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Research Article

Two Capacitor Problem with a LTI Capacitor and a Capacitor Modelled Using Conformal Fractional Order Derivative

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Abstract: Fractional order circuit elements have been started to model different types of circuit elements, circuits and systems in the last decades. There are different types of fractional derivatives. Recently, a new simple fractional derivative method called “conformable fractional derivative” has been brought out. It is simpler than other fractional derivatives and has already been used to model supercapacitors. It is important to model the new circuit elements and analyze the circuits containing them so that they can be exploited at their full potential. Two capacitor problem is a famous problem in physics and circuit theory. In this study, a new two capacitor problem a circuit which consists of an LTI capacitor and a supercapacitor which has been modelled with conformable fractional derivative have been examined. The differential equations which describe the circuit have been derived. The circuit current is found explicitly however the voltages of the capacitors do not have analytical solutions. That’s why they are solved numerically.

Keywords: *Circuit Analysis, Circuit Modelling, Circuit Theory, Energy Analysis, Fractional Order Derivative.*

LTI Kapasitor ve Konformal Kesirli Mertebeden Türev Kullanılarak Modellenmiş Kapasitör ile İki Kapasitör Problemi

Özet: Kesirli mertebeden devre elemanları, son yıllarda farklı tipteki devre elemanlarını, devreleri ve sistemleri modellemeye başlanmıştır. Farklı kesirli türev türleri vardır. Son zamanlarda, "uyumlu kesirli türev” adı verilen yeni bir basit kesirli türev yöntemi ortaya çıkmıştır. Diğer kesirli türevlerden daha basittir ve süperkapasitörleri modellemek için zaten kullanılmıştır. Yeni devre elemanlarını modellemek ve onları içeren devreleri analiz etmek, böylece tam potansiyellerinde kullanılabilmeleri için önemlidir. İki kapasitör problemi, fizikte ve devre teorisinde ünlü bir problemdir. Bu çalışmada, bir LTI kondansatör ve bir süperkapasitörden oluşan ve uyumlu fraksiyonel türev ile modellenen yeni bir iki kondansatör problemi incelenmiştir. Devreyi tanımlayan diferansiyel denklemler türetilmiştir. Devre akımı açıkça bulunur, ancak kapasitörlerin voltajlarının analitik çözümleri yoktur. Bu yüzden sayısal olarak çözülürler.

Anahtar Kelimeler: *Devre Analizi, Devre Modelleme, Devre Teorisi, Enerji Analizi, Kesirli Mertebeden Türev*

1. Introduction

Fractional derivatives (FDs) have first been considered by L’Hospital [1]. It has become a branch in applied mathematics [2]. The self-taught Oliver Heaviside used FDs to analyze electrical transmission lines circa 1890 [3]. Since the last half of the 20th century, the fractional Calculus has found applications in many different fields [4-5]. The FDs have been used to model circuit elements such as capacitors, inductors and memristors [6-10]. Analog applications such as filters, controllers and oscillators which are based on the fractional order circuit elements do exist in the literature [4-5, 8-9, 11-14]. Such circuit elements have also been used to model systems [4-5, 8-9, 11-14]. There are different types of FDs [15]. In [16], a new and simpler FD called “the conformable fractional derivative” (CFD) have been suggested. The CFD can be expressed using the familiar limit definition of the derivative of a function and it is simpler than the other FD definitions and it has a different description [16]. A conformal fractional derivative is actually not a fractional derivative when the other FDs are considered: it is simply a first derivative multiplied by a fractional power of the independent variable. This new theory is improved in [17]. This new definition is a natural extension of the classical derivative and it has also the advantage of being different from other FDs. For example, many classical theorems of calculus can be easily used to analyze the systems which are based on the CFDs. The CFD has a very important property: while the Riemann-Liouville FD of a constant is not zero, the CFD of a constant is zero. Due to these important and unusual properties compared to the other FDs, the CFD has emerged as a hot research area. The CFD is also physically interpretable while the other FDs are not [18]. Usages of FDs in electrical circuits have been inspected in [19]. Analytical solutions of electrical circuits have been examined in [20-22]. FD-based supercapacitor models do exist in literature [23-26]. An electric circuit containing a supercapacitor modeled with the CFD has been inspected in [27]. Analysis of a parallel resonance circuit with a CFD capacitor is made using Simulink in [28].

Two capacitor paradox is an interesting problem and it is given in many remarkable course books and articles which are based on the basic electrical principles and applications [29-30]. Unlike some other paradoxes in science, this paradox is commonly revisited and examined in circuit theory [29-31]. The total energy and charge for two capacitor paradox is detailed in [32]. It is regarded as a missing parameter problem [30]. It is examined to find where the missing energy has disappeared [32-33]. There are different approaches to the problem [34]. Even the radiation from the two-capacitor problem has been examined in [35]. A two capacitor problem has been examined in [36] when one of the capacitors is replaced with a memcapacitor. To the best of our knowledge, a CFD capacitor and an LTI capacitor have not been examined in the literature yet. In this study, a CFD capacitor, an LTI capacitor and a series resistor without a power supply have been examined together. The differential equation which describes the circuit is given and it is examined whether it has an analytical solution or not. The circuit waveforms are drawn with Matlab™.

The paper is arranged as the follows. The definition of the CFD and a CFD based-capacitor model is given in the second section. The analysis of the two-capacitor problem without loss and when one of the capacitors is replaced with a CFD-based supercapacitor is done in the third section. The analysis of the two-capacitor problem with a series resistor added is done in the fourth section. The paper is finalized with the conclusions section.

2. The Conformal Fractional Derivative and the CFD Capacitor Constitutive Law

For $0 < \alpha \le 1$ and t > 0, the Conformal Fractional Derivative (CFD) is described by Khalil et al as the follows [16]:

$$D_\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = f^{'}(t)t^{1-\alpha} = \frac{df(t)}{dt}t^{1-\alpha} \quad (1)$$

More information about CFD can be found in conformable fractional calculus [15-18].

If a capacitor can be modeled using CFD [27], its constitutive law can be expressed as:

$$i_C(t) = C_\alpha \frac{d^\alpha v_C(t)}{dt^\alpha} \quad (2)$$

Where $i_C(t)$, $v_C(t)$ and C_α are CFD capacitor current, CFD capacitor voltage and CFD capacitor coefficient, respectively.

3. Two Capacitor Problem with a CFD Capacitor an LTI Capacitor

One of the two capacitor problems, often given in the textbooks, consists of just two LTI capacitors without a capacitor. A circuit which consists of only an LTI capacitor and a CFD capacitor is shown in Figure 1. The circuit is lossless since it has no resistance. Such a circuit can be thought as a modified two capacitor problem.

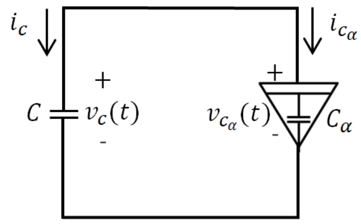

Figure 1. The CFD capacitor connected to a parallel capacitor

When Kirchhoff's laws are used for the circuit shown in Figure 1, the following equations are obtained:

$$i_c(t) + i_{c_\alpha}(t) = 0 \text{ and } v_c(t) = v_{c_\alpha}(t) \quad (3)$$

$$C\frac{dv_c(t)}{dt} + C_\alpha \frac{dv_c(t)}{dt}t^{1-\alpha} = (C + C_\alpha t^{1-\alpha})\frac{dv_c(t)}{dt} = 0 \quad (4)$$

$$\frac{dv_c(t)}{dt} = 0 \quad (5)$$

By taking integration of each side, the CFD capacitor voltage is found as:

$$\int dv_c(t) = \int 0dt \quad (6)$$

$$v_C(t) = K \quad (7)$$

Where K is the integration constant.

If the initial condition $v_C(0)$ is used at t=0, the integration constant is found as

$$v_c(0) = K \quad (8)$$

The CFD capacitor is found as

$$v_c(t) = v_c(0) \quad (9)$$

This means that there is no current flowing in the circuit while it was not the case with a memcapacitor and an LTI capacitor in [36].

4. Two Capacitor Problem with a CFD Capacitor, an LTI Capacitor, and a Resistor

A circuit which consists of a resistor, an LTI capacitor and a CFD capacitor is shown in Figure 2. The differential equation which describes the circuit is found and solved in this section.

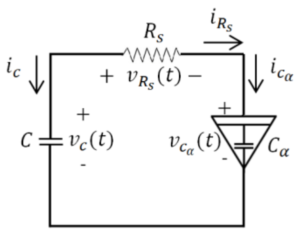

Figure 2. The CFD capacitor and a series resistor, which is connected a parallel capacitor

When Kirchhoff's laws are applied to the circuit, the voltage and current equations are written as:

$$i_{R_s} = i_{c_\alpha} \text{ and } i_c = -i_{c_\alpha} \tag{10}$$

$$v_{R_s}(t) + v_{C_\alpha}(t) = v_c(t) \tag{11}$$

$$R_s i_{C_\alpha} + v_{C_\alpha}(t) = v_c(t) \tag{12}$$

$$R_s C_\alpha \frac{dv_{c_\alpha}(t)}{dt} t^{1-\alpha} + v_{c_\alpha}(t) = v_c(t) \tag{13}$$

If Eq. (10) and Eq. (13) are combined, the differential equation of $v_{c_\alpha}(t)$ is written in the Eq. (15);

$$C_\alpha \frac{dv_{c_\alpha}(t)}{dt} t^{1-\alpha} = -C \frac{dv_c(t)}{dt} \tag{14}$$

$$-\frac{C_\alpha}{C} \frac{dv_{c_\alpha}(t)}{dt} t^{1-\alpha} = \frac{d}{dt}\left(R_s C_\alpha t^{1-\alpha} \frac{dv_{c_\alpha}(t)}{dt} \right) + \frac{dv_{c_\alpha}(t)}{dt} \tag{15}$$

$$-\frac{C_\alpha}{C} \frac{dv_{c_\alpha}(t)}{dt} t^{1-\alpha} = R_s C_\alpha \left((1-\alpha) t^{-\alpha} \frac{dv_{c_\alpha}(t)}{dt} + t^{1-\alpha} \frac{d^2 v_{c_\alpha}(t)}{dt^2} \right) + \frac{dv_{c_\alpha}(t)}{dt} \tag{16}$$

$$R_s C_\alpha t^{1-\alpha} \frac{d^2 v_{c_\alpha}(t)}{dt^2} + \left(R_s C_\alpha (1-\alpha) t^{-\alpha} + \frac{C_\alpha}{C} t^{1-\alpha} + 1 \right) \frac{dv_{c_\alpha}(t)}{dt} = 0 \tag{17}$$

By arranging both sides, the equation is turned into the second order differential equation of the form $y'' + p(x)y' = q(x)$;

$$\frac{d^2 v_{c_\alpha}(t)}{dt^2} + \left((1-\alpha) t^{-1} + \frac{1}{R_s C} + \frac{t^{\alpha-1}}{R_s C_\alpha} \right) \frac{dv_{c_\alpha}(t)}{dt} = 0 \tag{18}$$

When the substitution is used as $u = dv_{c_\alpha}(t)/dt$, the equation is converted into;

$$u' + \left((1-\alpha) t^{-1} + \frac{1}{R_s C} + \frac{t^{\alpha-1}}{R_s C_\alpha} \right) u = 0 \tag{19}$$

Although Eq. (18) looks like a second order differential equation, actually a first order differential equation, Eq. (19), describes the circuit to be examined in Figure 2.

$$\begin{aligned} &\frac{u'}{u} + \left((1-\alpha) t^{-1} + \frac{1}{R_s C} + \frac{t^{\alpha-1}}{R_s C_\alpha} \right) = 0 \\ &\frac{u'}{u} = -\left((1-\alpha) t^{-1} + \frac{1}{R_s C} + \frac{t^{\alpha-1}}{R_s C_\alpha} \right) \end{aligned} \tag{20}$$

By taking the integration of each side of the equation, Eq. (20) turns into

$$\int \frac{u'}{u} dt = \int \left(\frac{(1-\alpha)}{t} + \frac{1}{R_s C} + \frac{t^{\alpha-1}}{R_s C_\alpha} \right) dt \tag{21}$$

$$\ln(u) = (1-\alpha)\ell n(t) + \frac{t}{CR_s} + \frac{t^\alpha}{\alpha R_s C_\alpha} + A \tag{22}$$

where A is the integration constant.

Using exponential function, Eq. (22) turns into:

$$\begin{aligned} &u = e^{\int \left(\frac{(1-\alpha)}{t} + \frac{1}{R_s C} + \frac{t^{\alpha-1}}{R_s C_\alpha} \right) dt + A} = e^{(1-\alpha)\ell n(t) + \frac{t}{CR_s} + \frac{t^\alpha}{\alpha R_s C_\alpha}} e^A \\ &u = Be^{(1-\alpha)\ell n(t) + \frac{t}{CR_s} + \frac{t^\alpha}{\alpha R_s C_\alpha}} \end{aligned} \tag{23}$$

Where $B = e^A$,

If the exponential term is arranged, the variable $u(t)$ is found as:

$$u(t) = Bt^{\alpha-1} e^{-\left(\frac{t}{CR_s} + \frac{t^\alpha}{\alpha R_s C_\alpha} \right)} \tag{24}$$

Then, the CFD capacitor voltage can be obtained as:

$$u(t) = \frac{dv_{c_\alpha}(t)}{dt} \rightarrow dv_{c_\alpha}(t) = u(t)dt \rightarrow v_{c_\alpha}(t) = \int u(t)dt \tag{25}$$

$$v_{c_\alpha}(t) = B\int_0^t t^{\alpha-1} e^{-\left(\frac{t}{CR_s} + \frac{t^\alpha}{\alpha R_s C_\alpha} \right)} dt + D \tag{26}$$

Where D is the related integral constant.

Using the initial CFD voltage, $v_{c_\alpha}(0)$

$$v_{c_\alpha}(0) = B\int_0^0 t^{\alpha-1} e^{-\left(\frac{t}{CR_s} + \frac{t^\alpha}{\alpha R_s C_\alpha} \right)} dt + D = 0 + D = D \tag{27}$$

Therefore, D is found as:

$$D = v_{c_\alpha}(0) \tag{28}$$

The CFD capacitor voltage turns into

$$v_{c_\alpha}(t) = B\int_0^t t^{\alpha-1} e^{-\left(\frac{t}{CR_s} + \frac{t^\alpha}{\alpha R_s C_\alpha} \right)} dt + v_{c_\alpha}(0) \tag{29}$$

The integral in Eq. (29) is not analytically solvable. Using a series solution such as the following, the integral can be solved as an infinite series given as follows.

$$\begin{aligned} &t^{\alpha-1} e^{(-bt-ct^\alpha)} = \\ &t^{\alpha-1} \sum_{k=0}^{\infty} \frac{(-bt-ct^\alpha)^{-1+2k}(2k-bt-ct^\alpha)}{(2k)!} \end{aligned} \tag{30}$$

However, application of such a series would not be practical.

To find B, Eq. (11) can be used. Remembering that the CFD capacitor current is equal to the resistor current:

$$i_{C_\alpha} = C_\alpha \frac{dv_{c_\alpha}(t)}{dt} t^{1-\alpha} = C_\alpha B t^{\alpha-1} e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} t^{1-\alpha} = C_\alpha B e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} \quad (31)$$

Then, the resistor voltage can be expressed as:

$$v_{R_s}(t) = R_s i_{C_\alpha} = R_s C_\alpha B e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} \quad (32)$$

When Eq. (12) is used, the LTI capacitor voltage can be solved in the following way:

$$v_c(t) = R_s i_{C_\alpha} + v_{C_\alpha}(t) \quad (33)$$

$$v_c(t) = R_s C_\alpha \frac{dv_{c_\alpha}(t)}{dt} t^{1-\alpha} + v_{c_\alpha}(t) \quad (34)$$

$$v_c(t) = R_s C_\alpha B e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} + B\int_0^t t^{\alpha-1} e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} dt + v_{c_\alpha}(0) \quad (35)$$

The integration constant B can be found using the initial conditions at t=0:

$$v_c(0) = R_s C_\alpha B + v_{c_\alpha}(0)$$
$$B = \left(v_c(0) - v_{c_\alpha}(0)\right) / \left(R_s C_\alpha\right) \quad (36)$$

Then, the capacitor current, the resistor voltage and the CFD capacitor voltage are given as:

$$i_{R_s}(t) = i_{C_\alpha} = \left(v_c(0) - v_{c_\alpha}(0)\right) e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} / R_s \quad (37)$$

Then, the resistor voltage is written in Eq. (37):

$$v_{R_s}(t) = R_s i_{C_\alpha} = R_s \left(v_c(0) - v_{c_\alpha}(0)\right) e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} \quad (38)$$

The LTI capacitor voltage is found as:

$$v_c(t) = \left(v_c(0) - v_{c_\alpha}(0)\right) e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} + \left(v_c(0) - v_{c_\alpha}(0)\right) / \left(R_s C_\alpha\right) \int_0^t t^{\alpha-1} e^{-\left(\frac{t}{CR_s}+\frac{t^\alpha}{\alpha R_s C_\alpha}\right)} dt + v_{c_\alpha}(0) \quad (39)$$

A Matlab code has been written to calculate the capacitor voltages. The circuit waveforms are calculated for three different alpha values when $v_c(0) = 1\ V$, $R_s = 1\Omega$, $C = 1\ F$ and $C_\alpha = 1\ F/s^{1-\alpha}$ are used. They are shown in Figures 3 and 4. Moreover, the resistor's voltage and current are also shown in Figures 5 and 6. It is interesting to note that the capacitor currents or the resistor current are found explicitly while the capacitor voltages are not.

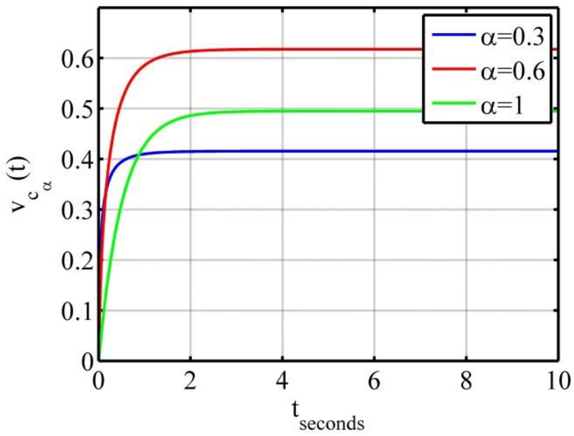

Figure 3. The CFD capacitor voltage

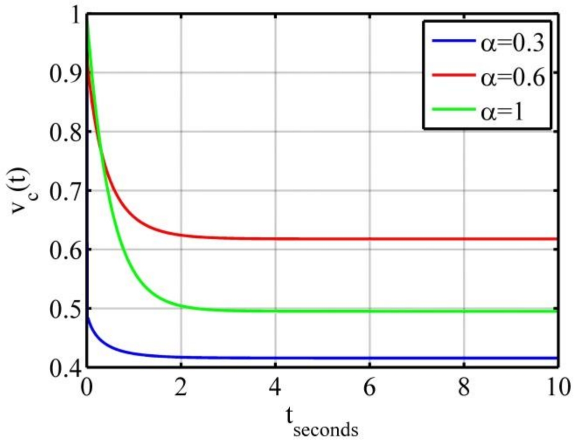

Figure 4. The LTI capacitor voltage

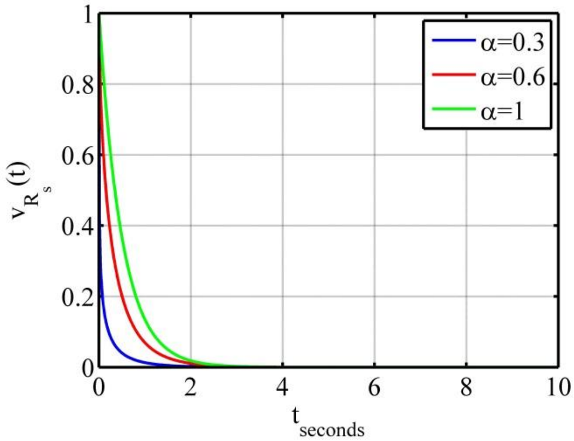

Figure 5. The resistor voltage

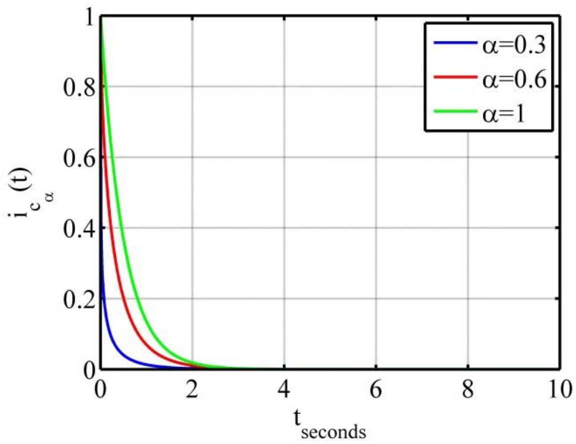

Figure 6. The CFD capacitor or the series resistor current

5. Conclusion

Supercapacitors or ultra-capacitors are becoming cheaper and more common each day. They cannot be modelled using the same constitutive law of an LTI capacitor. Therefore, simple and robust models are needed for their modelling and analysis in the circuits they are used. Only then, such capacitors can be fully exploited. Some supercapacitors could be modelled using fractional-order derivatives. In this paper, the two-capacitor problem has been examined when one of the capacitors has been replaced with a CFD capacitor. The solution of the current has been found explicitly but the voltages of the capacitors are shown to be calculable using numerical analysis. The results reported in this paper may find usage in the circuits where the LTI and the CFD capacitors are used together.

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