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by

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ABSTRACT: Many authors [1], [2], [3], [4] considered the problems under different weak conditions which imply the continuity of the functions. In this section, we will consider convex functions on a commutative topological group with a root function.

Definition: Suppose that G is a commutative group. A root function is a function $r: G \to G$ such that $r(xy) = r(x)r(y)$ and $r(x^2) = x$ holds for all x,y in G.

A real-valued function f defined on G is called a convex function if it is satisfies the following inequality

$$2f(r(xy)) \leq f(x) + f(y) \qquad \text{for all } x, y \text{ in } G$$

Proposition 1: Suppose G satisfies the following two conditions:

(a) G is a commutative topological group with a root function r.

(b) For any neighbourhood U of the identity e of G, $r(U)$ is also a neighbourhood of e and $r(U) \subset U$.

Let f be a convex function on G. If f is bounded above in some neighbourhood of some point of G, then f is locally bounded above on G.

Proof: By (a) and (b), r is a topological automorphism of G. Suppose f is bounded above in some neighbourhood U of e by M. Let z be any point of G then $zr(U)$ is a neighbourhood of z. For any x in U, we have.

$$f[zr(x)] = f[r(z^2)r(x)] = f[r(z^2x)]$$

$$f[r(z^2x)] \leq \tfrac{1}{2}[f(z^2) + f(x)] \leq \tfrac{1}{2} f(z^2) + \tfrac{1}{2} M$$

Thus f is bounded above in a neighbourhood yr(U) of y. Suppose f is bounded above by M on some neighbourhood cU of $c \neq e$, where U is a neighbourhood of e define $h(x) = f(cx)$. Then we have

$$h[r(xy)] = f[cr(xy)] = f[r(c^2)\, r(xy)] = f[r(cx\; cy)]$$
$$\leq \tfrac{1}{2}\, [f(cx) + f(cy)] = \tfrac{1}{2}\, [h(x) + h(y)]$$

Then h is a convex function which is bounded above in the neighbourhood U of e.So h is bounded above in some neighbourhood of any point of G. So is f.

Proposition 2: f, G as in proposition 1. If f is bounded above on some set A which is 2nd category and almost open, then f is bounded above on some open set.

Proof: Since r is a topological automorphism, r(A) is a 2 nd category and almost open set: r(A) r (A) contains an open set U. For x in U we have $x = a_1\, a_2$ $(a_1,\ a_2 \in A)$ and

$$f\,[r(a_1)\, r(a_2)] \leq \tfrac{1}{2}\, [f(a_1) + f(_2)\,] \leq \text{Sup}\ \{f(a) \mid a \in A\} < +\infty$$

Theorem: Suppose G satisfies the conditions (a) and (b) of the proposition 1.

Let f be a convex function on G. If f is bounded above on a 2 nd category and almost open set A of G, then f is continuous on G.

Proof: By proposition 2 f is bounded on some open set and by proposition 1 is bounded on some neighbourhood of each point of G.

ÖZET

Mevzuubahis problem, sürekliliği intac eden çeşitli zayıf şartlar gözönünde tutularak ele alınmıştır. Burada ise, bir kök fonksiyonu haiz komütatif topolojik grup üzerinde konveks fonksivonlar ele alınmıştır.

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