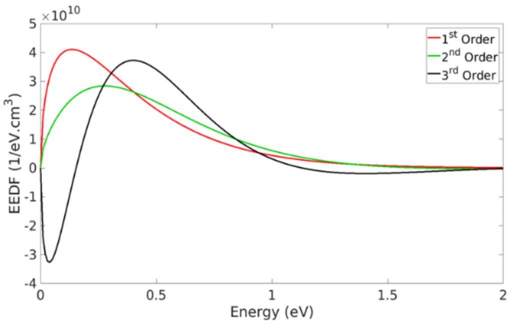

FIGURE 9. EEDF by Maxwellian Optimization.

Comparing the results in Fig. 6 with the literature, one may conclude that in polynomial fitting at high energies artificial oscillations of EEDF appear [8]. However, in Maxwellian Optimization the artificial oscillations disappear due to the exponential damping (see Eq. 3).

For non-Maxwellian distributions, it is reasonable to use another set of function. In Fig. 10, Maxwellian and Druyvestein distribution functions of electron energy having 1 eV mean energy are depicted. It is clearly seen from the figure that the electrons with high energies are less probable in Druyvestein plasma compared to the Maxwellian one since the non-elastic collisions of electrons with atoms and ions lead to the decreasing number of electrons with high energy. If this is the case, one should replace the exponential in (3) by $e^{(-(E/E_0)^2)}$.

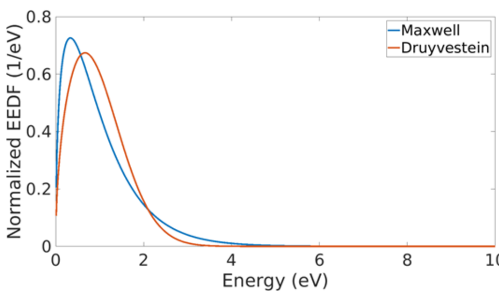

FIGURE 10. Normalized Maxwellian and Druyvestien EEDF.

4. CONCLUSIONS

We present an optimization procedure for the estimation of the Maxwellian like distribution functions by using exponential polynomial functions where finite data set of electron current is modeled as a continuous analytic function so that a more detailed description of plasma is attained. The optimization parameters are the energy scale, the order and the coefficients of expansion. EEDF is obtained by a straightforward differentiation of the modeled analytic function of electric current with respect to the energy twice. The results obtained by this optimization are compared with the ones obtained by using Savitzky-Golay filtering method and the polynomial optimization method. The results indicate that filtering data may be disadvantageous as the useful information encoded in the experimental data may be lost in the filtering process. In addition, based on the results one can conclude that it is vital to choose the appropriate model function to begin with for the optimization procedure to be satisfactory. The order of the approximation is defined by selecting the best fitting physical result. It has been shown that the second order polynomial fit yields the best fitting curve to the Maxwellian optimization.

5. ACKNOWLEDGMENT

This research is funded by TÜBİTAK UME.

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