International Journal of Contemporary Educational Research (IJCER)

www.ijcer.net

Problem Posing with Third-grade Children: Examining the Complexity of Problems

Tuğrul KAR[1], Tuğba ÖÇAL[2], Mehmet Fatih ÖÇAL[2], Ömer DEMİRCİ[3]

[1]Recep Tayyip Erdoğan University
[2]Ağrı İbrahim Çeçen University
[3]Erzincan Binali Yıldırım University

To cite this article:

Kar, T., Öçal, T., Öçal, M. F. & Demirci, Ö. (2021). Problem posing with third-grade children: Examining the complexity of problems. *International Journal of Contemporary Educational Research, 8*(1), 54-71. DOI: https://doi.org/10.33200/ijcer.820714

International Journal of Contemporary Educational Research

Volume 8, Number 1, March 2021, 54-71 ISSN: 2148-3868

Problem Posing with Third-grade Children: Examining the Complexity of Problems

Tuğrul KAR[1*], Tuğba ÖÇAL[2], Mehmet Fatih ÖÇAL[2], Ömer DEMİRCİ[3]

[1]Recep Tayyip Erdoğan University

[2]Ağrı İbrahim Çeçen University

[3]Erzincan Binali Yıldırım University

Abstract

The purpose of this study was to investigate what factors third-grade students took into consideration when posing problems for their peers and how these factors affected the mathematical complexities of the problems. Free and semi-structured problem-posing tasks were given to 27 third-grade students, and the problems they created for their peers were analyzed in terms of their semantic structure and arithmetic complexity. According to the findings of the study, there was a statistically significant difference between the semantic structures of the problems in both tasks created for the more mathematically proficient student, but there was no difference between their arithmetic complexities. In addition, according to the qualitative findings of the study, the magnitude of the numbers, the operation types, the number of operations used, and the interests of the students were taken into consideration in posing problems for students with low and high levels of mathematical ability.

Keywords: Arithmetic complexity, Arithmetic operations, Problem-posing, Semantic structure, Word problem

Introduction

Problem posing, also referred as problem finding, problem formulating, and problem creating (Singer & Voica, 2013), has recently taken interest of mathematics education researchers. Problem posing is an open-ended cognitive activity and is considered to be an important component of inquiry-based learning (Silver, 1997); therefore, it solidifies its importance within mathematics education. Whether problem posing is considered as a means of instruction (to engage students in learning important concepts and skills and to enhance their problem-solving competence) or as an object of instruction (to develop students' proficiency in posing mathematics problems), it should be included in the classroom assessment activities in both situations (Silver & Cai, 2005). This is because the problems that students pose reveal significant information about their mathematical understanding (Xie & Masingila, 2017).

In some problem-posing studies (e.g., Bonotto & Santo, 2015; Cai, 2003; Ellerton, 1986) participants are expected to pose complex/difficult problems. These kinds of directives have important potential for students' learning (Chen, Van Dooren, Chen & Verschaffel, 2007). Problem posing improves students' problem solving and creative thinking skills (Silver, 1997) because participants are questioning deeply mathematical structure of the tasks (Kar, Özdemir, Öçal, Güler & İpek 2019; Xie & Masingila, 2017) and think about the solution of the problem (Cai, 2003) during this process. In this context, expecting the students to pose complex problems contributes to their active use of the thinking, reasoning, and justification skills emphasized in the documents of National Council of Teachers of Mathematics (NCTM, 2000), and giving them opportunities to think in-depth about the mathematical structures in the proposed tasks. At the same time, these skills are among the components of mathematical proficiency. As such, it is recommended to conduct an in-depth investigation of a problem situation to improve them (Kilpatrick, Swafford & Findell, 2001).

One of the questions that Cai, Hwang, Jiang and Silber (2015) asked for investigating problem posing was as follows: "How do different characteristics of problem situations affect subjects' problem posing?" (p. 9). The conducted studies indicated that problem-posing performance (i.e., mathematical validity and solvability) differs according to the special conditions of the problem-posing tasks (e.g., being in free, semi-structured, and

[*] Corresponding Author: *Tuğrul Kar, karrtugrul@gmail.com*

structured forms), even though they aimed at measuring the same skill (Christou, Mousoulides, Pittalis, Pitta-Pantazi & Sriraman, 2005; Çetinkaya & Soybaş, 2018; Kılıç, 2013; Silber & Cai, 2017; Stickles, 2011). Therefore, the task format should be taken into consideration while making evaluation about students' problem-posing performances or other skills such as problem solving related to the problem posing. In addition, it is an important question to be answered whether there are other factors affecting the quality of the problems posed by students. In this context, previous studies have included directives such as "pose a problem for your friend" (e.g., Cankoy, 2014; Kopparla et al., 2019; Lowrie, 1999). Do such directives effectively allow addressing the students' complex problem-posing performances? More specifically, when third-grade students are asked to "pose a problem for your friend," does the particular friend's mathematical ability affect the complexity of the problem posed? This study aimed to investigate what factors third-grade students took into consideration when posing problems based on such directives and how these factors affected the mathematical complexities of the problems.

Theoretical Framework

Problem Posing and the Classification of Problem-Posing Tasks

Problem posing is defined as the generation of new problems or the reformulation of existing problems (Silver, 1994). In the literature (Christou, et al., 2005; Silver, 1994; Stoyanova & Ellerton, 1996), various criteria were taken into consideration for developing different problem-posing frameworks. Stoyanova and Ellerton (1996) described the widely accepted classifications within these frameworks as free, semi-structured, and structured situations in which students are asked to pose problems that are appropriate for the given situation. These questions can be posed without any limitations in free situations (e.g., "pose a difficult problem for your friends"). In semi-structured situations, students are given open-ended scenarios (e.g., "pose a problem using data from a graph"). In structured situations, well-structured tasks are presented (e.g., "pose a problem appropriate for the following equation: $(25 + 12) - 17 = ?$").

Bonotto and Santo (2015) stated that free and semi-structured problem-posing tasks encourage creative thinking and thus stimulate students' problem posing. In addition, Lowrie (1999) emphasized that free problem-posing tasks increase students' motivation, while Stoyanova and Ellerton (1996) indicated that students are more likely to reflect their own experiences through these tasks. Thus, in line with these views, our activities were designed with the assumption that they would allow students to reflect their own knowledge and creativity while posing problems. In the light of these explanations, more data will be available related to these types of activities and how students think about them when posing problems for their friends.

Mathematical Complexity of Word Problems

Responses to problem-posing tasks are evaluated in different ways depending on the purpose and the scope of the study. One of the most common approaches used in the evaluation of problems is mathematical complexity (e.g., Marshall, 1995; Silver & Cai, 1996, 2005). Mathematical complexity refers to the kinds of cognitive demands required to solve a problem; the problems whose solutions are more difficult are considered to be more complex (Lee & Heyworth, 2000). As such, complexity is one of the crucial features of posed problems and it reflects students' mathematical understanding and cognitive processes (Kwek, 2015). In the problem-posing process, the concepts in the task are analyzed in-depth and are linked to different mathematical concepts. Furthermore, the validity of the problem is questioned mathematically (Kar et al., 2019; Xie & Masingila, 2017). In this context, the attempts to pose complex problems can make students more active in terms of mathematical thinking and reasoning and, at the same time, they develop their problem-solving skills. Additionally, the attempts to pose more complex problems foster students' creative thinking skills (Silver, 1997) and give students opportunities to make connections between mathematics and their interests, which is often not the case in solving routine problems (Cai et al., 2020). Therefore, expecting students to pose complex problems makes an important contribution to their mathematics learning.

The complexities of the problems that students pose can be determined using many different perspectives. One way to determine the complexity of a word problem is changing the type and the number of operations included in it. This kind of approach is defined in the literature as arithmetic complexity (Leung & Silver, 1997; Silver & Cai, 1996). Although the number of steps gives an idea of the arithmetic complexity of the problem, it has a significant limitation. Leung and Silver (1997) reported that multi-step problems are more difficult than single-step problems, but a problem with five steps does not necessarily have to be more difficult than a problem with four. This is because, due to the absence of solutions, we cannot determine which problem (the one with four steps or the one with five steps) is more difficult than the other. Therefore, the single-step and multi-step distinction is taken into consideration in the analysis of arithmetic complexity. For example, Bonotto and Santo (2015) examined the problems posed in a situation requiring the use of real-life artefacts. They analyzed valid problems posed by fifth-grade students according to their arithmetic complexity. The researchers found that more than three-fourth of the problems posed by the students were multi-step problems. In another study, Chen et al. (2007) asked Chinese fourth- grade students to pose easy, moderately difficult, and difficult problems related to an open-ended story and they analyzed the arithmetical complexity of their responses according to whether they were single or multi-step. Students posed more complex problems in cases where they were expected to pose difficult problems. The results of this study suggested that the application of the activity had an influence on the mathematical complexity of the problems.

Another common approach taken for determining the complexity of word problems is semantic structures which refers to the relations among the quantitative data in the problem's text. These structures influence problem-solving performance (Bernardo, 1999; Marshall, 1995; Mulligan & Mitchelmore, 1997; Yeap & Kaur, 2001). Due to the way of mathematical relationships presented in the problems may make it difficult for students to form mental representations for the mathematical structure of the problem, it may increase the possibility of making mistakes in selecting solution strategies (Bernardo, 1999). Marshall (1995) divided word problems into five semantic structures—change, group, compare, restate, and vary—according to the relationship among the numeric quantities rather than their context or synthetic features. This classification of word problems is situation-based rather than operation-based (Yeap & Kaur, 2001). According to this classification, problems involving more semantic structures are thought to be more complex than those involving fewer structures (Marshall, 1995; Silber & Cai, 2017; Silver & Cai, 1996). The sample problems reflecting Marshall's (1995) five sematic structures were presented in Table 1.

Table 1. Examples for semantic structures in Marshall's (1995, p. 72) classification

Types	Sample problem
Change	Stan had 35 stamps in his stamp collection. His uncle sent him 8 more as a birthday present. How many stamps are there in his collection now?
Group	In Mr. Harrison's third-grade class, there were 18 boys and 17 girls. How many children are there in Mr. Harrison's class?
Compare	Bill walks a mile in 15 minutes. His brother Tom walks the same distance in 18 minutes. Which one is the faster walker?
Restate	At the pet store there are twice as many kittens as puppies in the store window. There are 8 kittens in the window. How many puppies are there also in the window?
Vary	Mary bought a package of gum that had 5 sticks of gum in it. How many sticks would she have if she bought 3 packages of gum?

Change refers to a difference in the quantity of a single item over a period of time. Marshall (1995) stated that there are three important numbers in this situation: the amount prior to the change, the extent of the change, and the resulting amount after the change has occurred. In the first problem shown in Table 1, Stan's 35 stamps at the beginning represent the amount prior to the change and the 8 additional stamps sent to him as a birthday gift represent the extent of the change. Accordingly, the total number of stamps in the collection represents the resulting amount after the change. In a *group* situation, there is a meaningful combination of small groups within a larger group. Marshall (1995) stated that there must be three or more numbers in this situation: the size of each subgroup and the size of the group as a result of their combination. In the second problem shown in Table 1, the 18 boys and 17 girls in the classroom represent the small groups, while total classroom size represents the bringing together of these groups to form a larger group. A *compare* situation exists whenever two things are contrasted to determine which of them is larger or smaller (Marshall, 1995). The third problem in Table 1 is about determining the faster person by comparing how long Bill and his brother take to walk a length of one-mile. *Restate* refers to the relationship between two items at a specific point in time. This relationship occurs at a certain time of the story, but a wider context cannot be generalized. Marshall (1995) emphasized that these relations should be expressed with numerical values as well as statements like existence of twice as great as, three more than, or one half of. In the fourth problem in Table 1, the expression "there are twice as many kittens as puppies in the store window" represents this condition. In a *vary* situation, the relationship between two things does not dependent on a specific time. In this structural relationship, although the numbers of the variables decrease or increase, the relationship is preserved. The situation derives its name from the fact that if one varies the amount of one thing, the amount of the second changes systematically as a function of the known relationship (Marshall, 1995). The last problem in Table 1 represents the functional relationship between one package of gum and the sticks of gum it holds with the expression: "a package of gum that had 5 sticks of gum in it." In this problem, there are fixed five sticks of gum for each package. The number of the sticks of the gums increase depending on the increase in the number of the packages. Therefore, the relationship between the numbers of the packages and sticks of the gums will be preserved.

Silver and Cai (1996) asked primary school students to pose three problems for an open-ended story and analyzed the problems according to Marshall's (1995) classification of semantic structures. It was determined that more than 60% of the posed problems contained at least two semantic structures, and the problems posed as a second response were more complex than the first ones. Yeap and Kaur (2000) investigated the relationship between third- and fifth-grade students' problem-solving and -posing performances according to grade levels and activity types. The posed problems were also analyzed using Marshall's (1995) classification. The findings of this study indicated that fifth-grade students posed more complex problems than third-grade students did, and that the problem-posing activity types affected the problems' semantic structures. Papadopoulos and Patsiala (2019) asked fourth grade students to pose problems for open-ended tasks (e.g., "Peter has 75 cents…", p. 4) without any external intervention in the first phase and benefitting from the what-if technique in the second phase; they noted the issues to which teachers called attention in the last phase. The mathematical complexities of the problems were analyzed according to Marshall's (1995) schema. It was determined that less than half of the posed problems in the first and second phases involved two or three semantic relations, while this rate increased to more than 60% in the last phase. This study reveals that systematic teaching on problem posing contributes to the development of the mathematical complexity of problems.

In this study, Marshall's (1995) situation-based classification was used. One of the strengths of this classification is that it enables the analysis of problems posed in free and semi-structured activities as were used in this study. Another strength is the opportunity to make statistical comparisons (e.g., Silver & Cai, 1996; Yeap & Kaur, 2001). This is because this classification focuses on the nature of the relations among the mathematical quantities rather than on the operation types. For example, a change in the semantic structure refers to a difference in the quantity of a single item over a period of time in this schema (Marshall, 1995). If an operation-oriented classification is taken into consideration, this would be classified as a separate semantic structure if there was a decrease over time and as a join semantic structure if there was an increase. Although the arithmetical operations are different, the relations between mathematical quantities have a similar structure. Therefore, Marshall (1995) classified both of these two situations under the category of a change in semantic structure.

The Factors Affecting Posing Complex Problems

One of the factors affecting the complexity of problem-posing performance is the task format (e.g., Geçici & Aydın, 2020; Leung & Silver, 1997; Silber & Cai, 2017; Silver & Cai, 1996). Participants posed mathematically more complex problems in structured problem-posing activities compared to those in free activities (e.g., Silber & Cai, 2017) and in the tasks containing specific numbers compared with those without specific numbers (e.g., Leung & Silver, 1997). Another factor that affects the complexity of problem-posing performance is the manner in which the activities are applied. For example, Chen et al. (2007) determined that when fourth-grade Chinese students were asked to pose easy, moderately difficult, and difficult problems for an open-ended story, the arithmetic complexity of the problems was greater in the case of posing difficult problem. Chapman (2012) indicated that the extent to which problem-posing is perceived as sense-making influences the complexity of the problems posed. For example, from the paradigmatic perspective, problem-posing is the creation of a problem with a universal interpretation, a particular solution, and an existence independent of the problem solver (Chapman, 2012). This perspective results in posing simple problems.

Students were asked to pose a problem for a particular person (e.g., for a friend, for a student with high mathematics ability) in a limited number of studies. In these studies, students were directed, for example, to "pose a (difficult) problem for your friend". This type of directive is seen as an important way both to motivate students and to understand their mathematical abilities (Cankoy, 2014; Lowrie, 1999, 2002). Winograd (1997), working with primary school students, indicated that students were highly motivated to pose interesting and difficult problems for their classmates, and they did not lose interest in the process of sharing their problems in the classroom environment. Ellerton (1986) stated that asking students to "pose a difficult problem for your friends" may affect students' success in performing the problem-posing task because the students' focus may be shifted to external factors. When using these directives, students do not always pose problems for their close friends, but they also pose problems for those who like to solve difficult problems (Lowrie, 1999). These results indicate that, during the problem-posing process, problem posers also focus on how the person who will solve will perceive the components of the problem.

Although there are studies that direct students to pose problems for their friends (e.g., Cankoy, 2014; Kopparla et al., 2019; Lowrie & Whitland, 2000; Yeap & Kaur, 2000), there has thus far not been a study examining the situation that is explored here. For example, two activities in Yeap and Kaur's (2000) study directing third- and fifth-grade students to pose problems were as follows: "…Your problem must have the numbers 3, 5 and 36. You can use more numbers, if you like" and "…The answer to the question in the problem must be 10." In both activities, students were given the following instruction: "Write a mathematics word problem for a friend to solve" (p. 606). The complexity of the problems posed for these activities were determined by means of Marshall's (1995) classification of semantic structures. The students posed more complex problems by using the numbers 3, 5, and 36 in the problem-posing activity. Although the reasons behind the success difference were not explored in the study, the presentation of the activity might be a factor. Additionally, students could pose problems in each activity by considering the different mathematical abilities of their friends, which might also result in differences in the complexities of the problems.

Lowrie and Whitland (2000) asked third-grade students to pose problems for second and fourth graders. They found that the third-grade students considered number magnitude, operation complexity, the type of mathematics concepts, and students' interests while posing problems. Students decreased the magnitude of the numbers for second graders and tended to use numbers with higher magnitudes for fourth graders in their problems. The researchers also found that some students posed problems using the content of the third-grade mathematics curriculum with the aim of helping the second graders improve their ability. The results of this study indicate that there might be differences between the problems posed for different grade levels in terms of arithmetical complexity. On the other hand, this result was not determined via quantitative approaches and, additionally, this study did not focus on the semantic structures of the problems. Thus, this study does not provide data on the factors that students consider in posing problems for their classmates. Cankoy (2014) conducted a five-week study investigating the effect of interlocked and traditional problem-posing instructions on fifth-grade students' problem-posing performance. A free problem-posing activity was used in the study and students were expected to pose problems for their friends. The posed problems were analyzed according to the dimensions of solvability (whether they were solvable or unsolvable), reasonability (whether they were reasonable or unreasonable), and mathematical structure (whether they were result unknown or start unknown). The results indicated that interlocked problem-posing instruction improved students' problem-posing performance more than traditional instruction.

When examining the aforementioned studies, some (Cankoy, 2014; Kopparla et al., 2019) did not focus on the mathematical complexities of the posed problems. In the studies focusing on the complexities of the problems, on the other hand, the performances of student groups were compared according to activity types (Yeap & Kaur, 2000), or students were asked to pose problems for different students in different grade levels (Lowrie & Whitland, 2000). Such studies do not provide explanations about the factors that students consider when posing problems for their classmates. Moreover, Lowrie (1999) pointed out that students could pose problems not only for their close friends but also for their friends who like to solve difficult problems. This explanation implies that students can adjust the complexity of the problems by considering the friend for whom the problem is posed. However, the literature reviewed reveals that whether students adjusted the complexities of the problems for their friend was not tested by means of quantitative approaches. As Silver (2009) indicated, complex problems sharpen students' mathematical thinking and reasoning skills. In this regard, quantitative approaches can provide strong results about students' tendencies in posing more complex problems for their friends having higher mathematics success. Therefore, this approach can provide experimental data based suggestions for their use in learning environments. In addition, students were asked to pose problems for their friends in the experimental studies on problem posing (e.g., Cankoy, 2014; Kopparla et al., 2019). In the pre-test and post-test stages of such experimental studies, the fact that the student wrote problems by taking into account his/her different friends may have affected the complexity of the posed problems. Therefore, the results of the present study may deepen our understanding of an important variable to be considered in creating experimental designs and interpreting students' problem-posing performance.

Although students are capable of posing problems, we have very limited in-depth understanding about how they think and what type of situations they take into consideration while posing problems (Cai, et al., 2015; Cai & Leikin, 2020). For example, the results of Lee's (2020) study showed that only a small portion were related to problem posing (62/17456 about 0.4%) among the research published in 13 academic journals particularly related to mathematics education, and only four studies were interested in the students' thinking processes. Supporting this study with qualitative approaches as well as quantitative approaches will give insight into what factors students take into account when writing problems for their friends, the role of the complexity of the problem as one of these factors, and what kind of arrangements they make to adjust the complexity of the problem. Therefore, the results of this study will deepen our understanding about how students think when they pose problems for their friends. This study is aiming to fill this gap in the literature.

Research Questions and Hypotheses

The present study attempted to answer the following research questions:

1. Is there any statistical difference among the arithmetical and semantic complexities of the problems that third-grade students pose for their classmates?
2. How do third-grade students think when posing problems for their classmates?

When students pose problems, besides focusing on the mathematical structures of the activity (e.g., number magnitude, number and types of operations) (Lowrie & Whitland, 2000), they also consider many other factors including the interests of the person or the group to be posed for (Chapman, 2012; Lowrie & Whitland, 2000) and the association of the problem with daily life (Rosli et al., 2015; Winograd, 1997). According to the hypothesis of this study, therefore, it is assumed that there will be no difference in the semantic complexity of the problems that students pose for different classmates. A similar assumption is made about the arithmetical complexity of the problems.

Method

Sample

This study was carried out with a primary school teacher working in a public school in Turkey and 27 third-graders. The students were 9–10 years old. The teacher had 13 years of teaching experience while the study was implemented and was studying for his master's degree at the time. All the students were educated in their native language, and their school was one of the popular schools in the city center. The socio-economic and educational levels of the students' parents were relatively similar and were medium or high in general.

The Problem-Posing Experience of the Teacher and Student Selection

One of the authors of this study and the teacher had been discussing the teaching of primary school mathematics in regular meetings for more than a year. In these meetings, they discussed methods for teaching primary school mathematics, problem-solving and problem-posing skills, and the meaning of mathematical understanding.In some meetings, they discussed the definition and the importance of problem posing, the classification of problem-posing tasks, and the types of analytical schemas used. In addition, some problem-posing tasks were carried out by the teacher in his classes, and his observations were discussed in the meetings. Thus, it was ensured that the teacher conducting this study had experience in problem-posing activities. In these meetings, some studies (e.g., Cai & Ding, 2017; Kilpatrick et al., 2001) related to the nature of mathematical understanding and its components were discussed. For example, Kilpatrick et al. (2001) identified five components of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Thus, the primary school teacher conducting this study had knowledge about mathematical understanding.

Undoubtedly, students' abilities are best recognized by the teacher who is with them daily. Furthermore, there are no written exams at the third-grade level. Therefore, the primary school teacher was expected to divide the students into four groups according to their mathematical understanding. Then, he was expected to choose two students from the four groups: the one from the group with the highest mathematical understanding and the one from the group with the lowest mathematical understanding. He considered criteria such as conceptual understanding, procedural fluency, and strategic competence in selecting the students. The teacher chose Kerem from the group with the highest mathematical understanding and Ecrin from the group with lowest mathematical understanding (pseudonyms have been used). The teacher explained the mathematical understanding of Ecrin and Kerem as follows:

> For Kerem: The most important characteristics of Kerem are being capable of mental processing and solving problems in different ways. He makes the operations faster and establishes the connections between the mathematical expressions in the problems and the necessary operations better, compared to his peers. He is also the head of the math club of our class.
>
> For Ecrin: She is a third-grade student, but lacks some mathematics knowledge related to first- and second-grade mathematics content. She is far behind in rhythmic counting compared to her friends, she forgets the mathematics subjects she has learned in previous years, she is very weak in mental processes, and she often makes mistakes in arithmetic operations. She especially has difficulty in solving two-step problems.

Student Background

A free problem-posing (FPP) task was implemented in the fall period. At the time of the implementation, the teacher stated that he taught addition and subtraction with three-digit numbers and included problem-solving activities that required the use of these operations. Furthermore, he stated that the problems were limited to what students learned in second grade in terms of multiplication and division, such as multiplying numbers up to 10 by one, two, three, four, and five, and dividing numbers up to 20 without remainders. The teacher emphasized that more complex multiplication and division operations are mainly included in the third- and fourth-grade curricula, and noted that he had not yet started teaching multiplication and division with two-digit and larger numbers at the time of the FPP task. A semi-structured problem-posing (SSPP) task was implemented approximately one month after the FPP task. During the implementation of the task, the teacher had just started teaching multiplication. At this stage, he stated that he conducted activities by reminding students of the multiplication subjects found mainly in the second-grade curriculum.

Problem-Posing Activities and Administration Procedure

In this study, FPP and SSPP tasks (see Figure 1) were given to students. The students were asked to pose a problem for their two friends, determined by their teacher, and each of the problem-posing tasks was completed in about 20 minutes. The teacher gave the students the impression that Ecrin and Kerem were randomly selected and their mathematics success was not mentioned to the class during the process. The teacher stated that students could find it difficult to follow the written directions so the teacher suggest that it would be more appropriate to explain what students have to do verbally before the implementation. In line with these opinions, no instructions were included in the tasks; instead, the teacher gave explanations to the students during the implementation process. The explanation given during the FPP task was as follows:

> In this lesson, you will pose problems for each other. I will randomly select two of your friends, and you'll pose a problem for each of them. Then, we will continue to practice our activity for different friends. I want you to pose one problem each for Kerem and Ecrin. You can start with whomever you want.

In the SSPP task, students were given an open-ended task associated with daily life (see Figure 1). This problem-posing task was derived from the study by Silver and Cai (2005), who stated that this activity can be used for classroom assessment of activities involving the addition and subtraction of two-digit numbers. Considering the purpose of this study, there were many reasons for choosing this task. First, because the teacher was still in the early stages of teaching multiplication and division, he stated that it would be more appropriate to use a data set unrelated to multiplication. Second, expanding the amount of data in the open-ended task would limit the problems to be posed. This would create obstacles to flexible thinking in differentiating problems. During the implementation phase, the students were given the following instruction:

> I'll give you an incomplete problem sentence. Imagine that you wrote the problem up to this point. You will write math problems for your friends by completing them as you wish. I want you to write one math problem each for Ecrin and Kerem. You can start with whomever you want.

FPP Task	**SSPP Task**
Write one math problem for Ecrin and Kerem.	By beginning with the information given below, write one math problem for Ecrin and Kerem. *Ayşe has 34 marbles, Burak has 27 marbles and Zeynep has 23 marbles.*
My friend's name for whom I posed the problem:	My friend's name for whom I posed the problem:
My friend's name for whom I posed the problem:	My friend's name for whom I posed the problem:

Figure 1. Problem-posing tasks and instructions given to the students.

The FPP and SSPP activities were not limited to posing problems only for Ecrin and Kerem. If that were the case, the other students in the classroom might conjecture that Ecrin and Kerem were not randomly selected. As a result of the discussions among the authors and the classroom teacher, it was decided that it would be more appropriate to have other students be involved in the activities, too. Therefore, before posing problems for Ecrin and Kerem in the FPP activity, similar processes were carried out for different students. In the SSPP activity, students were asked to pose problems for Ecrin and Kerem first and then to pose problems for some other randomly selected students. Thus, this process aimed to prevent students' from conjecturing that Ecrin and Kerem were deliberately selected.

Semi-Structured Interview Process

Semi-structured interviews with six students were carried out after each activity to better understand how students were thinking when they were posing problems. The teacher's opinions were taken into account in the selection of the students. The teacher was expected to identify six students reflecting the mathematical success of the classroom using the same selection criteria used for selecting Ecrin and Kerem. In the selection of Ecrin and Kerem, the teacher had divided the students into groups. Using a similar approach, the teacher was asked to think of the students as three groups with low, medium, and high levels of mathematical understanding and choose two students from each group. It was emphasized that the selected students should be open to communication to allow an in-depth investigation and enable the researcher to gather rich data in the interviews. Using this approach, the teacher chose six students.

The classroom teacher routinely organizes short meetings with some students at the end of the day to talk about their activities during the day. It was decided that the semi-structured interviews would be conducted at these meetings to hopefully prevent the other students from developing possible prejudices against the students interviewed. The interview process was shaped around the students' explanations of how each problem was posed. In the interviews, students were not asked any questions indicating or implying Ecrin or Kerem's mathematics success. It was thought that the students might emphasize Ecrin and Kerem's mathematics successes when explaining how they posed their problems. In such cases, it was decided that students would be asked only to explain their thoughts in more depth. It was also predicted that some students would pose problems for the other student based on the problem posed for either Ecrin or Kerem. This is because in the literature, it is emphasized that students give chained responses to problem-posing activities (e.g., Silver & Cai, 1996). It was also taken into consideration that third-grade students could write systematically complicated problems for their friends and provide explanations about the differences between these problems in interviews. In these cases, it was decided to ask the student for a more detailed explanation to understand the differences between the posed problems. We decided to use the following questions in this regard: Are these two problems different? If so, what are the differences? What are the factors that are effective in posing different problems?

Data Analysis

Since it would not be appropriate for Ecrin and Kerem to pose problems for themselves, their responses were not included in the analysis. Thus, the responses of 25 students were analyzed for both tasks. The problems that the students had posed were analyzed first for mathematical viability. Problems that could not be solved or that contained errors were evaluated as non-viable problems. The viable problems were then analyzed according to their semantic structures and arithmetic complexities (see Figure 2).

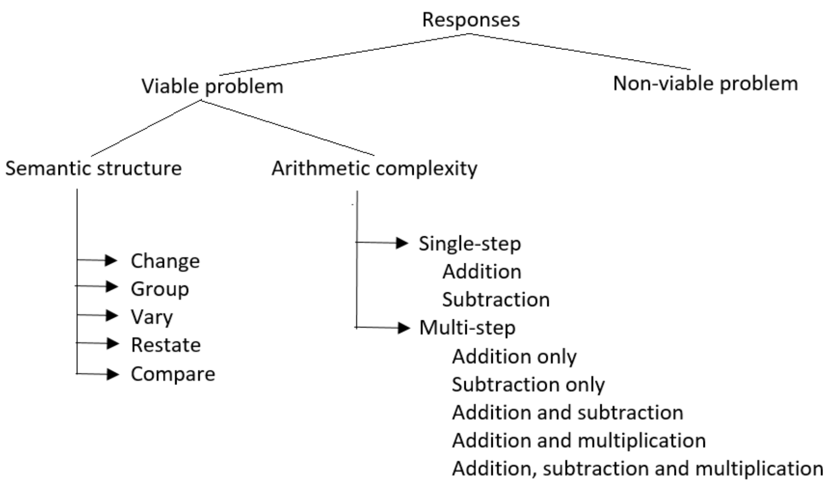

Figure 2. Schema related to the analysis of the problems posed by the students.

According to Figure 2, the mathematical complexity of the all valid problems was first analyzed according to semantic structures developed by Marshall (1995): *change*, *group*, *compare*, *restate*, and *vary* (a detailed description is presented in the section on the mathematical complexity of word problems). *Compare* and *vary* situations were not observed in the problems posed by the students. In this context, semantic structures of the posed problems were analyzed according to the *change*, *group*, and *restate* situations, and the number of semantic structures in each problem posed was determined. For example, if there were three semantic structures in the form of *change*/*change*/*group*, the semantic complexity of the problem would be coded by receiving three points. In this way, the complexity score of the problems that each student posed for Ecrin and Kerem was calculated. S_{11} posed the following problem for Kerem in SSPP task: *Teacher asks these three students to find their total amount of marbles and, then give him 43 of them. Accordingly, how many marbles do three friends have left?* In this problem, the total number of marbles corresponds to *the group* semantic structure, and removing 43 marbles from the total number of marbles corresponds to the *change* semantic structure. Since there are two semantic structures for this problem, its semantic complexity was coded with 2 points. Sample responses and explanations for these semantic structures are presented in the findings section.

Secondly, the problems were analyzed according to their arithmetic complexity and whether they were single-step or multi-step problems (see Figure 2). Regardless of the type of operation, problems that could be solved in one step were coded with one point, and multi-step problems were coded with two points. In addition, single-step and multi-step problems in the analytical process were also classified according to the types of operations they contained. In FPP task, S_4 wrote the following problem for Kerem*: Kerem has 987 marbles. He lost 567 of them. Then, his father bought 333 marbles for him. At the end, how many marbles does Kerem have?* The solution to this problem can be reached by the following operations respectively; 987-567=420 and 420+333=753. This problem is considered to be a multi-step problem and coded with 2 since it contains more than one arithmetic operation.

In order to determine whether there was a difference in complexity between the problems posed for Kerem and Ecrin, a Wilcoxon signed-rank test was applied, as the data were not distributed normally. In addition, the effect size was calculated when there were statistically significant differences between the posed problems. According to Cohen's (1988) interpretation of effect size, anything greater than .5 is large, .5–.3 is moderate, .3–.1 is small, and anything smaller than .1 is trivial. The students' responses to the problem-posing tasks were analyzed separately by two researchers. The researchers compared their analyses and reached a consensus on the classifications that differed. In addition, the findings include a presentation of direct quotations from students' responses to the questions asked during the interviews in order to explain possible differences between the problems posed for Ecrin and Kerem. These findings can provide evidence for the statistically determined results.

Findings

General Distribution of Problems the Students Posed

All students posed problems for Ecrin and Kerem. Taking into consideration the mathematical viability of the problems, one problem written for Ecrin was evaluated as non-viable. The non-viable problem of student S_{22} is as follows: *There are 500 liras in my coin bank. If I collected this money in a day, how much would I have on the second day?* The problem does not include any data on the amount of money in the coin bank on the second day. S_{22} wrote 500 + 500 next to the problem. Therefore, it seems that he tried to write a problem about how many liras he would have if he saved 500 liras in his coin bank on the second day. However, it was considered non-viable due to the fact that the operation was not mentioned in the wording of the problem.

Semantic Structure

One or more of the *change*, *group*, and *restate* situations were present in the students' problems in the viable category. The maximum number of semantic situations identified were three in the FPP task and five in the SSPP task. Sample problems in these categories are given in Table 2.

Table 2. Mathematical problem samples and their semantic structures

Ecrin	**Kerem**
There are 29 students in class 3/A, 30 students in class 3/C, and 32 students in class 3/D. What is the sum of the students in the three classes? [S_9: Group, FPP]	There are exactly 199 sheeps in Ali Baba's farm. Ali Baba's neighbor also has 100 sheeps. Accordingly, what is the total number of sheeps Ali Baba and his neighbor have? [S_9: Group, FPP]
There are 20 sheeps in a farm. 10 sheeps in the farm got sick, how many sheep are left? [S_2: Change, FPP]	Kerem has 10 sheep, 9 geese, and 8 chicks. Five of the chicks get lost. How many animals are left? [S_2: Change/Group, FPP]
Ali and his family collected 200 kilograms of tea leaves in the first day. In the second day, they collected 600 kilograms of tea leaves and gave 100 kilograms to his uncle. Accordingly, how many kilograms of tea leaves do they have at the end? [S_7: Change/Group, FPP]	Ömer raised a total of 304 TL on feast day. After the feast day, his father gave Ömer 50 TL. His mother gave him 4 TL more money than his father did. Accordingly, how many TL does Ömer have? [S_7: Change/Restate/Group, FPP]
Accordingly, what is the total number of marbles Ayşe, Burak and Zeynep have? [S_3: Group, SSPP]	Ayşe has 34 marbles, Burak has 27 marbles, and Zeynep has 23 marbles. Zeynep loses 7 marbles. Burak loses 3 marbles. What is the total number of marbles between them? [S_3-Change/Change/Group, SSPP]
The number of Görkem's marble is equal to the total number of Ayşe, Burak, and Zeynep's marbles. Accordingly, how many marble does Görkem have? [S_{14}: Restate/Group, SSPP]	If we find Ayşe, Burak, and Zeynep's total number of marbles and multiply it by 5, what will be the total number of marbles? Please find. [S_{14}: Restate/Group, SSPP]

According to Table 2, the problem written by S_9 for Ecrin in the FPP task is to combine three different groups to form a larger group. Thus, the problem only has a *group* situation. Since the problem that S_9 wrote for Kerem is to bring 199 and 100 sheeps together, it includes *group* situation. The problem written by S_2 for Ecrin in the FPP task includes the *change* situation since a certain part of the group is separated from it. In the problem that S_2 wrote for Kerem, the expression *5 of the chicks get lost* emphasizes *change*; in the last case, the total number of chicks, sheep, and geese emphasizes the *group* situation. In the problem written by S_7 in the FPP task for Ecrin, it includes *group* situation since it is about combining 600 kilograms and 200 kilogram of tea leaves. The problem includes *the change* situation due to the separation of 100 kilograms from the total amount of tea leaves. In the problem that S_7 wrote for Kerem, the expression *after the feast day, his father gave Ömer 50 TL* emphasizes *change*; the expression *his mother gave him 4 TL more money than his father did* emphasizes *restate*, as it denotes the relationship between quantities at a certain time. In the final case, the amounts of money are combined, thus emphasizing a *group* situation.

According to Table 2, S_3's problem for Ecrin in the SSPP task, which asks to calculate the total number of marbles between three people, is a *group* situation. In the problem written for Kerem by S_3, the loss of marbles by Zeynep and Burak emphasizes a *change* and the question of the total number of marbles the three have emphasizes a *group* situation. Lastly, in the problem written by S_{14} for Ecrin, the total number of marbles for Ayşe, Burak, and Zeynep includes *the group* situation and since Görkem's number of marbles is expressed over the total number of marbles, it includes *restate* situation. Since S_{14} used the same structure as the problem structure he wrote for Ecrin, the problem again includes both *the group* and *the restate* situations. The distribution of the problems posed for Ecrin and Kerem according to the number of semantic structures is given in Table 3.

Table 3. Distribution of posed problems according to the number of semantic structures

Number of Semantic Structures	FPP Task		SSPP Task	
	Ecrin	Kerem	Ecrin	Kerem
Zero	1(4)*	-	-	-
One	21(84)**	17(68)	17(68)	4(16)
Two	3(12)	6(24)	6(24)	15(60)

Three	-	2(8)	2(8)	5(16)
Four	-	-	-	-
Five	-	-	-	1(4)
Total	25(100)	25(100)	25(100)	25(100)

* Since one student could not write a viable problem for Ecrin, the number of semantic structures is considered to be zero.
** The data was calculated in frequency (percentage) over the number of students

According to Table 3, in the FPP task there were no more than two semantic structures in the problems written for Ecrin and no more than three semantic structures in the problems written for Kerem. In addition, only one semantic relation was observed in 84% of the problems posed for Ecrin, and this rate decreased to 68% for Kerem. In the SSPP task, up to five semantic structures were observed in the problems posed for Kerem. When the distributions are compared, 68% of the problems written for Ecrin included one semantic structure and 60% of the problems written for Kerem contained two semantic structures. Furthermore, according to the distributions in the table, more problems involving three or more semantic structures were written for Kerem.

According to the data in Table 3, students tended to produce more semantically complex problems for Kerem compared to Ecrin. According to the results of the Wilcoxon signed-rank test conducted to determine whether there was a statistically significant difference between the complexities of the problems, it was determined that there was in fact a significant difference on behalf of Kerem in free and semi-structured tasks (see, Table 4). The effect size of the differences in both problem-posing tasks was large.

Table 4. Wilcoxon signed-rank test results for the semantic complexity scores

Ecrin-Kerem	n	Rank mean	Rank total	z	p	r
SSPP task						
Negative ranks	1	6.00	6.00	-3.343	.001	.67
Positive ranks	15	8.67	130.00			
FPP task						
Negative ranks	0	.00	.00	-2.530	.011	.51
Positive ranks	7	4.00	28.00			

$^*p<.05$

Arithmetic Complexity

The classification of the problems written for Ecrin and Kerem according to their arithmetic complexity is presented in Table 5.

Table 5. Distribution of arithmetic complexity of posed problems

Arithmetic Complexity	FPP Task		SSPP Task	
	Ecrin	Kerem	Ecrin	Kerem
Zero-step	1(4)*	-	-	-
Single-step				
Addition	3(12)**	5(20)	1(4)	1(4)
Subtraction	17(68)	12(48)	1(4)	-
Multi-step				
Only addition	1(4)	1(4)	18(72)	9(36)
Only subtraction	1(4)	1(4)	1(4)	-
Addition and subtraction	2(8)	6(24)	4(16)	12(48)
Addition and multiplication	-	-	-	2(8)
Addition, Subtraction and multiplication	-	-	-	1(4)
Total	25(100)	25(100)	25(100)	25(100)

* Since one student could not write a viable problem for Ecrin, arithmetic complexity is considered to be zero.
** The data was calculated in frequency (percentage) over the number of students

According to Table 5, 80% and 68% of the FPP task problems written for Ecrin and Kerem, respectively, are single-step problems. In the FPP task, more than two thirds of the problems written for Ecrin and about half of the problems written for Kerem involve single-step subtraction. Considering the distribution of multi-step problems, four problems were written for Ecrin and eight problems were written for Kerem. In the SSPP task, one and two of the problems written for Kerem and Ecrin, respectively, are single-step problems. Regarding the distribution of multi-step problems, while the problems written for Ecrin included addition and subtraction only, the problems written for Kerem also included multiplication. In addition, multi-step problems including only addition were written at a rate of 72% for Ecrin, whereas multi-step problems including addition and subtraction were written at a rate of 48% for Kerem.

According to the results of the Wilcoxon signed-rank test conducted to determine whether there is a statistically significant difference between the arithmetic complexities of the problems, it was determined that there is no significant difference in the FPP task and the SSPP tasks (see, Table 6).

Table 6. Wilcoxon signed-rank test results for the arithmetic complexity scores

Ecrin-Kerem	n	Rank mean	Rank total	z	p	r
SSPP task						
Negative rank	1	2.00	2.00	-0.577	.564	
Positive rank	2	2.00	4.00			
FPP task						
Negative rank	1	4.00	4.00	-1.890	.059	
Positive rank	6	4.00	24.00			

*p<.05

Findings Related to The Factors Students Took into Consideration When Posing Problems

The data obtained from the semi-structured interviews show that students considered Ecrin and Kerem's mathematical understanding when writing problems. Five of the students stated that they were trying to write easy problems for Ecrin and difficult problems for Kerem. It was determined that they changed the number and type of operations or the magnitude of the numbers in order to make the problems more difficult for Kerem. For example, while S_6 asked Ecrin for the total number of marbles, he wrote the following problem for Kerem.

> Ayşe has 34 marbles, Burak has 27 marbles, and Zeynep has 23 marbles. The number of marbles Mert has is 2 times the number of marbles Zeynep has. Then, Mert gives Zeynep 11 marbles, since he had more. How many marbles do Mert and Ayşe have? [S_6: Restate/Change/Group]

S_6 stated that he took into consideration the mathematical understanding and changed the operation type accordingly:

Researcher (R):	How did you think while writing problems for Ecrin?
S_6:	Ecrin is not good at operations. I wrote her an easier, simple, and quick problem.
R:	How did you think while writing problems for Kerem?
S_6:	Kerem is very good at mathematics. He does all the operations very fast. I wrote more difficult questions for him.
R:	Are there any differences between the problems you wrote?
S_6:	There are differences, but there are a few things that are the same. There is addition in both problems. As for the difference, there is multiplication in Kerem's problem.

S_{15} asked for the total number of marbles in a problem posed for Ecrin, while doubling the total number of marbles for Kerem in the SSPP task. The solution to the problem written for Ecrin can be reached with two addition steps (34 + 27 + 23 = 84). However, the solution to the problem written for Kerem is reached with two addition steps (34 + 27 + 23 = 84) and one multiplication step (2 × 84 = 168). Both problems were considered to be multi-step, as they included more than one arithmetic operation. However, the types of operations they had were different. The student's explanation for this difference was as follows: *Because Ecrin is bad at addition, I asked her an addition problem to improve herself. Kerem is the president of mathematics club. He does operations fast. I added multiplication to make his question harder*.

S_{21} took the number magnitudes and numbers of operations into account in both problems posed for Ecrin and Kerem. But she chose one of them according to the task type. For example, the problems posed for Ecrin and Kerem in SSPP task were as follow:

If Ayşe, Burak and Zeynep combine their marbles, how many marbles will there be? (for Ecrin)

If Ayşe losts her 18 marbles, how many marbles will they all have? (for Kerem)

S_{21} wrote a problem for Ecrin that can be solved by addition operation. The problem she wrote for Kerem requires subtraction as well as addition. In this respect, it increased the number of operation types in the problem written for Kerem. In her explanations, S_{21} pointed out that Ecrin's mathematics knowledge was weak and Kerem's mathematics knowledge was good. Thus, while writing a problem, she emphasized that she wrote an easy problem for Ecrin and a difficult problem for Kerem. In addition, she stated that she increased the number of operations in order to make the problem more difficult. The interview with S_{21} is as follows:

R: How did you think while posing the problem for Ecrin?
S_{21}: Ecrin is not good in mathematics. For this reason, I wrote a simpler problem for Ecrin. .
R: How did you think while writing the problem for Kerem?
S_{21}: Kerem is very good in mathematics. For this reason, I wrote a difficult problem for Kerem.
R: Can you explain the difficulty differences?
S_{21}: I posed a simpler problem for Ecrin. But I posed a harder problem for Kerem. Ecrin's problem involves less operation, while the Kerem's problem involves more operations.

Similarly, in the FPP task, S_{21} took into account the difference in mathematics successes between the students. Unlike the SSPP task, the magnitude of the numbers changed in this task instead of increasing the number of operations. In addition, S_{21} emphasized that she likes marbles and, therefore, creates the story of the problem by taking into account the situations she likes. The two problems posed by S_{21} and the interview conducted was as follows:

Yaprak had 29 marbles. She gave 18 of them to her friend Öykü. How many marbles did Yaprak have left? (for Ecrin)

There were 999 cows in Ali Baba's farm. Ali Baba sold 786 cows. How many cows left were there for Ali Baba? (for Kerem)

R: What did you think when posing problem?
S_{21}: In the problems I wrote, the quality and whether it can really be solvable.
R: You paid attention. So, was there any difference between the problems you posed for Kerem and Ecrin?
S_{21}: Yes, there is.
R: What is it?
S_{21}: I asked Kerem's problem harder. This is because Kerem is good at mathematics. But, since Ecrin is somehow worse, I asked simpler problem.
R: So, what was the difference for you?
S_{21}: I asked only from two-digit numbers for Ecrin. The reason why I asked from marbles is that I like marbles. But, I asked problem for Kerem from three-digit numbers.

S_1 stated that he only took into account the magnitude of the numbers while differentiating the problems in the FPP task. He posed a problem with two-digit numbers for Ecrin, but posed a problem with three-digit numbers for Kerem because of his higher mathematical ability level. The problems written by the student and his explanation were as follows:

My friend has 55 marbles. He gave me 44 of 55 marbles. How many marbles does my friend have now? (for Ecrin)

I have 650 toy cars. I gave Kerem 499 toy cars. How many toy cars do I have left? (for Kerem)

> Explanation: Ecrin cannot do addition very well, so I wrote the problem with small numbers. Kerem does addition and subtraction fast.

In the interviews, it was determined that S_{23} took Ecrin's and Kerem's interests into consideration in addition to the mathematical structures while writing the problems. He posed a problem for Kerem about reading a book which used three-digit numbers and was solved by subtraction. He made the following statement about the problem: *Kerem likes reading books very much, so I wrote the question about reading books.* On the other hand, S_{24} took only Ecrin's and Kerem's interests into account while writing problems. She wrote problems including subtraction for both Ecrin and Kerem. The explanations she gave for the problems he wrote were as follows: *I asked Ecrin the number of hairgrips as she likes hairgrips. … I asked Kerem this question because he likes cars.*

Discussion and Conclusions

The worldwide recommendations for the reform of school mathematics suggest that problem posing has an important role (Chen, Van Dooren & Verschaffel, 2015). Studies on problem posing have not been yet one of the main subject of mathematics education research, and that more research is needed about students' cognitive processes (Cai et al., 2015). The purpose of this study was to expand the field's knowledge about students' understanding related to problem posing by examining what factors third grade students took into consideration when posing problems for their peers, and how these factors affected the complexities of the problems.

It was determined that all third grade primary school students wrote problems for Ecrin and Kerem for both tasks and almost all of the problems written were valid problems. These results indicate that students' performance of writing valid problems is high. Only one problem written for Ecrin at the FPP task was considered to be mathematically invalid. The student was unable to express the addition operation in the story of the problem. However, he was able to express the addition operation in the problem he wrote for Kerem. These data indicate that the student had difficulty in verbally expressing mathematical expressions. Problem-posing performance is also affected by writing ability (e.g., Çetinkaya & Soybaş, 2018; Kwek, 2015; Özgen, Aydın, Geçici & Bayram, 2019). For example, Kwek (2015) found that seventh-grade students were not generally aware of the difference between writing for a problem and writing for a solution. He concluded that the resulting unsolvable mathematical problems on inequality reflected the students' difficulties with enriching the content of the problem statements. Similarly, Özgen et al. (2019), who investigated eight grade students' performances to different problem posing tasks, indicated that students' inability to write and explain what they think significantly affected their success in problem posing. However, in our study, there was only one invalid problem resulting from a student's writing ability, indicating that this is not an important issue. Stickles (2011) indicated that the complexity of data used in the activity has an effect on problem-posing performance. Thus, not presenting complex relational data in the problem-posing activities might be the reason that invalid problems did not arise from the students' writing skills. In addition, students might have given up writing some problems due to possible difficulties experienced when expressing their thoughts. Although no such explanation was encountered in the interviews conducted with six students, this situation might have been encountered by some other students. This should be investigated in more detail with qualitative approaches.

The studies (e.g., Kar, 2015; Kılıç, 2013; Luo, 2009) investigating the semantic structures of the problems posed by students shows that some semantic structures were more prominent than others. For example, Kılıç (2013) determined that fourth and fifth grade students had tendencies to pose problems regarding the meaning of the combining for addition and the meaning of separation for subtraction. This study, parallel to the results in the related literature, found that some semantic structures were used more compared to the others. In both problem-posing tasks, the problems posed for Ecrin and Kerem were observed to utilize the *change*, *group*, and *restate* semantic structures. In this study, the limited mathematical background of the students might be the main reason why they could not write problems that involved the *compare* and *vary* structures. According to Marshall (1995), *compare* situations require determining whether the larger or smaller value is expected when expressions such as "quicker," "longer," "better buy," or "less costly" are used, and *vary* situations require functional thinking. Accordingly, *compare* situations can be created by means of addition and subtraction operations. Although students received instructions for addition and subtraction operations, the reason why they did not pose problems involving *compare* situations could be due to the fact that these kinds of semantic structures are not sufficiently included in their learning environment. It is not possible to explain this situation with the findings of this study. Therefore, further studies should investigate the semantic structures of the problems that are posed in mathematics lessons and found in textbooks. However, the fact that the *vary* situation is expressed through multiplicative relations and the students were still at the beginning stage of learning multiplication reveals why it was not seen in the problems. Furthermore, Yeap and Kaur (2001) stated that it is more difficult for third- and fifth-grade students to solve problems of *restate* situations compared to *group* situations for one-step problems. In this study, although the mathematics backgrounds of the third-grade students were quite limited, the fact that they wrote problems including the *restate* situation indicates their flexible thinking skills.

It was determined that there was a difference between the semantic structures of the problems posed for Ecrin and Kerem in both problem-posing tasks, and their effect size was at the large level. These results suggest that students write more semantically complex problems for peers that they consider to be more mathematically proficient. In addition, the students posed problems including more semantic structures in the SSPP task compared to the FPP task. Presenting open-ended stories in the SSPP task increased the number of semantic structures by further encouraging the linking of data.

Considering the arithmetic complexities of the problems posed for Ecrin and Kerem, it was found that there were more similarities than differences between the problems. The fact that a large percentage of the problems written for Ecrin and Kerem were single-step problems in the FPP task and multi-step problems in the SSPP task resulted in no statistically significant differences in arithmetic complexity. The main reason for changing the number of operations in the FPP and SSPP tasks was the format of the tasks. The fact that the number of marbles of each of the three persons was given in advance in the SSPP task directed the students to ask for the total number of marbles. This type of problem was preferred in the problems written for Ecrin; therefore, they were multi-step problems. The students' tendency to write problems that were more difficult for Kerem because of his success caused them to further develop the problems they wrote for Ecrin. Therefore, the problems written for Kerem were also multi-step problems. In addition, although the statistical test results did not show a significant difference between the arithmetical complexities of the problems posed for Kerem and Ecrin, problems involving multiplication operation were posed only for Kerem. These results indicate that the students did take into consideration the mathematical understanding of their classmate when posing problems.

According to the qualitative findings of this study, some results were obtained from the explanations regarding the problems posed for Ecrin and Kerem. First, the difference in mathematical success between Ecrin and Kerem was a dominant criterion during the problem-posing process. Moreover, Ecrin's and Kerem's interests were another common aspect that students took into consideration. Some students wrote problems after considering both factors. In such situations, the idea of posing more complex problems for Kerem was preserved and the context of the problem was determined according to the students' interests. In this aspect, the humanistic perspective (Chapman, 2012), in which the interests of the problem posers were taken into account, was utilized while posing the problems. Secondly, while posing complex problems, the students mainly considered the type and the number of operation and the magnitude of the numbers. In addition, changing the operation type was taken into consideration more than increasing the number of operations in posing more complex problems. While writing a problem for Kerem, the students tried to add a new type of operation to the operations required for solving the problem written for Ecrin. In this respect, it was understood from the students' perspective that the more complex problem meant the problems involving more and different types of operations.

Furthermore, Chapman (2012) indicated that producing problems should have a purpose of contributing to the students' learning (the utilitarian perspective). This perspective was also observed in third-grade students' problems posed for Ecrin and Kerem. Pointing out the operation type that Ecrin struggles with, some students wrote problems that included it in order to help her improve. Similarly, since Kerem was better in mathematics, students wrote problems with more operation types or larger numbers for him. In this way, the students pointed out that Kerem could practice more to improve further. Finally, unlike the perspectives that Chapman (2012) discussed, a new understanding was determined among third-grade students. According to this understanding, some students posed simpler problems for their friends in order for them not to experience feelings of failure while solving them. This was evident in the problems posed for Ecrin, because the students understood that her mathematics success was low.

This study can offer many potential contributions to the use of problem posing in mathematics education. If the problems and exercises posed are too easy, then opportunities for students to develop more sophisticated approaches are delayed (Downton & Sullivan, 2017). Students' posing of problems with different numbers and types of semantic relations for their friends can be turned into opportunities by teachers to enrich the learning environment, exposing students to many different word problems. Analyzing problems involving many more semantic relations gives opportunities to make more inquiries among the data provided by problems in the classroom environment. Additionally, in his famous book, *How to Solve It*, Polya (1957) indicates that "the student should also be able to point out the principal parts of the problem, the unknown, the data, the condition" (p. 6). In this context, students' sharing of problems involving different semantic relations can help them to understand word problems and, therefore, contribute to their problem-solving ability.

Teachers should think about not only the mathematical aspects but also the pedagogical aspects of the activities they use. Problems or activities should help teachers to learn about their students' mathematical thinking (Crespo & Sinclair, 2008). Teachers can use the activities of this study to determine students' mathematical understanding and the contexts in which they are interested. The semi-structured interviews indicated that students changed the magnitude of the number and the number and type of operations for students with different mathematical understanding. Thus, the problems written by students for their friends will give teachers an idea about the kinds of problems that students perceive as easy or difficult. Moreover, some students stated in the semi-structured interviews that they posed problems about topics in which their friends were interested. By means of such activities, teachers will be able to determine their students' interests and benefit from this knowledge in their teaching agenda.

This study may also aid researchers who are working in the field of problem posing. In this study, the complexities of the problems posed by the students for their friends differed. Therefore, it was understood that the activity directives were also effective in influencing the students' problem-posing performance in addition to the designed instructional methods or activity format. Students have been asked to pose problems for their friends in experimental (e.g., Cankoy, 2014; Kopparla et al., 2019) and correlational studies on problem posing (e.g., Silver & Cai, 1996; Yeap & Kaur, 2000). However, no explanation was given for situation (i.e., posing problems for their friends) that might have an effect on the problem-posing performance difference. Therefore, mathematics education researchers are advised to consider the effect of the "pose problems for your friends" directive on the complexity while designing problem-posing studies or interpreting the results of similar studies.

Any generalizability of the results of the current study will be limited for several reasons. The study was conducted with 27 third-grade students. Increasing the sample size would provide more reliable results for statistical analysis. In this study, it was determined that students mainly utilized the humanistic and utilitarian perspectives—as mentioned in Chapman's (2012) study—when posing problems. In future studies, conducting interviews with more students will be able to provide more information about the perspectives that students utilize when posing different problems. In this study, one of the students was chosen by the teacher from the students with high mathematical performance while the other was chosen from those with low performance. This situation is another limitation of this study. There is a need to investigate whether there is a difference between the complexities of the problems posed for those whose mathematical performance is similar and for those whose mathematical performance is dissimilar. Additionally, having the teacher make the determination about Ecrin's and Kerem's mathematical understanding can be seen as another limitation of the study. Determining students' mathematical understanding using a more objective assessment tool may make a stronger contribution to the validity of the results of the study. Finally, this study included two problem-posing tasks in free and semi-structured formats. Three non-relational numerical data figures were included in the SSPP task, taking into consideration the students' mathematical level. Problem-posing tasks that can be amended by changing the amount of data will be able to provide more enlightening information about how problems are posed for students of different ability levels.

References

Bernardo, A. B. I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19*(2), 149-163.

Bonotto, C., & Santo, L. D. (2015). On the relationship between problem posing, problem solving, and creativity in the primary school. In F. M. Singer, N. F. Ellerton, & J. Cai (Eds.), *Mathematical problem posing. From research to effective practice* (pp. 103-123). New York: Springer.

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: An exploratory study. *International Journal of Mathematical Education in Science and Technology, 34*(5), 719-737.

Cai, J., Chen, T., Li, X., Xu, R., Zhang, S., Hu, Y., … Song, N. (2020). Exploring the impact of a problem-posing workshop on elementary school mathematics teachers' conceptions on problem posing and lesson design. *International Journal of Educational Research, 102, 1-12.* DOI: https://doi.org/10.1016/j.ijer.2019.02.004

Cai, J., & Ding, M. (2017). On mathematical understanding: Perspectives of experienced Chinese mathematics teachers. *Journal of Mathematics Teacher Education, 20*(1), 5-29.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-posing research in mathematics education: some answered and unanswered questions. In F. M. Singer, N. F. Ellerton, & J. Cai (Eds.), *Mathematical problem posing. From research to effective practice* (pp. 3-34). New York: Springer.

Cai, J., & Leikin, R. (2020). Affect in mathematical problem posing: Conceptualization, advances, and future directions for research. *Educational Studies in Mathematics*. 105, 287-301. doi:10.1007/s10649-020-10008-x

Cankoy, O. (2014). Interlocked problem posing and children's problem posing performance in free structured situations. *International Journal of Science and Mathematics Education*, *12*(1), 219-238.

Chapman, O. (2012). Prospective elementary school teachers' ways of making sense of mathematical problem posing. *PNA: Revista de Investigación en Didáctica de la Matemática*, *6*(4), 135–146.

Chen, L., Van Dooren, W., Chen, Q., & Verschaffel, L. (2007). The relationship between posing and solving arithmetic word problems among Chinese elementary school children. *Research in Mathematics Education*, *11*(1), 1-31.

Chen, L. C., Van Dooren, W., & Verschaffel, L. (2015). Enhancing the development of Chinese fifth-graders' problem-posing and problem-solving abilities, beliefs, and attitudes: A design experiment. In F. M. Singer, N. F. Ellerton, & J. Cai (Eds.), *Mathematical problem posing. From research to effective practice* (pp. 309-329). New York: Springer.

Christou, C., Mousoulides, N., Pittalis, M., Pitta-Pantazi, D., & Sriraman, B. (2005). An empirical taxonomy of problem posing processes. *Zentralblatt Für Didaktik Der Mathematik- International Journal on Mathematics Education*, *37*(3), 149–158.

Cohen, J. (1988). *Statistical power analysis for the behavioral science.* Hillsdale, NJ: Erlbaum.

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education, 11*(5), 395–415.

Çetinkaya, A., & Soybaş, D. (2018). An investigation of problem posing skills of elementary school 8th grade students. *Journal of Theoretical Educational Science, 11*(1), 169-200.

Downton, A., & Sullivan, P. (2017). Posing complex problems requiring multiplicative thinking prompts students to use sophisticated strategies and build mathematical connections. *Educational Studies in Mathematics, 95*(3), 303-328.

Ellerton, N. F. (1986). Children's made-up mathematics problems: A new perspective on talented mathematicians. *Educational Studies in Mathematics*, *17*(3), 261–271.

Geçici, M. E., & Aydın, M. (2020). Determining the geometry problem posing performances of eighth grade students in different problem posing situations. *International Journal of Contemporary Educational Research*, *7*(1), 1-17. DOI: https://doi.org/10.33200/ijcer.575063

Kar, T., Özdemir, E., Öçal, M. F., Güler, G., ve İpek, A. S. (2019). Indicators of prospective mathematics teachers' success in problem solving: the case of creativity in problem posing. M. Graven, H. Venkat, A. Essien, ve P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, s. 456-463). Pretoria, South Africa: PU.

Kar, T. (2015). Analysis of problems posed by sixth-grade middle school students for the addition of fractions in terms of semantic structures. *International Journal of Mathematical Education in Science and Technology, 46*(6), 879-894.

Kılıç, Ç. (2013). Determining performance of elementary students related to problem posing activities requiring four arithmetical operations with natural numbers. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi*, *20*, 256-274.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academies Press.

Kopparla, M., Biçer, A., Vela, K., Lee, Y., Bevan, D., Kwon, H., … Capraro, R. M. (2019). The Effects of problem-posing intervention types on elementary students' problem-solving. *Educational Studies*, *45*(6), 1–18.

Kwek, M. L. (2015). Using problem posing as a formative assessment tool. In F. M. Singer, N. F. Ellerton, & J. Cai (Eds.), *Mathematical problem posing. From research to effective practice* (pp. 273-292). New York: Springer.

Lee, S. Y. (2020). Research Status of mathematical problem posing in mathematics education journals. *International Journal of Science and Mathematics Education*. 1-17. https://doi.org/10.1007/s10763-020-10128-z

Lee, F. L., & Heyworth, R. (2000). Problem complexity: A measure of problem difficulty in algebra by using computer. *Education Journal*, *28*(1), 85–107.

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal*, *9*(1), 5-24.

Lowrie, T. (1999). Free problem posing: Year 3/4 students constructing problems for friends to solve. In J. Truran & K. Truran (Eds.), *Making a Difference: Proceedings of the 22nd Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 328–335). Sydney: MERGA.

Lowrie, T. (2002). Designing a framework for problem posing: Young children generating open-ended tasks. *Contemporary Issues in Early Childhood, 3*(3), 354-364.

Lowrie, T., & Whitland, J. (2000). Problem posing as a tool for learning, planning and assessment in the primary school. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th Conference of the Psychology of Mathematics Education* (pp. 247-254). Japan: Hiroshima.

Luo, F. (2009). Evaluating the effectiveness and insights of preservice elementary teachers' abilities to construct word problems for fraction multiplication. *Journal of Mathematics Education, 2*(1), 83–98.

Marshall, S. P. (1995). *Schemas in problem solving.* New York, NY: Cambridge University Press.

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's ıntuitive models of multiplication and division. *Journal for Research in Mathematics Education, 28*(3), 309-330.

National Council of Teachers of Mathematics (2000). *Principles and standard for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Özgen, K., Aydın, M., Geçici, M. E., & Bayram, B. (2019). Investigation of eighth grade students' skills in problem-posing. *International Journal for Mathematics Teaching and Learning, 20*(1), 106-130.

Papadopoulos, I., & Patsiala, N. (2019). *Capturing problem posing landscape in a grade-4 classroom: A pilot study*. Paper presented at the Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht, Netherlands.

Polya, G. (1957). *How to solve it: A new aspect of mathematical method.* Princeton, N.J.: Princeton University Press.

Rosli, R., Capraro, M. M., Goldsby, D., Gonzalez, E. G., Onwuegbuzie, A. J., & Capraro, R. M. (2015). Middle grade preservice teachers' mathematical problem solving and problem posing. In F. M. Singer, N. Ellerton, & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 333-354). New York: Springer.

Silber, S., & Cai, J. (2017). Pre-service teachers' free and structured mathematical problem posing. *International Journal of Mathematical Education in Science and Technology, 48*(2), 163-184.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics. 14*(1), 19–28.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *Zentralblatt für Didaktik der Mathematik, 29*(3), 75–80.

Silver, E. A. (2009). Cross-national comparisons of mathematics curriculum materials: what might we learn? *Zentralblatt Für Didaktik Der Mathematik- International Journal on Mathematics Education, 41*(6), 827–832.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521-539.

Silver, E. A., & Cai, J. (2005). Assessing students' mathematical problem posing. *Teaching Children Mathematics, 12*(3), 129-135.

Singer, F. M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational Studies in Mathematics, 83*(1), 9–26.

Stickles, P. R. (2011). An analysis of secondary and middle school teachers' mathematical problem posing. *Investigations in Mathematics Learning, 3*(2), 1-34.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing. In P. Clarkson (Ed.), *Technology in mathematics education*. Melbourne, Australia: Mathematics Education Research Group of Australasia

Winograd, K. (1997). Ways of sharing student-authored story problems. *Teaching Children Mathematics, 4*(1), 40-47.

Xie, J., & Masingila, J. O. (2017). Examining interactions between problem posing and problem solving with prospective primary teachers: A case of using fractions. *Educational Studies in Mathematics, 96*(1), 101-118.

Yeap, B. H., & Kaur, B. (2000). Exploring the relationship between mathematical problem posing and problem solving, type of task and grade level. In J. Ee, B. Kaur, N. H. Lee, & B. H. Yeap (Eds.), *New 'Literacies': Educational response to a knowledge-based society: Proceedings of the ERA-AME-AMIC Joint Conference* (pp. 605-611). Singapore: Educational Research Association.

Yeap, B. H., & Kaur, B. (2001). Semantic characteristics that make arithmetic word problems difficult. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and Beyond: Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia Incorporated* (pp. 555-562). Sydney: MERGA.