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Gradation of continuity for mappings between L-soft topological spaces

L-esnek topolojik uzaylar arasındaki dönüşümler için sürekliliğin derecelendirmesi

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Abstract
In this article, we aim to present the degrees of continuity, closedness and openness for a soft mapping which is defined between L-soft topological spaces, where L is a complete DeMorgan algebra. We propose the gradation of continuity for a soft mapping with the help of the soft closure operators and by considering the fuzzy soft inclusion which depends on the lattice implication. We also observe many characterizations and properties of the degree of the continuity. Then, we present the degree of openness for a soft mapping with help of the soft interior operators. At the end, we investigate the relations among the proposed concepts; the degree of continuity, closedness and openness in a natural way.

Keywords: Closure, Continuity, Fuzzy soft set, L-soft topology, Openness, Soft maping

Öz
Bu çalışmada, L bir tam DeMorgan cebiri olmak üzere, L-esnek topolojik uzaylar arasında tanımlanan esnek dönüşümler için süreklilik, kapalılık ve açıklığın derecelendirmesini sunmayı amaçladık. Esnek kapanış operatörleri yardımıyla ve kafes gerektirme işlemine dayanan bulanık esnek içerme bağıntısının da dikkate alınmasıyla esnek bir dönüşüm için sürekliliğin derecelendirmesini ifade ettik. Ayrıca sürekliliğin bu derecelendirmesinin birçok karakterizasyonunu ve özelliğini gözlemledik. Daha sonra, esnek iç operatörlerinin yardımıyla esnek dönüşümler için açıklığın derecelendirmesini verdik. En sonunda, ifade edilen yapılar olan sürekliliğin, kapalılığın ve açıklığın derecelendirmeleri arasındaki ilişkileri doğal bir yolla inceledik.

***Anahtar kelimeler**: Kapanış, Süreklilik, Bulanık esnek küme, L-esnek topoloji, Açıklık, Esnek dönüşüm*

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1. Introduction

1. Giriş

The soft set theory, described by Molodtsov (1999), is one of the mathematical methods that aims to identify phenomena and concepts of ambiguous, undefined and imprecise meaning. According to this definition, a soft set is a parameterized family of classical sets. This means that parameters play the key role in this definition. Since the objects obtained from the experiments, human decisions, the datas in the computer sciences and so on, depend on some parameters, this new perspective idea drew attention of pure mathematicians as well as researchers in the area of applied mathematics. In the general topology, a set is open or not open, and this idea is based on the two-valued logic. Hovewer, in the fuzzy topology a set (or a fuzzy set) is open to some degree, and this idea is based on the fuzzy logic (or so called many valued logic) which gives some belongingness degrees to the elements of the sets. This way of gradation is very useful in many areas, since the real life problems are not black or white, they have greyness and fuzzy logic helps us to model these kinds of phenomena. The birth of the fuzzy soft set theory (Maji et al., 2001) which is gained by combining the soft set theory and the fuzzy set theory (Zadeh, 1965), has accelareted the investigations in many directions. The idea of fuzzy soft set theory is based on the parametric gradation of belongingness. So, it is a more suitable tool for the real life modellings. Inspring by this idea, mathematicians working in the pure sciences embedded these set theories to their own branches. Up to now, lots of spectacular and creative researches about the theories of soft sets and the fuzzy soft sets have been considered by some scholars (Roy & Maji, 2007; Çetkin, 2019; Terepeta, 2019; Kocinac et al., 2021; Al-jarrah et al., 2022; Çetkin, 2022).

The concept of continuous function is basic to much of mathematics since it is a special function between some structured spaces. Despite the sets (or fuzzy sets) have some openness degrees in the fuzzy topological spaces, being a continuous map, an open or a closed map are defined as in the classical case. In order to make the structures more compatible with the spirit of the fuzzy theory, Pang (2014) defined the graded continuity and openness for mappings between L-fuzzifying topological spaces. He initiated to give some degrees of continuous mappings and open mappings in the corresponding spaces. Later, the degrees of continuous mappings and open mappings between L-fuzzy topological spaces have been presented by Liang and Shi (2014). Further, the concept of L-continuity between L-topological spaces has been presented and some different characterizations have been described (Xiu & Li, 2019).

By inspired from the former theories, our main intention is to consider the continuous (open and closed) mappings between L-soft topological spaces, in the view of many valued logic by giving some degrees to what extent the mappings are contiuous (open and closed). Hence, in this study, we deal with the formulation of the gradation of continuity for soft mappings between L-soft topological spaces. In this manner, we propose some operators denoted by "$Cont, Close, Open$", respectively, which assigns each soft mapping to some value of the underlying lattice L, which shows “the degree” of continuity, closedness and the openness of the given soft mapping, respectively. In this way, each soft mapping can be regarded as continuous, closed or open to some degrees. Hence, we obtain a compatible continuity definition in the observed structured spaces.

2. Preliminaries

2. Ön bilgiler

Let X be a nonempty set and L be a completely distributive DeMorgan algebra, i.e., completely distributive lattice with an order reversing involution $':L \to L$. The smallest element and the largest element in L is denoted by $0_L, 1_L$, respectively. L^X denotes the set of all L-fuzzy subsets of X. For more details about lattices, one can see (Gierz et al., 1980; Liu & Luo, 1997).

The binary operation $\mapsto$ on the complete DeMorgan algebra L is given by

$$\alpha \mapsto \beta = \vee \{\gamma \in L \mid \alpha \wedge \gamma \leq \beta \}$$

For all $\alpha, \beta, \gamma, \delta \in L$ and $\{\alpha_i\}_i, \{\beta_i\}_i \subseteq L$, the followings are valid:

(1) $\alpha \mapsto \beta \geq \gamma$ iff $\alpha \wedge \gamma \leq \beta$.
(2) $\alpha \mapsto \beta = 1_L \Leftrightarrow \alpha \leq \beta$.
(3) $\alpha \mapsto \bigwedge_i \beta_i = \bigwedge_i (\alpha \mapsto \beta_i)$.
(4) b$(\bigvee_i \alpha_i) \mapsto \beta = \bigwedge_i (\alpha_i \mapsto \beta)$.
(5) $(\alpha \mapsto \gamma) \wedge (\gamma \mapsto \beta) \leq \alpha \mapsto \beta$.
(6) $\alpha \leq \beta$ implies $\gamma \mapsto \alpha \leq \gamma \mapsto \beta$.
(7)) $\alpha \leq \beta$ implies $\beta \mapsto \gamma \leq \alpha \mapsto \gamma$.

Let E be an arbitarary nonempty set viewed on the set of parameters. The parameterized version of an L-fuzzy set is called an L-fuzzy soft set and it is defined as follows.

Definition 2.1. (Maji et al., 2001; Çetkin & Aygün, 2014) f is called an L-fuzzy soft set on X, where f is a mapping from E into the set of all L-fuzzy sets, L^X. This means that, $f_e \coloneqq f(e): X \to L$ is an L-fuzzy set on X, for each parameter $e \in E$. The fanily of all L-fuzzy soft sets on X is denoted by $(L^X)^E$.

Definition 2.2. (Ahmad & Kharal, 2009; Çetkin, 2014) Let f, g be two L-fuzzy soft sets on X, then the set-theoretical operations are as follows:

(1) f is called an L-fuzzy soft subset of g and denoted by $f \sqsubseteq g$, if $f_e \leq g_e$, for each $e \in E$. f, g are called equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.
(2) the union of f and g, is an L-fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.
(3) the intersection of f and g, is an L-fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.
(4) the complement of an L-fuzzy soft set f, is denoted by f', where $f': E \to L^X$ is denied by $f'_e(x) = \big(f_e(x)\big)'$, for each $e \in E$ and $x \in X$. It is clear that $(f')' = f$.

Definition2.3. (Çetkin, 2014)

(1) An L-fuzzy soft set f on X, is called a null (empty) L-fuzzy soft set $on\ X$, denoted by $\tilde{0}_X$, if $f_e(x) = 0_L$, for each $e \in E$ and $x \in X$.
(2) An L-fuzzy soft set f on X, is called an absolute (universal) L-fuzzy soft set $on\ X$, denoted by $\tilde{1}_X$, if $f_e(x) = 1_L$, for each $e \in E$ and $x \in X$.

Definition 2.4. (Kharal & Ahmad, 2009; Aygünoğlu & Aygün, 2009; Çetkin, 2014) Let $\varphi : X_1 \to X_2$ and $\varphi : E_1 \to E_2$ be two crisp functions, where E_1 and E_2 are the parameter sets for the classical sets X_1 and X_2, respectively. Then the pair $(\varphi, \psi): (X_1, E_1) \to (X_2, E_2)$ (which is denoted by $\varphi_\psi \coloneqq (\varphi, \psi)$, for short) is said to be a soft mpping from X_1 to X_2. Then the image and the inverse image (pre-image) are defined by follows.

(1) Let f be an L-fuzzy soft set on X_1, then its image under φ_ψ is an L-fuzzy soft set on X_2,
$\varphi_\psi(f)_k(y) = \bigvee_{y=\varphi(x)} \bigvee_{k=\psi(e)} f_e(x)$, for each $k \in E_2$ and $y \in X_2$.

(2) Let g be an L-fuzzy soft set on X_2, then its pre-image under φ_ψ is an L-fuzzy soft set on X_1,
$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}\big(\varphi(x)\big)$, for each $e \in E_1$ and $x \in X_1$.

(3) If φ and ψ are both surjective (injective), then the soft mapping φ_ψ is called surjective (injective).

(4) Let φ_ψ be a soft mapping from X_1 to X_2, and $\varphi^*_{\psi^*}$ be a soft mapping from X_2 to X_3. Then the composition $\varphi^*_{\psi^*} \circ \varphi_\psi$ is a soft mapping from X_1 to X_3, and it is defined as follows $\big(\varphi^*_{\psi^*} \circ \varphi_\psi\big) = (\varphi^* \circ \varphi)_{\psi^* \circ \psi}$.

Proposition 2.5. (Kharal & Ahmad, 2009; Çetkin, 2014) Let $\varphi_\psi: (X_1, E_1) \to (X_2, E_2)$ be a soft mapping. Then the followings are satisfied for each $f, f_1, f_2 \in (L^{X_1})^{E_1}$ and $g, g_1, g_2 \in (L^{X_2})^{E_2}$,

(1) $f_1 \sqsubseteq f_2$ implies $\varphi_\psi(f_1) \sqsubseteq \varphi_\psi(f_2)$.
(2) $g_1 \sqsubseteq g_2$ implies $\varphi_\psi^{-1}(g_1) \sqsubseteq \varphi_\psi^{-1}(g_2)$.

(3) $f \sqsubseteq \varphi_\psi^{-1}\left(\varphi_\psi(f)\right)$, the equality holds if φ_ψ is injective.
(4) $\varphi_\psi\left(\varphi_\psi^{-1}(g)\right) \sqsubseteq g$, the equality holds if φ_ψ is surjective.

Definition 2.6. (Çetkin, 2019) The fuzzy soft inclusion $[\tilde{\sqsubseteq}] : (L^X)^E \times (L^X)^E \to L$ is defined by the following equaliy

$$[f \tilde{\sqsubseteq} g] = \wedge_{x\in X} \wedge_{e\in E}\left(f_e'(x) \vee g_e(x)\right)$$

or equivalently,

$$[f \tilde{\sqsubseteq} g] = \wedge_{x\in X} \wedge_{e\in E}\left(f_e(x) \mapsto g_e(x)\right).$$

Lemma 2.7. Let φ_ψ be a mapping from an L-soft space (X_1, E_1) to an L-soft space (X_2, E_2). Then the fuzzy soft inclusion satisfies the following conditions for each $f, g, h \in (L^{X_1})^{E_1}$ and $u, v \in (L^{X_2})^{E_2}$,

(1) $[f \tilde{\sqsubseteq} g] = 1_L \iff f \sqsubseteq g$.
(2) $f \sqsubseteq g$ implies $[f \tilde{\sqsubseteq} h] \geq [g \tilde{\sqsubseteq} h]$
(3) $f \sqsubseteq g$ implies $[h \tilde{\sqsubseteq} f] \leq [h \tilde{\sqsubseteq} g]$
(4) $[f \tilde{\sqsubseteq} g] \wedge [g \tilde{\sqsubseteq} h] \leq [f \tilde{\sqsubseteq} h]$
(5) $[f \tilde{\sqsubseteq} g] \leq [\varphi_\psi (f) \tilde{\sqsubseteq} \varphi_\psi (g)]$
(6) $[u \tilde{\sqsubseteq} v] \leq [\varphi_\psi^{-1} (u) \tilde{\sqsubseteq} \varphi_\psi^{-1} (v)]$.

Proof. It is straightforward from the proeprties of the implication and Definition 2.6.

Definition 2.8. (Tanay & Kandemir, 2011) Let τ be the collection of L-fuzzy soft sets on X. Then τ is said to be an L-soft topology on X if

(T1) $\tilde{0}_X, \tilde{1}_X \in \tau$
(T2) $f, g \in \tau$ implies $f \sqcap g \in \tau$.
(T3) $\{f_i\}_{i\in\Gamma} \subseteq \tau$ implies $\sqcup_{i\in\Gamma} f_i \in \tau$.

Then the pair (X, τ) is called an L-soft topological space. Every member of τ is called an L-soft open set, and if $g' \in \tau$, then the L-soft set $g \in (L^X)^E$ is called an L-soft closed set.

A soft mapping $\varphi_\psi: (X_1, \tau^1) \to (X_2, \tau^2)$ is called continuous between L-soft topological spaces if $\varphi_\psi^{-1}(g) \in \tau^1$ for all $g \in \tau^2$.

Definition 2.9. (Varol & Aygün, 2012) A map $cl : (L^X)^E \to (L^X)^E$ is said to be an L-soft closure operator on X if the following conditions are satisfied

(SC1) $cl(\tilde{0}_X) = \tilde{0}_X$
(SC2) $cl(f \sqcup g) = cl(f) \sqcup cl(g)$, for each $f, g \in (L^X)^E$
(SC3) $f \sqsubseteq cl(f)$, for each $f \in (L^X)^E$

If additionally it satisfies
(SC4) $cl(cl(f)) = cl(f)$, for each $f \in (L^X)^E$

then the map cl is a topological L-soft closure operator on X. For any topological L-soft closure operator on X, the collection

$\tau = \{f \in (L^X)^E \mid cl(f') = f'\}$ is an L-soft topology on X in which closure of f coincides with $cl(f)$. And if τ is an L-soft topology on X, then

$cl(f) = \sqcap \{g \in (L^X)^E \mid f \sqsubseteq g \text{ and } g' \in \tau\}$ defines an L-soft closure operator on X. (Çetkin & Aygün, 2016)

Definition 2.10. Let $\mathcal{C}$ be the collection of L-fuzzy soft sets on X. Then the collection $\mathcal{C}$ is said to be an L-soft cotopology on X if

(CT1) $\tilde{0}_X,\ \tilde{1}_X \in \mathcal{C}$
(CT2) $f, g \in \mathcal{C}$ implies $f \sqcup g \in \mathcal{C}$.
(CT3) $\{f_i\}_{i\in\Gamma} \subseteq \mathcal{C}$ implies $\sqcap_{i\in\Gamma} f_i \in \mathcal{C}$.

For an L-soft cotopology on X, the pair $(X, \mathcal{C})$ is called an L-soft cotopological space.
A soft mapping $\varphi_\psi: (X_1, \mathcal{C}^1) \to (X_2, \mathcal{C}^2)$ is called continuous between L-soft cotopological spaces if $\varphi_\psi^{-1}(g) \in \mathcal{C}^1$ for all $g \in \mathcal{C}^2$.

A mapping $\varphi_\psi: (X_1, \mathcal{C}^1) \to (X_2, \mathcal{C}^2)$ is called closed between L-soft cotopological spaces if $\varphi_\psi(f) \in \mathcal{C}^2$ for all $f \in \mathcal{C}^1$.

Definition 2.11. A map $int: (L^X)^E \to (L^X)^E$ is said to be an L-soft interior operator on X if the following conditions are satisfied

(SI1) $int(\tilde{1}_X) = 1_X$
(SI2) $int(f \sqcap g) = int(f) \sqcap int(g)$, for each $f, g \in (L^X)^E$
(SI3) $int(f) \sqsubseteq f$, for each $f \in (L^X)^E$
(SI4) $int(int(f)) = int(f)$, for each $f \in (L^X)^E$

Then the pair (X, int) is called an L-soft interior space. A mapping $\varphi_\psi: (X_1, int^1) \to (X_2, int^2)$ is called continuous between L-soft interior spaces if it is provided that

$\varphi_\psi(int^1(f)) \sqsubseteq int^2\left(\varphi_\psi(f)\right)$, for each $f \in (L^X)^E$.

It is easily observed that there is close relationship between L-soft topological spaces and L-soft interior operators. In fact these two concepts are equivalent in the following sense,
If τ is an L-soft topology on X, then

$int^\tau(f) = \sqcup\{g \in (L^X)^E \mid g \subseteq f \text{ and } g \in \tau\}\}$

defines an L-soft interior operator on X. And conversely, if $int: (L^X)^E \to (L^X)^E$ is an L-soft interior operator on X, then $\tau^{int} = \{f \in (L^X)^E \mid int(f) = f\}$ defines an L-soft topology on the same set. In addition, $\tau^{int^\tau} = \tau$ and $int^{\tau^{int}} = int$. If we consider these two concepts in the categorical meaning, then one can see the similar correspondence between the morphisms described between the objects below. So that, there is one-to-one correspondence between the objects and the morphisms of the considered structures. Then one can conclude that, these two concepts are same in the categorical aspect.

Theorem 2.12. (Georgiou et al., 2013) Let $(X_1, \tau^1), (X_2, \tau^2)$ be two L-soft topological spaces. Then the following conditions are equivalent.

(1) $\varphi_\psi: (X_1, \tau^1) \to (X_2, \tau^2)$ is continuous.
(2) $g \notin \tau^2$ implies $\varphi_\psi^{-1}(g) \notin \tau^1$.
(3) $\varphi_\psi(cl(f)) \sqsubseteq cl\left(\varphi_\psi(f)\right)$, for each $f \in (L^{X_1})^{E_1}$.
(4) $cl\left(\varphi_\psi^{-1}(g)\right) \sqsubseteq \varphi_\psi^{-1}(cl(g))$, for each $g \in (L^{X_2})^{E_2}$.

Theorem 2.13. If (X, τ) is an L-soft topological space, the collection $\mathcal{C} = \{f \mid f' \in \tau\}$ constitutes an L-soft cotopological space $(X, \mathcal{C})$. Additionally, $cl(f) = \left(int(f')\right)'$, for each $f \in (L^X)^E$.

Proof. Straighforward and therefore omitted.

3. Degrees of continuity, closedness and openness for soft mappings

3. Esnek dönüşümler için süreklilik, kapalılık ve açıklığın dereceleri

In this section, we define the degrees of continuity and closedness for a soft mapping between L-soft topological spaces with the help of the fuzzy soft inclusion. Later, we define the degree of openness for a given soft mapping with the help of the interior operator characterization. We investigate some properties and characteristics of the presented concepts.

Definition 3.1. Let $(X_1, \mathcal{C}^1)$, $(X_2, \mathcal{C}^2)$ be two L-soft cotopological spaces. Then

(1) the degree of continuity for $\varphi_\psi: (X_1, \mathcal{C}^1) \to (X_2, \mathcal{C}^2)$ is defined by

$$Cont(\varphi_\psi) = \bigwedge_{f \in (L^{X_1})^{E_1}} \left[\varphi_\psi(cl(f)) \,\tilde{\sqsubseteq}\, cl\left(\varphi_\psi(f)\right)\right]$$

(2) the degree of closedness for $\varphi_\psi: (X_1, \mathcal{C}^1) \to (X_2, \mathcal{C}^2)$ is defined by

$$Close(\varphi_\psi) = \bigwedge_{f \in (L^{X_1})^{E_1}} \left[cl\left(\varphi_\psi(f)\right) \,\tilde{\sqsubseteq}\, \varphi_\psi(cl(f))\right]$$

Remark 3.2. (1) If $Cont(\varphi_\psi) = 1_L$, then $\varphi_\psi(cl(f)) \sqsubseteq cl\left(\varphi_\psi(f)\right)$ for each $f \in (L^X)^E$, which is an equavalent condition of the continuity of φ_ψ (see Theorem 2.12).

(2) If $Close(\varphi_\psi) = 1_L$, then $cl\left(\varphi_\psi(f)\right) \sqsubseteq \varphi_\psi(cl(f))$ for each $f \in (L^X)^E$, which is the equivalent form of the closedness for the soft mapping φ_ψ defined between L-soft cotopological spaces.
Now, let us give some characterizations of the degree of continuity for a soft mapping.

Theorem 3.3. Let $(X_1, \mathcal{C}^1)$, $(X_2, \mathcal{C}^2)$ be two L-soft cotopological spaces and φ_ψ be a soft mapping which is defined from $(X_1, \mathcal{C}^1)$ to $(X_2, \mathcal{C}^2)$. Then the following is satisfied.

$$Cont(\varphi_\psi) = \bigwedge_{g \in (L^{X_2})^{E_2}} \left[\varphi_\psi\left(cl\left(\varphi_\psi^{-1}(g)\right)\right) \,\tilde{\sqsubseteq}\, cl(g)\right].$$

Proof. From Definition 3.1 (2), we have that

$$\begin{aligned}
&Cont(\varphi_\psi) = \wedge_{f \in (L^{X_1})^{E_1}} \left[\varphi_\psi(cl(f)) \,\tilde{\sqsubseteq}\, cl\left(\varphi_\psi(f)\right)\right] \\
&\leq \wedge_{g \in (L^{X_2})^{E_2}} \left[\varphi_\psi\left(cl\left(\varphi_\psi^{-1}(g)\right)\right) \,\tilde{\sqsubseteq}\, cl\left(\varphi_\psi\left(\varphi_\psi^{-1}(g)\right)\right)\right] \\
&\leq \wedge_{g \in (L^{X_2})^{E_2}} \left[\varphi_\psi\left(cl\left(\varphi_\psi^{-1}(g)\right)\right) \,\tilde{\sqsubseteq}\, cl(g)\right] \\
&\leq \wedge_{f \in (L^{X_1})^{E_1}} \left[\varphi_\psi\left(cl\left(\varphi_\psi^{-1}(\varphi_\psi(f))\right)\right) \,\tilde{\sqsubseteq}\, cl\left(\varphi_\psi(f)\right)\right] \\
&\leq \wedge_{f \in (L^{X_1})^{E_1}} \left[\varphi_\psi(cl(f)) \,\tilde{\sqsubseteq}\, cl\left(\varphi_\psi(f)\right)\right].
\end{aligned}$$

The above implications gives the desired equality.

Theorem 3.4. Let $(X_1, \mathcal{C}^1)$, $(X_2, \mathcal{C}^2)$ be two L-soft cotopological spaces and φ_ψ be a soft mapping which is defined from $(X_1, \mathcal{C}^1)$ to $(X_2, \mathcal{C}^2)$. Then the following is satisfied.

$$Cont(\varphi_\psi) = \bigwedge_{f \in (L^{X_1})^{E_1}} \left[cl(f) \,\tilde{\sqsubseteq}\, \varphi_\psi^{-1}\left(cl\left(\varphi_\psi(f)\right)\right)\right].$$

Proof. First let us consider the fuzzy soft inclusion as follows:

$$\left[\varphi_\psi(cl(f)) \,\tilde{\sqsubseteq}\, cl\left(\varphi_\psi(f)\right)\right] = \wedge_{y \in X_2} \wedge_{k \in E_2} \left(\varphi_\psi(cl(f))_k(y) \mapsto cl\left(\varphi_\psi(f)\right)_k(y)\right)$$

$= \bigwedge_{y\in X_2} \bigwedge_{k\in E_2} \left(\bigvee_{k=\psi(e)} \bigvee_{y=\varphi(x)} cl(f)_e(x) \mapsto cl\left(\varphi_\psi(f)\right)_k (y) \right)$
$= \bigwedge_{y\in X_2} \bigwedge_{k\in E_2} \bigwedge_{k=\psi(e)} \bigwedge_{y=\varphi(x)} \left(cl(f)_e(x) \mapsto cl\left(\varphi_\psi(f)\right)_{\psi(e)} (\varphi(x)) \right)$
$= \bigwedge_{y\in X_2} \bigwedge_{k\in E_2} \bigwedge_{k=\psi(e)} \bigwedge_{y=\varphi(x)} \left(cl(f)_e(x) \mapsto \varphi_\psi^{-1}\left(cl\left(\varphi_\psi(f)\right)\right)_e (x) \right)$
$= \bigwedge_{x\in X_1} \bigwedge_{e\in E_1} \left(cl(f)_e(x) \mapsto \varphi_\psi^{-1}\left(cl\left(\varphi_\psi(f)\right)\right)_e (x) \right)$
$= \left[cl(f) \sqsubseteq \varphi_\psi^{-1}\left(cl\left(\varphi_\psi(f)\right)\right)\right].$

The observation given above and Definition 3.1 (1), imply the fact that

$$Cont(\varphi_\psi) = \bigwedge_{f\in (L^{X_1})^{E_1}} \left[cl(f) \sqsubseteq \varphi_\psi^{-1}\left(cl\left(\varphi_\psi(f)\right)\right)\right]$$

as claimed.

Theorem 3.5. Let $(X_1, \mathcal{C}^1)$, $(X_2, \mathcal{C}^2)$ be two L-soft cotopological spaces and φ_ψ, be a soft mapping which is defined from $(X_1, \mathcal{C}^1)$ to $(X_2, \mathcal{C}^2)$. Then the following is satisfied.

$$Cont(\varphi_\psi) = \bigwedge_{g\in (L^{X_2})^{E_2}} \left[cl(\varphi_\psi^{-1}(g)) \sqsubseteq \varphi_\psi^{-1}(cl(g))\right].$$

Proof. By considering the soft mapping and fuzzy soft inclusion properties, we obtain the following implication

$\bigwedge_{g\in (L^{X_2})^{E_2}} \left[cl\left(\varphi_\psi^{-1}(g)\right) \sqsubseteq \varphi_\psi^{-1}(cl(g))\right]$
$\leq \bigwedge_{f\in (L^{X_1})^{E_1}} \left[cl(\varphi_\psi^{-1}(\varphi_\psi(f))) \sqsubseteq \varphi_\psi^{-1}\left(cl\left(\varphi_\psi(f)\right)\right)\right]$
$\leq \bigwedge_{f\in (L^{X_1})^{E_1}} \left[cl(f) \sqsubseteq \varphi_\psi^{-1}\left(cl\left(\varphi_\psi(f)\right)\right)\right] = Cont(\varphi_\psi)$, by Theorem 3.4.

In order to prove the converse implication, let us consider the following

$\left[cl\left(\varphi_\psi^{-1}(g)\right) \sqsubseteq \varphi_\psi^{-1}(cl(g))\right] = \bigwedge_{x\in X_1} \bigwedge_{e\in E_1} \left(cl\left(\varphi_\psi^{-1}(g)\right)_e (x) \mapsto \varphi_\psi^{-1}(cl(g))_e(x)\right)$
$= \bigwedge_{x\in X_1} \bigwedge_{e\in E_1} \left(cl\left(\varphi_\psi^{-1}(g)\right)_e (x) \mapsto cl(g)_{\psi(e)}(\varphi(x))\right)$
$\geq \bigwedge_{y\in X_2} \bigwedge_{k\in E_2} \left(\bigvee_{y=\varphi(x)} \bigvee_{k=\varphi(e)} cl\left(\varphi_\psi^{-1}(g)\right)_e (x) \mapsto cl(g)_k(y)\right)$
$= \bigwedge_{y\in X_2} \bigwedge_{k\in E_2} \left(\varphi_\psi\left(cl\left(\varphi_\psi^{-1}(g)\right)\right)_k (y) \mapsto cl(g)_k(y)\right)$
$= \left[\varphi_\psi\left(cl\left(\varphi_\psi^{-1}(g)\right)\right) \sqsubseteq cl(g)\right].$

By Theorem 3.3, this implies the fact that $\bigwedge_{g\in (L^{X_2})^{E_2}} \left[cl\left(\varphi_\psi^{-1}(g)\right) \sqsubseteq \varphi_\psi^{-1}(cl(g))\right] \geq Cont(\varphi_\psi)$.
This completes the proof.

Theorem 3.6. Let $(X_1, \mathcal{C}^1)$, $(X_2, \mathcal{C}^2)$ and $(X_3, \mathcal{C}^3)$ be the L-soft cotopological spaces. Then for the soft mappings $\varphi_\psi: (X_1, \mathcal{C}^1) \to (X_2, \mathcal{C}^2)$ and $\varphi^*_{\psi^*}: (X_2, \mathcal{C}^2) \to (X_3, \mathcal{C}^3)$, the following conditions are satisfied.
(1) $Cont(\varphi_\psi) \wedge Cont(\varphi^*_{\psi^*}) \leq Cont(\varphi^*_{\psi^*} \circ \varphi_\psi)$.
(2) $Close(\varphi_\psi) \wedge Close(\varphi^*_{\psi^*}) \leq Close(\varphi^*_{\psi^*} \circ \varphi_\psi)$.

Proof. We give only the proof of (1), since the second condition is proved by considering the similar observations.

$$
\begin{aligned}
& Cont(\varphi_\psi) \wedge Cont(\varphi^*_{\psi^*}) \\
& = \wedge_{f\in(L^{X_1})^{E_1}} \left[\varphi_\psi(cl(f)) \tilde{\sqsubseteq} cl\left(\varphi_\psi(f)\right)\right] \wedge \wedge_{g\in(L^{X_2})^{E_2}} \left[\varphi^*_{\psi^*}(cl(g)) \tilde{\sqsubseteq} cl\left(\varphi^*_{\psi^*}(g)\right)\right] \\
& \leq \wedge_{f\in(L^{X_1})^{E_1}} \left[\varphi_\psi(cl(f)) \tilde{\sqsubseteq} cl\left(\varphi_\psi(f)\right)\right] \wedge \wedge_{g\in(L^{X_2})^{E_2}} \left[cl(g) \tilde{\sqsubseteq} \left(\varphi^*_{\psi^*}\right)^{-1}\left(cl\left(\varphi^*_{\psi^*}(g)\right)\right)\right] \\
& \leq \wedge_{f\in(L^{X_1})^{E_1}} \left\{\left[\varphi_\psi(cl(f)) \tilde{\sqsubseteq} cl\left(\varphi_\psi(f)\right)\right] \wedge \left[cl\left(\varphi_\psi(f)\right) \tilde{\sqsubseteq} \left(\varphi^*_{\psi^*}\right)^{-1}\left(cl\left(\varphi^*_{\psi^*}\left(\varphi_\psi(f)\right)\right)\right)\right]\right\} \\
& \leq \wedge_{f\in(L^{X_1})^{E_1}} \left[\varphi_\psi(cl(f)) \tilde{\sqsubseteq} \left(\varphi^*_{\psi^*}\right)^{-1}\left(cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right)\right)\right] \\
& \leq \wedge_{f\in(L^{X_1})^{E_1}} \left[(\varphi^*_{\psi^*} \circ \varphi_\psi)(cl(f)) \tilde{\sqsubseteq} cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right)\right] \\
& = Cont(\varphi^*_{\psi^*} \circ \varphi_\psi).
\end{aligned}
$$

Theorem 3.7. Let $\varphi_\psi: (X_1, \mathcal{C}^1) \to (X_2, \mathcal{C}^2)$ and $\varphi^*_{\psi^*}: (X_2, \mathcal{C}^2) \to (X_3, \mathcal{C}^3)$ be two soft mappings between L-soft cotopological spaces, where $\varphi^*_{\psi^*}$ is injective. Then we hve

$$Close(\varphi^*_{\psi^*} \circ \varphi_\psi) \wedge Cont(\varphi^*_{\psi^*}) \leq Close(\varphi_\psi).$$

Proof. By the injectivity of the soft mapping $\varphi^*_{\psi^*}$, we have $\left(\varphi^*_{\psi^*}\right)^{-1}\left(\varphi^*_{\psi^*}(g)\right) = g$, for all $g \in (L^{X_2})^{E_2}$. Then from this fact, we gain the following implications

$$
\begin{aligned}
& Close(\varphi^*_{\psi^*} \circ \varphi_\psi) \wedge Cont(\varphi^*_{\psi^*}) \\
& = \wedge_{f\in(L^{X_1})^{E_1}} \left[cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right) \tilde{\sqsubseteq} (\varphi^*_{\psi^*} \circ \varphi_\psi)(cl(f))\right] \wedge \wedge_{g\in(L^{X_2})^{E_2}} \left[\varphi^*_{\psi^*}(cl(g)) \tilde{\sqsubseteq} cl\left(\varphi^*_{\psi^*}(g)\right)\right] \\
& \leq \wedge_{f\in(L^{X_1})^{E_1}} \left[cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right) \tilde{\sqsubseteq} (\varphi^*_{\psi^*} \circ \varphi_\psi)(cl(f))\right] \wedge \\
& \qquad \wedge_{f\in(L^{X_1})^{E_1}} \left[\varphi^*_{\psi^*}\left(cl(\varphi_\psi(f))\right) \tilde{\sqsubseteq} cl\left(\varphi^*_{\psi^*}(\varphi_\psi(f))\right)\right] \\
& = \bigwedge_{f\in(L^{X_1})^{E_1}} \left\{\left[cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right) \tilde{\sqsubseteq} (\varphi^*_{\psi^*} \circ \varphi_\psi)(cl(f))\right] \wedge \left[\varphi^*_{\psi^*}\left(cl\left(\varphi_\psi(f)\right)\right) \tilde{\sqsubseteq} cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right)\right]\right\} \\
& \leq \wedge_{f\in(L^{X_1})^{E_1}} \left[\varphi^*_{\psi^*}\left(cl\left(\varphi_\psi(f)\right)\right) \tilde{\sqsubseteq} (\varphi^*_{\psi^*} \circ \varphi_\psi)(cl(f))\right] \\
& \leq \wedge_{f\in(L^{X_1})^{E_1}} \left[cl\left(\varphi_\psi(f)\right) \tilde{\sqsubseteq} \left(\varphi^*_{\psi^*}\right)^{-1}\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(cl(f))\right)\right] \\
& = \wedge_{f\in(L^{X_1})^{E_1}} \left[cl\left(\varphi_\psi(f)\right) \tilde{\sqsubseteq} \varphi_\psi(cl(f))\right] = Close(\varphi_\psi).
\end{aligned}
$$

Hence, the proof is completed.

Theorem 3.8. Let $\varphi_\psi: (X_1, \mathcal{C}^1) \to (X_2, \mathcal{C}^2)$ and $\varphi^*_{\psi^*}: (X_2, \mathcal{C}^2) \to (X_3, \mathcal{C}^3)$ be two soft mappings between L-soft cotopological spaces, where φ_ψ is surjective. Then we hve

$$Close(\varphi^*_{\psi^*} \circ \varphi_\psi) \wedge Cont(\varphi_\psi) \leq Close(\varphi^*_{\psi^*}).$$

Proof. By the surjectivity of the soft mapping φ_ψ, we have $\varphi_\psi\left(\varphi_\psi^{-1}(g)\right) = g$, for all $g \in (L^{X_2})^{E_2}$. Then from this fact, we gain the following implications

$$
\begin{aligned}
& Close(\varphi^*_{\psi^*}) = \wedge_{g\in(L^{X_2})^{E_2}} \left[cl\left(\varphi^*_{\psi^*}(g)\right) \tilde{\sqsubseteq} \varphi^*_{\psi^*}(cl(g))\right] \\
& = \wedge_{g\in(L^{X_2})^{E_2}} \left[cl\left(\varphi^*_{\psi^*}\left(\varphi_\psi\left(\varphi_\psi^{-1}(g)\right)\right)\right) \tilde{\sqsubseteq} \varphi^*_{\psi^*}\left(cl\left(\varphi_\psi\left(\varphi_\psi^{-1}(g)\right)\right)\right)\right] \\
& \geq \wedge_{f\in(L^{X_1})^{E_1}} \left[cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right) \tilde{\sqsubseteq} \varphi^*_{\psi^*}(cl(\varphi_\psi(f))\right] \\
& \geq \wedge_{g\in(L^{X_2})^{E_2}} \left[cl\left(\varphi^*_{\psi^*}(g)\right) \tilde{\sqsubseteq} \varphi^*_{\psi^*}(cl(g))\right] = Close(\varphi^*_{\psi^*}).
\end{aligned}
$$

This implies the following equality (*),

$$Close(\varphi^*_{\psi^*}) = \wedge_{f\in(L^{X_1})^{E_1}} \left[cl\left((\varphi^*_{\psi^*} \circ \varphi_\psi)(f)\right) \tilde{\sqsubseteq} \varphi^*_{\psi^*}(cl(\varphi_\psi(f))\right] \tag{1}$$

We also have that

$$
\begin{aligned}
&Close(\varphi^*_{\psi^*}\circ\varphi_\psi)\wedge Cont(\varphi_\psi)\\
&=\wedge_{f\in(L^{X_1})^{E_1}}\left[cl\left((\varphi^*_{\psi^*}\circ\varphi_\psi)(f)\right)\,\tilde{\sqsubseteq}\,(\varphi^*_{\psi^*}\circ\varphi_\psi)\big(cl(f)\big)\right]\wedge\wedge_{f\in(L^{X_1})^{E_1}}\left[\varphi_\psi\big(cl(f)\big)\,\tilde{\sqsubseteq}\,cl\left(\varphi_\psi(f)\right)\right]\\
&\le\wedge_{f\in(L^{X_1})^{E_1}}\left[cl\left((\varphi^*_{\psi^*}\circ\varphi_\psi)(f)\right)\,\tilde{\sqsubseteq}\,(\varphi^*_{\psi^*}\circ\varphi_\psi)\big(cl(f)\big)\right]\wedge\wedge_{f\in(L^{X_1})^{E_1}}\left[\varphi^*_{\psi^*}\left(\varphi_\psi\big(cl(f)\big)\right)\,\tilde{\sqsubseteq}\,\varphi^*_{\psi^*}\left(cl\left(\varphi_\psi(f)\right)\right)\right]\\
&\le\wedge_{f\in(L^{X_1})^{E_1}}\left[cl\left((\varphi^*_{\psi^*}\circ\varphi_\psi)(f)\right)\,\tilde{\sqsubseteq}\,\varphi^*_{\psi^*}\left(cl\left(\varphi_\psi(f)\right)\right)\right]\quad\text{(From the equality (1))}\\
&=\wedge_{g\in(L^{X_2})^{E_2}}\left[cl\left(\varphi^*_{\psi^*}(g)\right)\,\tilde{\sqsubseteq}\,\varphi^*_{\psi^*}\big(cl(g)\big)\right]=Close(\varphi^*_{\psi^*}).
\end{aligned}
$$

Hence, the proof is completed.

The degree of continuity is computed not only for a soft mapping which is defined between L-soft cotopological spaces but also for a soft mapping which is defined between L-soft topological spaces. We mean that the degree to what entend the continuity of a soft mapping can also be defined by means of the interior operator. Analogously, the openness degree of a soft mapping is described in the following way.

Theorem 3.9. Let (X_1,τ^1), (X_2,τ^2) be two L-soft topological spaces. Then the following is also true for the degree of continuity for the soft mapping $\varphi_\psi:(X_1,\tau^1)\to(X_2,\tau^2)$

$$Cont(\varphi_\psi)=\bigwedge_{g\in(L^{X_2})^{E_2}}\left[\varphi_\psi^{-1}\big(int(g)\big)\,\tilde{\sqsubseteq}\,int\left(\varphi_\psi^{-1}(g)\right)\right]$$

Proof. The equivalence is obtained from Theorem 2.13 and Definition 3.1.

Definition 3.10. Let (X_1,τ^1), (X_2,τ^2) be two L-soft topological spaces. Then the degree of openness for the soft mapping $\varphi_\psi:(X_1,\tau^1)\to(X_2,\tau^2)$ is defined by

$$Open(\varphi_\psi)=\bigwedge_{f\in(L^{X_1})^{E_1}}\left[\varphi_\psi\big(int(f)\big)\,\tilde{\sqsubseteq}\,int\left(\varphi_\psi(f)\right)\right]$$

Remark 3.11. If $Open(\varphi_\psi)=1_L$, then $\varphi_\psi\big(int(f)\big)\sqsubseteq int\left(\varphi_\psi(f)\right)$ is valid for each $f\in(L^{X_1})^{E_1}$. This is exactly the equivalent form of the openness of a soft mapping defined between soft topological spaces.

Theorem 3.12. Let $(X_1,\tau^1),(X_2,\tau^2)$ and (X_3,τ^3) be L-soft topological spaces and $\varphi_\psi:(X_1,\tau^1)\to(X_2,\tau^2)$, $\varphi^*_{\psi^*}:(X_2,\tau^2)\to(X_3,\tau^3)$ be two soft mappings. The the following conditions are satisfied.

(1) $Cont(\varphi_\psi)\wedge Cont(\varphi^*_{\psi^*})\le Cont(\varphi^*_{\psi^*}\circ\varphi_\psi)$.

(2) $Open(\varphi_\psi)\wedge Open(\varphi^*_{\psi^*})\le Open(\varphi^*_{\psi^*}\circ\varphi_\psi)$.

Proof. One can see the proof similarly to that of Theorem 3.6.

Theorem 3.13. Let $(X_1,\tau^1),(X_2,\tau^2)$ and (X_3,τ^3) be L-soft topological spaces and $\varphi_\psi:(X_1,\tau^1)\to(X_2,\tau^2)$, $\varphi^*_{\psi^*}:(X_2,\tau^2)\to(X_3,\tau^3)$ be two soft mappings. If the soft maping φ_ψ is surjective, then the following is obtained

$$Open(\varphi^*_{\psi^*}\circ\varphi_\psi)\wedge Cont(\varphi_\psi)\le Open(\varphi^*_{\psi^*}).$$

Proof. Since the soft mapping φ_ψ is surjective, then $\varphi_\psi\left(\varphi_\psi^{-1}(g)\right)=g$, for each $g\in(L^{X_2})^{E_2}$. From this fact, we gain that

$$
\begin{aligned}
&\wedge_{h\in(L^{X_2})^{E_2}}\left[\varphi^*_{\psi^*}\big(int(h)\big)\,\tilde{\sqsubseteq}\,int\left(\varphi^*_{\psi^*}(h)\right)\right]\\
&=\wedge_{h\in(L^{X_2})^{E_2}}\left[\varphi^*_{\psi^*}\left(int\left(\varphi_\psi\left(\varphi_\psi^{-1}(h)\right)\right)\right)\,\tilde{\sqsubseteq}\,int\left(\varphi^*_{\psi^*}\left(\varphi_\psi\left(\varphi_\psi^{-1}(h)\right)\right)\right)\right]
\end{aligned}
$$

$\geq \bigwedge_{f\in(L^{X_1})^{E_1}} \left[\varphi^*_{\psi^*}\left(int\left(\varphi_\psi(f)\right)\right) \tilde{\sqsubseteq} int\left(\varphi^*_{\psi^*}\left(\varphi_\psi(f)\right)\right)\right]$
$\geq \bigwedge_{h\in(L^{X_2})^{E_2}} \left[\varphi^*_{\psi^*}\left(int(h)\right) \tilde{\sqsubseteq} int\left(\varphi^*_{\psi^*}(h)\right)\right]$

The above observation implies the following

$$\bigwedge_{h\in(L^{X_2})^{E_2}} \left[\varphi^*_{\psi^*}\left(int(h)\right) \tilde{\sqsubseteq} int\left(\varphi^*_{\psi^*}(h)\right)\right] = \bigwedge_{f\in(L^{X_1})^{E_1}} \left[\varphi^*_{\psi^*}\left(int\left(\varphi_\psi(f)\right)\right) \tilde{\sqsubseteq} int\left(\varphi^*_{\psi^*}\left(\varphi_\psi(f)\right)\right)\right]$$

In order to get the proof, let us consider the above fact as follows:

$Open\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right) \wedge Cont\left(\varphi_\psi\right)$
$= \bigwedge_{f\in(L^{X_1})^{E_1}}\left[(\varphi^*_{\psi^*}\circ\varphi_\psi)\left(int(f)\right) \tilde{\sqsubseteq} int\left((\varphi^*_{\psi^*}\circ\varphi_\psi)(f)\right)\right] \wedge \bigwedge_{g\in(L^{X_2})^{E_2}} \left[\varphi_\psi^{-1}\left(int(g)\right) \tilde{\sqsubseteq} int\left(\varphi_\psi^{-1}(g)\right)\right]$
$\leq \bigwedge_{f\in(L^{X_1})^{E_1}}\left[\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)\left(int(f)\right) \tilde{\sqsubseteq} int\left(\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)(f)\right)\right]$
$\wedge \bigwedge_{f\in(L^{X_1})^{E_1}} \left[\varphi^*_{\psi^*}\left(int\left(\varphi_\psi(f)\right)\right) \tilde{\sqsubseteq} (\varphi^*_{\psi^*}\circ\varphi_\psi)\left(int(f)\right)\right]$
$\leq \bigwedge_{f\in(L^{X_1})^{E_1}}\left[\varphi^*_{\psi^*}\left(int\left(\varphi_\psi(f)\right)\right) \tilde{\sqsubseteq} int\left(\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)(f)\right)\right]$ (By the above equality)
$= \bigwedge_{g\in(L^{X_2})^{E_2}}\left[\varphi^*_{\psi^*}\left(int(g)\right) \tilde{\sqsubseteq} int\left(\varphi^*_{\psi^*}(g)\right)\right] = Open\left(\varphi^*_{\psi^*}\right)$.
This completes the proof.

Theorem 3.14. Let $(X_1,\tau^1), (X_2,\tau^2)$ and (X_3,τ^3) be L-soft topological spaces and $\varphi_\psi: (X_1,\tau^1) \to (X_2,\tau^2)$, $\varphi^*_{\psi^*} : (X_2,\tau^2) \to (X_3,\tau^3)$ be two soft mappings. If the soft maping $\varphi^*_{\psi^*}$ is injective, then we get the following

$$Open\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right) \wedge Cont\left(\varphi^*_{\psi^*}\right) \leq Open\left(\varphi_\psi\right).$$

Proof. Since the soft mapping $\varphi^*_{\psi^*}$ is injective, then $\left(\varphi^*_{\psi^*}\right)^{-1}\left(\varphi^*_{\psi^*}(g)\right) = g$, for each $g \in (L^{X_2})^{E_2}$. From this fact, it is seen that

$Open\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right) \wedge Cont\left(\varphi^*_{\psi^*}\right)$
$= \bigwedge_{f\in(L^{X_1})^{E_1}}\left[\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)\left(int(f)\right) \tilde{\sqsubseteq} int\left(\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)(f)\right)\right]$
$\wedge \bigwedge_{h\in(L^{X_2})^{E_2}} \left[\left(\varphi^*_{\psi^*}\right)^{-1}\left(int(h)\right) \tilde{\sqsubseteq} int\left(\left(\varphi^*_{\psi^*}\right)^{-1}(h)\right)\right]$
$\leq \bigwedge_{f\in(L^{X_1})^{E_1}}\left[\left(\varphi^*_{\psi^*}\right)^{-1}\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)\left(int(f)\right) \tilde{\sqsubseteq} \left(\varphi^*_{\psi^*}\right)^{-1}\left(int\left(\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)(f)\right)\right)\right]$
$\wedge \bigwedge_{f\in(L^{X_1})^{E_1}} \left[\left(\varphi^*_{\psi^*}\right)^{-1}\left(int\left(\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)(f)\right)\right) \tilde{\sqsubseteq} int\left(\left(\varphi^*_{\psi^*}\right)^{-1}\left(\left(\varphi^*_{\psi^*}\circ\varphi_\psi\right)(f)\right)\right)\right]$
$= \bigwedge_{f\in(L^{X_1})^{E_1}} \left[\varphi_\psi\left(int(f)\right) \tilde{\sqsubseteq} int\left(\varphi_\psi(f)\right)\right] = Open\left(\varphi_\psi\right)$.

This completes the proof.

Definition 3.15. Let (X_1,τ^1), (X_2,τ^2) be two L-soft topological spaces and $\varphi_\psi: (X_1,\tau^1) \to (X_2,\tau^2)$ be a bijective soft mapping. Then the degree $Hom\left(\varphi_\psi\right)$ to which φ_ψ is a homeomorphism is defined by $Hom\left(\varphi_\psi\right) = Cont\left(\varphi_\psi\right) \wedge Open\left(\varphi_\psi\right)$.

Under the light of the above discussions, one can infer the following results.

Corollary 3.16. Let $(X_1,\tau^1), (X_2,\tau^2)$ and (X_3,τ^3) be L-soft topological spaces and $\varphi_\psi: (X_1,\tau^1) \to (X_2,\tau^2)$, $\varphi^*_{\psi^*} : (X_2,\tau^2) \to (X_3,\tau^3)$ be two bijective soft mappings. Then the followings are satisfied.
(1) $Hom\left(\varphi_\psi\right) \wedge Hom\left(\varphi^*_{\psi^*}\right) \leq Hom(\varphi^*_{\psi^*}\circ\varphi_\psi)$.
(2) $Hom\left(\varphi_\psi\right) = Cont\left(\varphi_\psi\right) \wedge Cont\left(\varphi_\psi^{-1}\right) = Cont\left(\varphi_\psi\right) \wedge Close\left(\varphi_\psi\right)$.

4. Conclusion

4. Sonuç

In the present study, we proposed the gradation of continuity, closedness and openness for the soft mappings to some degrees. The perspective of gradation of the openness of sets, spaces and also mappings between some structured spaces yields researchers efficiently applications to the daily life modellings. Since (fuzzy) soft sets and (fuzzy) soft spaces are natural effective tools to reflect and model the real phenomena, we found it reasonable to investigate the degrees of soft mappings between graded soft topological spaces. For further research, we hope to investigate the relations and the properties of the graded soft mappings which are defined between compact, connected and separated soft topological spaces to some degrees. In addition, for future work, we aim to propose the parametric gradation of topologcal structures which are not defined so far, and the special mappings between the corresponding spaces such as graded mappings between soft bornological spaces, soft uniform spaces, soft proximity spaces and so on.

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Author contribution

Yazar katkısı

All authors contributed equally to this work. They all read and approved the last version of the manuscript.

Declaration of ethical code

Etik beyanı

The author of this article, declares that the material and the methods used in this study do not require ethical committee approval and/or special legal permission.

Conflicts of interest

Çıkar çatışması beyanı

The authors declare that they have no conflict of interest.

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