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Modeling and Analysis of Social Networks Based On Petri Net Theory

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Abstract— With the increasing use of social networks recently, studies in this area have also increased. These studies are generally related to social network analysis and modeling. The social networks are modeled using graphs; however, graphs are static structures and cannot model the dynamic properties of social networks. Petri nets are a graphical structure and are used in the modeling of dynamic structures. In this study, Petri nets was used to model the dynamic behavior of social networks. The model obtained was analyzed in case of behavioral and structural properties and the major properties of the model were determined.

Keywords : *social networks, petri nets, graphs*

1. Introduction

Online social networks have been rapidly developed in recent years. These allow people to produce and share various content. Online social networks originated from e-mail and has been the most widely used applications after the rapid development of the Internet and smart devices [1]–[4]. Most people share information continuously using social networks, so social network analysis has been used to analyze relationships between users. This paper aims to model and analyze the behavior of users in social networks.

Petri nets are a mathematical and graphical modeling tool used to analyze various systems. They are suitable tool for describing and studying systems that are distributed, asynchronous, concurrent, parallel, and/or stochastic [5]. Petri nets have places and transitions and these are interconnected with directional arrows. Petri nets are also called Place Transition(PT) nets [6]. The places represent conditions, and the transitions represent events. The formal definition of a Petri net can be given as follows [5]:

$$PN = (P, T, F, W, M_0)$$

P: places

T: transitions

F: arcs between transitions and places

W: weight function

M_0: initial marking vector

In graphical representation of a Petri net model, places are shown as circles and transitions as bars. Arcs are labeled with their weights. The places marked with tokens and tokens shown as black dots. The tokens indicate number of data items available in places. If a place has at least one token then it would be enabled and a transition can be fired. A token is removed from input place and added to output place when a transition is fired. So, the tokens travel through the petri net when transitions are fired.

There are many properties and problems in dynamic systems that can be analyzed with Petri nets. Some of those that can be used in social network models are Reachability, Liveness, Reversibility, Boundedness and Persistence [5]. These properties will also be analyzed in the next sections.

Structural properties of a Petri net are independent from the initial marking of the Petri nets and can be analyzed from the incidence matrix of the net. The dynamic behavior of Petri nets can be analyzed by matrix equations [5].

Incidence Matrix: For a Petri net with m places and n transitions, the incidence matrix $A = [a_{ij}]$ is a nxm matrix and its entry is given by

$$a_{ij} = a_{ij}^{+} - a_{ij}^{-} \tag{1}$$

where $a_{ij}^{+} = w(i,j)$ is the weight of the arc that goes from transition i to output place j and $a_{ij}^{-} = w(j,i)$ is the weight of the arc that to transition i from input place j. In this equation, a_{ij}^{+} represents the number of tokens added to place j, a_{ij}^{-} represents the number of tokens removed from place j and a_{ij} represents the number of tokens changed in place j when transition i fires once.

When writing matrix equations, M_k marking is written as an $m\ x\ 1$ column vector and the *j*th entry of M_k indicates the number of tokens in place *j*, after the *k*th firing in firing sequence. *k*th firing is written as u_k control vector that is an $n\ x\ 1$ column vector. The u_k column vector consists of one nonzero entry and the rest is 0. This nonzero entry in the *i*th position denotes that transition *i* fires at the *k*th firing. In A incidence matrix, *i*th row indicates the change of marking as the result of firing transition i, and the following state equation can be written [5]:

$$M_k = M_{k-1} + A^T u_k, \quad k = 1,2,3, \ldots \tag{2}$$

If there is a firing sequence $\{u_1, u_2, u_3, \ldots, u_d\}$ that transform M_0 to destination marking M_d, so M_d is reachable from M_0, the state equation can be written as

$$M_d = M_0 + A^T \sum_{k=1}^{d} u_k \tag{3}$$

Social networks can be defined as a collection of social or interpersonal relationships within a social group. In order to better understand the social concepts, Barnes [7] has started with the work he has done and the work of other researchers and has gained speed. After this stage, the number of theoretical and experimental studies increased and the diversity increased. Social networks are used to explain the characteristics and behavior of individuals: they are used to analyze the social processes of large and small groups. The basic data of social networks can be defined as a set of social units, for example individuals can be defined as pairs connected by a certain social connection [8]. Examples include a group of friends, a group of employees at a workplace, a group of people in a tribe, etc. Graphs are used to model and analyze these examples.

In this paper, we propose a Petri net model of relation between two users in social networks. Then we analyzed some behavioral and structural properties of this dynamic model. Thanks to incidence matrix of Petri nets, we can describe and analyze the dynamic behavior of the model mathematically.

2. Materials and Methods

It is known that a graph is a static model / structure whose properties cannot mimic the social network, since social networks are dynamic structures. Petri nets are used to model dynamic systems. That is why the communication between users in social network systems can be modeled using Petri nets. In this way, the properties of the social network as behavioral and structural properties will be analyzed in subsections.

The social network can be modeled as Petri nets, and any individual can be modeled as a sub-Petri net. Figure 1 is a model for two individuals' model.

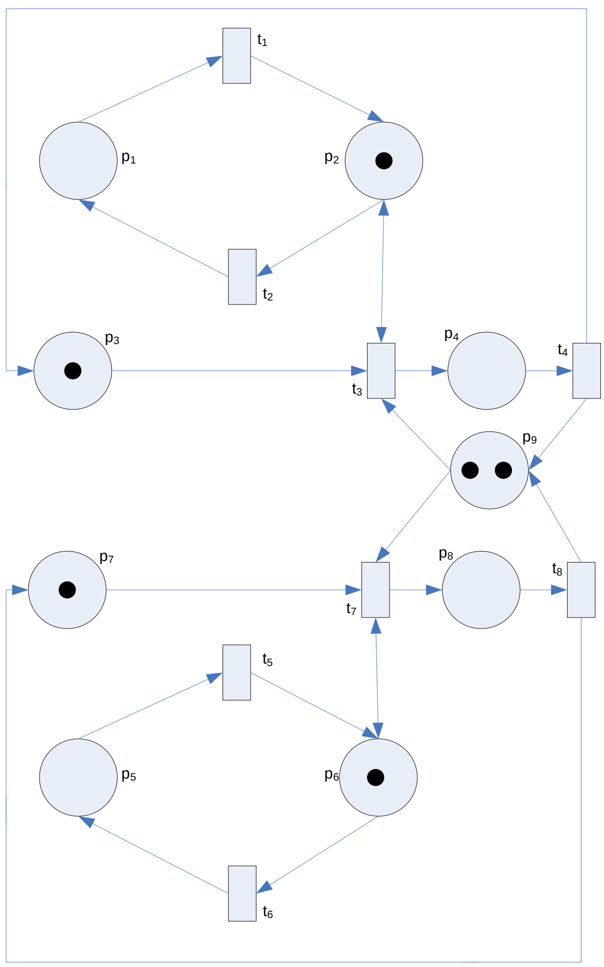

Figure 1. Petri net model of the social network for two users.

After the Petri net model of this social network is obtained, the model is analyzed structurally by the following steps.

The set of places in this model is

$$P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9\}$$

and the set of transitions is

$$T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}$$

The Incidence Matrix $= [a_{ij}]$, where $a_{ij} = a_{ij}^{+} - a_{ij}^{-}$, is 8 x 9 matrix and obtained as following from Eq.(1):

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mp 1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mp 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

The column vector of initial marking in this model is

$$M_0 = [0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 2]^T$$

Suppose that first user wants to generate a message to deliver to other users in social network. If the user is online (place P_2 has tokens) and able to send messages (place P_3 has tokens) and there are available shared links (place P_9 has tokens), then transition T_3 will be activated. After transition T_3 being fired, one token will be removed from each of places P_2, $P_{3,}$ P_9 and added to place P_4. After place P_4 has token, transition T_4 will be activated and will be ready to forward data to shared links. After transition T_4 being fired, one token will be removed from places P_4 and added to places P_3, P_9.

The steps of this simulation can be obtained by matrix equations in structurally analysis also. We used the Eq.(3) to find status of model after every transition is fired. Suppose that we want to find status of model and find result of marking all places after transition T_3 fired. In this case firing vector can be written as

$$u = [0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T$$

and from Eq.(3) we calculate result of marking as follow:

$$M_1 = [0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 2]^T$$

After this brief analyzing the net model, Figure 1 will be analyzed based on behavioral and structural properties.

2.1. Behavioral Properties

The main aim of modeling any entity with Petri net requires analyzing dynamical properties of entity. Due to this case, the properties of the net such as reachability, boundedness, liveness, reversibility and home state, coverability and persistence are analyzed in this subsection.

2.1.1. Reachability

Reachability is a fundamental dynamic property of any system. Reachability means that starting any marking Mi and ending in another marking Mj. This means that marking Mj is reachable from marking Mi. For example, Figure 1 depicts a Petri net model for two users in the network. The initial marking M_0=(0 1 1 0 0 1 1 0 2) and reachability tree of the net is seen in Figure 2. Reachability tree contains all possible reachable states in the net.

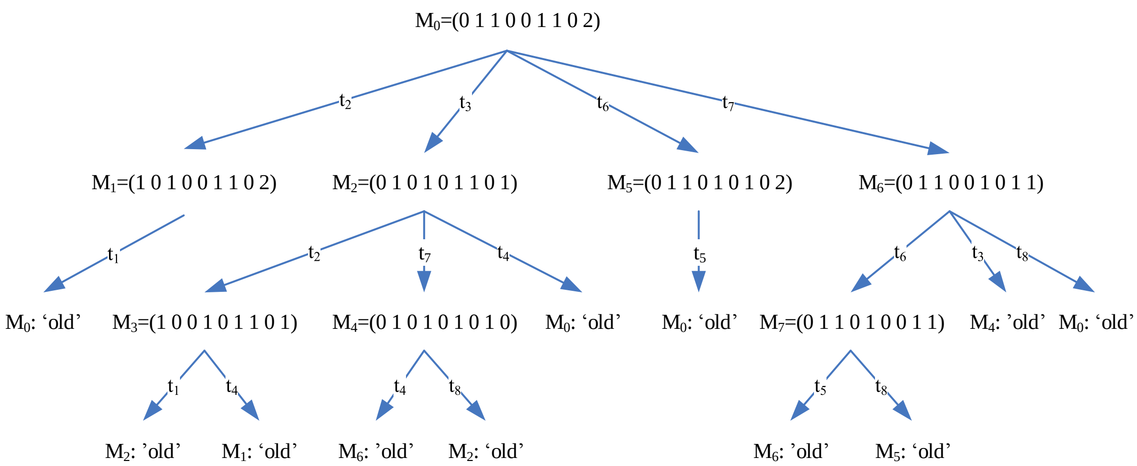

Figure 2. Reachability tree of the net in Fig. 1.

2.1.2. Boundedness

Boundedness means that each place has maximum capacity of tokens. Figure 2 depicts that each place does not exceed 2 tokens for any markings in the net, so this net is 2-bounded. This means that there is no overflows in the places regardless of which transitions are fired.

2.1.3. Liveness

The liveness means that the net is a deadlock free net. It is possible to fire any transitions regardless of which marking has been reached as seen in Figure 2. There are always live transitions at any marking of the net.

2.2. Structural Properties

The structural properties of the net such as structural liveness, controllability, structural boundedness, repetitive and consistency are analyzed in this subsection.

2.2.1. Structural Liveness

If there is a liveness marking for this model, it is said that Petri net model is structurally live. The Petri net in Figure 1 is a liveness net, since all transitions have chance to fire infinitely in any firing sequence. Whole network for two individuals is also liveness as seen in Figure 2. Figure 2 illustrates that any individual may want to get off-line marking (logout), this situation does not affect the remaining individuals and they can be active in the net; this means that network is still alive.

2.2.2. Controllability

A Petri net is said to be controllable, if a marking is reachable from all other remaining markings. The obtained Petri net for an individual is fully controllable since all markings are reachable from each other. Figure 2 depicts that each possible markings are reachable from each other. An individual may want to get off-line marking; this does not affect the remaining individuals and they can perform their activities in the network.

2.2.3. Structural Boundedness

If the multiplication of incidence matrix with a vector of positive integers concludes in zero or negative value, Petri net is structurally bounded. Incidence matrix for net in Figure 1 will be as follow.

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mp 1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mp 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} \leq 0$$

The solution of this linear equation system is {0,0,0,0,0,0,0,0}. So, the Petri net model of the social network is not structurally bounded since there are upper bound for any individual to propagate messages.

3. Results and Discussion

Social networks are often modeled using graphs; however, graphs are static models and cannot model the dynamic properties of the social networks. Therefore, we modeled the social network using the Petri net and made some analyzes on this model. The properties of the Petri net model of the social network can be summarized as in Table 1.

Table 1. Major properties obtained from the model.

Behavioral Properties		Structural Properties	
Reachability	YES	Structural Liveness	YES
Bounded	YES	Controllability	YES
Liveness (Safe)	YES	Structurally Boundedness	NO

All these properties show that this model does not get in deadlock case (it is deadlock free) and there are no any restrictions on propagated messages of individuals. Another important point is that each step in the Petri net model can be achieved using mathematical equations.

4. REFERENCES

[1] M. Li, X. Wang, K. Gao, and S. Zhang, "A Survey on Information Diffusion in Online Social Networks: Models and Methods," Information, vol. 8, no. 4, p. 118, 2017.

[2] L. Liu, B. Qu, B. Chen, A. Hanjalic, and H. Wang, "Modeling of Information Diffusion on Social Networks with Applications to WeChat," 2017.

[3] T. Wang, J. He, and X. Wang, "An information spreading model based on online social networks," Phys. A Stat. Mech. its Appl., vol. 490, pp. 488–496, 2018.

[4] A. Guille, H. Hacid, C. Favre, and D. A. Zighed, "Information diffusion in online social networks," ACM SIGMOD Rec., vol. 42, no. 1, p. 17, 2013.

[5] T. Murata, "Petri Nets: Properties, Analysis and Applications," Proc. IEEE, vol. 77, no. 4, pp. 541–580, 1989.

[6] A. Pinna, R. Tonelli, M. Orrú, and M. Marchesi, "A petri nets model for blockchain analysis," Comput. J., vol. 61, no. 9, pp. 1374–1388, 2018.

[7] J. A. Barnes, "Class and Committees in a Norwegian Island Parish," Hum. Relations, vol. 7, no. 1, pp. 39–58, 1954.

[8] L. C. Freeman, "Social networks and the structure experiment," Res. Methods Soc. Netw. Anal., pp. 11–40, 1989.