



## Series Solution of Modelling the Pollution

NURETTİN DOĞAN 

*Department of Computer Engineering, Faculty of Technology, Selçuk University, Konya, Turkey.*

Received: 13-02-2021 • Accepted: 17-04-2021

**ABSTRACT.** In this article, an approximate analytical series solution of Modeling the pollution of a lake system is given. Laplace-Adomian decomposition method (LADM) was used for the serial solution. LADM with different input values has been used for the solution. The solutions obtained by LADM were compared with the results obtained by Fourth Order Runge Method. In addition, the results are compared with residual errors. These comparisons show that LADM is a powerful method for obtaining approximate solutions to the problem of modeling the pollution of a lake system.

*2010 AMS Classification:* 34A99

**Keywords:** Modeling the pollution, Laplace-Adomian decomposition method, approximate solution.

### 1. INTRODUCTION

Water is a source of life for all living things. Therefore the correct use of water resources is one of the most important problems of today. Modern life is threatening wildlife and water resources. Water pollution occurs as a result of discharging wasted water without any treatment process. Throughout the planet, water contamination impacts drinking water, waterways, reservoirs and oceans. Human wellbeing and the nature are subsequently affected by this. It leads cause of deaths and diseases worldwide. Therefore, studies related to water pollution are very important.

The simulation of multiple systems is represented by ordinary non-linear differential equations. Mathematical modeling and numerical simulations of these models are very important in recent researches. In recent years modeling the pollution of a system of lakes draw attention. Biazar at al. solved a system of three lakes contamination model in [2], linked by channels flowing between them. Fig. 1 shows this model of system.

Each lake is called a wide compartment in this model and the interconnecting channel is considered to be pipes between the compartments. In the channels or pipes, the direction of flow is illustrated by the arrows in the diagram. In the first pool, a pollutant is added, where  $p(x)$  denotes the rate at which the pollutant per unit time reaches the lake. The function  $p(x)$  may be constant or may change over time. It is really important to know the amount of contamination in each lake at any moment.

In this model  $y_i(x)$  denotes the amount of the pollutant in lake  $i$  at any time  $x \geq 0$ , where  $i = 1, 2, 3$ . The pollutant in each lake is believed to be spread evenly across the lake by some mixing mechanism and the amount of water  $V_i$  in lake  $i$  remains unchanged for each of the lakes.

So in lake  $i$ , the concentration of the contaminant is given at any time by

$$c_i(x) = \frac{y_i(x)}{V_i}.$$

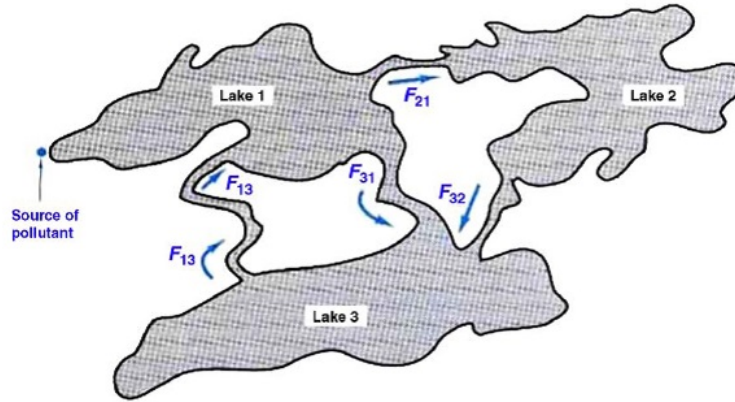


FIGURE 1. System of three lakes with interconnecting channels. A pollutant enters the first lake at the indicated source [2].

Each lake is initially assumed to be clean, so  $y_i(0) = 0$  for each  $i = 1, 2, 3$ . The flow rate from lake  $i$  to lake  $j$ , for modelling the lake system's dynamic behavior is denoted by constant  $F_{ji}$ . Direction of the pollutant flow from lake  $i$  into lake  $j$  is characterized by

$$r_{ji}(x) = F_{ji} \cdot c_i(x) = \frac{F_{ji}}{V_i} y_i(x)$$

at any time  $x$ . We're going to observe that

$$\text{Pollutant change rate} = \text{input rate} - \text{output rate}.$$

The following system of first-order equations, the application of this principle to each lake is obtained. [2]:

$$\frac{dy_1}{dx} = \frac{F_{13}}{V_3} y_3(x) + p(x) - \frac{F_{31}}{V_1} y_1(x) - \frac{F_{21}}{V_1} y_1(x), \quad (1.1)$$

$$\frac{dy_2}{dx} = \frac{F_{21}}{V_1} y_1(x) - \frac{F_{32}}{V_2} y_2(x),$$

$$\frac{dy_3}{dx} = \frac{F_{31}}{V_1} y_1(x) + \frac{F_{32}}{V_2} y_2(x) - \frac{F_{13}}{V_3} y_3(x).$$

The rate of inflow and outflow of the lake must be the same. Therefore, the volume of each lake stays constant. So we have the following criteria :

$$\text{Lake 1: } F_{13} = F_{21} + F_{31},$$

$$\text{Lake 2: } F_{21} = F_{32},$$

$$\text{Lake 3: } F_{31} + F_{32} = F_{13}.$$

To investigate the suggested model in literature, many techniques have been used. To solve this problem, He's Variational Iteration Method [10], Homotopy Perturbation Method [11] and Variational Iteration Method [3] have been used. The aim of this paper is to extend the use of LADM [1, 4–9, 12] in order to obtain estimated solution of the pollution model.

## 2. SOLUTION PROCEDURE

We apply the LADM to the pollution model in this section. We remember that the Laplace transform of  $y'_i(x)$  is defined by  $x$

$$L\{y'_i\} = sL\{y_i\} - y_i(0) \quad ; \quad i = 1, 2, \dots, n$$

to solve this pollution model by using the LADM. The following system is produced by applying the Laplace transform to both sides of (1.1).

$$\begin{aligned}
 L\{y_1(x)\} &= \frac{y_1(0)}{s} + \frac{1}{s}L\{p(x)\} + \frac{F_{13}}{V_3} \frac{1}{s}L\{y_3(x)\} - \frac{F_{31}}{V_1} \frac{1}{s}L\{y_1(x)\} - \frac{F_{21}}{V_1} \frac{1}{s}L\{y_1(x)\}, \\
 L\{y_2(x)\} &= \frac{y_2(0)}{s} + \frac{F_{21}}{V_1} \frac{1}{s}L\{y_1(x)\} - \frac{F_{32}}{V_2} \frac{1}{s}L\{y_2(x)\}, \\
 L\{y_3(x)\} &= \frac{y_3(0)}{s} + \frac{F_{31}}{V_1} \frac{1}{s}L\{y_1(x)\} + \frac{F_{32}}{V_2} \frac{1}{s}L\{y_2(x)\} - \frac{F_{13}}{V_3} \frac{1}{s}L\{y_3(x)\}.
 \end{aligned}
 \tag{2.1}$$

If we substitute  $y_1 = \sum_{k=0}^{\infty} y_{1k}$ ,  $y_2 = \sum_{k=0}^{\infty} y_{2k}$  and  $y_3 = \sum_{k=0}^{\infty} y_{3k}$  in (2.1) we obtain the followings

$$\begin{aligned}
 L\left\{\sum_{k=0}^{\infty} y_{1k}\right\} &= \frac{y_1(0)}{s} + \frac{1}{s}L\{p(x)\} + \frac{F_{13}}{V_3} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{3k}\right\} - \frac{F_{31}}{V_1} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{1k}\right\} - \frac{F_{21}}{V_1} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{1k}\right\}, \\
 L\left\{\sum_{k=0}^{\infty} y_{2k}\right\} &= \frac{y_2(0)}{s} + \frac{F_{21}}{V_1} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{1k}\right\} - \frac{F_{32}}{V_2} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{2k}\right\}, \\
 L\left\{\sum_{k=0}^{\infty} y_{3k}\right\} &= \frac{y_3(0)}{s} + \frac{F_{31}}{V_1} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{1k}\right\} + \frac{F_{32}}{V_2} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{2k}\right\} - \frac{F_{13}}{V_3} \frac{1}{s}L\left\{\sum_{k=0}^{\infty} y_{3k}\right\}.
 \end{aligned}$$

Using the LADM, we get the following process.

$$\begin{aligned}
 L\{y_{10}\} &= \frac{y_1(0)}{s} + \frac{1}{s}L\{p(x)\}, \quad L\{y_{1k+1}\} = \frac{F_{13}}{V_3} \frac{1}{s}L\{y_{3k}\} - \frac{F_{31}}{V_1} \frac{1}{s}L\{y_{1k}\} - \frac{F_{21}}{V_1} \frac{1}{s}L\{y_{1k}\}, \\
 L\{y_{20}\} &= \frac{y_2(0)}{s}, \quad L\{y_{2k+1}\} = \frac{F_{21}}{V_1} \frac{1}{s}L\{y_{1k}\} - \frac{F_{32}}{V_2} \frac{1}{s}L\{y_{2k}\}, \\
 L\{y_{30}\} &= \frac{y_3(0)}{s}, \quad L\{y_{3k+1}\} = \frac{F_{31}}{V_1} \frac{1}{s}L\{y_{1k}\} + \frac{F_{32}}{V_2} \frac{1}{s}L\{y_{2k}\} - \frac{F_{13}}{V_3} \frac{1}{s}L\{y_{3k}\}.
 \end{aligned}
 \tag{2.2}$$

$y_{i0}$  is given by applying the inverse Laplace transform to the first part of (2.2). The use of  $y_{i0}$  helps one to evaluate  $y_{i1}$ . The determination of  $y_{i0}$  and  $y_{i1}$  leads to the determination of  $y_{i2}$ , and so on. Proceeding in this way, the components of  $y_{ik}, k \geq 0$  are completely determined upon using the second part of (2.2). Immediately after using the equations

$y_1 = \sum_{k=0}^{\infty} y_{1k}$ ,  $y_2 = \sum_{k=0}^{\infty} y_{2k}$  and  $y_3 = \sum_{k=0}^{\infty} y_{3k}$ , the series solution follows.

### 3. APPLICATIONS

There are three different mathematical models proposed for the pollution of a lake. These models are used to estimate the contamination that may occur in reality very closely. The impulse, step and sinusoidal input are the input models [2].

**3.1. Impulse Input.** For pollutants that have immediately been released into the lake, the impulse input model is used. There is a spike of impulse input functions and everywhere else equals zero. The time during which the contaminant was dumped is seen by the spike. At time zero, assuming the impulse is always, after some time has passed, the model shows the level of contamination. The key points are that the lake continues with an initial accumulation of contaminants, and that no pollutant reaches the lake until the model has finished [2].

The dumping of one barrel of gasoline into a lake at time zero would be an example. One quick strong waste dumping, without any response to the lake. Therefore, input  $p(x)$  is equal to 100 units for a period of 10 units of time, see Fig. 2.

$$\begin{aligned}
 V_1 &= 2900 \text{ mi}^3, \quad V_2 = 850 \text{ mi}^3, \quad V_3 = 1180 \text{ mi}^3, \\
 F_{21} &= 18 \text{ mi}^3/\text{year}, \quad F_{32} = 18 \text{ mi}^3/\text{year}, \quad F_{31} = 20 \text{ mi}^3/\text{year}, \quad F_{13} = 38 \text{ mi}^3/\text{year}
 \end{aligned}$$

So the System (1.1) would be [2]

$$\begin{aligned}
 \frac{dy_1}{dx} &= \frac{38}{1180}y_3(x) + 100 - \frac{20}{2900}y_1(x) - \frac{18}{2900}y_1(x), \\
 \frac{dy_2}{dx} &= \frac{18}{2900}y_1(x) - \frac{18}{850}y_2(x), \\
 \frac{dy_3}{dx} &= \frac{20}{2900}y_1(x) + \frac{18}{850}y_2(x) - \frac{38}{1180}y_3(x), \\
 y_1(0) &= 0, \quad y_2(0) = 0, \quad y_3(0) = 0.
 \end{aligned}
 \tag{3.1}$$

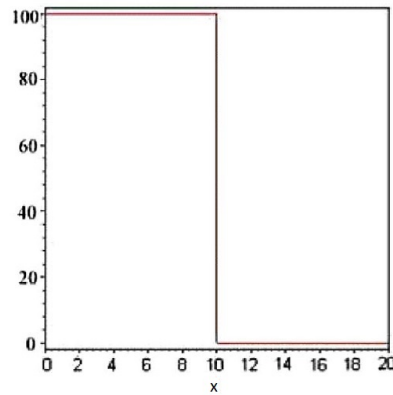


FIGURE 2. Impulse input graph [2].

Table 1 provides the results of the solution (3.1) and shows the absolute errors obtained by using the fourth-order approach of Runge Kutta and LADM with just 10 terms. With LADM, we obtained a reasonable approximation .

**Table 1**

**Absolute errors obtained by the fourth order method of Runge Kutta and LADM for the impulse input model.**

$i$	$x_i$	$ y_1(x_i) - RKM $	$ y_2(x_i) - RKM $	$ y_3(x_i) - RKM $
0	0	0	0	0
1	0.01	3.279322702E-9	1.771872540E-9	1.507450477E-9
2	0.02	6.558182220E-9	3.543234768E-9	3.014947435E-9
3	0.03	9.835210868E-9	5.313439606E-9	4.521771394E-9
4	0.04	8.192236223E-9	4.425516560E-9	3.766719502E-9
5	0.05	8.189531719E-9	4.423805971E-9	3.765724466E-9
6	0.06	8.186788136E-9	4.422072136E-9	3.764715709E-9
7	0.07	8.184046330E-9	4.420338772E-9	3.763707081E-9
8	0.08	8.181989308E-9	4.419039538E-9	3.762950857E-9
9	0.09	8.179783961E-9	4.417644289E-9	3.762138656E-9
10	0.10	8.177499566E-9	4.416201359E-9	3.761298576E-9

To see the solution's accuracy, the residual error for the model is defined as

$$\begin{aligned}
 resy_1 &= \frac{d\tilde{y}_1}{dx} - \left( \frac{38}{1180}\tilde{y}_3(x) + 100 - \frac{20}{2900}\tilde{y}_1(x) - \frac{18}{2900}\tilde{y}_1(x) \right), \\
 resy_2 &= \frac{d\tilde{y}_2}{dx} - \left( \frac{18}{2900}\tilde{y}_1(x) - \frac{18}{850}\tilde{y}_2(x) \right), \\
 resy_3 &= \frac{d\tilde{y}_3}{dx} - \left( \frac{20}{2900}\tilde{y}_1(x) + \frac{18}{850}\tilde{y}_2(x) - \frac{38}{1180}\tilde{y}_3(x) \right)
 \end{aligned}$$

where the LADM solutions for  $y_1$ ,  $y_2$  and  $y_3$  are  $\tilde{y}_1$ ,  $\tilde{y}_2$  and  $\tilde{y}_3$ , respectively. Table 2 presents the residual errors with just 10 terms.

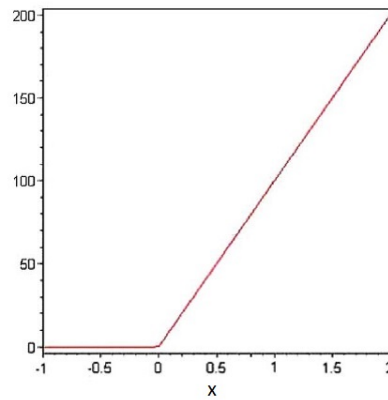


FIGURE 3. Step input graph [2].

**Table 2**  
**Residual errors in the impulse input model obtained by LADM.**

$i$	$x_i$	$resy_1$	$resy_2$	$resy_3$
0	0	0	0	0
1	0.01	6.38031E-45	1.49229E-45	4.88802E-45
2	0.02	1.30669E-41	3.0562E-42	1.00107E-41
3	0.03	1.13025E-39	2.64354E-40	8.65899E-40
4	0.04	2.6761E-38	6.2591E-39	2.05019E-38
5	0.05	3.11538E-37	7.28655E-38	2.38673E-37
6	0.06	2.31476E-36	5.41397E-37	1.77336E-36
7	0.07	1.2616E-35	2.95074E-36	9.66522E-36
8	0.08	5.48064E-35	1.28186E-35	4.19878E-35
9	0.09	2.00221E-34	4.68295E-35	1.53391E-34
10	0.10	6.38031E-34	1.49229E-34	4.88802E-34

**3.2. Step Input.** For contaminants that enter the lake at a steady concentration and rate, the step input model is used and continues indefinitely in the same way. At time zero, there is no pollutant concentration. The important points for the step input are that the input suddenly increases at time zero and that after time zero, essentially the input remains constant. The graph in Fig. 3 represents the  $p(x)$  graph [2].

For instance, when a factory starts production at time zero, waste discharge begins at a fixed rate and concentration (the pollutant is not reacting with the lake). We now assume  $p(x) = 100.x$ , so

$$\begin{aligned} \frac{dy_1}{dx} &= \frac{38}{1180}y_3(x) + (100x) - \frac{20}{2900}y_1(x) - \frac{18}{2900}y_1(x), \\ \frac{dy_2}{dx} &= \frac{18}{2900}y_1(x) - \frac{18}{850}y_2(x), \\ \frac{dy_3}{dx} &= \frac{20}{2900}y_1(x) + \frac{18}{850}y_2(x) - \frac{38}{1180}y_3(x), \\ y_1(0) &= 0, y_2(0) = 0, y_3(0) = 0. \end{aligned}$$

will be the system (1.1) with parameters defined in the Impulse Input Section [2]. **Table 3** provides the results of the Step Input solution and shows the absolute errors obtained by using LADM with just 10 terms and the Runge Kutta solution. With LADM, we obtained a reasonable approximation. The residual errors are given in **Table 4** for the Step Input Model.

**Table 3**  
**Results for Model Step Input.**

$i$	$x_i$	$ y_1(x_i) - RKM $	$ y_2(x_i) - RKM $	$ y_3(x_i) - RKM $
0	0	0	0	0
1	0.01	9.527303321E-10	5.442136859E-9	6.047305602E-9
2	0.02	1.692552535E-8	3.027378594E-9	3.361434626E-9
3	0.03	1.693817476E-8	3.034303813E-9	3.367158827E-9
4	0.04	1.695126052E-8	3.041463734E-9	3.373084669E-9
5	0.05	1.696432304E-8	3.048610545E-9	3.379000384E-9
6	0.06	1.695281782E-8	3.042485136E-9	3.373620621E-9
7	0.07	1.694677859E-8	3.039312809E-9	3.370753598E-9
8	0.08	1.694347002E-8	3.037615796E-9	3.369142118E-9
9	0.09	1.694016538E-8	3.035920854E-9	3.367532322E-9
10	0.10	1.693686258E-8	3.034227400E-9	3.365923171E-9

**Table 4**  
**The residual errors obtained by LADM for step input model**

$i$	$x_i$	$resy_1$	$resy_2$	$resy_3$
0	0	0	0	0
1	0.01	5.31692E-48	1.24357E-48	4.07335E-48
2	0.02	2.17781E-44	5.09367E-45	1.66845E-44
3	0.03	2.82563E-42	6.60885E-43	2.16475E-42
4	0.04	8.92032E-41	2.08637E-41	6.83395E-41
5	0.05	1.29808E-39	3.03606E-40	9.94471E-40
6	0.06	1.15738E-38	2.70698E-39	8.8668E-39
7	0.07	7.35931E-38	1.72126E-38	5.63804E-38
8	0.08	3.65376E-37	8.54576E-38	2.79919E-37
9	0.09	1.50166E-36	3.51221E-37	1.15043E-36
10	0.10	5.31692E-36	1.24357E-36	4.07335E-36

**3.3. Sinusoidal Input.** For pollutants periodically added to the lake, the sinusoidal input model is used for (see Fig. 4). For an overall concentration, pollution reaches the system and then varies regularly across the average. Sinusoidal input changes  $p(x)$  to a more useful one

$$p(x) = p_i \cdot \left( 1 + a \cdot \sin\left(\frac{2\pi \cdot x}{X}\right) \right).$$

For the sinusoidal model, the additional variables are defined as follows:

- $a$  the normalize amplitude (always in-between 0 and 1),
- $X$  the period of fluctuations,
- $p_i$  the average input concentration of pollutant.

$p_i(1 + a)$  for the peaks and  $p_i(1 - a)$  for the valleys are the actual amplitude of the term. The normalized amplitude should stay in-between zero and one; otherwise, the concentration of input is negative and make no sense. For a sinusoidal model, the key argument is that over time the concentration increases, that the change in input concentration must be periodic, and that this change should be presented to the average concentration by a sinusoidal function. See Fig. 4 [2].

An example would be a dumping waste processing plant generating more production due to hours of service during the day than at night; thus a periodic input. Eventually, the lake concentration converges to the contaminant's average

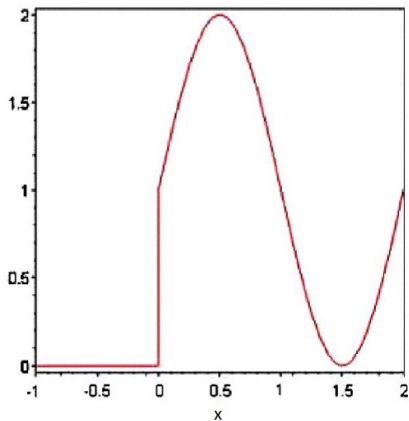


FIGURE 4. Sinusoidal input graph [2].

input concentration. If  $p(x) = 1 + \sin(x)$  is replaced in (1.1) the following system in [2] obtained as

$$\begin{aligned} \frac{dy_1}{dx} &= \frac{38}{1180}y_3(x) + [1 + \sin(x)] - \frac{20}{2900}y_1(x) - \frac{18}{2900}y_1(x), \\ \frac{dy_2}{dx} &= \frac{18}{2900}y_1(x) - \frac{18}{850}y_2(x), \\ \frac{dy_3}{dx} &= \frac{20}{2900}y_1(x) + \frac{18}{850}y_2(x) - \frac{38}{1180}y_3(x), \\ y_1(0) &= 0, y_2(0) = 0, y_3(0) = 0. \end{aligned}$$

Table 5 provides the results of the sinusoidal input solution and shows the absolute errors achieved by using LADM with just 10 terms and the Runge Kutta solution respectively. With LADM, we obtained a reasonable approximation . The residual errors are shown in Table 6 for sinusoidal input model.

**Table 5**  
**Results for Sinusoidal input model**

$i$	$x_i$	$ y_1(x_i) - RKM $	$ y_2(x_i) - RKM $	$ y_3(x_i) - RKM $
0	0	0	0	0
1	0.01	1.374177553E-9	4.999656610E-10	5.601765325E-10
2	0.02	3.683751231E-9	9.988435393E-10	1.119230613E-9
3	0.03	6.825411250E-9	1.497171754E-9	1.677759968E-9
4	0.04	5.876943036E-9	1.246767404E-9	1.397327097E-9
5	0.05	6.291624402E-9	1.245941687E-9	1.396536622E-9
6	0.06	6.705999082E-9	1.245056395E-9	1.395681601E-9
7	0.07	7.120030579E-9	1.244119763E-9	1.394769420E-9
8	0.08	6.910662459E-9	1.243513596E-9	1.394192161E-9
9	0.09	6.840454439E-9	1.242847674E-9	1.393555594E-9
10	0.10	6.838823896E-9	1.242144063E-9	1.392880581E-9

**Table 6****The residual errors obtained by LADM for sinusoidal input model**

$i$	$x_i$	$resy_1$	$resy_2$	$resy_3$
0	0	0	0	0
1	0.01	3.08149E-33	0	0
2	0.02	0	0	0
3	0.03	3.08149E-33	7.70372E-34	0
4	0.04	0	0	0
5	0.05	3.08149E-33	7.70372E-34	0
6	0.06	3.08149E-33	7.70372E-34	3.08149E-33
7	0.07	0	0	3.08149E-33
8	0.08	0	0	3.08149E-33
9	0.09	0	0	0
10	0.10	0	0	3.08149E-33

## 4. CONCLUSION

With reliable results, the LADM, which was used to solve the pollution of a lake system model, appears to be quite simple to use. The numerical results obtained by LADM compared with the solution of the Fourth Order Runge Kutta Method. Furthermore residual errors are given for showing the accuracy. In particular, the current approach offers exceptional precision for small time values  $x$ . The results show that the LADM is an efficient mathematical method for solving the pollution of a lake system model. Mathematica 7 was used to do all of the computations in the present study.

## ACKNOWLEDGEMENT

Part of this article was presented as a summary with Ömer AKIN at the 13th Mathematics Symposium in 2015. I would like to thank the referees for their contribution in eliminating the typos in this article.

## CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this article.

## REFERENCES

- [1] Babolian, E., Biazar, J., Vahidi, A.R., *A new computational method for Laplace transforms by decomposition method*, Applied Mathematics and Computation, **150**(2004), 841–846.
- [2] Biazar, J., Farrokhi, L., Islam, M.R., *Modeling the pollution of a system of lakes*, Applied Mathematics and Computation, **178**(2006), 423–430.
- [3] Biazar, J., Shahbala, M., Ebrahimi, H., *VIM for Solving the Pollution Problem of a System of Lakes*, Hindawi Publishing Corporation, Journal of Control Science and Engineering, **2010**(2010), Article ID 829152, 6 pages, doi:10.1155/2010/829152.
- [4] Doğan, N., *Solution Of The System Of Ordinary Differential Equations By Combined Laplace Transform-Adomian Decomposition Method*, Mathematical and Computational Applications An International Journal, **17**(2012), 203-211.
- [5] Doğan, N., Akin, Ö., *Series solution of epidemic model*, TWMS Journal of Applied and Engineering Mathematics, **2**(2)(2012), 238-244.
- [6] Doğan, N., *Numerical treatment of the model for HIV infection of CD4+T cells by using Multi-Step Laplace Adomian Decomposition Method*, Discrete Dynamics in Nature and Society, **2012**(2012), Article ID 976352, 11 pages, doi:10.1155/2012/976352.
- [7] Doğan, N., *Numerical solution of Chaotic Genesio System with Multi-Step Laplace Adomian Decomposition Method*, Kuwait Journal of Science, **40**(2013).
- [8] Khuri, S.A., *A Laplace Decomposition Algorithm Applied To A Class Of Nonlinear Differential Equations*, Journal of Applied Mathematics, **1**(4)(2001), 141–155.
- [9] Kiyamaz, O., *An Algorithm for Solving Initial Value Problems Using Laplace Adomian Decomposition Method*, Applied Mathematical Sciences, **3**(30)(2009), 1453 – 1459.
- [10] Merdan, M., *He's Variational Iteration Method Applied To The Solution Of Modelling The Pollution Of A System Of Lakes*, Dumlupınar Üniversitesi Fen Bilimleri Enstitüsü Dergisi, **18**(Nisan 2009), 59–70.
- [11] Merdan, M., *Homotopy Perturbation Method For Solving Modelling The Pollution Of A System Of Lakes*, SDU Journal Of Science (E-Journal), **4**(1)(2009), 99-111.
- [12] Wazwaz, A., *The combined Laplace transform-Adomian decomposition method for handling nonlinear Volterra integro-differential equations*, Applied Mathematics and Computation, **216**(2010), 1304–1309.