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Fair Endowment Calculation for Nursing Homes Using Actuarial Techniques: A Turkey Experience

Aktüeryal Teknikleri Kullanılarak Huzurevleri İçin Adil Bağış Hesaplaması: Türkiye Örneği

Abstract

Background: Nursing homes continue to play an important role in elderly services, including offering physical, psychological, and social support. For this reason, it is crucial to ensure their sustainable financial management. Aims: This study aims to build a model that incorporates factors such as age, gender, and real rate of return, all of which affect the minimum amount of the one-time endowment. Methods: In this study, the minimum endowment is calculated using actuarial techniques by considering the age and gender of the nursing home resident along with the real rate of return for endowments. Our model incorporates a probability of spending calculated using mortality rates from Turkey Life Tables (TRH-2010) and a 2% real rate of return. Results: The expected value of ₺1 spent each year as long as the individual lives varies with age and gender. For a 60-year-old female, this expected value is over 0.99 (i.e., 99% probability of spending ₺1 during the year), whereas it falls below 0.50 for an 81-year-old, and 0.10 for a 90-year-old. For a 60-year-old male, the expected value is about 0.99, which falls below 0.50 for a 78-year-old, and 0.10 for an 89-year-old. Thus, the customary endowment is insufficient for female elderly persons below the age of 71 and male elderly persons below the age of 68. Conclusion: Many factors can affect the fair amount of a one-time endowment. Failure to take these factors into account may seriously jeopardize the fairness and sustainability of elderly services.

Öz

Arka plan: Huzurevleri, fiziksel, psikolojik ve sosyal destek de dâhil olmak üzere yaşlı hizmetlerinde önemli bir rol oynar. Bu nedenle, huzurevlerinin sürdürülebilir finansal yönetimlerini sağlamak çok önemlidir. Amaç: Bu çalışma, huzurevlerinde tek seferlik bağış miktarının adil ve doğru belirlenmesinde yaş, cinsiyet ve gerçek getiri oranı gibi faktörleri içeren bir model oluşturmayı amaçlamaktadır. Yöntem: Bu çalışmada asgari bağış, huzurevinde ikamet eden kişinin yaşı ve cinsiyeti ile bağışların reel getiri oranı dikkate alınarak aktüeryal teknikler kullanılarak hesaplanmıştır. Modelimiz, Türkiye Yaşam Tablolarından (TRH-2010) alınan ölüm oranları kullanılarak hesaplanan bir harcama olasılığını ve% 2'lik bir reel getiri oranını içermektedir. Bulgular: Bireysel yaşamlar yaşa ve cinsiyete göre değiştiği sürece her yıl harcanan 1 TL'nin beklenen değeri. 60 yaşında bir kadın için bu beklenen değer 0,99'un üzerindedir (yani yıl içinde 1 TL harcama olasılığı% 99), 81 yaşında ise 0,50'nin ve 90 yaşında 0,10'un altına düşmektedir. 60 yaşında bir erkek için beklenen değer yaklaşık 0,99'dur ve 78 yaşında birisi için 0,50'nin ve 89 yaşında birisi için 0,10'un altına düşmektedir. Bu nedenle, 71 yaşın altındaki kadın yaşlılar ve 68 yaşın altındaki erkek yaşlılar için geleneksel bağış yetersizdir. Sonuç: Bir kerelik bağışın adil miktarını birçok faktör etkileyebilir. Bu faktörlerin hesaba katılmaması, yaşlı hizmetlerinin adaletini ve sürdürülebilirliğini tehlikeye atabilir.

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Introduction

Old age is not a new phenomenon, but many societies are witnessing a steadily rising average age, leading to issues regarding sustainability of the government pension system as well as staggering healthcare costs. Spending programs for the elderly are at the forefront of the problems faced by both developed and developing countries. For this reason, countries are developing new policy proposals to reduce the cost of aging, improve the elderly's quality of health, and avoid social security vulnerabilities (Gokbunar et all, 2016)

In recent years, despite the proliferation of home care services and living spaces, private alternative policies and practices are emerging. Nursing homes, the first line of elderly services in Turkey, date back to Seljuk origin (circa 100 A.D.). They still play an important role within the framework of social security and social services programs for the elderly (Ardahan, 2010).

Old age leads to many problems such as chronic diseases, disabilities, and dependence on other people. Elderly people can suffer from the loss of physical and mental capacity, become dependent on others, and face increasing psychological problems. They might fail to take their prescribed medicine regularly and might be unable to perform adequate personal care due to chronic diseases and disabilities (Lau et all, 2005). This may cause the elderly to be dependent on others for personal and medical care, to the point of possibly requiring institutions or individuals to care for them. Nursing home care involves ensuring the continuity of the physical, mental, and social capacity of elderly residents for as long as possible (Wagner et all, 2001). In addition, nursing homes should not only provide clinical care but also meet the social and psychological needs of their residents (Glass, 1991). For this reason, it is crucial to ensure sustainable financial management for these nursing homes.

Nursing homes have been affected by neoliberal economic movements that emerged globally at the beginning of 1980s, and they have been increasingly financed by the private sector rather than the public sector. This is a natural consequence of the demand for higher quality services. Nursing home services that have primarily been funded by the public sector have progressed toward a structure in which costs are increasingly covered by individuals. As a result, individuals have started to purchase insurance to pay for these expenses (Çadır, 2017).

Elderly people accepted to nursing homes have two payment options (Turkey Official Newspaper, 2001). The first option is monthly payments, the amount of which is determined by the Ministry of Family and Social Policies. The monthly fee received from an elderly person residing in a single room in a nursing home operated by Kizilay (Turkish Red Crescent) is ₺1,750 for 2017 excluding VAT. This fee was used as the monthly fee in our study. The second option is a one-time endowment that is expected to compensate for the care costs of the elderly person for the rest of her/his life. In 2017, a nursing home operated by Kizilay required a one-time endowment equivalent to an upfront payment for 10 years.

This study aims to build a model that incorporates factors such as age, gender, and real rate of return, all of which affect the minimum amount of the one-time endowment.

1. Materials and Method

1.1. Method of Study

The minimum amount of endowment needed is calculated using actuarial techniques and considering the age and gender of the elderly person as well as the real rate of return for endowments. Our model incorporates the probability of spending calculated using mortality rates from Turkey Life Tables (TRH-2010) and a 2% real rate of return. We utilized R 3.4.2 and R Studio Desktop 1.1.442 software packages for our computations. The R script is included in Appendix A.

1.2. Limitations

Apart from these three factors, estimates of an individual's current and future health status can also affect the costs and, therefore, the minimum amount of endowment. For example, the cost of a bedridden patient and cost of an elderly person able to self-care will be very different. As another example, the life expectancy of a patient who has had a heart attack will naturally be shorter than that of a healthy individual. In addition, extensive data are needed to study the impact of these factors. We plan to incorporate some of these factors into our model in our future studies due to and the availability of appropriate data and the scope of this study.

1.3. Factors Affecting Minimum Endowment Amount

Age: This factor indicates the age at which the elderly person is admitted to the nursing home, which is the most important factor affecting the minimum endowment amount. The life expectancy of a person aged 60 and another person aged 90 are very different. According to Turkey Life Tables (TRH-2010), the life expectancy of a 60-year-old female is 20.79 years whereas the life expectancy of a 90-year-old is only 3.29 years (Hacettepe University, 2010).

Gender: Women live longer than men on average. According to WHO data, in 2015, the life expectancy at birth is 73.7 years for women and 69.1 years for men. In Europe, which has the longest life expectancy in the world, the difference is even higher. Life expectancy at birth in Europe is 80.2 years for women but merely 73.2 years for men (WHO,2016).

According to Turkey Life Tables (TRH-2010), the life expectancy of a 60-year-old woman is 20.79 years, whereas the life expectancy of a male of the same age is 17.62 years. This difference is more important for individuals who are admitted to nursing homes at a relatively young age, albeit with a declining difference in life expectancies at higher ages.

Real Rate of Return: As the real rate of return increases, the initial endowment will be invested at a higher rate, and the minimum amount expected to compensate for the care costs of the elderly person for the rest of her/his life turn out to be lower. The real rate of return is assumed to be 2%. For future studies, we plan to conduct a sensitivity analysis to determine the impact of the real rate of return on the minimum amount of initial endowment. For instance, historical data covering the last 10 years may be utilized to get a more realistic estimate of the real rate of return. Hence, despite having the real rate of return as a variable in our model, our current study focuses on the impact of age and gender.

2. Results

2.1. Life Expectancy Calculations

Turkey Female and Males Life Tables (TRH-2010) are given in Table 1. Turkey Female Life (TRH-2010) and Table 2. Turkey Male Life (TRH-2010), respectively. The explanations of the symbols in the table are as follows (Strauss and Shavelle, 2010).

Table 1. Turkey Female Life (TRH-2010)

Age x	q_x	p_x	l_x	d_x	e_x	e_x at x	Age x	q_x	p_x	l_x	d_x	e_x	e_x at x
0	0.008161	0.991839	100,000.00	816.11	78.02	78.02	**50**	0.002650	0.997350	96,638.68	256.06	29.74	79.74
1	0.000278	0.999722	99,183.89	27.57	77.66	78.66	**51**	0.002840	0.997160	96,382.62	273.75	28.82	79.82
2	0.000235	0.999765	99,156.31	23.32	76.68	78.68	**52**	0.003129	0.996871	96,108.87	300.72	27.90	79.90
3	0.000202	0.999798	99,132.99	20.07	75.70	78.70	**53**	0.003517	0.996483	95,808.16	336.94	26.98	79.98
4	0.000187	0.999813	99,112.92	18.50	74.72	78.72	**54**	0.004006	0.995994	95,471.21	382.44	26.08	80.08
5	0.000143	0.999857	99,094.41	14.17	73.73	78.73	**55**	0.004401	0.995599	95,088.77	418.45	25.18	80.18
6	0.000116	0.999884	99,080.25	11.53	72.74	78.74	**56**	0.004735	0.995265	94,670.32	448.28	24.29	80.29
7	0.000100	0.999900	99,068.72	9.89	71.75	78.75	**57**	0.005225	0.994775	94,222.04	492.34	23.40	80.40
8	0.000093	0.999907	99,058.82	9.26	70.76	78.76	**58**	0.005875	0.994125	93,729.70	550.63	22.52	80.52
9	0.000097	0.999903	99,049.57	9.62	69.76	78.76	**59**	0.006688	0.993312	93,179.07	623.14	21.65	80.65
10	0.000094	0.999906	99,039.95	9.31	68.77	78.77	**60**	0.007251	0.992749	92,555.94	671.16	20.79	80.79
11	0.000084	0.999916	99,030.64	8.34	67.78	78.78	**61**	0.007724	0.992276	91,884.78	709.70	19.94	80.94
12	0.000084	0.999916	99,022.30	8.37	66.78	78.78	**62**	0.008609	0.991391	91,175.08	784.97	19.09	81.09
13	0.000095	0.999905	99,013.94	9.39	65.79	78.79	**63**	0.009923	0.990077	90,390.11	896.96	18.26	81.26
14	0.000115	0.999885	99,004.55	11.41	64.79	78.79	**64**	0.011685	0.988315	89,493.14	1,045.69	17.43	81.43
15	0.000135	0.999865	98,993.13	13.37	63.80	78.80	**65**	0.013220	0.986780	88,447.45	1,169.28	16.63	81.63
16	0.000148	0.999852	98,979.76	14.66	62.81	78.81	**66**	0.014533	0.985467	87,278.18	1,268.37	15.85	81.85
17	0.000162	0.999838	98,965.11	16.05	61.82	78.82	**67**	0.016337	0.983663	86,009.80	1,405.18	15.08	82.08
18	0.000177	0.999823	98,949.06	17.54	60.83	78.83	**68**	0.018672	0.981328	84,604.62	1,579.70	14.32	82.32
19	0.000193	0.999807	98,931.51	19.14	59.84	78.84	**69**	0.021583	0.978417	83,024.92	1,791.93	13.58	82.58
20	0.000210	0.999790	98,912.37	20.76	58.85	78.85	**70**	0.024463	0.975537	81,232.99	1,987.24	12.87	82.87
21	0.000226	0.999774	98,891.62	22.31	57.86	78.86	**71**	0.027224	0.972776	79,245.75	2,157.41	12.18	83.18
22	0.000241	0.999759	98,869.30	23.83	56.88	78.88	**72**	0.030523	0.969477	77,088.34	2,352.96	11.51	83.51
23	0.000256	0.999744	98,845.48	25.30	55.89	78.89	**73**	0.034440	0.965560	74,735.38	2,573.88	10.85	83.85
24	0.000271	0.999729	98,820.18	26.73	54.90	78.90	**74**	0.039082	0.960918	72,161.50	2,820.20	10.22	84.22
25	0.000282	0.999718	98,793.45	27.90	53.92	78.92	**75**	0.044930	0.955070	69,341.30	3,115.50	9.62	84.62
26	0.000295	0.999705	98,765.55	29.09	52.93	78.93	**76**	0.051247	0.948753	66,225.81	3,393.88	9.05	85.05

27	0.000311	0.999689	98,736.46	30.68	51.95	78.95	**77**	0.057276	0.942724	62,831.93	3,598.77	8.51	85.51
28	0.000331	0.999669	98,705.77	32.69	50.97	78.97	**78**	0.062975	0.937025	59,233.16	3,730.18	8.00	86.00
29	0.000356	0.999644	98,673.09	35.09	49.98	78.98	**79**	0.068250	0.931750	55,502.98	3,788.10	7.50	86.50
30	0.000372	0.999628	98,638.00	36.67	49.00	79.00	**80**	0.075940	0.924060	51,714.88	3,927.21	7.01	87.01
31	0.000385	0.999615	98,601.33	37.98	48.02	79.02	**81**	0.086117	0.913883	47,787.67	4,115.31	6.55	87.55
32	0.000411	0.999589	98,563.35	40.53	47.04	79.04	**82**	0.095750	0.904250	43,672.36	4,181.63	6.12	88.12
33	0.000450	0.999550	98,522.82	44.32	46.06	79.06	**83**	0.104485	0.895515	39,490.73	4,126.17	5.71	88.71
34	0.000501	0.999499	98,478.50	49.35	45.08	79.08	**84**	0.111664	0.888336	35,364.56	3,948.94	5.32	89.32
35	0.000536	0.999464	98,429.15	52.71	44.10	79.10	**85**	0.122720	0.877280	31,415.62	3,855.31	4.93	89.93
36	0.000562	0.999438	98,376.44	55.25	43.12	79.12	**86**	0.139435	0.860565	27,560.31	3,842.87	4.54	90.54
37	0.000613	0.999387	98,321.19	60.29	42.15	79.15	**87**	0.156215	0.843785	23,717.43	3,705.02	4.20	91.20
38	0.000690	0.999310	98,260.91	67.83	41.17	79.17	**88**	0.171981	0.828019	20,012.42	3,441.75	3.88	91.88
39	0.000793	0.999207	98,193.08	77.88	40.20	79.20	**89**	0.184245	0.815755	16,570.67	3,053.07	3.59	92.59
40	0.000860	0.999140	98,115.19	84.41	39.23	79.23	**90**	0.196392	0.803608	13,517.60	2,654.75	3.29	93.29
41	0.000911	0.999089	98,030.78	89.27	38.26	79.26	**91**	0.215417	0.784583	10,862.85	2,340.04	2.97	93.97
42	0.001015	0.998985	97,941.51	99.41	37.30	79.30	**92**	0.239333	0.760667	8,522.81	2,039.79	2.64	94.64
43	0.001174	0.998826	97,842.10	114.83	36.34	79.34	**93**	0.270550	0.729450	6,483.02	1,753.98	2.32	95.32
44	0.001387	0.998613	97,727.27	135.54	35.38	79.38	**94**	0.313514	0.686486	4,729.04	1,482.62	1.99	95.99
45	0.001574	0.998426	97,591.73	153.57	34.43	79.43	**95**	0.370141	0.629859	3,246.42	1,201.63	1.67	96.67
46	0.001725	0.998275	97,438.16	168.07	33.48	79.48	**96**	0.445527	0.554473	2,044.79	911.01	1.36	97.36
47	0.001918	0.998082	97,270.09	186.60	32.54	79.54	**97**	0.559928	0.440072	1,133.78	634.83	1.05	98.05
48	0.002154	0.997846	97,083.49	209.13	31.60	79.60	**98**	0.747798	0.252202	498.95	373.11	0.75	98.75
49	0.002433	0.997567	96,874.36	235.68	30.67	79.67	**99**	1.000000	0.000000	125.84	125.84	0.50	99.50

Table 2. Turkey Male Life (TRH-2010)

Age x	q_x	p_x	l_x	d_x	e_x	e_x at x	Age	q_x	p_x	l_x	d_x	e_x	e_x at x
0	0.019533	0.980467	100,000.00	1,953.31	71.93	71.93	**50**	0.005159	0.994841	92,248.01	475.94	25.79	75.79
1	0.000888	0.999112	98,046.69	87.08	72.35	73.35	**51**	0.005558	0.994442	91,772.07	510.08	24.93	75.93
2	0.000776	0.999224	97,959.61	76.03	71.42	73.42	**52**	0.006163	0.993837	91,261.98	562.44	24.06	76.06
3	0.000686	0.999314	97,883.58	67.17	70.47	73.47	**53**	0.006979	0.993021	90,699.55	633.00	23.21	76.21
4	0.000642	0.999358	97,816.41	62.84	69.52	73.52	**54**	0.008014	0.991986	90,066.55	721.77	22.37	76.37
5	0.000552	0.999448	97,753.58	54.00	68.57	73.57	**55**	0.008968	0.991032	89,344.78	801.27	21.54	76.54
6	0.000430	0.999570	97,699.57	42.00	67.60	73.60	**56**	0.009810	0.990190	88,543.51	868.63	20.74	76.74
7	0.000355	0.999645	97,657.57	34.64	66.63	73.63	**57**	0.010834	0.989166	87,674.87	949.90	19.94	76.94
8	0.000327	0.999673	97,622.92	31.92	65.66	73.66	**58**	0.012050	0.987950	86,724.97	1,045.07	19.15	77.15
9	0.000347	0.999653	97,591.00	33.84	64.68	73.68	**59**	0.013470	0.986530	85,679.90	1,154.15	18.38	77.38
10	0.000335	0.999665	97,557.16	32.66	63.70	73.70	**60**	0.014781	0.985219	84,525.75	1,249.42	17.62	77.62
11	0.000291	0.999709	97,524.50	28.40	62.72	73.72	**61**	0.016036	0.983964	83,276.33	1,335.41	16.88	77.88
12	0.000295	0.999705	97,496.10	28.77	61.74	73.74	**62**	0.017600	0.982400	81,940.92	1,442.13	16.14	78.14
13	0.000347	0.999653	97,467.33	33.78	60.76	73.76	**63**	0.019498	0.980502	80,498.79	1,569.56	15.42	78.42
14	0.000446	0.999554	97,433.56	43.43	59.78	73.78	**64**	0.021763	0.978237	78,929.24	1,717.70	14.72	78.72
15	0.000551	0.999449	97,390.13	53.71	58.80	73.80	**65**	0.024068	0.975932	77,211.54	1,858.30	14.04	79.04
16	0.000626	0.999374	97,336.42	60.90	57.84	73.84	**66**	0.026344	0.973656	75,353.23	1,985.10	13.37	79.37
17	0.000690	0.999310	97,275.52	67.14	56.87	73.87	**67**	0.028939	0.971061	73,368.13	2,123.20	12.72	79.72
18	0.000745	0.999255	97,208.38	72.42	55.91	73.91	**68**	0.031899	0.968101	71,244.93	2,272.62	12.08	80.08
19	0.000790	0.999210	97,135.96	76.76	54.95	73.95	**69**	0.035280	0.964720	68,972.31	2,433.34	11.47	80.47
20	0.000858	0.999142	97,059.20	83.24	53.99	73.99	**70**	0.039232	0.960768	66,538.97	2,610.44	10.87	80.87
21	0.000932	0.999068	96,975.96	90.37	53.04	74.04	**71**	0.043486	0.956514	63,928.53	2,779.99	10.29	81.29
22	0.000973	0.999027	96,885.60	94.26	52.09	74.09	**72**	0.047834	0.952166	61,148.54	2,925.00	9.73	81.73
23	0.000981	0.999019	96,791.34	94.91	51.14	74.14	**73**	0.052306	0.947694	58,223.54	3,045.44	9.20	82.20
24	0.000955	0.999045	96,696.43	92.33	50.19	74.19	**74**	0.056931	0.943069	55,178.10	3,141.34	8.68	82.68
25	0.000919	0.999081	96,604.09	88.75	49.24	74.24	**75**	0.063283	0.936717	52,036.76	3,293.05	8.17	83.17
26	0.000905	0.999095	96,515.35	87.31	48.28	74.28	**76**	0.071007	0.928993	48,743.71	3,461.13	7.69	83.69
27	0.000906	0.999094	96,428.03	87.40	47.33	74.33	**77**	0.078297	0.921703	45,282.58	3,545.49	7.24	84.24
28	0.000924	0.999076	96,340.63	89.00	46.37	74.37	**78**	0.084963	0.915037	41,737.09	3,546.13	6.81	84.81
29	0.000957	0.999043	96,251.64	92.11	45.41	74.41	**79**	0.090677	0.909323	38,190.96	3,463.05	6.40	85.40
30	0.000973	0.999027	96,159.52	93.56	44.45	74.45	**80**	0.099237	0.900763	34,727.91	3,446.30	5.99	85.99
31	0.000978	0.999022	96,065.96	94.00	43.50	74.50	**81**	0.111418	0.888582	31,281.61	3,485.35	5.59	86.59
32	0.001010	0.998990	95,971.97	96.93	42.54	74.54	**82**	0.123215	0.876785	27,796.27	3,424.93	5.23	87.23
33	0.001068	0.998932	95,875.03	102.38	41.58	74.58	**83**	0.133970	0.866030	24,371.34	3,265.03	4.90	87.90
34	0.001152	0.998848	95,772.66	110.32	40.62	74.62	**84**	0.142406	0.857594	21,106.31	3,005.67	4.57	88.57
35	0.001202	0.998798	95,662.34	115.02	39.67	74.67	**85**	0.153331	0.846669	18,100.64	2,775.38	4.25	89.25
36	0.001236	0.998764	95,547.31	118.05	38.72	74.72	**86**	0.170400	0.829600	15,325.25	2,611.42	3.93	89.93
37	0.001320	0.998680	95,429.26	125.95	37.77	74.77	**87**	0.189071	0.810929	12,713.84	2,403.82	3.64	90.64
38	0.001456	0.998544	95,303.31	138.72	36.81	74.81	**88**	0.208788	0.791212	10,310.01	2,152.61	3.37	91.37
39	0.001643	0.998357	95,164.59	156.37	35.87	74.87	**89**	0.227740	0.772260	8,157.41	1,857.76	3.12	92.12
40	0.001769	0.998231	95,008.22	168.05	34.93	74.93	**90**	0.241418	0.758582	6,299.64	1,520.85	2.90	92.90
41	0.001861	0.998139	94,840.17	176.49	33.99	74.99	**91**	0.253833	0.746167	4,778.79	1,213.02	2.66	93.66
42	0.002048	0.997952	94,663.67	193.87	33.05	75.05	**92**	0.271552	0.728448	3,565.78	968.29	2.39	94.39
43	0.002331	0.997669	94,469.81	220.18	32.12	75.12	**93**	0.302863	0.697137	2,597.48	786.68	2.10	95.10
44	0.002710	0.997290	94,249.63	255.42	31.19	75.19	**94**	0.368996	0.631004	1,810.80	668.18	1.80	95.80
45	0.003020	0.996980	93,994.21	283.87	30.27	75.27	**95**	0.444244	0.555756	1,142.62	507.6	1.55	96.55
46	0.003269	0.996731	93,710.34	306.36	29.36	75.36	**96**	0.480228	0.519772	635.02	304.95	1.40	97.40

47	0.003630	0.996370	93,403.98	339.04	28.46	75.46	**97**	0.501158	0.498842	330.06	165.41	1.23	98.23
48	0.004104	0.995896	93,064.94	381.93	27.56	75.56	**98**	0.540448	0.459552	164.65	88.98	0.96	98.96
49	0.004693	0.995307	92,683.01	435.00	26.67	75.67	**99**	1.000000	0.000000	75.67	75.67	0.50	99.50

$\boldsymbol{l_x}$ is the survivorship function, namely the number of persons alive at age x. For example, of the original 100,000 females in the hypothetical cohort, l_{50} = 96,639 (or 96.639%) live to age 50. These values are computed recursively from the m_x values using the following formula:

$$l_{x+1} = l_x e^{-m_x} \quad (1)$$

with l_0, the radix of the table, arbitrarily set to 100,000. For example

$$l_2 = l_1 e^{-m_1} = 99{,}184 e^{-0.000278} = 99{,}156 \ (1)$$

$\boldsymbol{d_x}$ is number of deaths in the interval (x, x+1) for persons alive at age x, computed as follows:

$$d_x = l_x - l_{x+1} \ (2)$$

For example, of the l_{50} = 96,639 persons alive at age 50

$$d_{50} = l_{50} - l_{51} = 96{,}639 - 96{,}383 = 256 \ (2)$$

Thus, according to the formula, 256 persons died prior to age 51. $\boldsymbol{q_x}$ is the probability of dying at age x, also known as the (age-specific) risk of death. Generally, these are derived using the formula

$$q_x = 1 - e^{-m_x} = 1 - p_x \ (3)$$

under the assumption that the instantaneous mortality rate, or force of mortality, remains constant throughout the age interval from x to x+1, whereas p_x is the probability of living at age x. By construction, q_x is also equal to d_x/l_x. Thus, for example

$$q_{50} = \frac{d_{50}}{l_{50}} = \frac{256}{96{,}639} = 0.002650 \ (3)$$

$\boldsymbol{m_x}$ is the mortality rate at age x. Generally, these quantities are estimated from the data and are the sole input to the life table. That is, all other quantities are determined once the m_x values are specified. By construction

$$m_x = \frac{d_x}{L_x} \ (4)$$

where the number of deaths at age x is divided by the number of person-years at risk at age x. Note that the mortality rate, m_x, and the probability of death, q_x, are not identical. For a one-year interval, they will be close in value, but m_x will always be larger.

$\boldsymbol{L_x}$ is midpoint survivorship, i.e., total number of person-years lived by the cohort from age x to x+1 (Princee, 2016). This is the sum of years lived by the l_{x+1} persons who survive the interval, and d_x persons who die during the interval. The former contribute exactly 1 year each, while the latter contribute, on average, approximately half a year, so that

$$L_x = l_{x+1} + 0.5 \times d_x \ (5)$$

This approximation assumes that deaths occur, on average, half way in the age interval x to x+1.

It is also possible to view L_x as the average number of persons alive during the interval x to x+1:

$$L_x = l_{x+1} + 0.5 \times d_x = l_{x+1} + 0.5 \times (l_x - l_{x+1}) = \frac{l_x + l_{x+1}}{2} \ (5)$$

$\boldsymbol{T_x}$ is total number of person-years lived by the cohort from age x until all members of the cohort have died. This is the sum of numbers in the Lx column from age x to the last row in the table. $\boldsymbol{e_x}$ is the (remaining) life expectancy of persons alive at age x, computed as follows:

$$e_x = \frac{T_x}{l_x} \ (6)$$

For example, at age 50, the life expectancy is

$$e_{50} = \frac{T_{50}}{l_{50}} = \frac{2{,}873{,}915}{96{,}639} = 29.74 \ (6)$$

Finally, $\boldsymbol{e_x}$ **at** $\boldsymbol{x}$ is average life expectancy for persons at age x:

$$e_x \text{ at } x = e_x + x \ (7)$$

For example, at age 50, the life expectancy is

$$e_{50} \text{ at } 50 = e_{50} + 50 = 29.74 + 50 = 79.74 \ (7)$$

2.2. Minimum Amount of One-Time Endowment Needed to Cover Lifetime Expenses

The minimum amount of the one-time endowment needed to cover lifetime expenses is assumed to be equal to the actuarial present value of monthly payments determined by the Ministry of Family and Social Policies. In this calculation, we prefer to use real rather than nominal values in order to avoid estimation of annual inflation figures. This necessitates the use of the real rate of return in the computation of discount factors. Accordingly, we have also implicitly assumed that the costs of taking care of elderly and, hence, the monthly payments increase at the rate of inflation. We compute the actuarial present value of ₺1 spent each year as long as the elderly person lives. This actuarial present value corresponds to the minimum multiple needed to be used in the computation of one-time endowment. For instance, if the actuarial present value turns out to be 10, then the nursing home needs to demand a one-time endowment that is equivalent to an upfront payment for at least 10 years.

Under these assumptions, the actuarial present value of ₺1 spent each year as long as the elderly lives is given by (MIT, 2005):

$\text{Actuarial Present Value of ₺}1 = \sum_{k=1}^{\omega-x} {}_{k}L_x \, v^{(k-0.5)}$ (8)

where

$\boldsymbol{\omega}$ = highest age in the mortality table;[1]

$\boldsymbol{x}$ = current age of the elderly;

${}_{k}\boldsymbol{L}_{\boldsymbol{x}}$ = probability of spending ₺1 for an elderly who is at the age of x in the kth year after her/his acceptance to the nursing home;

$\boldsymbol{v}$ = discount factor;

${}_{k}\boldsymbol{L}_{\boldsymbol{x}}$ is given by

$$ {}_{k}L_x = \frac{L_{x+k-1}}{l_x} $$

i.e., the average number of persons alive during the interval $x+k-1$ to $x+k$ divided by the number of persons alive at age x.

It is required by law to be over 60 years old to apply to live in a nursing home. For this reason, our results are tabulated for ages 60 and above. As an example, for ages 60 and 70, we show the average number of persons alive during the interval $x+k$–1 to $x+k$, the number of persons alive at age x, the probability of spending ₺1 for an elderly who is at the age of x in the kth year after her/his acceptance to the nursing home, the discount factor for each year k, and the calculated actuarial present value of ₺1 spent each year as long as the elderly lives in Table 3 and Table 4.

[1] The highest age in TRH-2010 is 100.

Table 3. Actuarial Present Value Calculation for Age 60

k	L_{60+k-1}		l_{60}		${}_kL_{60}$		v^k	Actuarial Present Value	
	Female	Male	Female	Male	Female	Male		Female	Male
1	92,220	83,901	92,556	84,526	0.9964	0.9926	0.9901	16.56	14.36
2	91,530	82,609	92,556	84,526	0.9889	0.9773	0.9707		
3	90,783	81,220	92,556	84,526	0.9808	0.9609	0.9517		
4	89,942	79,714	92,556	84,526	0.9718	0.9431	0.9330		
5	88,970	78,070	92,556	84,526	0.9613	0.9236	0.9147		
6	87,863	76,282	92,556	84,526	0.9493	0.9025	0.8968		
7	86,644	74,361	92,556	84,526	0.9361	0.8797	0.8792		
8	85,307	72,307	92,556	84,526	0.9217	0.8554	0.8620		
9	83,815	70,109	92,556	84,526	0.9056	0.8294	0.8451		
10	82,129	67,756	92,556	84,526	0.8873	0.8016	0.8285		
11	80,239	65,234	92,556	84,526	0.8669	0.7718	0.8123		
12	78,167	62,539	92,556	84,526	0.8445	0.7399	0.7963		
13	75,912	59,686	92,556	84,526	0.8202	0.7061	0.7807		
14	73,448	56,701	92,556	84,526	0.7936	0.6708	0.7654		
15	70,751	53,607	92,556	84,526	0.7644	0.6342	0.7504		
16	67,784	50,390	92,556	84,526	0.7324	0.5962	0.7357		
17	64,529	47,013	92,556	84,526	0.6972	0.5562	0.7213		
18	61,033	43,510	92,556	84,526	0.6594	0.5148	0.7071		
19	57,368	39,964	92,556	84,526	0.6198	0.4728	0.6933		
20	53,609	36,459	92,556	84,526	0.5792	0.4313	0.6797		
21	49,751	33,005	92,556	84,526	0.5375	0.3905	0.6663		
22	45,730	29,539	92,556	84,526	0.4941	0.3495	0.6533		
23	41,582	26,084	92,556	84,526	0.4493	0.3086	0.6405		
24	37,428	22,739	92,556	84,526	0.4044	0.2690	0.6279		
25	33,390	19,603	92,556	84,526	0.3608	0.2319	0.6156		
26	29,488	16,713	92,556	84,526	0.3186	0.1977	0.6035		
27	25,639	14,020	92,556	84,526	0.2770	0.1659	0.5917		
28	21,865	11,512	92,556	84,526	0.2362	0.1362	0.5801		
29	18,292	9,234	92,556	84,526	0.1976	0.1092	0.5687		
30	15,044	7,229	92,556	84,526	0.1625	0.0855	0.5576		
31	12,190	5,539	92,556	84,526	0.1317	0.0655	0.5466		
32	9,693	4,172	92,556	84,526	0.1047	0.0494	0.5359		
33	7,503	3,082	92,556	84,526	0.0811	0.0365	0.5254		
34	5,606	2,204	92,556	84,526	0.0606	0.0261	0.5151		
35	3,988	1,477	92,556	84,526	0.0431	0.0175	0.5050		
36	2,646	889	92,556	84,526	0.0286	0.0105	0.4951		
37	1,589	483	92,556	84,526	0.0172	0.0057	0.4854		
38	816	247	92,556	84,526	0.0088	0.0029	0.4759		
39	312	120	92,556	84,526	0.0034	0.0014	0.4665		
40	63	38	92,556	84,526	0.0007	0.0004	0.4574		

Table 4. Actuarial Present Value Calculation for Age 70

k	L_{70+k-1}		l_{70}		${}_kL_{70}$		v^k	Actuarial Present Value	
	Female	Male	Female	Male	Female	Male		Female	Male
1	80,239	65,234	81,233	66,539	0.9878	0.9804	0.9901	11.00	9.45
2	78,167	62,539	81,233	66,539	0.9623	0.9399	0.9707		
3	75,912	59,686	81,233	66,539	0.9345	0.8970	0.9517		
4	73,448	56,701	81,233	66,539	0.9042	0.8521	0.9330		
5	70,751	53,607	81,233	66,539	0.8710	0.8057	0.9147		
6	67,784	50,390	81,233	66,539	0.8344	0.7573	0.8968		
7	64,529	47,013	81,233	66,539	0.7944	0.7066	0.8792		
8	61,033	43,510	81,233	66,539	0.7513	0.6539	0.8620		
9	57,368	39,964	81,233	66,539	0.7062	0.6006	0.8451		
10	53,609	36,459	81,233	66,539	0.6599	0.5479	0.8285		
11	49,751	33,005	81,233	66,539	0.6125	0.4960	0.8123		
12	45,730	29,539	81,233	66,539	0.5629	0.4439	0.7963		
13	41,582	26,084	81,233	66,539	0.5119	0.3920	0.7807		
14	37,428	22,739	81,233	66,539	0.4607	0.3417	0.7654		
15	33,390	19,603	81,233	66,539	0.4110	0.2946	0.7504		
16	29,488	16,713	81,233	66,539	0.3630	0.2512	0.7357		
17	25,639	14,020	81,233	66,539	0.3156	0.2107	0.7213		
18	21,865	11,512	81,233	66,539	0.2692	0.1730	0.7071		
19	18,292	9,234	81,233	66,539	0.2252	0.1388	0.6933		
20	15,044	7,229	81,233	66,539	0.1852	0.1086	0.6797		
21	12,190	5,539	81,233	66,539	0.1501	0.0832	0.6663		
22	9,693	4,172	81,233	66,539	0.1193	0.0627	0.6533		
23	7,503	3,082	81,233	66,539	0.0924	0.0463	0.6405		
24	5,606	2,204	81,233	66,539	0.0690	0.0331	0.6279		
25	3,988	1,477	81,233	66,539	0.0491	0.0222	0.6156		
26	2,646	889	81,233	66,539	0.0326	0.0134	0.6035		
27	1,589	483	81,233	66,539	0.0196	0.0073	0.5917		
28	816	247	81,233	66,539	0.0100	0.0037	0.5801		
29	312	120	81,233	66,539	0.0038	0.0018	0.5687		
30	63	38	81,233	66,539	0.0008	0.0006	0.5576		

These calculations are performed for all female and male elderly persons above the age of 60. The results are shown in Table 5.

Table 5. Actuarial Present Value

Age	Female	Male	Age	Female	Male
60	16.56	14.36	**80**	6.36	5.49
61	16.00	13.84	**81**	5.97	5.15
62	15.43	13.33	**82**	5.61	4.84
63	14.86	12.83	**83**	5.26	4.55
64	14.30	12.32	**84**	4.93	4.27
65	13.74	11.83	**85**	4.58	3.99
66	13.19	11.34	**86**	4.25	3.70
67	12.63	10.85	**87**	3.95	3.44
68	12.08	10.38	**88**	3.67	3.19
69	11.53	9.91	**89**	3.40	2.97
70	11.00	9.45	**90**	3.13	2.77
71	10.48	9.00	**91**	2.84	2.55
72	9.97	8.56	**92**	2.54	2.31
73	9.46	8.14	**93**	2.24	2.03
74	8.97	7.72	**94**	1.93	1.74
75	8.49	7.31	**95**	1.63	1.51
76	8.03	6.91	**96**	1.33	1.37
77	7.60	6.54	**97**	1.03	1.20
78	7.18	6.19	**98**	0.74	0.95
79	6.77	5.84	**99**	0.50	0.50

Our first and most important observation is that age has the highest impact on the minimum amount of the initial one-time endowment required to cover lifetime expenses. At age 60, this corresponds to an upfront payment for 16.56 and 14.36 years for female and male elderly persons, respectively. Similarly, these required payments correspond to 11.00 and 9.45 years for age 70, respectively, for female and male elderly persons. These findings also demonstrate that gender has a considerable impact on the required minimum initial endowment.

For example, nursing homes operated by the Turkish Red Crescent required a monthly payment of ₺1,750 for a single room in 2017. Using this amount as a basis for monthly costs, the minimum amount of initial one-time endowment should be at least ₺347,714 and ₺301,467 for female and male elderly persons who are at the age of 60. For age 70, these figures correspond to ₺231,077 and ₺198,346. However, these nursing homes demand a flat rate of ₺210,000 without consideration of the age or gender of the elderly resident. Hence, a customary endowment at the time of acceptance that covers 10 years of expenses is insufficient for female elderly below the age of 71 and male elderly below the age of 68.

3. Discussion

Two different payment methods are available for elderly people admitted to a nursing home in Turkey. Regardless of method, the payment amount will be of interest to all stakeholders, such as the elderly and their relatives, nursing institutions, governments, etc. Currently, individual-specific payment amounts are not usually computed. This is understandable to some extent for social and moral reasons. Nevertheless, it is highly probable that innovations in payment methods for nursing homes will occur in the near future given the aging of the population in many countries along with increasing expectations for service differentiation and growing financial pressures on individuals, nursing institutions, and publicly financed social security systems.

Applying a flat rate of initial endowment regardless of age and gender is inequitable yet potentially threatening to the solvency of nursing institutions. As demonstrated, the life expectancy for a 75-year-old man is 8.17 years, which is less than the half of the life expectancy for a 60-year old man, which is 17.62 years. For this reason, taking a flat rate from elderly persons above a certain age is unfair. For social reasons, it may be desirable to provide a transfer of resources from the young to elderly and/or from the healthy to the sick. However, failure to take factors such as age and gender into account in pricing may seriously jeopardize the fairness and sustainability of elderly services. For this reason, it is crucial to use more equitable and fair payment models for nursing homes.

Other factors that affect the care costs of the elderly have also been examined in the literature. Welch et al. examined the impact of Alzheimer's disease on the cost and length of stay in nursing homes. The median length of stay for Alzheimer's patients was 2.75 years, over 10 times the national median length of stay for all diagnoses, and nursing home charges were estimated to be between $35,000 and $52,000 per patient (Welch et all, 1992). Other studies focused on the impact of institutional characteristics on costs and pricing rather than the individual characteristics of the elderly. For instance, in studies conducted by Birnbaum et al. and McKay, the effects of chain ownership on nursing home costs have been examined. Chain ownership resulted in lower costs due to economies of scale (McKay, 1991). Hazra, Rudisill and Gulliford have investigated the determinants of health care costs in the senior elderly and found that annual health care costs increased from 80 years (£2972 in men, £2603 in women) to 97 (men; £4721) or 98 years (women; £3963), before declining. Costs were significantly elevated in the last year of life but this effect declined with age, from £10,027 in younger octogenarians to £7021 in centenarians. This decline was steeper in participants with comorbidities or impairments; £14,500 for 80–84-year-olds and £6752 for centenarians with 7+ impairment (Hazra, Rudisill and Gulliford, 2018).

Kemper and Murtaugh found that the probability of nursing home use increases sharply with age at death: 17% for age 65 to 74, 36% for age 75 to 84, and 60% for age 85 to 94. According to this study, it is projected that more women than men will enter nursing homes (52% vs. 33%), and among them, more women than men will have total lifetime nursing home use of 5 years or more (25% vs. 13%) (Kemper and Murtaugh, 1991). These results are similar to ours. Kemper and Murtaugh also examined the impact of ethnic differences on the cost and length of staying in nursing homes and found significant differences between persons of black and white ethnic backgrounds.

Some studies examined the effect of disability status on mortality in addition to the effects of age and gender. For example, in the simulation study conducted by Rasoanaivo, disabled persons were assumed to have a mortality rate three times higher than that of the whole population (Rasoanaivo, 2001). In a study conducted by Lew and Garfınkel, cigarette use and obesity were found to be significant factors affecting mortality (Lew and Garfınkel, 1987). In an other study Furlan and Fehlings have examined the impact of age on mortality, impairment, and disability among adults with acute traumatic spinal cord injury and found that mortality rates among older people (≥65 years) were significantly greater than those of younger individuals at 6 weeks, at 6 months, and at 1 year following spinal cord injury . Among survivors, age was not significantly correlated with motor recovery or change in pain scores in the acute and chronic stages after spinal cord injury based on regression analyses adjusted for major confounders (Furlan and Fehlings, 2009).

It should be noted that some macroeconomic assumptions such as the real rate of return may also have a significant impact on the results of this study. Considering the low-interest-rate environment existing after the global crisis of 2008, a 2% real rate of return may be too optimistic. A lower real rate of return will automatically translate into a required higher initial one-time endowment.

Conclusion and Evaluation

Some reasons may make it difficult to enforce age and, particularly, gender differences in pricing. However, it is imperative for nursing institutions to take these factors into account even if a flat rate is charged. In a hypothetical setting, an institution where the occupants are mostly relatively younger women should charge a much higher flat rate compared to another institution where the occupants are relatively older men. The administration of such a pricing policy is critical for the financial sustainability of nursing institutions.

Hence, the results of this study have two potential uses. First, it may be possible to use individual-specific pricing in a social setting where use of factors such as age and gender is acceptable. Second, even if individual-specific pricing is not possible for social and/or legal reasons, nursing institutions can utilize similar models for financial planning purposes to ensure long-term sustainability. The model can be extended to account for additional resources such as extra physical space and additional employees for varying demographic compositions of nursing home inhabitants.

For future studies, we intend to explore the impact of other individual-specific factors such as smoking and drinking habits and/or obesity if sufficient amounts of reliable data can be obtained. We also plan to elaborate on the impact of macroeconomic factors such as the real rate of return.

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Appendix A – R Script

```
setwd("C:/Users/USER/Downloads")
# read data from csv files which include mortality tables for female and male elderly
femaledata=read.csv(file="kadinmortalite0.txt",header=TRUE,sep=";")
maledata=read.csv(file="erkekmortalite0.txt",header=TRUE,sep=";")
# assumption for real rate of return
rror=.02
# for each age x, calculate
# Lx+k-1(average number of persons alive during the interval x+k-1 to x+k),
# kLx (probability of spending ₺1 for an elderly who is at the age of x in the k-th year
# after her/his acceptance to the nursing home)
# vk (discount factor)
# APV (actuarial present value)
Lxk1=array(dim=c(2,99,40))
kLx=array(dim=c(2,99,40))
```

```
APV=array(dim=c(2,99))
for(age in 60:99) {
 vk=array(0,40)
 sum1=0
 sum2=0
 for(k in 1:(100-age)) {
  Lxk1[1,age,k]=(femaledata[age+k,4]+femaledata[age+k+1,4])/2
  Lxk1[2,age,k]=(maledata[age+k,4]+maledata[age+k+1,4])/2
  kLx[1,age,k]=Lxk1[1,age,k]/femaledata[age+1,4]
  kLx[2,age,k]=Lxk1[2,age,k]/maledata[age+1,4]
  vk[k]=1/(1+rror)^(k-0.5)
  sum1=sum1+kLx[1,age,k]*vk[k]
  sum2=sum2+kLx[2,age,k]*vk[k]
 }
 APV[1,age]=sum1
 APV[2,age]=sum2
```