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# Classes of bi-Starlike and bi-Convex Bounded Functions with Complex Order

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**Abstract**

In this paper, estimates for second and third coefficients of certain classes of bi-starlike and bi-convex bounded functions with complex order in the open unit disk are determined, and certain special cases are also indicated.

***Keywords:*** *Analytic function, bi-univalent, bi-starlike, bi-convex, bounded functions.*
***2010 Mathematics Subject Classification:*** *30C45*

## 1. Introduction

Let $\mathscr{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Let $\mathscr{S}^*(\alpha)$ and $\mathscr{C}(\alpha)$ $(0 \leq \alpha < 1)$ denote the subclasses of $\mathscr{A}$ that consists, respectively, of starlike of order $\alpha$ and convex of order $\alpha$ in $\mathbb{U}$ (see [13]).
Let $\mathscr{P}(M)$ denote the class of functions $p(z)$ analytic in $\mathbb{U}$ satisfying the properties $p(0) = 1$ and

$$|p(z) - M| < M, \tag{1.2}$$

for a fixed $M, M > \frac{1}{2}$. The class $\mathscr{P}(M)$ was introduced and investigated by Libera and Livingston [10]. Singh and Singh [14] introduced the class $\mathscr{S}_M^*$ bounded starlike functions $f \in \mathscr{A}$ satisfying the following inequality

$$\left| \frac{zf'(z)}{f(z)} - M \right| < M \quad (z \in \mathbb{U}). \tag{1.3}$$

Nasr and Aouf [12] defined the class $\mathscr{F}(b, M)$ $\left(b \in \mathbb{C}^* = \mathbb{C} \backslash \{0\}, M > \frac{1}{2}\right)$, of bounded starlike functions of complex order, for $f(z)/z \neq 0, z \in \mathbb{U}$ and fixed $M$, satisfying the following inequality

$$\left| \frac{b - 1 + \frac{zf'(z)}{f(z)}}{b} - M \right| < M \quad (z \in \mathbb{U}). \tag{1.4}$$

We note that $\mathscr{F}\left(\cos\lambda\ e^{-i\lambda}, M\right) = \mathscr{F}_{\lambda,M}$ $\left(|\lambda| < \frac{\pi}{2}, M > \frac{1}{2}\right)$ is the class of bounded $\lambda-$spirallike functions defined by Kulshestha [8] and $\mathscr{F}\left((1-\alpha)\cos\lambda\ e^{-i\lambda}, M\right) = \mathscr{F}_{\lambda,M}(\alpha)$ $\left(|\lambda| < \frac{\pi}{2}, 0 \leq \alpha < 1, M > \frac{1}{2}\right)$ is the class of bounded $\lambda-$spirallike functions $f(z)$ of order $\alpha$ (see Aouf [3, with $p = 1$] and Aouf [2]).
Also, Nasr and Aouf [11] defined the class $\mathscr{G}(b, M)$ $\left(b \in \mathbb{C}^*, M > \frac{1}{2}\right)$, of bounded convex functions of complex order, for $f'(z) \neq 0, z \in \mathbb{U}$ and fixed $M$, satisfying

$$\left| \frac{b + \frac{zf''(z)}{f'(z)}}{b} - M \right| < M \quad (z \in \mathbb{U}). \tag{1.5}$$

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We note that $\mathscr{G}(\cos\lambda\ e^{-i\lambda},M)=\mathscr{G}_{\lambda,M}\left(|\lambda|<\frac{\pi}{2},M>\frac{1}{2}\right)$ is the class of bounded Robertson functions investigated by Kulshestha [8] and $\mathscr{G}\left((1-\alpha)\cos\lambda\ e^{-i\lambda},M\right)=\mathscr{G}_{\lambda,M}(\alpha)\ \left(|\lambda|<\frac{\pi}{2},0\leq\alpha<1,M>\frac{1}{2}\right)$ is the class of bounded Robertson functions of order $\alpha$ investigated by Aouf [3, with $p=1$] and Aouf [2].

Let $\mathscr{S}$ denote the class of all functions in $\mathscr{A}$ which are univalent in $\mathbb{U}$. It is well known that every function $f\in\mathscr{S}$ has an inverse $f^{-1}$, defined by

$$f^{-1}(f(z))=z\quad(z\in\mathbb{U})$$

and

$$f\left(f^{-1}(w)\right)=w\quad\left(|w|<r_0(f);r_0(f)\geq\frac{1}{4}\right),$$

where

$$f^{-1}(w)=w-a_2w^2+\left(2a_2^2-a_3\right)w^3-\left(5a_2^3-5a_2a_3+a_4\right)w^4+\ldots. \tag{1.6}$$

A function $f\in\mathscr{A}$ is said to be bi-univalent in $\mathbb{U}$ if both $f$ and $f^{-1}$ are univalent in $\mathbb{U}$. Let $\sum$ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1.1).

Using the class $\mathscr{P}(M)$, we now introduce the following subclasses of $\sum$ as follows:

**Definition 1.1.** *A function $f\in\sum$ is said to be in the class $\mathscr{S}_{\sum}(b,M)$ if it satisfies the following condition:*

$$1+\frac{1}{b}\left[\frac{zf'(z)}{f(z)}-1\right]\in\mathscr{P}(M) \tag{1.7}$$

*and*

$$1+\frac{1}{b}\left[\frac{wg'(w)}{g(w)}-1\right]\in\mathscr{P}(M)\ , \tag{1.8}$$

*where $g=f^{-1},b\in\mathbb{C}^*$ and $M>\frac{1}{2}$.*

**Definition 1.2.** *A function $f\in\sum$ is said to be in the class $\mathscr{C}(b,M)$ if it satisfies the following condition:*

$$1+\frac{1}{b}\frac{zf''(z)}{f'(z)}\in\mathscr{P}(M) \tag{1.9}$$

*and*

$$1+\frac{1}{b}\frac{wg''(w)}{g'(w)}\in\mathscr{P}(M)\ , \tag{1.10}$$

*where $g=f^{-1},b\in\mathbb{C}^*$ and $M>\frac{1}{2}$.*

Taking additional choices of $b$ and $M$, the classes $\mathscr{S}_{\sum}(b,M)$ and $\mathscr{C}_{\sum}(b,M)$ reduces to the following subclasses of $\sum$:

(i) $\mathscr{S}_{\sum}(b;\infty)=\mathscr{S}_{\sum}(b)$
$=\left\{f\in\sum:\Re\left\{1+\frac{1}{b}\left(\frac{zf'(z)}{f(z)}-1\right)\right\}>0\text{ and }\Re\left\{1+\frac{1}{b}\left(\frac{wg'(w)}{g(w)}-1\right)\right\}>0\right\};$

(ii) $\mathscr{C}_{\sum}(b;\infty)=\mathscr{C}_{\sum}(b)$
$=\left\{f\in\sum:\Re\left\{1+\frac{1}{b}\frac{zf''(z)}{f'(z)}\right\}>0\text{ and }\Re\left\{1+\frac{1}{b}\frac{wg''(w)}{g'(w)}\right\}>0\right\};$

(iii) $\mathscr{S}_{\sum}(1;M)=\mathscr{S}_{\sum}(M)$
$=\left\{f\in\sum:\frac{zf'(z)}{f(z)}\in\mathscr{P}(M)\text{ and }\frac{wg'(w)}{g(w)}\in\mathscr{P}(M)\right\};$

(iv) $\mathscr{C}_{\sum}(1;M)=\mathscr{C}_{\sum}(M)$
$=\left\{f\in\sum:1+\frac{zf''(z)}{f'(z)}\in\mathscr{P}(M)\text{ and }1+\frac{wg''(w)}{g'(w)}\in\mathscr{P}(M)\right\};$

(v) $\mathscr{S}_{\sum}(1-\eta;\infty)=\mathscr{S}_{\sum}(\eta)\,(0\leq\eta<1)$ (see [5] and [16])
$=\left\{f\in\sum:\Re\left\{\frac{zf'(z)}{f(z)}\right\}>\eta\text{ and }\Re\left\{\frac{wg'(w)}{g(w)}\right\}>\eta\right\};$

(vi) $\mathscr{C}_{\sum}(1-\eta;\infty)=\mathscr{C}_{\sum}(\eta)\,(0\leq\eta<1)$ (see [5] and [16])
$=\left\{f\in\sum:\Re\left\{1+\frac{zf''(z)}{f'(z)}\right\}>\eta\text{ and }\Re\left\{1+\frac{wg''(w)}{g'(w)}\right\}>\eta\right\};$

(vii) $\mathscr{S}_{\sum}\left((1-\alpha)e^{-i\lambda}\cos\lambda;M\right)=\mathscr{S}_{\sum}^{\lambda}(\alpha,M)\left(|\lambda|<\frac{\pi}{2},0\leq\alpha<1\right)$
$=\left\{f\in\sum:\frac{e^{i\lambda}\frac{zf'(z)}{f(z)}-\alpha\cos\lambda-i\sin\lambda}{(1-\alpha)\cos\lambda}\in\mathscr{P}(M)\text{ and }\frac{e^{i\lambda}\frac{wg'(w)}{g(w)}-\alpha\cos\lambda-i\sin\lambda}{(1-\alpha)\cos\lambda}\in\mathscr{P}(M)\right\};$

(viii) $\mathscr{C}_{\sum}\left((1-\alpha)e^{-i\lambda}\cos\lambda;M\right)=\mathscr{C}_{\sum}^{\lambda}(\alpha,M)\left(|\lambda|<\frac{\pi}{2},0\leq\alpha<1\right)$
$=\left\{f\in\sum:\frac{e^{i\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)-\alpha\cos\lambda-i\sin\lambda}{(1-\alpha)\cos\lambda}\in\mathscr{P}(M)\text{ and }\right.$
$\left.\frac{e^{i\lambda}\left(1+\frac{wg''(w)}{g'(w)}\right)-\alpha\cos\lambda-i\sin\lambda}{(1-\alpha)\cos\lambda}\in\mathscr{P}(M)\right\};$

(ix) $\mathscr{S}_\Sigma\left((1-\alpha)e^{-i\lambda}\cos\lambda;\infty\right)=\mathscr{S}_\Sigma^\lambda(\alpha)\left(|\lambda|<\frac{\pi}{2},0\le\alpha<1\right)$
$=\left\{f\in\Sigma:\Re\left\{e^{i\lambda}\frac{zf'(z)}{f(z)}\right\}>\alpha\cos\lambda \text{ and } \Re\left\{e^{i\lambda}\frac{wg'(w)}{g(w)}\right\}>\alpha\cos\lambda\right\};$

(x) $\mathscr{C}_\Sigma\left((1-\alpha)e^{-i\lambda}\cos\lambda;\infty\right)=\mathscr{C}_\Sigma^\lambda(\alpha)\left(|\lambda|<\frac{\pi}{2},0\le\alpha<1\right)$
$=\left\{f\in\Sigma:\Re\left\{e^{i\lambda}\left(1+\frac{zf''(z)}{f'(z)}\right)\right\}>\alpha\cos\lambda \text{ and }\right.$
$\left.\Re\left\{e^{i\lambda}\left(1+\frac{wg''(w)}{g'(w)}\right)\right\}>\alpha\cos\lambda\right\}.$

Srivastava et al. [15], Ali et al. [1], Frasin and Aouf [6], Goyal and Goswami [7] and many others (see [4], [9] and [5]) have introduced and investigated subclasses of bi-univalent functions and obtained bounds for the initial coefficients.
In order to establish our main results, we need the following lemma.

**Lemma 1.3.** *[10] If $p(z)=1+\sum\limits_{n=1}^{\infty}p_nz^n\in\mathscr{P}(M)$ in $\mathbb{U}$ for fixed $M$, $M>\frac{1}{2}$. Then*

$$|p_n|\le 1+m\qquad\left(m=1-\frac{1}{M}\right).\tag{1.11}$$

*The result is sharp.*

In the present paper, we estimates on the coefficients for second and third coefficients of the subclasses $\mathscr{S}_\Sigma(b,M)$ and $\mathscr{C}_\Sigma(b,M)$.

## 2. Main results

Unless otherwise mentioned, we assume throughout this section that $g=f^{-1}$, $b\in\mathbb{C}^*$ and $m=1-\frac{1}{M}\left(M>\frac{1}{2}\right)$.

**Theorem 2.1.** *Let $f(z)$ given by (1.1) belongs to the class $\mathscr{S}_\Sigma(b;M)$, then*

$$|a_2|\le\min\left\{|b|(1+m);\sqrt{|b|(1+m)}\right\}\tag{2.1}$$

*and*

$$|a_3|\le|b|(1+m)\min\left\{1;\frac{1+|b|(1+m)}{2};\frac{1+3|b|(1+m)}{2}\right\}.\tag{2.2}$$

*The result is sharp.*

*Proof.* If $f\in\mathscr{S}_\Sigma(b;M)$, according to the Definition 1.1 we have

$$1+\frac{1}{b}\left[\frac{zf'(z)}{f(z)}-1\right]=p(z)\tag{2.3}$$

and

$$1+\frac{1}{b}\left[\frac{wg'(w)}{g(w)}-1\right]=q(w)\tag{2.4}$$

where $p(z),q(w)\in\mathscr{P}(M)$. Using the fact that the functions $p(z)$ and $q(w)$ have the following Taylor expansions

$$p(z)=1+p_1z+p_2z^2+\dots,\tag{2.5}$$

and

$$q(w)=1+q_1w+q_2w^2+\dots.\tag{2.6}$$

Since

$$1+\frac{1}{b}\left[\frac{zf'(z)}{f(z)}-1\right]=1+\frac{1}{b}a_2z+\frac{1}{b}\left(2a_3-a_2^2\right)z^2+\dots.\tag{2.7}$$

From (1.6), we have

$$1+\frac{1}{b}\left[\frac{zg'(w)}{g(w)}-1\right]=1-\frac{1}{b}a_2w-\frac{1}{b}\left(2a_3-3a_2^2\right)w^2+\dots.\tag{2.8}$$

from (2.5) and (2.6) combined with (2.7) and (2.8), it follows that

$$\frac{1}{b}a_2=p_1,\tag{2.9}$$

$$\frac{1}{b}\left(2a_3-a_2^2\right)=p_2,\tag{2.10}$$

$$-\frac{1}{b}a_2 = q_1, \tag{2.11}$$

$$-\frac{1}{b}\left(2a_3 - 3a_2^2\right) = q_2. \tag{2.12}$$

Now, from (2.10) and (2.12) we deduce that

$$a_2^2 = \frac{b}{2}(q_2 + p_2), \tag{2.13}$$

and

$$a_3 = \frac{b}{4}(3p_2 + q_2). \tag{2.14}$$

From (2.9) and (2.10) we get

$$a_3 = \frac{b}{2}\left(p_2 + p_1^2 b\right), \tag{2.15}$$

while from (2.11) and (2.12) we deduce that

$$a_3 = \frac{b}{2}\left[-q_2 + 3q_1^2 b\right]. \tag{2.16}$$

Combining (2.9) and (2.13) for the computation of the upper-bound of $|a_2|$, and (2.14), (2.15) and (2.16) for the computation of $|a_3|$, by using Lemma 1.3 we easily find the estimates of our theorem. Finally, the assertions (2.1) and (2.2) of Theorem 2.1 are sharp in view of the fact that assertion (1.11) of Lemma 1.3 is sharp. □

By similarly applying the method of proof of Theorem 2.1, we easily get the following theorem.

**Theorem 2.2.** *Let $f(z)$ given by (1.1) belongs to the class $\mathscr{C}_{\Sigma}(b;M)$, then*

$$|a_2| \leq \min\left\{\frac{|b|(1+m)}{2}; \sqrt{\frac{|b|(1+m)}{2}}\right\} \tag{2.17}$$

*and*

$$|a_3| \leq \frac{|b|(1+m)}{6}\min\{4; 1+|b|(1+m); 1+2|b|(1+m)\}. \tag{2.18}$$

*The result is sharp.*

Taking $M = \infty$ in Theorems 2.1 and 2.2, we obtain

**Corollary 2.3.** *(i) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{S}_{\Sigma}(b)$, then*

$$|a_2| \leq \min\left\{2|b|; \sqrt{2|b|}\right\}$$

*and*

$$|a_3| \leq |b|\min\{2; 1+2|b|; 1+6|b|\}.$$

*The result is sharp.*
*(ii) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{C}_{\Sigma}(b)$, then*

$$|a_2| \leq \min\left\{|b|; \sqrt{|b|}\right\}$$

*and*

$$|a_3| \leq \frac{|b|}{3}\min\{4; 1+2|b|; 1+4|b|\}.$$

*The result is sharp.*

Taking $b = 1$ in Theorems 2.1 and 2.2, we obtain the following corollary.

**Corollary 2.4.** *(i) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{S}_{\Sigma}(M)$, then*

$$|a_2| \le \min\left\{1+m; \sqrt{1+m}\right\}$$

*and*

$$|a_3| \le (1+m)\min\left\{1; \frac{2+m}{2}; \frac{4+3m}{2}\right\}.$$

*The result is sharp.*
*(ii) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{C}_{\Sigma}(M)$, then*

$$|a_2| \le \min\left\{\frac{1+m}{2}; \sqrt{\frac{1+m}{2}}\right\}$$

*and*

$$|a_3| \le \frac{1+m}{6}\min\{4; 2+m; 3+2m\}.$$

*The result is sharp.*

Taking $M=\infty$ and $b=1-\eta$ $(0\le\eta<1)$ in Theorems 2.1 and 2.2, we obtain the following result.

**Corollary 2.5.** *(i) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{S}_{\Sigma}(\eta)$, then*

$$|a_2| \le \min\left\{2(1-\eta); \sqrt{2(1-\eta)}\right\}$$

*and*

$$|a_3| \le (1-\eta)\min\{2; 3-2\eta; 7-6\eta\}.$$

*The result is sharp.*
*(ii) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{C}_{\Sigma}(\eta)$, then*

$$|a_2| \le \min\left\{1-\eta; \sqrt{1-\eta}\right\}$$

*and*

$$|a_3| \le \frac{1-\eta}{3}\min\{4; 1+2(1-\eta); 1+4(1-\eta)\}.$$

*The result is sharp.*

Taking $b=(1-\alpha)e^{-i\lambda}\cos\lambda \left(|\lambda|<\frac{\pi}{2}, 0\le\alpha<1\right)$ in Theorems 2.1 and 2.2, we obtain the following corollary.

**Corollary 2.6.** *(i) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{S}_{\Sigma}^{\lambda}(\alpha,M)$, then*

$$|a_2| \le \min\left\{(1+m)(1-\alpha)\cos\lambda; \sqrt{(1+m)(1-\alpha)\cos\lambda}\right\}$$

*and*

$$|a_3| \le (1+m)(1-\alpha)\cos\lambda \min\left\{1; \frac{1+(1+m)(1-\alpha)\cos\lambda}{2}; \frac{1+3(1+m)(1-\alpha)\cos\lambda}{2}\right\}.$$

*The result is sharp.*
*(ii) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{C}_{\Sigma}^{\lambda}(\alpha,M)$, then*

$$|a_2| \le \min\left\{\frac{(1+m)(1-\alpha)\cos\lambda}{2}; \sqrt{\frac{(1+m)(1-\alpha)\cos\lambda}{2}}\right\}$$

*and*

$$|a_3| \le \frac{(1+m)(1-\alpha)\cos\lambda}{6}\min\{4; 1+(1+m)(1-\alpha)\cos\lambda; 1+2(1+m)(1-\alpha)\cos\lambda\}.$$

*The result is sharp.*

Taking $b=(1-\alpha)e^{-i\lambda}\cos\lambda \left(|\lambda|<\frac{\pi}{2}, 0\le\alpha<1\right)$ and $M=\infty$ in Theorems 2.1 and 2.2, we obtain the following corollary.

**Corollary 2.7.** *(i) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{S}_{\Sigma}^{\lambda}(\alpha)$, then*

$$|a_2| \leq \min\left\{2(1-\alpha)\cos\lambda; \sqrt{2(1-\alpha)\cos\lambda}\right\}$$

*and*

$$|a_3| \leq (1-\alpha)\cos\lambda \min\{2; 1+2(1-\alpha)\cos\lambda; 1+6(1-\alpha)\cos\lambda\}.$$

*The result is sharp.*
*(ii) Let $f(z)$ given by (1.1) belongs to the class $\mathscr{C}_{\Sigma}^{\lambda}(\alpha)$, then*

$$|a_2| \leq \min\left\{(1-\alpha)\cos\lambda; \sqrt{(1-\alpha)\cos\lambda}\right\}$$

*and*

$$|a_3| \leq \frac{(1-\alpha)\cos\lambda}{3} \min\{4; 1+2(1-\alpha)\cos\lambda; 1+4(1-\alpha)\cos\lambda\}.$$

*The result is sharp.*

Taking $\alpha = \lambda = 0$ in Corollary 2.7, we obtain the following examples.

**Example 2.8.** *Let $f(z)$ given by (1.1) belongs to the class $\mathscr{S}_{\Sigma}$, then*

$$|a_2| \leq \sqrt{2}$$

*and*

$$|a_3| \leq 2.$$

*The result is sharp, where*

$$\mathscr{S}_{\Sigma} = \left\{f \in \sum : \Re\left\{\frac{zf'(z)}{f(z)}\right\} > 0 \text{ and } \Re\left\{\frac{wg'(w)}{g(w)}\right\} > 0\right\}.$$

**Example 2.9.** *Let $f(z)$ given by (1.1) belongs to the class $\mathscr{C}_{\Sigma}$, then*

$$|a_2| \leq 1$$

*and*

$$|a_3| \leq 1.$$

*The result is sharp, where*

$$\mathscr{C}_{\Sigma} = \left\{f \in \sum : \Re\left\{1+\frac{zf''(z)}{f'(z)}\right\} > 0 \text{ and } \Re\left\{1+\frac{wg''(w)}{g'(w)}\right\} > 0\right\}.$$

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The author contributed to the writing of this paper. The author read and approved the final manuscript.

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