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Authors: Hami GÜNDOĞDU, Ömer Faruk GÖZÜKIZIL
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New Kinds of Solutions to (3+1)-Dimensional mKDV and mBBM Equations

Hami GÜNDOĞDU*[1], Ömer Faruk GÖZÜKIZIL[2]

Abstract

In this work, we consider (3+1) dimensional nonlinear partial differential equations, namely modified KdV and Benjamin-Bona-Mahony equations. Different types of solutions to these equations are derived by Jacobi elliptic sine function expansion method. Besides that, we introduce new types of solutions for two more modified forms of given equations. The gained solutions include exact, singular, periodic, and kink solutions. It is stated that some conditions related to the coefficients provide us with the existence of the gained solutions.

Keywords: Elliptic sine function method, (3+1)-Dimensional mKdV equation, (3+1)-Dimensional mBBM Equation

1. INTRODUCTION

Nonlinear partial differential equations, shortly NLPDEs, have a substantial space in most branches of science and engineering. They have helped us by describing scientific problems in nonlinear optics, solid-state physics, plasma waves, fluid mechanics, chemical kinematics, plasma physics and some other areas. In these fields, a considerable amount of information can be gathered if travelling wave solutions of NLPDEs are obtained. These solutions allow us to understand distinct types of scientific events, not been explained yet. In the literature, it is generally seen that (1+1) and (2+1) dimensional NLPDEs have been studied by researchers. Moreover, many methods have been discovered to acquire the exact solutions to these kinds of NLPDEs. The tanh-coth method [1], the tanh-sech method [2], the sech-csch method [3], the modified extended tanh-function method [4], the sine-cosine method [5], generalized hyperbolic function method [6], the (G'/G) expansion method [7], the exp-function method [8], and Jacobi elliptic function expansion method [9] can be given as examples of numerical, analytical and theoretical methods. Eventually, it is possible to gain the solutions of NLPDEs by using one of these methods.

*Corresponding Author: hamigundogdu@sakarya.edu.tr
[1] Sakarya University, Department of Mathematics, 54187, Serdivan, Turkey.
ORCID: https://orcid.org/0000-0002-7042-1885
[2] Sakarya University, Department of Mathematics, 54187, Serdivan, Turkey.
E-Mail: farukg@sakarya.edu.tr ORCID: https://orcid.org/0000-0002-5975-6430

In some models, the encountered problems may not be represented by (1+1) or (2+1) dimensional NLPDEs. Yet, they may be modelled by (3+1) or higher dimensional equations. Under these circumstances, getting solutions of NLPDEs is not straightforward. There is a fact that (3+1) dimensional equations symbolize real-world problems. Therefore, it encourages scientists to pay more attention to these types of equations. In addition to (3+1) dimensional ones, higher dimensional NLPDEs is quite attractive for not only physicists but also mathematicians.

We examine, in this work, modified KdV(mKdV) and modified Benjamin-Bona-Mahony (mBBM) equations which have considerable importance in (1+1) dimensional equations. Moreover, these NLPDEs make data available and convenient to use in some substantial areas such as fluid mechanics, solitary waves, electro-dynamics and so on. The famous KdV equation was introduced in [10] as in the following:

$$u_t + uu_x + u_{xxx} = 0. \tag{1}$$

This equation has been used for modelling shallow water waves of small amplitude and large wavelength. In addition to that, some important physical, chemical and even biological events, for instance, acoustic solitons in plasma, blood pressure pulses and internal gravity waves have modelled by equation (1).

Having an important role to describe some significant scientific models lead scientists to do more works on KdV equation. That is why Benjamin, Bona and Mahony come up with the regularized long-wave equation, also called BBM equation as an alternative to equation (1). BBM equation is given in [11] by

$$u_t + u_x + uu_x - u_{xxt} = 0. \tag{2}$$

It is seen that there is a slight difference between equation (1) and (2). u_{xxx} is replaced by $-u_{xxt}$. The equation (2) has been used to analyze the surface waves of large wavelength in liquids, acoustic gravity waves, incompressible fluids and anharmonic crystals, hydromagnetic waves in cold plasma and some others.

In the literature, modified forms of the equations, mentioned above, have been given by

$$u_t + u^2u_x + u_{xxx} = 0, \tag{3}$$

and

$$u_t + u_x + u^2u_x - u_{xxt} = 0. \tag{4}$$

The equations given in (3) and (4) are called modified KdV (mKdV) and modified BBM (mBBM) equations, respectively. There are numerous studies on mKdV and mBBM equations. The existence of the solutions, exact solutions by many methods, new solitary solutions, approximate explicit solutions of these equations, and more examples can be found in the open literature.

(3+1) dimensional NLPDEs are of plenty improvements in most fields of modern sciences. Therefore, a great deal of equations in this dimension have been considered and analyzed. Moreover, Hereman [12] have introduced the (3+1) dimensional mKdV equation in the following form:

$$u_t + 6u^2u_x + u_{xyz} = 0. \tag{5}$$

In the same manner, two types of mKdV equations have been given in [13] as follows:

$$u_t + 6u^2u_y + u_{xyz} = 0, \tag{6}$$

and

$$u_t + 6u^2u_z + u_{xyz} = 0. \tag{7}$$

Introducing new kinds of (3+1) dimensional mKdV equation has provided Wazwaz with the motivation for bringing new types of (3+1) mBBM equation. As a result,

$$u_t + u_x + u^2u_y - u_{xzt} = 0, \tag{8}$$

$$u_t + u_z + u^2u_x - u_{xyt} = 0, \tag{9}$$

and

$$u_t + u_y + u^2u_z - u_{xxt} = 0 \tag{10}$$

have been presented in the same paper.

Our purpose is to derive different types of solutions to (3+1) dimensional equations given in (5) and (8) under the conditions that they exist. Jacobi elliptic sine function expansion method (sn-ns method), proposed by Liu et al. [14], is used to achieve our aim. Because, this method is one of the most respectable methods, see its efficiency [15]-[18]. This method gives the singular solution, periodic solution, exact solution and kink solution.

2. MAIN PRINCIPLES OF THE METHOD

In this chapter, the fundamental principles of the method have been given. With the help of this method, we have looked for the travelling wave solutions of (3+1) dimensional nonlinear partial differential equation given in the general form

$$P(u, u_t, u_x, u_y, u_z, u_{xy}, u_{xzt}, \ldots) = 0. \tag{11}$$

Firstly, the wave transformation for (3+1) dimension is considered as follows:

$$u(x,y,z,t) = v(\xi),\ \xi = \alpha x + \beta y + \gamma z - \omega t, \tag{12}$$

where α, β, γ are arbitrary constants and ω stands for the dispersion relation.

Then, putting the ordinary derivatives of $v(\xi)$ instead of the partial derivatives of $u(x,y,z,t)$ turns (11) into the following ordinary differential equation(ODE) with respect to the variable ξ as in the following:

$$Q(v, v', v'', \ldots) = 0, \tag{13}$$

with Q being a polynomial with respect to functions v, v', v'',...

In this method, the travelling wave solutions to (13) are investigated in the following form:

$$v(\xi) = a_0 + \sum_{j=1}^{n} a_j sn^j(\xi \mid m) + b_j ns^j(\xi \mid m) \tag{14}$$

where n, the balancing constant, is desired positive parameter, sn is the Jacobi elliptic *sine* function and ns represents the inverse of sn. In (13), equating the power of highest-order of linear and nonlinear terms to each other gives n.

Putting (14) into the equation (13) offers an equation in the power of sn and ns. All the coefficients of the same power of sn and ns in the resulting equation are collected. After that, these coefficients should be equal to zero. Therefore, it provides us with a system of algebraic equations involving the parameters, a_k, b_k, $(k = 0,\ldots,n)$ and ω. Solving the algebraic system yields the desired parameters. Then, putting these parameters into (14) gives the analytic solutions, depended in the variable ξ, in a closed-form. Then, putting the wave transformation $\xi = \alpha x + \beta y + \gamma z - \omega t$ back into the solutions in (14) gives us the desired solutions.

3. SOLUTIONS OF (3+1) MKDV AND MBBM EQUATIONS

Our purpose is to obtain the solutions of different types of (3+1) mKdV and mBBM equations given in (5)-(10) in this section. In this regard, the method mentioned above has been applied to these equations, respectively.

3.1. Solutions of mKdV Equations

We consider the mKdV equations given in (5)-(7). Firstly, we obtain the solutions of equation (5). Using the transformations in (12) turns the equation (5) into the following ODE:

$$-\omega v' + 6\alpha v^2 v' + \alpha\beta\gamma v''' = 0. \tag{15}$$

Taking integration of the equation (15) and considering the integration constant as zero yields the following equation:

$$-\omega v + 2\alpha v^3 + \alpha\beta\gamma v'' = 0. \tag{16}$$

It can be rewritten as follows:

$$v''-(\frac{w}{\alpha\beta\gamma})v+(\frac{2}{\beta\gamma})v^3=0. \quad (17)$$

It has the form of Duffing Equation with $p = -\frac{w}{\alpha\beta\gamma}$, $q = \frac{2}{\beta\gamma}$ and $r = 0$.

Now, we are to determine the balancing constant n. For this purpose, we equalize the highest order of linear term and the highest power of the nonlinear term. Then, it gives $n = 1$.

Afterwards, the solution is taken in the following form:

$$v(\xi)=a_0+\sum_{j=1}^{1}a_j sn^j(\xi\,|\,m)+b_j ns^j(\xi\,|\,m) \quad (18)$$

or

$$v(\xi)=a_0+a_1 sn(\xi\,|\,m)+b_1 ns(\xi\,|\,m) \quad (19)$$

Inserting $v(\xi)$ into the equation (17) yields the following algebraic system:

$sn^2: 3a_0a_1^2q=0,$

$sn^1: a_1^3q+2k^2m^2a_1=0,$

$sn^0: 3a_1^2b_1^2q+3a_0^2a_1b_1-k^2m^2a_1b_1 -k^2a_1b_1+pa_1b_1=0,$

$ns^1: 6a_0a_1b_1q+a_0^3q+a_0p=0,$

$ns^2: b_1^3q+2k^2b_1=0.$

Solving this system, we obtain the following unknown parameters:

$$a_0=0, a_1=\pm\frac{m\sqrt{-2p}}{\sqrt{(m^2+6m+1)q}}, b_1=\pm\frac{\sqrt{-2p}}{\sqrt{(m^2+6m+1)q}}, k=\frac{\sqrt{p}}{\sqrt{(m^2+6m+1)}},$$

$$a_0=0, a_1=\pm\frac{m\sqrt{-2p}}{\sqrt{(m^2-6m+1)q}}, b_1=\pm\frac{\sqrt{-2p}}{\sqrt{(m^2-6m+1)q}}, k=\frac{\sqrt{p}}{\sqrt{(m^2-6m+1)}},$$

$$a_0=0, a_1=\pm\frac{m\sqrt{-2p}}{\sqrt{(m^2+1)q}}, b_1=0, k=\frac{\sqrt{p}}{\sqrt{(m^2+1)}},$$

$$a_0=0, a_1=0, b_1=\pm\frac{m\sqrt{-2p}}{\sqrt{(m^2+1)q}}, k=\frac{\sqrt{p}}{\sqrt{(m^2+1)}}.$$

Inserting these coefficients in the expansion (18) gives us the solutions in the following forms:

$$v_1(\xi)=\pm\frac{\sqrt{-2p}}{\sqrt{(m^2+6m+1)q}}[msn(\frac{\sqrt{p}}{\sqrt{(m^2+6m+1)}}\xi,m) +ns(\frac{\sqrt{p}}{\sqrt{(m^2+6m+1)}}\xi,m)],$$

$$v_2(\xi)=\pm\frac{\sqrt{-2p}}{\sqrt{(m^2-6m+1)q}}[msn(\frac{\sqrt{p}}{\sqrt{(m^2-6m+1)}}\xi,m) +ns(\frac{\sqrt{p}}{\sqrt{(m^2-6m+1)}}\xi,m)],$$

$$v_3(\xi)=\pm\frac{m\sqrt{-2p}}{\sqrt{(m^2+1)q}}sn(\frac{\sqrt{p}}{\sqrt{(m^2+1)}}\xi,m),$$

$$v_4(\xi)=\pm\frac{m\sqrt{-2p}}{\sqrt{(m^2+1)q}}ns(\frac{\sqrt{p}}{\sqrt{(m^2+1)}}\xi,m).$$

In order to get the exact solutions, we let $m \to 1$. We therefore have

$$v_1(\xi)=\pm\sqrt{\frac{-2p}{q}}\coth(k\xi,m),\; k=\sqrt{\frac{p}{2}},$$

$$v_2(\xi)=\pm\sqrt{\frac{2p}{q}}\operatorname{csch}(k\xi,m),\; k=\sqrt{-p},$$

$$v_3(\xi)=\pm\sqrt{\frac{-p}{q}}\tanh(k\xi,m),\; k=\sqrt{\frac{p}{2}},$$

where $p = -\frac{w}{\alpha\beta\gamma}$ and $q = \frac{2}{\beta\gamma}$.

To be specific, we take $k = \pm 1$. Then, we have the following solutions:

$u_1(x,y,z,t) = \pm\sqrt{-\beta\gamma}\coth(\alpha x+\beta y+\gamma z+2\alpha\beta\gamma t),$

$u_2(x,y,z,t) = \pm\sqrt{-\beta\gamma}\cot(\alpha x+\beta y+\gamma z-2\alpha\beta\gamma t),$

$u_3(x,y,z,t) = \pm\sqrt{-\beta\gamma}\operatorname{csch}(\alpha x+\beta y+\gamma z-\alpha\beta\gamma t),$

$u_4(x,y,z,t) = \pm\sqrt{-\beta\gamma}\,\csc(\alpha x + \beta y + \gamma z + \alpha\beta\gamma t)$,

$u_5(x,y,z,t) = \pm\sqrt{-\beta\gamma}\,\tanh(\alpha x + \beta y + \gamma z + 2\alpha\beta\gamma t)$,

$u_6(x,y,z,t) = \pm\sqrt{-\beta\gamma}\,\tan(\alpha x + \beta y + \gamma z - 2\alpha\beta\gamma t)$.

Under the condition of $\beta\gamma < 0$, the above solutions exist. Furthermore, u_1, u_3 and u_4 provide us with singular solutions. u_2 is the exact solution. u_6 gives the periodic solution. Eventually, the kink solution is derived from u_5.

Here we obtain the solutions of equation (6). Using the wave transformation we have

$$-\omega v' + 6\beta v^2 v' + \alpha\beta\gamma v''' = 0. \qquad (20)$$

Integrating it and considering the integration constant as zero yields the following equation:

$$-\omega v + 2\beta v^3 + \alpha\beta\gamma v'' = 0. \qquad (21)$$

From the same process above, we gain the solutions for the equation (6) as follows:

$u_1(x,y,z,t) = \pm\sqrt{-\alpha\gamma}\,\coth(\alpha x + \beta y + \gamma z + 2\alpha\beta\gamma t)$,

$u_2(x,y,z,t) = \pm\sqrt{-\alpha\gamma}\,\cot(\alpha x + \beta y + \gamma z - 2\alpha\beta\gamma t)$,

$u_3(x,y,z,t) = \pm\sqrt{-\alpha\gamma}\,\text{csch}(\alpha x + \beta y + \gamma z - \alpha\beta\gamma t)$,

$u_4(x,y,z,t) = \pm\sqrt{-\alpha\gamma}\,\csc(\alpha x + \beta y + \gamma z + \alpha\beta\gamma t)$,

$u_5(x,y,z,t) = \pm\sqrt{-\alpha\gamma}\,\tanh(\alpha x + \beta y + \gamma z + 2\alpha\beta\gamma t)$,

$u_6(x,y,z,t) = \pm\sqrt{-\alpha\gamma}\,\tan(\alpha x + \beta y + \gamma z - 2\alpha\beta\gamma t)$.

These solutions exist for $\alpha\gamma < 0$. For equation (6), exact solution, periodic solution, kink solution, and the singular solutions are acquired.

Now we look for the solutions to the last equation (7) of mKDV equations. Using the wave transformation we have

$$-\omega v + 2\gamma v^3 + \alpha\beta\gamma v'' = 0. \qquad (22)$$

Following the same process above gives the desired solutions for the equation (7) as follows:

$u_1(x,y,z,t) = \pm\sqrt{-\alpha\beta}\,\coth(\alpha x + \beta y + \gamma z + 2\alpha\beta\gamma t)$,

$u_2(x,y,z,t) = \pm\sqrt{-\alpha\beta}\,\cot(\alpha x + \beta y + \gamma z - 2\alpha\beta\gamma t)$,

$u_3(x,y,z,t) = \pm\sqrt{-\alpha\beta}\,\text{csch}(\alpha x + \beta y + \gamma z - \alpha\beta\gamma t)$,

$u_4(x,y,z,t) = \pm\sqrt{-\alpha\beta}\,\csc(\alpha x + \beta y + \gamma z + \alpha\beta\gamma t)$,

$u_5(x,y,z,t) = \pm\sqrt{-\alpha\beta}\,\tanh(\alpha x + \beta y + \gamma z + 2\alpha\beta\gamma t)$,

$u_6(x,y,z,t) = \pm\sqrt{-\alpha\beta}\,\tan(\alpha x + \beta y + \gamma z - 2\alpha\beta\gamma t)$.

These solutions exist in the case of $\alpha\beta < 0$. Here we obtain different kinds of solutions to equation (7) such as exact solution, periodic solution, kink solution, and singular solutions.

3.2. Solutions of mBBM Equations

We consider the mBBM equation (8) and two more alternative forms of mBBM equation given in (9) and (10).

At first, we consider the mBBM equation (8). By using the transformation

$$-\omega v' + \alpha v' + \beta v^2 v' + \alpha\gamma\omega v''' = 0. \quad (23)$$

Integrating both sides of the equation (23) and taking the integration constant as zero satisfies the following equation:

$$-\omega v + \alpha v + (\beta/3)\, v^3 + \alpha\beta\gamma v''' = 0. \quad (24)$$

We can rewrite the equation (24) as follows:

$$v'' + \left(\frac{\alpha - w}{\alpha\gamma w}\right)v + \frac{\beta}{3\alpha\gamma w}v^3 = 0. \quad (25)$$

It is in the form of Duffing Equation with $p = \frac{\alpha - w}{\alpha\gamma w}$, $q = \frac{\beta}{3\alpha\gamma w}$, and $r = 0$.

It is seen that mKdv and mBBm equations have similar structures. As a result, the procedure given above can be certainly followed here, as well. At the end of some calculations, we gain the desired solutions of the different forms of mBBM equation (8), (9), and (10).

For the equation (8), we have the following solutions:

$$u_1(x,y,z,t) = \pm\frac{\alpha}{\beta}\sqrt{-\frac{6\beta\gamma}{1+2\alpha\gamma}}\coth\left(\alpha x + \beta y + \gamma z - \frac{\alpha}{1+2\alpha\gamma}t\right),\ \frac{\beta\gamma}{1+2\alpha\gamma} < 0,$$

$$u_2(x,y,z,t) = \pm\frac{\alpha}{\beta}\sqrt{-\frac{6\beta\gamma}{1-2\alpha\gamma}}\cot\left(\alpha x + \beta y + \gamma z - \frac{\alpha}{1-2\alpha\gamma}t\right),\ \frac{\beta\gamma}{1-2\alpha\gamma} < 0,$$

$$u_3(x,y,z,t) = \pm\frac{\alpha}{\beta}\sqrt{-\frac{6\beta\gamma}{1-\alpha\gamma}}\operatorname{csch}\left(\alpha x + \beta y + \gamma z - \frac{\alpha}{1-\alpha\gamma}t\right),\ \frac{\beta\gamma}{1-\alpha\gamma} < 0,$$

$$u_4(x,y,z,t) = \pm\frac{\alpha}{\beta}\sqrt{-\frac{6\beta\gamma}{1+2\alpha\gamma}}\csc\left(\alpha x + \beta y + \gamma z - \frac{\alpha}{1+2\alpha\gamma}t\right),\ \frac{\beta\gamma}{1+2\alpha\gamma} < 0,$$

$$u_5(x,y,z,t) = \pm\frac{\alpha}{\beta}\sqrt{-\frac{6\beta\gamma}{1+2\alpha\gamma}}\tanh\left(\alpha x + \beta y + \gamma z - \frac{\alpha}{1+2\alpha\gamma}t\right),\ \frac{\beta\gamma}{1+2\alpha\gamma} < 0,$$

$$u_6(x,y,z,t) = \pm\frac{\alpha}{\beta}\sqrt{-\frac{6\beta\gamma}{1-2\alpha\gamma}}\tan\left(\alpha x + \beta y + \gamma z - \frac{\alpha}{1-2\alpha\gamma}t\right),\ \frac{\beta\gamma}{1-2\alpha\gamma} < 0.$$

For the equation (9), we obtain the following solutions:

$$u_1(x,y,z,t) = \pm\sqrt{-\frac{6\beta\gamma}{1+2\alpha\gamma}}\coth\left(\alpha x + \beta y + \gamma z - \frac{\gamma}{1+2\alpha\gamma}t\right),\ \frac{\beta\gamma}{1+2\alpha\gamma} < 0,$$

$$u_2(x,y,z,t) = \pm\sqrt{-\frac{6\beta\gamma}{1-2\alpha\gamma}}\cot\left(\alpha x + \beta y + \gamma z - \frac{\gamma}{1-2\alpha\gamma}t\right),\ \frac{\beta\gamma}{1-2\alpha\gamma} < 0,$$

$$u_3(x,y,z,t) = \pm\sqrt{-\frac{6\beta\gamma}{1-\alpha\gamma}}\operatorname{csch}\left(\alpha x + \beta y + \gamma z - \frac{\gamma}{1-\alpha\gamma}t\right),\ \frac{\beta\gamma}{1-\alpha\gamma} < 0,$$

$u_4(x,y,z,t) = \pm\sqrt{-\frac{6\beta\gamma}{1+2\alpha\gamma}}\csc(\alpha x + \beta y + \gamma z - \frac{\gamma}{1+2\alpha\gamma}t),\ \frac{\beta\gamma}{1+2\alpha\gamma} < 0,$

$u_5(x,y,z,t) = \pm\sqrt{-\frac{6\beta\gamma}{1+2\alpha\gamma}}\tanh(\alpha x + \beta y + \gamma z - \frac{\gamma}{1+2\alpha\gamma}t),\ \frac{\beta\gamma}{1+2\alpha\gamma} < 0,$

$u_6(x,y,z,t) = \pm\sqrt{-\frac{6\beta\gamma}{1-2\alpha\gamma}}\tan(\alpha x + \beta y + \gamma z - \frac{\gamma}{1-2\alpha\gamma}t),\ \frac{\beta\gamma}{1-2\alpha\gamma} < 0.$

For the equation (10), we gain the following solutions:

$u_1(x,y,z,t) = \pm\frac{\alpha}{\gamma}\sqrt{-\frac{6\beta\gamma}{1+2\alpha^2}}\coth(\alpha x + \beta y + \gamma z - \frac{\beta}{1+2\alpha^2}t),\ \frac{\beta\gamma}{1+2\alpha^2} < 0,$

$u_2(x,y,z,t) = \pm\frac{\alpha}{\gamma}\sqrt{-\frac{6\beta\gamma}{1-2\alpha^2}}\cot(\alpha x + \beta y + \gamma z - \frac{\beta}{1-2\alpha^2}t),\ \frac{\beta\gamma}{1-2\alpha^2} < 0,$

$u_3(x,y,z,t) = \pm\frac{\alpha}{\gamma}\sqrt{-\frac{6\beta\gamma}{1-\alpha^2}}\operatorname{csch}(\alpha x + \beta y + \gamma z - \frac{\beta}{1-\alpha^2}t),\ \frac{\beta\gamma}{1-\alpha^2} < 0,$

$u_4(x,y,z,t) = \pm\frac{\alpha}{\gamma}\sqrt{-\frac{6\beta\gamma}{1+\alpha^2}}\csc(\alpha x + \beta y + \gamma z - \frac{\beta}{1+\alpha^2}t),\ \frac{\beta\gamma}{1+\alpha^2} < 0,$

$u_5(x,y,z,t) = \pm\frac{\alpha}{\gamma}\sqrt{-\frac{6\beta\gamma}{1+2\alpha^2}}\tanh(\alpha x + \beta y + \gamma z - \frac{\beta}{1+2\alpha^2}t),\ \frac{\beta\gamma}{1+2\alpha^2} < 0,$

$u_6(x,y,z,t) = \pm\frac{\alpha}{\gamma}\sqrt{-\frac{6\beta\gamma}{1-2\alpha^2}}\tan(\alpha x + \beta y + \gamma z - \frac{\beta}{1-2\alpha^2}t),\ \frac{\beta\gamma}{1-2\alpha^2} < 0.$

The exact, singular, periodic and kink solutions of the mBBM equations (8), (9), and (10) are obtained with the conditions in which they exist.

4. MAIN RESULTS

In our work, some other forms of modified KdV and BBM equations in addition to well-known ones have been investigated. The principal purpose is to acquire different kinds of solutions to these equations. So, we have applied to the Jacobi elliptic sine function expansion method for achieving this goal. Eventually, the solutions are obtained. It is guaranteed the existence of solutions under some conditions, related to the coefficients, α, β *and* γ.

In [13], Wazwaz obtained the solutions of the equations (5) and (8) by using some methods such as the sech-csch, tanh-coth, the sec-csc and the tan-coth method. For the equations (6), (7), (9) and (10), Wazwaz used the sech-csch and tanh-coth method to get the soliton and the kink solution. In this paper, we have just benefited from the sn-ns method.

In addition to one-dimensional NLPDEs, it is seen that the method is completely applicable, suitable and useful for solving higher order dimensional NLPDEs. This method allows us to get the solutions in different forms of functions such as Jacobi elliptic, hyperbolic and trigonometric functions. Because of this reason, there is no need for using a lot of distinct methods to obtain different types of solutions.

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