



Research Article

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2^k FACTORIAL EXPERIMENTS IN RELIABILITY ANALYSIS FOR WEIBULL AND LOG-NORMAL DISTRIBUTIONS

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ABSTRACT: The life times of the components of a product are often analysed in quality control processes. Design of Experiment is mainly used to achieve quality but its' application to life times are less common. Life times always associate with Reliability. This study integrates experimental design specifically a two-level factorial experimental design and simulation models to determine the important the stress factors subjected to different conditions which significantly effects the life times of a product. The main purpose is to optimize the response by extending the lifetimes of a product. The Factors considered are temperature and voltage and their magnitude of the power are set at two levels. Simulation design of Weibull and log-normal distributions are used to generate failure times and maximum likelihood (ML) applied in the estimation of parameters.

Keywords: Reliability; Simulation; Weibull distribution; Log-normal distribution; Factorial design.

1. INTRODUCTION

Technology advancement, budget constrain and restrictive testing time has translated to high reliability of products [1]. Hence, manufactures strive to remain competitive and completely relevant to the global market by meeting consumer's demands in terms of quality expectations. Likewise, the product lifetime is equally an increasing important characteristic [2] and manufactures need to understand the expected life times of their product under various operating conditions. As a result, manufacturing companies target new products to reduce the initial failures, minimize random failures and to increase product reliability. Lifetime of a product is not only a common concern to engineers but also to statisticians and is a key characteristic of product quality and reliability. It characterizes the time-span during which a product can be expected to operate safely under certain conditions and meet specified standards of performance.

A lot of efforts have been made by researchers to develop new reliability models [3]. Yu and Chen, 2014 developed reliability model of a complicated product with multiple failure modes under different stress conditions [4], Huairou 2008 [5] and Hu, Zhen and Du, Xiaoping, 2014 [6] developed a new lifetime cost optimization model to predict product lifetime and Beaudry, 1978 studied measures to reflect interaction between the reliability and performance characteristics of a product. In the review article it was described that, the distribution of lifetimes and how lifetimes depend on set experimental (predictor) variables among others [7].

According to Khan & Islam, 2012 [8] reliability is the percentage measure of degree to which product/system is in an operable and committable state at the point in time when it is needed. That is, the availability of a product or a system. Reliability is very important in that product and brand reputations are made or destroyed by their product reliability performance. Poor reliability (unreliability) causes adverse consequences and therefore a number of products or systems are serious threat. Unreliability may have implications for: Safety, reputation, good will, delays, profit margins, cost of repair and maintenance and competitiveness. Work to minimize failures, improve maintenance effectiveness, shorten repair times and meet customer and organization expectation has numerous benefits. Reliability has broad and important impact across the product life cycle. Therefore, the cost of unreliability can damage a company's image. Many manufactures have adopted reliability testing to meet and exceed the demands of customers [9]. Considering the reliability definition by Crowder, 2002 [10] as "probability of performing without failure a specified function under given conditions for a specified period of time." Therefore, reliability testing usually involves simulation of conditions under which the item will be used during its lifespan. Reliability does not compare the product to some predetermined specifications, such as the case with quality assurance, but rather investigates the performance over a predetermined period of time.

Traditionally, the lifetime of a product can be demonstrated by three different patterns of failures over time. These patterns are combined to produce a bathtub curve as shown in Fig. 1. The curves not only represent reliability performance of components or non-repaired items, but also observes the reliability performance of a large sample of homogeneous items entering the field at some start time (usually zero). If we observe the items over their lifetime without replacement then we can observe three distinct patterns or shapes. The three distinct patterns or periods are mathematically categorized as; a decreasing rate of failure, a constant rate of failure and an increasing rate of failure which in practice are known as infant mortality, useful life and wear out as shown in Fig. 1. During the infant mortality phase, the weaker components are removed during manufacturing process, pre-delivery testing or when an item comes into service. At the useful phase, new component has an equal probability of failure as the old component. Finally, the wear out failure which has increasing failure rate caused by weariness, fatigue and degradation [11]. As reliability (or part) of a product improves, cases of failed parts become less frequent in the field [12]. When a new product is introduced, the company ensures that initial failure is minimized, random failures are reduced during the expected working period of a product and finally to extend the product's lifetime.

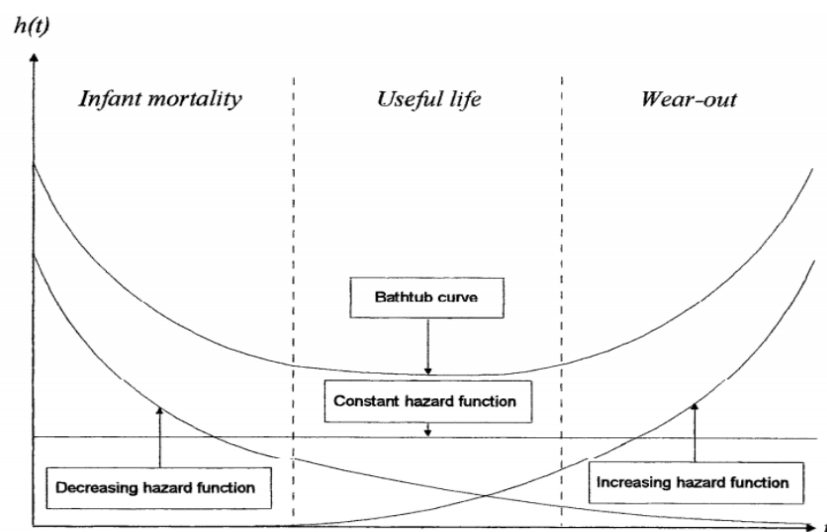


Figure 1. The Bathtub curve.

When design of experiment (DOE) is used for life testing, the response is life or failure time. Failure time distribution is the most widely used measure for reliability of a product. The distribution is constructed based on failure time data of a product. The failure time or lifetime of a product is described as a continuous random variable T . Its probability distribution function is characterized by cumulative density function, probability density function, reliability function or hazard function. The reliability function gives the probability of a product surviving up to time t while the hazard function also known as the failure rate function describes the probability of failure at the smallest interval $(t, \delta + t)$, given survival up to.

In this article we are concerned with finding factors that affect the lifetimes of a product and also increasing the lifetimes by conducting simulation with right censoring both for Weibull and log-normal distribution. Simulation model is developed based on these factor levels. Maximum likelihood method is used to estimate the parameters of both Weibull distribution and log-normal distribution. Finally, we find that combination of input-factor values that optimizes the response.

1.1. Maximum Likelihood Estimation (MLE)

Maximum likelihood estimate (MLE) can be described as follows: Given the model and its parameters, the MLE function is the probability (density) of a sample data seen as a function of the model parameters. Estimates of the model parameters are obtained by maximizing logarithm of the likelihood. Estimates are the values that maximizes probability of the sample data. Its main aim is to find combination of parameters β and η that maximize the probability of a given data. MLE estimates tend to predict long life with small samples.

The PDF of a log normal distribution is given by:

$$f(t/\mu, \sigma^2) = \frac{1}{\sigma t \sqrt{2\pi}} \exp\left(-\frac{(\log t - \mu)^2}{2\sigma^2}\right), \quad t > 0 \quad (1)$$

Where σ is the shape parameter and $\mu = T_{50}$, median (scale) parameter.

While the PDF of Weibull distribution is:

$$f\left(\frac{t}{\eta}, \beta\right) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{t}{\eta}\right)^\beta\right), \quad t > 0 \quad (2)$$

η is the scale parameter and β is the shape parameter.

2. MATERIAL AND METHOD

A complete factorial design can become large even at two levels of each factors. An experiment with eight factors would require $2^5 = 32$ runs. In the case of reliability experiment, each run may be measured as in hours, days, months etc and may take several thousand hours, which would be practically infeasible to conduct 32 runs. Thus, more efficient methods of conducting experiments are needed. Fractional factorial designs can reduce the number of runs by choosing a subset or fraction of the complete factorial design. A 2^{5-2} factorial design would reduce the

number of runs by using a quarter fractional factorial experiment, $(1/4 (2^{5-2}))$ to eight runs. Although this provides an advantage of reducing the number of runs, the disadvantage is that many of the effects are hidden or confounded by the main effect factors that the experimenter deems the most important.

This assumes that many of the confounded effects are not significant and have very minimal effect or do not affect the response at all. Fractional factorial designs are particularly useful if it can be estimated which main effects and interactions are significant so that the remaining effects can be confounded [13].

The parameters are chosen to determine their corresponding influence on the lifetimes, which are:

- Voltage (V)
- Temperature (C)

These factors are to be studied in levels of high and low.

Table 1. Factors and factor levels used in the design.

Factor	Low(-)	High(+)
Temperature	38	43
Voltage	105	120

2.1. Simulation Test

A simulation of two factors temperature and voltage were investigated to determine the reliability of a product. A simulation of a 2^2 design with no interaction was conducted for Weibull distribution and log-normal distribution. The main objective is to identify the significant factors that affect the lifetimes of a product. Table 2 gives the layout of the design matrix.

Table 2. Layout of 2^2 design matrix.

Run	I	A	B	AB	Response
1	1	+	-	-	R ₁
2	1	-	+	-	R ₂
3	1	+	+	+	R ₃
4	1	-	-	+	R ₄

$$ln(\eta) = \alpha_0 + \alpha_1 T \text{ emperature} + \alpha_2 V \text{ oltage} + \alpha_{12}(T \text{ emperature})(V \text{ oltage}) \tag{3}$$

2.2. Simulation Results

The simulation results are summarized for Weibull distribution and given in Table 3. Similarly, the results for Log-normal distribution are summarized and given in Table 4.

Table 3. Weibull distribution analysis results.

Likelihood ratio test-table				
Model	Effect	DF	Ln(LKV)	P Value
Reduced	Temperature	1	-1378.3	<0.0001
	Voltage	1	-1281	0.0007
Full	-	4	-1377	-

Estimates of Parameters			
Predictor	Coef.	S.d Error	P Value
Intercept	4.8139	1.647	0.068
Temperature	0.0304	0.013	1.15E-02
Voltage	-0.0466	0.257	7.17E-03

Table 4. Log-normal distribution analysis results.

Likelihood ratio test - table				
Model	Effect	DF	Ln(LKV)	P Value
Reduced	Temperature	1	-1425	0.0001
	Voltage	1	-1654	0.0200
Full	-	4	-2144	-

Estimates of Parameters			
Predictor	Coef.	S.d Error	P Value
Intercept	5.0753	0.0432	0.0067
Temperature	0.0998	0.077	0.0231
Voltage	-0.1006	0.0266	0.0002

The layout of table 3 and table 4 are similar to the ANOVA table. This makes it easy to read for those who are familiar with ANOVA. From the P value column, temperature and voltage are important to the product life. The estimated relationship is:

$$\ln(\eta) = \alpha_0 + \alpha_1 T \text{ emperature} + \alpha_2 V \text{ oltage}$$

for weibull distribution and for log-normal distribution. The estimated shape parameter for the weibull distribution is 1.85.

3. CONCLUSION

Instant results and sequentially make agricultural experiments different from industrial experiments. For most industrial experiments results are always available instantly (days, hours e.t.c) and the results from each group can be acted upon to be used in the next experiments while in agricultural experiments, processes are always restricted during growing seasons. In addition, normal distribution which characterizes most experimental designs is not a logical distribution for lifetimes due to censoring. Due to censoring, analysis of variance (ANOVA) and least square method (LSM) cannot be used to improve reliability [14]. In this case, ANOVA method can only be applicable if suspensions are treated as failures and midpoint of interval data used as failure times.

This approximation gives wrong results and lead to wrong conclusions. Consequently, the use of ANOVA method on lifetimes data violates the normal distribution assumption among others. In addition, the commonly used ML estimate approach is considered to have an “estimability” problem. Testing for important effects in the model cannot be done by comparing the ML estimates with their corresponding standard errors because the ML estimates may be infinite. One factor with two levels experiment with censored observation and failure time data is given according to the factor levels. In this example it is observed that as the parameter tends toward infinity the likelihood function increase to the maximum and therefore concluded that ML estimate for the main effect tends to be infinite when the true factor effect is large [15].

The study aims at increasing the lifetimes of a product by simulation. The important factors are identified through the screening process and a second order (quadratic) response surface

methodology used to determine the optimal values. Weibull and lognormal models with an assumed scale parameter was used in the model simulation and maximum likelihood estimation for the parameter estimation. Temperature and voltage were found to be significant effects and therefore they play an important role in the lifetimes of a component.

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