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Intuitionistic Fuzzy Magnified Translation of PS-Algebra

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Abstract — In this paper, the concepts of intuitionistic fuzzy α-translation (IFAT), intuitionistic fuzzy α-multiplication (IFAM), and intuitionistic fuzzy magnified $\beta\alpha$-translation (IFMBAT) are introduced in the setup of PS-algebra. Some properties of PS-ideal and PS-subalgebra are investigated by applying the concepts of IFAT, IFAM, and IFMBAT. Intersection and union of intuitionistic fuzzy PS-ideals are explained through results and examples.

Keywords — *Intuitionistic fuzzy α-translation, intuitionistic fuzzy α-multiplication, intuitionistic fuzzy PS-ideal, intuitionistic fuzzy PS-subalgebra, intuitionistic fuzzy magnified $\beta\alpha$-translation*

Mathematics Subject Classification (2020) — 03B20, 06F99

1. Introduction

Zadeh [1] introduced the idea of fuzzy set in 1965. The deep study of fuzzy subsets and its applications to different mathematical structures developed the fuzzy mathematics. Fuzzy algebra is a significant branch of fuzzy mathematics. Idea of Fuzzy set has been applied to different algebraic structures like groups, rings, modules, vector spaces and topologies. In this way, Iseki and Tanaka [2] introduced the idea of BCK-algebra in 1978. Iseki [3] introduced the idea of BCI-algebra in 1980 and it is clear that the class of BCK-algebra is a proper sub class of the class of BCI-algebra. Lee et al. [4] discussed the fuzzy translation, (normalized, maximal) fuzzy extension and fuzzy multiplication of fuzzy subalgebra in BCK/BCI-algebra. Relationship among fuzzy translation, (normalized, maximal) fuzzy extension and fuzzy multiplication are also investigated. Ansari and Chandramouleeswaran [5] introduced the notion of fuzzy translation, fuzzy extension and fuzzy multiplication of fuzzy β ideals of β-algebra and investigated some of their properties. Priya and Ramachandran [6, 7] introduced the class of PS-algebra. Lekkoksung [8] concentrated on fuzzy magnified translation in ternary hemirings, which is a generalization of BCI / BCK/Q / KU / d-algebra. Senapati et al. [9] have done much work on intuitionistic fuzzy H-ideals in BCK/BCI-algebra. Jana et al. [10] wrote on intuitionistic fuzzy G-algebra. Senapati et al. [11] discussed fuzzy translations of fuzzy H-ideals in BCK/BCI-algebra. Atanassov [12] introduced intuitionistic fuzzy set. Senapati [13] investigated the relationship among intuitionistic fuzzy translation, intuitionistic fuzzy extension and intuitionistic fuzzy multiplication in B-algebra. Kim and Jeong [14] introduced the intuitionistic fuzzy structure of B-algebra. Senapati et al. [15] introduced the cubic subalgebras and cubic closed ideals of B-algebras. Senapati et al. [16] discussed the fuzzy dot subalgebra and fuzzy dot ideal of B-algebras. Priya and Ramachandran [17] worked on fuzzy translation and fuzzy multiplication in PS-algebra. Chandramouleeswaran et al. [18]

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worked on fuzzy translation and fuzzy multiplication in BF/BG-algebra. Jun and Kim [19] worked on intuitionistic fuzzification of the concept of subalgebras and ideals in BCK-algebras.

Purpose of this paper is to introduce the idea of intuitionistic fuzzy α translation (IFAT), intuitionistic fuzzy α multiplication (IFAM) and intuitionistic fuzzy magnified $\beta\alpha$ translation (IFMBAT) in PS-algebra. Some of their properties are investigated in depth by using the idea of intuitionistic fuzzy PS ideal (IFID) and intuitionistic fuzzy PS subalgebra (IFSU).

2. Preliminaries

In this section, we present some basic definitions, that are helpful to understand the paper.

Definition 2.1. [3] An algebra $(Y;*,0)$ of type (2,0) is called a BCI-algebra if it satisfies the following conditions:

i. $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$

ii. $t_1 * (t_1 * t_2) \leq t_2$

iii. $t_1 \leq t_1$

iv. $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$

v. $t_1 \leq 0 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, $\forall\, t_1, t_2, t_3 \in Y$

Definition 2.2. [1] An algebra $(Y;*,0)$ of type (2,0) is called a BCK-algebra if it satisfies the following conditions:

i. $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$

ii. $t_1 * (t_1 * t_2) \leq t_2$

iii. $t_1 \leq t_1$

iv. $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$

v. $0 \leq t_1 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$

Definition 2.3. [7] A nonempty set S with a constant 0 and having binary operation $*$ is called PS-algebra if it satisfies the following conditions:

i. $t_1 * t_1 = 0$

ii. $t_1 * 0 = 0$

iii. $t_1 * t_2 = 0$ and $t_2 * t_1 = 0 \Rightarrow t_1 = t_2$, $\forall\, t_1, t_2 \in Y$

Definition 2.4. [7] Let S be a nonempty subset of PS-algebra Y, then S is called a PS subalgebra of Y if $t_1 * t_2 \in S$, $\forall\, t_1, t_2 \in S$.

Definition 2.5. [7] Let Y ba a PS-algebra and I is a subset of Y, then I is called a PS ideal of Y if it satisfies following conditions:

i. $0 \in I$

ii. $t_2 * t_1 \in I$ and $t_2 \in I \rightarrow t_1 \in I$

Definition 2.6. [6] Let Y be a PS-algebra. A fuzzy set B of Y is called a fuzzy PS ideal of Y if it satisfies the following conditions:

i. $\mu(0) \geq \mu(t_1)$

ii. $\mu(t_1) \geq \min\{\mu(t_2 * t_1), \mu(t_2)\}$, for all $t_1, t_2 \in Y$

2.1. Fuzzy and Intuitionistic Fuzzy Logics

Definition 2.7. [1] Let Y be the group of objects denoted generally by t_1. Then, a fuzzy set B of Y is defined as $B = \{< t_1, \mu_B(t_1) > \mid t_1 \in Y\}$, where $\mu_B(t_1)$ is called the membership value of t_1 in B and $\mu_B(t_1) \in [0, 1]$.

Definition 2.8. [6] A fuzzy set B of PS-algebra Y is called a fuzzy PS subalgebra of Y if $\mu(t_1 * t_2) \geq \min\{\mu(t_1), \mu(t_2)\}$, $\forall\ t_1, t_2 \in Y$.

Definition 2.9. [4,5] Let a fuzzy subset B of Y and $\alpha \in [0, 1 - sup\{\mu_B(t_1) \mid t_1 \in Y\}]$. A mapping $(\mu_B)^T_\alpha \mid Y \in [0, 1]$ is said to be a fuzzy α translation of μ_B if it satisfies $(\mu_B)^T_\alpha(t_1) = \mu_B(t_1) + \alpha$, $\forall\ t_1 \in Y$.

Definition 2.10. [4,5] Let a fuzzy subset B of Y and $\alpha \in [0, 1]$. A mapping $(\mu_B)^M_\alpha \mid Y \rightarrow [0, 1]$ is said to be a fuzzy α multiplication of B if it satisfies $(\mu_B)^M_\alpha(t_1) = \alpha.(\mu_B)(t_1)$, $\forall\ t_1 \in Y$.

Definition 2.11. [12] An intuitionistic fuzzy set (IFS) B over Y is an object having the form $B = \{\langle t_1, \mu_B(t_1), \nu_B(t_1)\rangle \mid t_1 \in Y\}$, where $\mu_B(t_1) \mid Y \rightarrow [0, 1]$ and $\nu_B(t_1) \mid Y \rightarrow [0, 1]$, with the condition $0 \leq \mu_B(t_1) + \nu_B(t_1) \leq 1$, $\forall\ t_1 \in Y$. $\mu_B(t_1)$ and $\nu_B(t_1)$ represent the degree of membership and the degree of non-membership of the element t_1 in the set B respectively.

Definition 2.12. [12] Let $A = \{\langle t_1, \mu_A(t_1), \nu_A(t_1)\rangle \mid t_1 \in Y\}$ and $B = \{\langle t_1, \mu_B(t_1), \nu_B(t_1)\rangle \mid t_1 \in Y\}$ are two IFSs on Y. Then, intersection and union of A and B are indicated by $A \cap B$ and $A \cup B$ respectively and are given by

$$A \cap B = \{\langle t_1, \min(\mu_A(t_1), \mu_B(t_1)), \max(\nu_A(t_1), \nu_B(t_1))\rangle \mid t_1 \in Y\}$$

$$A \cup B = \{\langle t_1, \max(\mu_A(t_1), \mu_B(t_1)), \min(\nu_A(t_1), \nu_B(t_1))\rangle \mid t_1 \in Y\}$$

Definition 2.13. [14] An IFS $B = \{\langle t_1, \mu_B(t_1), \nu_B(t_1)\rangle \mid t_1 \in Y\}$ of Y is called an IFSU of Y if it satisfies these two conditions:

i. $\mu_B(t_1 * t_2) \geq \min\{\mu_B(t_1), \mu_B(t_2)\}$

ii. $\nu_B(t_1 * t_2) \leq \max\{\nu_B(t_1), \nu_B(t_2)\}$, $\forall\ t_1, t_2 \in Y$

Definition 2.14. [19] An IFS $B = \{\langle t_1, \mu_B(t_1), \nu_B(t_1)\rangle \mid t_1 \in Y\}$ of Y is said to be an IFID of Y if satisfies these conditions:

i. $\mu_B(0) \geq \mu_B(t_1), \nu_B(0) \leq \nu_B(t_1)$

ii. $\mu_B(t_1) \geq \min\{\mu_B(t_1 * t_2), \mu_B(t_2)\}$

iii. $\nu_B(t_1) \leq \max\{\nu_B(t_1 * t_2), \nu_B(t_2)\}$, for all $t_1, t_2 \in Y$

Definition 2.15. [8] Let μ be a fuzzy subset of Y, $\alpha \in$ [0,T] and $\beta \in [0, 1]$. A mapping $\mu^{MT}_{\beta\alpha} \mid Y \rightarrow$[0,1] is said to be fuzzy magnified $\beta\alpha$ translation of μ if it satisfies $\mu^{MT}_{\beta\alpha}(t_1) = \beta.\mu(t_1) + \alpha$, for all $t_1 \in Y$.

3. Intuitionistic Fuzzy Translation and Multiplication

For simplicity, we use the notion $B = (\mu_B, \nu_B)$ for the IFS $B = \{\langle t_1, \mu_B(t_1), \nu_B(t_1)\rangle \mid t_1 \in Y\}$. In this paper, we use $¥ = \inf\{\nu_B(t_1) \mid t_1 \in Y\}$ for any IFS $B = (\mu_B, \nu_B)$ of Y.

3.1. Intuitionistic Fuzzy Translation and Multiplication of PS Subalgebra

Definition 3.1. Let $B = (\mu_B, \nu_B)$ be an IFS of Y and let $\alpha \in [0, ¥]$. An object of the form $B^T_\alpha = ((\mu_B)^T_\alpha, (\nu_B)^T_\alpha)$ is called an IFAT of B, when $(\mu_B)^T_\alpha(t_1) = \mu_B(t_1) + \alpha$ and $(\nu_B)^T_\alpha(t_1) = \nu_B(t_1) - \alpha$, for all $t_1 \in Y$.

Example 3.2. Let $Y = \{0, 1, 2\}$ be a PS-algebra with the following Cayley table:

*	0	1	2
0	0	1	2
1	0	0	1
2	0	2	0

thus $(Y; *, 0)$ is a PS-algebra. Now, IFS $B = (\mu_B, \nu_B)$ is defined as

$$\mu_B(t_1) = \begin{cases} 0.2 & \text{if } t_1 \neq 1 \\ 0.4 & \text{if } t_1 = 1, \end{cases}$$

$$\nu_B(t_1) = \begin{cases} 0.6 & \text{if } t_1 \neq 1 \\ 0.3 & \text{if } t_1 = 1 \end{cases}$$

is an IFSU. HereÃ¼ ¥ $= inf\{\nu_B(t_1) \mid t_1 \in Y\} = 0.3$, choose $\alpha = 0.2$, then the mapping $B^T_{0.2} \mid Y \to [0, 1]$ is defined as

$$(\mu_B)^T_{0.2}(t_1) = \begin{cases} 0.4 & \text{if } t_1 \neq 1 \\ 0.6 & \text{if } t_1 = 1 \end{cases}$$

and

$$(\nu_B)^T_{0.2}(t_1) = \begin{cases} 0.4 & \text{if } t_1 \neq 1 \\ 0.1 & \text{if } t_1 = 1 \end{cases}$$

which imply that, $(\mu_B)^T_{0.2}(t_1) = \mu_B(t_1) + 0.2$ and $(\nu_B)^T_{0.2}(t_1) = \nu_B(t_1) - 0.2$, $\forall\, t_1 \in Y$ is an intuitionistic fuzzy (0.2) translation.

Theorem 3.3. Let B be an IFSU of Y and $\alpha \in [0, ¥]$. Then, IFAT B^T_α of B is an IFSU of Y.

Proof. Assume that, $t_1, t_2 \in Y$. Then,

$$\begin{aligned} (\mu_B)^T_\alpha(t_1 * t_2) &= \mu_A(t_1 * t_2) + \alpha \\ &\geq \min\{\mu_B(t_1), \mu_B(t_2)\} + \alpha \\ &= \min\{\mu_B(t_1) + \alpha, \mu_B(t_2) + \alpha\} \\ &= \min\{(\mu_B)^T_\alpha(t_1), (\mu_B)^T_\alpha(t_2)\} \end{aligned}$$

and

$$\begin{aligned} (\nu_B)^T_\alpha(t_1 * t_2) &= \nu_B(t_1 * t_2) - \alpha \\ &\leq \max\{\nu_B(t_1), \nu_B(t_2)\} - \alpha \\ &= \max\{\nu_B(t_1) - \alpha, \nu_B(t_2) - \alpha\} \\ &= \max\{(\nu_B)^T_\alpha(t_1), (\nu_B)^T_\alpha(t_2)\} \end{aligned}$$

Hence, IFAT B^T_α of B is an IFSU of Y. □

Theorem 3.4. Let B be an IFS of Y such that IFAT B^T_α of B is an IFSU of Y for some $\alpha \in [0, ¥]$. Then, B is an IFSU of Y.

PROOF. Let $B_\alpha^T = ((\mu_B)_\alpha^T, (\nu_B)_\alpha^T)$ be an IFSU of Y for some $\alpha \in [0, ¥]$ and $t_1, t_2 \in Y$. So, we have

$$\begin{aligned} \mu_B(t_1 * t_2) + \alpha &= (\mu_B)_\alpha^T(t_1 * t_2) \\ &\geq \min\{(\mu_B)_\alpha^T(t_1), (\mu_B)_\alpha^T(t_2)\} \\ &= \min\{\mu_B(t_1) + \alpha, \mu_B(t_2) + \alpha\} \\ &= \min\{\mu_B(t_1), \mu_B(t_2)\} + \alpha \end{aligned}$$

and

$$\begin{aligned} \nu_B(t_1 * t_2) - \alpha &= (\nu_B)_\alpha^T(t_1 * t_2) \\ &\leq \max\{(\nu_B)_\alpha^T(t_1), (\nu_B)_\alpha^T(t_2)\} \\ &= \max\{\nu_B(t_1) - \alpha, \nu_B(t_2) - \alpha\} \\ &= \max\{\mu_B(t_1), \nu_B(t_2)\} - \alpha \end{aligned}$$

which imply that, $\mu_B(t_1 * t_2) \geq \min\{\mu_B(t_1), \mu_B(t_2)\}$ and $\nu_B(t_1 * t_2) \leq \max\{\nu_B(t_1), \nu_B(t_2)\}$, for all $t_1, t_2 \in Y$. Hence, B is an IFSU of Y. □

Definition 3.5. Let B be an IFS of Y and $\alpha \in [0, 1]$. An object having the form $B_\alpha^M = (\mu_B)_\alpha^M, (\nu_B)_\alpha^M$ is called an IFAM of B. If $(\mu_B)_\alpha^M(t_1) = \alpha.\mu_B(t_1)$ and $(\nu_B)_\alpha^M(t_1) = \alpha.\nu_B(t_1)$, for all $t_1 \in Y$.

Example 3.6. Let $Y = \{0, 1, 2\}$ be a PS-algebra with the following Cayley table:

*	0	1	2
0	0	1	2
1	0	0	1
2	0	2	0

thus $(Y; *, 0)$ is a PS-algebra. Now IFS $B = (\mu_B, \nu_B)$ is defined as

$$\mu_B(t_1) = \begin{cases} 0.5 & \text{if } t_1 \neq 1 \\ 0.4 & \text{if } t_1 = 1, \end{cases}$$

$$\nu_B(t_1) = \begin{cases} 0.2 & \text{if } t_1 \neq 1 \\ 0.3 & \text{if } t_1 = 1 \end{cases}$$

is an IFSU, choose $\alpha = 0.2$, then the mapping $B_{(0.2)}^M \mid Y \to [0, 1]$ is defined by

$$(\mu_B)_{0.2}^M(t_1) = \begin{cases} 0.10 & \text{if } t_1 \neq 1 \\ 0.08 & \text{if } t_1 = 1 \end{cases}$$

and

$$(\mu_B)_{0.2}^M(t_1) = \begin{cases} 0.04 & \text{if } t_1 \neq 1 \\ 0.06 & \text{if } t_1 = 1 \end{cases}$$

which imply that, $(\mu_B)_{0.2}^M(t_1) = \mu_B(t_1).(0.2)$, $(\nu_B)_{0.2}^M(t_1) = \nu_B(t_1).(0.2)$, $\forall\ t_1 \in Y$ is an intuitionistic fuzzy (0.2) multiplication.

Theorem 3.7. Let IFS $B = (\mu_B, \nu_B)$ of Y and $\alpha \in [0, 1]$, if the IFAM B_α^M of B be an IFSU of Y. Then, B is an IFSU of Y.

PROOF. Assume that, B_α^M of B is an IFSU of Y for some $\alpha \in [0, 1]$. Now, for all $t_1, t_2 \in Y$, we have

$$\begin{aligned} \mu_B(t_1 * t_2).\alpha &= (\mu_B)_\alpha^M(t_1 * t_2) \\ &\geq \min\{(\mu_B)_\alpha^M(t_1), (\mu_B)_\alpha^M(t_2)\} \\ &= \min\{\mu_B(t_1).\alpha, \mu_B(t_2).\alpha\} \\ &= \min\{\mu_B(t_1), \mu_B(t_2)\}.\alpha \end{aligned}$$

and

$$\begin{aligned}\nu_B(t_1 * t_2).\alpha &= (\nu_B)^M_\alpha(t_1 * t_2)\\ &\le \max\{(\nu_B)^M_\alpha(t_1), (\nu_B)^M_\alpha(t_2)\}\\ &= \max\{\nu_B(t_1).\alpha, \nu_B(t_2).\alpha\}\\ &= \max\{\nu_B(t_1), \nu_B(t_2)\}.\alpha\end{aligned}$$

which imply that, $\mu_B(t_1 * t_2) \ge \min\{\mu_B(t_1), \mu_B(t_2)\}$ and $\nu_B(t_1 * t_2) \le \max\{\nu_B(t_1), \nu_B(t_2)\}$, for all $t_1, t_2 \in Y$. Hence, B is an IFSU of Y. □

Theorem 3.8. Let IFS $B = (\mu_B, \nu_B)$ of Y is an IFSU of Y and $\alpha \in [0, 1]$, then IFAM B^M_α of B is an IFSU of Y.

Proof. Suppose that, $B = (\mu_B, \nu_B)$ be an IFSU of Y. Then, for all $t_1, t_2 \in Y$, we have

$$\begin{aligned}(\mu_B)^M_\alpha(t_1 * t_2) &= \alpha.\mu(t_1 * t_2)\\ &\ge \alpha.\min\{(\mu_B)(t_1), (\mu_B)(t_2)\}\\ &= \min\{\alpha.\mu_B(t_1), \alpha.\mu_B(t_2)\}\\ &= \min\{(\mu_B)^M_\alpha(t_1), (\mu_B)^M_\alpha(t_2)\}\\ &\ge \min\{(\mu_B)^M_\alpha(t_1), (\mu_B)^M_\alpha(t_2)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^M_\alpha(t_1 * t_2) &= \alpha.\nu(t_1 * t_2)\\ &\le \alpha.\max\{(\nu_B)(t_1), (\nu_B)(t_2)\}\\ &= \max\{\alpha.\nu_B(t_1), \alpha.\nu_B(t_2)\}\\ &= \max\{(\mu_B)^M_\alpha(t_1), (\mu_B)^M_\alpha(t_2)\}\\ &\le \max\{(\nu_B)^M_\alpha(t_1), (\nu_B)^M_\alpha(t_2)\}\end{aligned}$$

which imply that, $\mu_B(t_1 * t_2) \ge \min\{\mu_B(t_1), \mu_B(t_2)\}$ and $\nu_B(t_1 * t_2) \le \max\{\nu_B(t_1), \nu_B(t_2)\}$, for all $t_1, t_2 \in Y$. Hence, B^M_α is an IFSU of Y. □

3.2. Intuitionistic Fuzzy Translation and Multiplication of PS Ideal

In this section, intuitionistic fuzzy α translation of IFID, intuitionistic fuzzy α multiplication of IFID, union and intersection of intuitionistic fuzzy translation of IFID are investigated through some results.

Theorem 3.9. If IFAT B^T_α of B is an intutionistic fuzzy PS ideal, then it fulfills the condition $(\mu_B)^T_\alpha(t_1 * (t_2 * t_1)) \ge (\mu_B)^T_\alpha(t_2)$ and $(\nu_B)^T_\alpha(t_1 * (t_2 * t_1)) \le (\nu_B)^T_\alpha(t_2)$.

Proof. Let IFAT B^T_α of B is an intutionistic fuzzy PS ideal. Then,

$$\begin{aligned}(\mu_B)^T_\alpha(t_1 * (t_2 * t_1)) &= \mu_B(t_1 * (t_2 * t_1)) + \alpha\\ &\ge \min\{\mu_B(t_2 * (t_1 * (t_2 * t_1))) + \alpha, \mu_B(t_2) + \alpha\}\\ &= \min\{\mu_B(0) + \alpha, \mu_B(t_2) + \alpha\}\\ &= \min\{(\mu_B)^T_\alpha(0), (\mu_B)^T_\alpha(t_2)\}\\ &= (\mu_B)^T_\alpha(t_2)\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^T_\alpha(t_1 * (t_2 * t_1)) &= \nu_B(t_1 * (t_2 * t_1)) - \alpha\\ &\le \max\{\nu_B(t_2 * (t_1 * (t_2 * t_1))) - \alpha, \nu_B(t_2) - \alpha\}\\ &= \max\{\nu_B(0) - \alpha, \nu_B(t_2) - \alpha\}\\ &= \max\{(\nu_B)^T_\alpha(0), (\nu_B)^T_\alpha(t_2)\}\\ &= (\nu_B)^T_\alpha(t_2)\end{aligned}$$

Hence, $(\mu_B)^T_\alpha(t_1 * (t_2 * t_1)) \geq (\mu_B)^T_\alpha(t_2)$ and $(\nu_B)^T_\alpha(t_1 * (t_2 * t_1)) \leq (\nu_B)^T_\alpha(t_2)$. □

Theorem 3.10. If B is an IFID of Y, then IFAT B^T_α of B is an IFID of Y, for all $\alpha \in [0, ¥]$.

Proof. Let B be an IFID of Y and $\alpha \in [0, ¥]$. Then, $(\mu_B)^T_\alpha(0) = \mu_B(0)+\alpha \geq \mu_B(t_1)+\alpha = (\mu_B)^T_\alpha(t_1)$ and $(\nu_B)^T_\alpha(0) = \nu_B(0) - \alpha \leq \nu_B(t_1) - \alpha = (\nu_B)^T_\alpha(t_1)$. Therefore,

$$\begin{aligned}(\mu_B)^T_\alpha(t_1) &= \mu_B(t_1) + \alpha,\\ &\geq \min\{\mu_B(t_1 * t_2), \mu_B(t_2)\} + \alpha\\ &= \min\{\mu_B(t_1 * t_2) + \alpha, \mu_B(t_2) + \alpha\}\\ &= \min\{(\mu_B)^T_\alpha(t_1 * t_2), (\mu_B)^T_\alpha(t_2)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^T_\alpha(t_1) &= \nu_B(t_1) - \alpha,\\ &\leq \max\{\nu_B(t_1 * t_2), \nu_B(t_2)\} - \alpha\\ &= \max\{\nu_B(t_1 * t_2) - \alpha, \nu_B(t_2) - \alpha\}\\ &= \max\{(\nu_B)^T_\alpha(t_1 * t_2), (\nu_B)^T_\alpha(t_2)\}\end{aligned}$$

for all $t_1, t_2 \in Y$ and $\alpha \in [0, ¥]$. Hence, B^T_α of B is an IFID of Y. □

Theorem 3.11. If B is an intutionistic fuzzy set of Y, such that IFAT B^T_α of B is an IFID of Y, for all $\alpha \in [0, ¥]$. Then, B is an IFID of Y.

Proof. Suppose B^T_α is an IFID of Y, where $\alpha \in [0, ¥]$ and $t_1, t_2 \in Y$ then,

$$\begin{aligned}\mu_B(0) + \alpha = (\mu_B)^T_\alpha(0) \geq (\mu_B)^T_\alpha(t_1) = \mu_B(t_1) + \alpha\\ \nu_B(0) - \alpha = (\nu_B)^T_\alpha(0) \leq (\nu_B)^T_\alpha(t_1) = \nu_B(t_1) - \alpha\end{aligned}$$

which imply, $\mu_B(0) \geq \mu_B(t_1)$ and $\nu_B(0) \leq \nu_B(t_1)$ now,

$$\begin{aligned}\mu_B(t_1) + \alpha = (\mu_B)^T_\alpha(t_1) &\geq \min\{(\mu_B)^T_\alpha(t_1 * t_2), (\mu_B)^T_\alpha(t_2)\}\\ &= \min\{\mu_B(t_1 * t_2) + \alpha, \mu_B(t_2) + \alpha\}\\ &= \min\{\mu_B(t_1 * t_2), \mu_B(t_2)\} + \alpha\end{aligned}$$

and

$$\begin{aligned}\nu_B(t_1) - \alpha = (\nu_B)^T_\alpha(t_1) &\leq \max\{(\nu_B)^T_\alpha(t_1 * t_2), (\nu_B)^T_\alpha(t_2)\}\\ &= \max\{\nu_B(t_1 * t_2) - \alpha, \nu_B(t_2) - \alpha\}\\ &= \max\{\nu_B(t_1 * t_2), \nu_B(t_2)\} - \alpha\end{aligned}$$

which imply that, $\mu_B(t_1) \geq \min\{\mu_B(t_1 * t_2), \mu_B(t_2)\}$ and $\nu_B(t_1) \leq \max\{\nu_B(t_1 * t_2), \nu_B(t_2)\}$, for all $t_1, t_2 \in Y$. Hence, B is an IFID of Y. □

Theorem 3.12. Let B be an IFID of Y for some $\alpha \in [0, ¥]$. Then, IFAT B^T_α of B is an IFSU of Y.

Proof. Assume that, $t_1, t_2 \in Y$, then

$$\begin{aligned}(\mu_B)^T_\alpha(t_1 * t_2) &= \mu_B(t_1 * t_2) + \alpha\\ &\geq \min\{\mu_B(t_2 * (t_1 * t_2)), \mu_B(t_2)\} + \alpha\\ &= \min\{\mu_B(0), \mu_B(t_2)\} + \alpha\\ &\geq \min\{\mu_B(t_1), \mu_B(t_2)\} + \alpha\\ &= \min\{\mu_B(t_1) + \alpha, \mu_B(t_2) + \alpha\}\\ &= \min\{(\mu_B)^T_\alpha(t_1), (\mu_B)^T_\alpha(t_2)\}\\ &\geq \min\{(\mu_B)^T_\alpha(t_1), (\mu_B)^T_\alpha(t_2)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^T_\alpha(t_1 * t_2) &= \nu_B(t_1 * t_2) - \alpha \\ &\leq \max\{\nu_B(t_2 * (t_1 * t_2)), \nu_B(t_2)\} - \alpha \\ &= \max\{\nu_B(0), \nu_B(t_2)\} - \alpha \\ &\leq \max\{\nu_B(t_1), \nu_B(t_2)\} - \alpha \\ &= \max\{\nu_B(t_1) - \alpha, \nu_B(t_2) - \alpha\} \\ &= \max\{(\nu_B)^T_\alpha(t_1), (\nu_B)^T_\alpha(t_2)\} \\ &\leq \max\{(\nu_B)^T_\alpha(t_1), (\nu_B)^T_\alpha(t_2)\}\end{aligned}$$

Hence, B^T_α is an IFSU of Y. □

Theorem 3.13. If IFAT B^T_α of B is an IFID of Y and $\alpha \in [0, ¥]$, then B is an IFSU of Y.

PROOF. Suppose that, B^T_α of B is an IFID of Y. Since

$$\begin{aligned}(\mu_B)(t_1 * t_2) + \alpha &= (\mu_B)^T_\alpha(t_1 * t_2) \\ &\geq \min\{(\mu_B)^T_\alpha(t_2 * (t_1 * t_2)), (\mu_B)^T_\alpha(t_2)\} \\ &= \min\{(\mu_B)^T_\alpha(0), (\mu_B)^T_\alpha(t_2)\} \\ &\geq \min\{(\mu_B)^T_\alpha(t_1), (\mu_B)^T_\alpha(t_2)\} \\ &= \min\{\mu_B(t_1) + \alpha, \mu_B(t_2) + \alpha\} \\ &= \min\{\mu_B(t_1), \mu_B(t_2)\} + \alpha\end{aligned}$$

then $\mu_B(t_1 * t_2) \geq \min\{\mu_B(t_1), \mu_B(t_2)\}$. Similarly, since

$$\begin{aligned}(\nu_B)(t_1 * t_2) - \alpha &= (\nu_B)^T_\alpha(t_1 * t_2) \\ &\leq \max\{(\nu_B)^T_\alpha(t_2 * (t_1 * t_2)), (\nu_B)^T_\alpha(t_2)\} \\ &= \max\{(\nu_B)^T_\alpha(0), (\nu_B)^T_\alpha(t_2)\} \\ &\leq \max\{(\nu_B)^T_\alpha(t_1), (\nu_B)^T_\alpha(t_2)\} \\ &= \max\{\nu_B(t_1) - \alpha, \nu_B(t_2) - \alpha\} \\ &= \max\{\nu_B(t_1), \nu_B(t_2)\} - \alpha\end{aligned}$$

then $\nu_B(t_1 * t_2) \leq \max\{\nu_B(t_1), \nu_B(t_2)\}$. Hence, B is an IFSU of Y. □

Theorem 3.14. Intersection of any two intuitionistic fuzzy translations of an intuitionistic fuzzy PS ideal B of Y is an intuitionistic fuzzy PS ideal of Y.

PROOF. Suppose, B^T_α and B^T_β are intuitionistic fuzzy translations of intuitionistic fuzzy PS ideal B of Y, where $\alpha, \beta \in [0, ¥]$ and $\alpha \leq \beta$, as we know that, B^T_α and B^T_β are intuitionistic fuzzy PS ideals of Y. Then,

$$\begin{aligned}((\mu_B)^T_\alpha \cap (\mu_B)^T_\beta)(t_1) &= \min\{(\mu_B)^T_\alpha(t_1), (\mu_B)^T_\beta(t_1)\} \\ &= \min\{\mu_B(t_1) + \alpha, \mu_B(t_1) + \beta\} \\ &= \mu_B(t_1) + \alpha \\ &= (\mu_B)^T_\alpha(t_1)\end{aligned}$$

and

$$\begin{aligned}((\nu_B)^T_\alpha \cap (\nu_B)^T_\beta)(t_1) &= \max\{(\nu_B)^T_\alpha(t_1), (\nu_B)^T_\beta(t_1)\} \\ &= \max\{\nu_B(t_1) - \alpha, \nu_B(t_1) - \beta\} \\ &= \nu_B(t_1) - \alpha \\ &= (\nu_B)^T_\alpha(t_1)\end{aligned}$$

Hence, $B^T_\alpha \cap B^T_\beta$ is an intuitionistic fuzzy PS ideal of Y. □

Theorem 3.15. Union of any two intuitionistic fuzzy translations of an IFID B of Y is an IFID of Y.

Proof. Suppose B^T_α and B^T_β are intuitionistic fuzzy translations of an IFID B of Y, where $\alpha, \beta \in [0, ¥]$ and $\alpha \leq \beta$, as we know that, B^T_α and B^T_β are intuitionistic fuzzy PS ideals of Y. Then,

$$\begin{aligned}((\mu_B)^T_\alpha \cup (\mu_B)^T_\beta)(t_1) &= \max\{(\mu_B)^T_\alpha(t_1), (\mu_B)^T_\beta(t_1)\} \\ &= \max\{\mu_B(t_1) + \alpha, \mu_B(t_1) + \beta\} \\ &= \mu_B(t_1) + \beta \\ &= (\mu_B)^T_\beta(t_1)\end{aligned}$$

and

$$\begin{aligned}((\nu_B)^T_\alpha \cup (\nu_B)^T_\beta)(t_1) &= \min\{(\nu_B)^T_\alpha(t_1), (\nu_B)^T_\beta(t_1)\} \\ &= \min\{\nu_B(t_1) - \alpha, \nu_B(t_1) - \beta\} \\ &= \nu_B(t_1) - \beta \\ &= (\nu_B)^T_\beta(t_1)\end{aligned}$$

Hence, $B^T_\alpha \cup B^T_\beta$ is an intuitionistic fuzzy PS ideal of Y. □

Theorem 3.16. Let B be an IFS of Y such that IFAM B^M_α of B is an IFID of Y for $\alpha \in (0, 1]$, then B is an IFID of Y.

Proof. Suppose that, B^M_α is an IFID of Y for $\alpha \in (0, 1]$ and $t_1, t_2 \in Y$. Then, $\alpha.\mu_B(0) = (\mu_B)^M_\alpha(0) \geq (\mu_B)^M_\alpha(t_1) = \alpha.\mu_B(t_1)$, so $\mu_B(0) \geq \mu_B(t_1)$ and $\alpha.\nu_B(0) = (\nu_B)^M_\alpha(0) \leq (\nu_B)^M_\alpha(t_1) = \alpha.\nu_B(t_1)$, so $\nu_B(0) \leq \nu_B(t_1)$. Since

$$\begin{aligned}\alpha.\mu_B(t_1) &= (\mu_B)^M_\alpha(t_1) \\ &\geq \min\{(\mu_B)^M_\alpha(t_1 * t_2), (\mu_B)^M_\alpha(t_2)\} \\ &= \min\{\alpha.\mu_B(t_1 * t_2), \alpha.\mu_B(t_2)\} \\ &= \alpha.\min\{\mu_B(t_1 * t_2), \mu_B(t_2)\}\end{aligned}$$

then $\mu_B(t_1) \geq \min\{\mu_B(t_1 * t_2), \mu_B(t_2)\}$. Similarly, since

$$\begin{aligned}\alpha.\nu_B(t_1) &= (\nu_B)^M_\alpha(t_1) \\ &\leq \max\{(\nu_B)^M_\alpha(t_1 * t_2), (\nu_B)^M_\alpha(t_2)\} \\ &= \max\{\alpha.\nu_B(t_1 * t_2), \alpha.\nu_B(t_2)\} \\ &= \alpha.\max\{\nu_B(t_1 * t_2), \nu_B(t_2)\}\end{aligned}$$

then $\nu_B(t_1) \leq \max\{\nu_B(t_1 * t_2), \nu_B(t_2)\}$. Hence, B is an IFID of Y. □

Theorem 3.17. If B is an IFID of Y, then IFAM B^M_α of B is an IFID of Y, for all $\alpha \in (0, 1]$.

Proof. Let B be an IFID of Y and $\alpha \in (0, 1]$, we have

$$\begin{aligned}(\mu_B)^M_\alpha(0) &= \alpha.\mu_B(0) \\ &\geq \alpha.\mu_B(t_1) \\ &= (\mu_B)^M_\alpha(t_1)\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^M_\alpha(0) &= \alpha.\nu_B(0)\\ &\leq \alpha.\nu_B(t_1)\\ &= (\nu_B)^M_\alpha(t_1)\end{aligned}$$

Moreover,

$$\begin{aligned}(\mu_B)^M_\alpha(t_1) &= \alpha.\mu_B(t_1)\\ &\geq \alpha.\min\{\mu_B(t_1 * t_2), \mu_B(t_2)\}\\ &= \min\{\alpha.\mu_B(t_1 * t_2), \alpha.\mu_B(t_2)\}\\ &= \min\{(\mu_B)^M_\alpha(t_1 * t_2), (\mu_B)^M_\alpha(t_2)\}\\ &\geq \min\{(\mu_B)^M_\alpha(t_1 * t_2), (\mu_B)^M_\alpha(t_2)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^M_\alpha(t_1) &= \alpha.\nu_B(t_1)\\ &\leq \alpha.\max\{\nu_B(t_1 * t_2), \nu_B(t_2)\}\\ &= \max\{\alpha.\nu_B(t_1 * t_2), \alpha.\nu_B(t_2)\}\\ &= \max\{(\nu_B)^M_\alpha(t_1 * t_2), (\nu_B)^M_\alpha(t_2)\}\\ &\leq \max\{(\nu_B)^M_\alpha(t_1 * t_2), (\nu_B)^M_\alpha(t_2)\}\end{aligned}$$

Hence, B^M_α of B is an IFID of Y, $\forall\ \alpha \in (0,1]$. □

Theorem 3.18. Let B be an IFID of Y and $\alpha \in [0,1]$. Then, IFAM B^M_α of B is an IFSU of Y.

PROOF. Suppose that, $t_1, t_2 \in Y$, we have

$$\begin{aligned}(\mu_B)^M_\alpha(t_1 * t_2) &= \alpha.\mu_B(t_1 * t_2)\\ &\geq \alpha.\min\{\mu_B(t_2 * (t_1 * t_2)), \mu_B(t_2)\}\\ &= \alpha.\min\{\mu_B(0), \mu_B(t_2)\}\\ &\geq \alpha.\min\{\mu_B(t_1), \mu_B(t_2)\}\\ &= \min\{\alpha.\mu_B(t_1), \alpha.mu_B(t_2)\}\\ &= \min\{(\mu_B)^M_\alpha(t_1), (\mu_B)^M_\alpha(t_2)\}\\ &\geq \min\{(\mu_B)^M_\alpha(t_1), (\mu_B)^M_\alpha(t_2)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^M_\alpha(t_1 * t_2) &= \alpha.\nu_B(t_1 * t_2)\\ &\leq \alpha.\max\{\nu_B(t_2 * (t_1 * t_2)), \nu_B(t_2)\}\\ &= \alpha.\max\{\nu_B(0), \nu_B(t_2)\}\\ &\leq \alpha.\max\{\nu_B(t_1), \nu_B(t_2)\}\\ &= \max\{\alpha.\nu_B(t_1), \alpha.\nu_B(t_2)\}\\ &= \max\{(\nu_B)^M_\alpha(t_1), (\nu_B)^M_\alpha(t_2)\}\\ &\leq \max\{(\nu_B)^M_\alpha(t_1), (\nu_B)^M_\alpha(t_2)\}\end{aligned}$$

Hence, B^M_α is an IFSU of Y. □

Theorem 3.19. If the IFAM B^M_α of B is an IFID of Y, for $\alpha \in (0,1]$. Then, B is an intuitionistic fuzzy PS-subalgebra of Y.

Proof. Assume that, B_α^M of B is an IFID of Y. Since

$$\begin{aligned}\alpha.(\mu_B)(t_1 * t_2) &= (\mu_B)_\alpha^M(t_1 * t_2)\\ &\geq \min\{(\mu_B)_\alpha^M(t_2 * (t_1 * t_2)), (\mu_B)_\alpha^M(t_2)\}\\ &= \min\{(\mu_B)_\alpha^M(0), (\mu_B)_\alpha^M(t_2)\}\\ &\geq \min\{(\mu_B)_\alpha^M(t_1), (\mu_B)_\alpha^M(t_2)\}\\ &= \min\{\alpha.\mu_B(t_1), \alpha.\mu_B(t_2)\}\\ &= \alpha.\min\{\mu_B(t_1), \mu_B(t_2)\}\end{aligned}$$

then $\mu_B(t_1 * t_2) \geq \min\{\mu_B(t_1), \mu_B(t_2)\}$. Similarly, since

$$\begin{aligned}\alpha.(\nu_B)(t_1 * t_2) &= (\nu_B)_\alpha^M(t_1 * t_2)\\ &\leq \max\{(\nu_B)_\alpha^M(t_2 * (t_1 * t_2)), (\nu_B)_\alpha^M(t_2)\}\\ &= \max\{(\nu_B)_\alpha^M(0), (\nu_B)_\alpha^M(t_2)\}\\ &\leq \max\{(\nu_B)_\alpha^M(t_1), (\nu_B)_\alpha^M(t_2)\}\\ &= \max\{\alpha.\nu_B(t_1), \alpha.\nu_B(t_2)\}\\ &= \alpha.\max\{\nu_B(t_1), \nu_B(t_2)\}\end{aligned}$$

then $\nu_B(t_1 * t_2) \leq \max\{\nu_B(t_1), \nu_B(t_2)\}$. Hence, B is an IFSU of Y. □

Theorem 3.20. Intersection of any two intuitionistic fuzzy multiplications of an IFID B of Y is an IFID of Y.

Proof. Suppose that, B_α^M and B_β^M are intuitionistic fuzzy multiplications of IFID B of Y, where $\alpha, \beta \in [0, 1]$ and $\alpha \leq \beta$, as we know that B_α^M and B_β^M are IFIDs of Y. Then,

$$\begin{aligned}((\mu_B)_\alpha^M \cap (\mu_B)_\beta^M)(t_1) &= \min\{(\mu_B)_\alpha^M(t_1), (\mu_B)_\beta^M(t_1)\}\\ &= \min\{\mu_B(t_1).\alpha, \mu_B(t_1).\beta\}\\ &= \mu_B(t_1).\alpha\\ &= (\mu_B)_\alpha^M(t_1)\end{aligned}$$

and

$$\begin{aligned}((\nu_B)_\alpha^M \cap (\nu_B)_\beta^M)(t_1) &= \max\{(\nu_B)_\alpha^M(t_1), (\nu_B)_\beta^M(t_1)\}\\ &= \max\{\nu_B(t_1).\alpha, \nu_B(t_1).\beta\}\\ &= \nu_B(t_1).\alpha\\ &= (\nu_B)_\alpha^M(t_1)\end{aligned}$$

Hence, $B_\alpha^M \cap B_\beta^M$ is IFID of Y. □

Theorem 3.21. Union of any two intuitionistic fuzzy multiplications of an IFID B of Y is an IFID of Y.

Proof. Suppose that, B_α^M and B_β^M are intuitionistic fuzzy multiplications of an IFID B of Y, where $\alpha, \beta \in [0, 1]$ and $\alpha \leq \beta$ and B_α^M and B_β^M are IFIDs of Y. Then,

$$\begin{aligned}((\mu_B)_\alpha^M \cup (\mu_B)_\beta^M)(t_1) &= \max\{(\mu_B)_\alpha^M(t_1), (\mu_B)_\beta^M(t_1)\}\\ &= \max\{\mu_B(t_1).\alpha, \mu_B(t_1).\beta\}\\ &= \mu_B(t_1).\beta\\ &= (\mu_B)_\beta^M(t_1)\end{aligned}$$

and

$$\begin{aligned}((\nu_B)^M_\alpha \cup (\nu_B)^M_\beta)(t_1) &= \min\{(\nu_B)^M_\alpha(t_1), (\nu_B)^M_\beta(t_1)\} \\ &= \min\{\nu_B(t_1).\alpha, \nu_B(t_1).\beta\} \\ &= \nu_B(t_1).\beta \\ &= (\nu_B)^M_\beta(t_1)\end{aligned}$$

Hence, $B^M_\alpha \cup B^M_\beta$ is IFID of Y. □

3.3. Intuitionistic Fuzzy Magnified $\beta\alpha$ Translation

In this section, the notion of intuitionistic fuzzy magnified $\beta\alpha$ translation IFMBAT is presented and investigated.

Definition 3.22. Let $B = (\mu_B, \nu_B)$ be an IFS of Y and $\alpha \in [0, ¥]$, $\beta \in [0, 1]$. An object having the form $B^{MT}_{\beta\,\alpha} = \{(\mu_B)^{MT}_{\beta\,\alpha}, (\nu_B)^{MT}_{\beta\,\alpha}\}$ is said to be an IFMBAT of B if it satisfies $(\mu_B)^{MT}_{\beta\,\alpha}(t_1) = \beta.\mu_B(t_1) + \alpha$ and $(\nu_B)^{MT}_{\beta\,\alpha}(t_1) = \beta.\nu_B(t_1) - \alpha$, $\forall\ t_1 \in Y$.

Example 3.23. Let $Y = \{0, 1, 2\}$ be a PS-algebra defined in example 2.1. A IFS $B = (\mu_B, \nu_B)$ of Y is defined as:

$$\mu_B(t_1) = \begin{cases} 0.3 & \text{if } t_1 \neq 2 \\ 0.5 & \text{if } t_1 = 2 \end{cases}$$

$$\nu_B(t_1) = \begin{cases} 0.6 & \text{if } t_1 \neq 2 \\ 0.4 & \text{if } t_1 = 2 \end{cases}$$

is an IFSU and $¥ = inf\{\nu_B(t_1) \mid t_1 \in Y\} = 0.4$, choose $\alpha = 0.1 \in [0, ¥]$ and $\beta = 0.3 \in [0, 1]$, then the mapping $B^{MT}_{(0.3)\,(0.1)} \mid Y \to [0, 1]$ is given as

$$(\mu_B)^{MT}_{(0.3)\,(0.1)}(t_1) = \begin{cases} (0.3)(0.3) + (0.1) = 0.19 & \text{if } t_1 \neq 2 \\ (0.3)(0.5) + (0.1) = 0.25 & \text{if } t_1 = 2 \end{cases}$$

and

$$(\nu_B)^{MT}_{(0.3)\,(0.1)}(t_1) = \begin{cases} (0.3)(0.6) - (0.1) = 0.08 & \text{if } t_1 \neq 2 \\ (0.3)(0.4) - (0.1) = 0.02 & \text{if } t_1 = 2 \end{cases}$$

which imply that, $(\mu_B)^{MT}_{(0.3)\,(0.1)}(t_1) = (0.3).\mu_B(t_1) + 0.1$ and $(\nu_B)^{MT}_{(0.3)\,(0.1)}(t_1) = (0.3).\nu_B(t_1) - 0.1$, $\forall\ t_1 \in Y$. Hence, $B^{MT}_{(0.3)\,(0.1)}$ is an intuitionistic fuzzy magnified (0.3)(0.1) translation.

Theorem 3.24. Let B be an intuitionistic fuzzy subset of Y, such that $\alpha \in [0, ¥]$, $\beta \in [0, 1]$ and a mapping $B^{MT}_{\beta\,\alpha} \mid Y \to [0, 1]$ is IFMBAT of B, if B is IFSU of Y. Then, $B^{MT}_{\beta\,\alpha}$ is IFSU of Y.

Proof. Let B be an IFS of Y, $\alpha \in [0, ¥]$, $\beta \in [0, 1]$ and a mapping $B^{MT}_{\beta\,\alpha} \mid Y \to [0, 1]$ is IFMBAT of B. Suppose B is an IFSU of Y. Then,

$$\begin{aligned}\mu_B(t_1 * t_2) &\geq \min\{\mu_B(t_1), \mu_B(t_2)\} \\ \nu_B(t_1 * t_2) &\leq \max\{\nu_B(t_1), \nu_B(t_2)\}\end{aligned}$$

Moreover,

$$\begin{aligned}(\mu_B)^{MT}_{\beta\,\alpha}(t_1 * t_2) &= \beta.\mu_B(t_1 * t_2) + \alpha \\ &\geq \beta.\min\{\mu_B(t_1), \mu_B(t_2)\} + \alpha \\ &= \min\{\beta.\mu_B(t_1) + \alpha, \beta.\mu_B(t_2) + \alpha\} \\ &= \min\{(\mu_B)^{MT}_{\beta\,\alpha}(t_1), (\mu_B)^{MT}_{\beta\,\alpha}(t_2)\} \\ &\geq \min\{(\mu_B)^{MT}_{\beta\,\alpha}(t_1), (\mu_B)^{MT}_{\beta\,\alpha}(t_2)\}\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^{MT}_{\beta\alpha}(t_1 * t_2) &= \beta.\nu_B(t_1 * t_2) - \alpha \\ &\leq \beta.\max\{\nu_B(t_1), \nu_B(t_2)\} - \alpha \\ &= \max\{\beta.\nu_B(t_1) - \alpha, \beta.\nu_B(t_2) - \alpha\} \\ &= \max\{(\nu_B)^{MT}_{\beta\alpha}(t_1), (\nu_B)^{MT}_{\beta\alpha}(t_2)\} \\ &\leq \max\{(\nu_B)^{MT}_{\beta\alpha}(t_1), (\nu_B)^{MT}_{\beta\alpha}(t_2)\}\end{aligned}$$

Hence, IFMBAT $B^{MT}_{\beta\alpha}$ is an IFSU of Y. □

Theorem 3.25. Let B be an IFS of Y, such that $\alpha \in [0, ¥]$, $\beta \in [0,1]$ and a mapping $B^{MT}_{\beta\alpha} \mid Y \to [0,1]$ is IFMBAT of B, if $B^{MT}_{\beta\alpha}$ is IFSU of Y. Then, B is an IFSU of Y.

Proof. Let B be an intuitionistic fuzzy subset of Y, where $\alpha \in [0, ¥]$, $\beta \in [0,1]$ and a mapping $B^{MT}_{\beta\alpha} \mid Y \to [0,1]$ is IFMBAT of B. Let $B^{MT}_{\beta\alpha} = \{(\mu_B)^{MT}_{\beta\alpha}, (\nu_B)^{MT}_{\beta\alpha}\}$ is an IFSU of Y, we have

$$\begin{aligned}\beta.\mu_B(t_1 * t_2) + \alpha &= (\mu_B)^{MT}_{\beta\alpha}(t_1 * t_2) \\ &\geq \min\{(\mu_B)^{MT}_{\beta\alpha}(t_1), (\mu_B)^{MT}_{\beta\alpha}(t_2)\} \\ &= \min\{\beta.\mu_B(t_1) + \alpha, \beta.\mu_B(t_2) + \alpha\} \\ &= \beta.\min\{\mu_B(t_2), \mu_B(t_1)\} + \alpha\end{aligned}$$

and

$$\begin{aligned}\beta.\nu_B(t_1 * t_2) - \alpha &= (\nu_B)^{MT}_{\beta\alpha}(t_1 * t_2) \\ &\leq \max\{(\nu_B)^{MT}_{\beta\alpha}(t_1), (\nu_B)^{MT}_{\beta\alpha}(t_2)\} \\ &= \max\{\beta.\nu_B(t_2) - \alpha, \beta.\nu_B(t_1) - \alpha\} \\ &= \beta.\max\{\nu_B(t_1), \nu_B(t_1)\} - \alpha\end{aligned}$$

which imply that, $\mu_B(t_1 * t_2) \geq \min\{\mu_B(t_1), \mu_B(t_2)\}$ and $\nu_B(t_1 * t_2) \leq \max\{\nu_B(t_1), \nu_B(t_2)\}$, for all $t_1, t_2 \in Y$. Hence, B is an IFSU of Y. □

Theorem 3.26. If B is an IFID of Y, then IFMBAT $B^{MT}_{\beta\alpha}$ of B is an IFID of Y, for all $\alpha \in [0, ¥]$ and $\beta \in (0,1]$.

Proof. Suppose that $B = (\mu_B, \nu_B)$ be an IFID of Y. Then,

$$\begin{aligned}(\mu_B)^{MT}_{\beta\alpha}(0) &= \beta.\mu_B(0) + \alpha \\ &\geq \beta.\mu_B(t_1) + \alpha \\ &= (\mu_B)^{MT}_{\beta\alpha}(t_1)\end{aligned}$$

and

$$\begin{aligned}(\nu_B)^{MT}_{\beta\alpha}(0) &= \beta.\nu_B(0) - \alpha \\ &\leq \beta.\nu_B(t_1) - \alpha \\ &= (\nu_B)^{MT}_{\beta\alpha}(t_1)\end{aligned}$$

Moreover, since

$$\begin{aligned}(\mu_B)^{MT}_{\beta\alpha}(t_1) &= \beta.\mu_B(t_1) + \alpha \\ &\geq \beta.\min\{\mu_B(t_1 * t_2), \mu_B(t_2)\} + \alpha \\ &= \min\{\beta.\mu_B(t_1 * t_2) + \alpha, \beta.\mu_B(t_2) + \alpha\} \\ &= \min\{(\mu_B)^{MT}_{\beta\alpha}(t_1 * t_2), (\mu_B)^{MT}_{\beta\alpha}(t_2)\}\end{aligned}$$

then $(\mu_B)^{MT}_{\beta\alpha}(t_1) \geq \min\{(\mu_B)^{MT}_{\beta\alpha}(t_1 * t_2), (\mu_B)^{MT}_{\beta\alpha}(t_2)\}$, for all $t_1, t_2 \in Y$ and $\forall\ \alpha \in [0, ¥]$, $\beta \in (0, 1]$. Similarly, since

$$\begin{aligned}(\nu_B)^{MT}_{\beta\alpha}(t_1) &= \beta.\nu_B(t_1) - \alpha \\ &\leq \beta.\max\{\nu_B(t_1 * t_2), \nu_B(t_2)\} - \alpha \\ &= \max\{\beta.\nu_B(t_1 * t_2) - \alpha, \beta.\nu_B(t_2) - \alpha\} \\ (\nu_B)^{MT}_{\beta\alpha}(t_1) &= \max\{(\nu_B)^{MT}_{\beta\alpha}(t_1 * t_2), (\nu_B)^{MT}_{\beta\alpha}(t_2)\}\end{aligned}$$

then $(\nu_B)^{MT}_{\beta\alpha}(t_1) \leq \max\{(\nu_B)^{MT}_{\beta\alpha}(t_1 * t_2), (\nu_B)^{MT}_{\beta\alpha}(t_2)\}$, for all $t_1, t_2 \in Y$ and $\forall\ \alpha \in [0, ¥]$, $\beta \in (0, 1]$. Hence, $B^{MT}_{\beta\alpha}$ of B is an IFID of Y. □

Theorem 3.27. If B is an intuitionistic fuzzy set of Y, such that IFMBAT $B^{MT}_{\beta\alpha}$ of B is an IFID of Y, for all $\alpha \in [0, ¥]$ and $\beta \in (0, 1]$. Then, B is an IFID of Y.

Proof. Suppose that IFMBAT $B^{MT}_{\beta\alpha}$ is an IFID of Y for some $\alpha \in [0, ¥]$, $\beta \in (0, 1]$ and $t_1, t_2 \in Y$, then

$$\begin{aligned}\beta.\mu_B(0) + \alpha &= (\mu_B)^{MT}_{\beta\alpha}(0) \\ &\geq (\mu_B)^{MT}_{\beta\alpha}(t_1) \\ &= \beta.\mu_B(t_1) + \alpha\end{aligned}$$

and

$$\begin{aligned}\beta.\nu_B(0) - \alpha &= (\nu_B)^{MT}_{\beta\alpha}(0) \\ &\leq (\nu_B)^{MT}_{\beta\alpha}(t_1) \\ &= \beta.\nu_B(t_1) - \alpha\end{aligned}$$

which imply that, $\mu_B(0) \geq \mu_B(t_1)$ and $\nu_B(0) \leq \nu_B(t_1)$. Now, we have

$$\begin{aligned}\beta.\mu_B(t_1) + \alpha &= (\mu_B)^{MT}_{\beta\alpha}(t_1) \\ &\geq \min\{(\mu_B)^{MT}_{\beta\alpha}(t_1 * t_2), (\mu_B)^{MT}_{\beta\alpha}(t_2)\} \\ &= \min\{\beta.\mu_B(t_1 * t_2) + \alpha, \beta.\mu_B(t_2) + \alpha\} \\ &= \beta.\min\{\mu_B(t_1 * t_2), \mu_B(t_2)\} + \alpha\end{aligned}$$

and

$$\begin{aligned}\beta.\nu_B(t_1) - \alpha &= (\nu_B)^{MT}_{\beta\alpha}(t_1) \\ &\leq \max\{(\nu_B)^{MT}_{\beta\alpha}(t_1 * t_2), (\nu_B)^{MT}_{\beta\alpha}(t_2)\} \\ &= \max\{\beta.\nu_B(t_1 * t_2) - \alpha, \beta.\nu_B(t_2) - \alpha\} \\ &= \beta.\max\{\nu_B(t_1 * t_2), \nu_B(t_2)\} - \alpha\end{aligned}$$

which imply that, $\mu_B(t_1) \geq \min\{\mu_B(t_1 * t_2), \mu_B(t_2)\}$ and $\nu_B(t_1) \leq \max\{\nu_B(t_1 * t_2), \nu_B(t_2)\}$, for all $t_1, t_2 \in Y$. Hence, B is an IFID of Y.

□

Theorem 3.28. Intersection of any two IFMBATs $B^{MT}_{\beta\alpha}$ of an IFID B of Y is an IFID of Y.

Proof. Suppose that, $B^{MT}_{\beta\alpha}$ and $B^{MT}_{\acute{\beta}\acute{\alpha}}$ are two IFMBATs of IFID B of Y, where $\alpha, \acute{\alpha} \in [0, ¥]$ and $\beta, \acute{\beta} \in (0, 1]$. Assume $\alpha \leq \acute{\alpha}$, and $\beta = \acute{\beta}$, then by Theorem 3.26, $B^{MT}_{\beta\alpha}$ and $B^{MT}_{\acute{\beta}\acute{\alpha}}$ are IFIDs of Y.

Therefore,

$$\begin{aligned}((\mu_B)^{MT}_{\beta\alpha} \cap (\mu_B)^{MT}_{\acute{\beta}\acute{\alpha}})(t_1) &= \min\{(\mu_B)^{MT}_{\beta\alpha}(t_1), (\mu_B)^{MT}_{\acute{\beta}\acute{\alpha}}(t_1)\} \\ &= \min\{\beta.\mu_B(t_1) + \alpha, \acute{\beta}.\mu_B(t_1) + \acute{\alpha}\} \\ &= \beta.\mu_B(t_1) + \alpha \\ &= (\mu_B)^{MT}_{\beta\alpha}(t_1)\end{aligned}$$

and

$$\begin{aligned}((\nu_B)^{MT}_{\beta\alpha} \cap (\nu_B)^{MT}_{\acute{\beta}\acute{\alpha}})(t_1) &= \max\{(\nu_B)^{MT}_{\beta\alpha}(t_1), (\nu_B)^{MT}_{\acute{\beta}\acute{\alpha}}(t_1)\} \\ &= \max\{\beta.\nu_B(t_1) - \alpha, \acute{\beta}.\nu_B(t_1) - \acute{\alpha}\} \\ &= \beta.\nu_B(t_1) - \alpha \\ &= (\nu_B)^{MT}_{\beta\alpha}(t_1)\end{aligned}$$

Hence, $B^{MT}_{\beta\alpha} \cap B^{MT}_{\acute{\beta}\acute{\alpha}}$ is IFID of Y. □

Theorem 3.29. Union of any two IFMBATs $B^{MT}_{\beta\alpha}$ of an IFID B of Y is an IFID of Y.

PROOF. Suppose that, $B^{MT}_{\beta\alpha}$ and $B^{MT}_{\acute{\beta}\acute{\alpha}}$ are two IFMBATs of IFID B of Y, where $\alpha, \acute{\alpha} \in [0, ¥]$ and $\beta, \acute{\beta} \in (0, 1]$. Assume $\alpha \leq \acute{\alpha}$, and $\beta = \acute{\beta}$, then by Theorem 3.26, $B^{MT}_{\beta\alpha}$ and $B^{MT}_{\acute{\beta}\acute{\alpha}}$ are IFIDs of Y. Therefore,

$$\begin{aligned}((\mu_B)^{MT}_{\beta\alpha} \cup (\mu_B)^{MT}_{\acute{\beta}\acute{\alpha}})(t_1) &= \max\{(\mu_B)^{MT}_{\beta\alpha}(t_1), (\mu_B)^{MT}_{\acute{\beta}\acute{\alpha}}(t_1)\} \\ &= \max\{\beta.\mu_B(t_1) + \alpha, \acute{\beta}.\mu_B(t_1) + \acute{\alpha}\} \\ &= \acute{\beta}.\mu_B(t_1) + \acute{\alpha} \\ &= (\mu_B)^{MT}_{\acute{\beta}\acute{\alpha}}(t_1)\end{aligned}$$

and

$$\begin{aligned}((\nu_B)^{MT}_{\beta\alpha} \cup (\nu_B)^{MT}_{\acute{\beta}\acute{\alpha}})(t_1) &= \min\{(\nu_B)^{MT}_{\beta\alpha}(t_1), (\nu_B)^{MT}_{\acute{\beta}\acute{\alpha}}(t_1)\} \\ &= \min\{\beta.\nu_B(t_1) - \alpha, \acute{\beta}.\nu_B(t_1) - \acute{\alpha}\} \\ &= \acute{\beta}.\nu_B(t_1) - \acute{\alpha} \\ &= (\nu_B)^{MT}_{\acute{\beta}\acute{\alpha}}(t_1)\end{aligned}$$

Hence, $B^{MT}_{\beta\alpha} \cup B^{MT}_{\acute{\beta}\acute{\alpha}}$ is IFID of Y. □

4. Conclusion

In this paper, IFAT, IFAM and IFMBAT of PS-algebra are discussed with the help of subalgebras and ideals. Moreover, IFMBAT of PS-algebra is studied, which gave us new line of thought to apply PS-algebra on some other sets. For future work, PS-algebra can be applied on interval valued intuitionistic fuzzy magnified translation, neutrosophic cubic magnified translation and T-neutrosophic cubic magnified translation.

Conflicts of Interest

The authors declare no conflict of interest.

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