

STELLAR MASS-RADIUS RELATION

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ABSTRACT

The stellar mass-radius relation is determined by using new data on absolute dimensions of the components of well-detached binary systems. The relation is compared with the published empirical and theoretical mass-radius relations. The location of the knee in the empirical relation is estimated in 1.66 solar mass. The results can be useful (i) in estimating the masses of MS stars which are not components of binary systems, (ii) in constraining the theoretical stellar models, (iii) in estimating the internal chemical composition and age of the MS binary components and (iv) in the light curve analysis of detached binary systems.

INTRODUCTION

The basic stellar data, like the mass, radius and luminosity rest almost exclusively on the data derived by the analysis of light and radial velocity curves for some well-behaved detected binaries. The available accurate data on absolute dimensions of binaries have been compiled by Popper (1980) and subsequently by Harmanec (1988). Such data provide a well-known case for the comparison of current theories of stellar evolution with empirical data.

Kopal (1959) used the data of detached binaries from the Catalogue of the Elements of Eclipsing Binaries (Kopal and Shapley, 1956) and obtained a good statistical definition of the mass-radius (MR) relation. The relation was piecewise linear with a break around two solar mass. The changing slope around two solar mass in the relation was interpreted as the change in behavior of massive and less massive stars.

By using data from visual binaries in addition to data from eclipsing detached binaries Habets and Heintze (1981) studied the empirical MR relation.

In his work Lacy (1977, 1979) paid special attention to the MR relation in order to compare empirical data with theoretical models of the main sequence (MS) stars and thus derive information concerning the mean internal chemical composition of MS stars. He found the best agreement for the metal content $z=0.04$ and indications of a knee in the logarithmic MR relation for the MS stars around 1.3 solar mass.

In a paper by Gimenez and Zamorano (1985), Popper's (1980) compilation of absolute stellar data have been used to form the MR relation for the MS stars. They compared the empirical relation with the theoretical predictions formed by the zero age MS models of Hejlesen (1980). They compared the results with the previous empirical relations and discussed the application of the relation to the study of the light curves of eclipsing binaries. They also indicated a knee in the logarithmic MR relation which was fixed around 1.8 solar mass.

To date, quite a number of advances have been made in both sides; observational and theoretical. In the observational side, systematic more accurate photometric and spectroscopic observations and sophisticated data analysis of eclipsing detached binaries yielded quite a significant number of new data on the absolute dimensions of component stars (Harmanec, 1988). In the theoretical side stellar physics has undergone appreciable revision since the publication of most extensive and most widely used theoretical stellar models (see. Ciardillo and Demarque 1977 and Mengel et al., 1979). First, the new generation of opacities has been provided in the Los Alamos Astrophysical Opacity Library. These includes new atomic data and important improvements to the method of calculating opacities and have been shown (see Magee, Merts and Hubner, 1975) to be $\geq 50\%$ larger than previously computed values (e.g. those of Cox and Stewart, 1970 a, b) over significant ranges of temperature and density. The low temperature opacities of Alexander, Johnson, and Rypma (1983) which include the contribution from the most important atomic, molecular, and particular absorbers, similarly represent a major advance over what has previously been available. As a result of the efforts of Kurucz (1979), Gustafsson et al. (1975) and Eriksson et al. (1981) considerable progress has been made in model atmosphere calculations. Thus, now more accurate theoretical estimates of the effective temperatures, bolometric corrections and color temperature relations are available for application to stars in almost all parts of the H-R diagram.

DATA AND THE EMPIRICAL MASS-RADIUS RELATION

In order to derive the MR relation by using the observational data, the mean observed masses and radii from table 3 of Harmanec (1988) have been used. These data cover the whole body of available data for normal MS stars between the spectral types 06 and M8. It was noted by Harmanec that although the available data give the best estimates of mean values used, there are still large uncertainties for stars of spectral types earlier than B2 and later than KO.

A plot of the mean masses (M) and radii (R) in logarithmic scale (see Figure 1) shows a clear correlation. A linear least square fit to all the points leads to

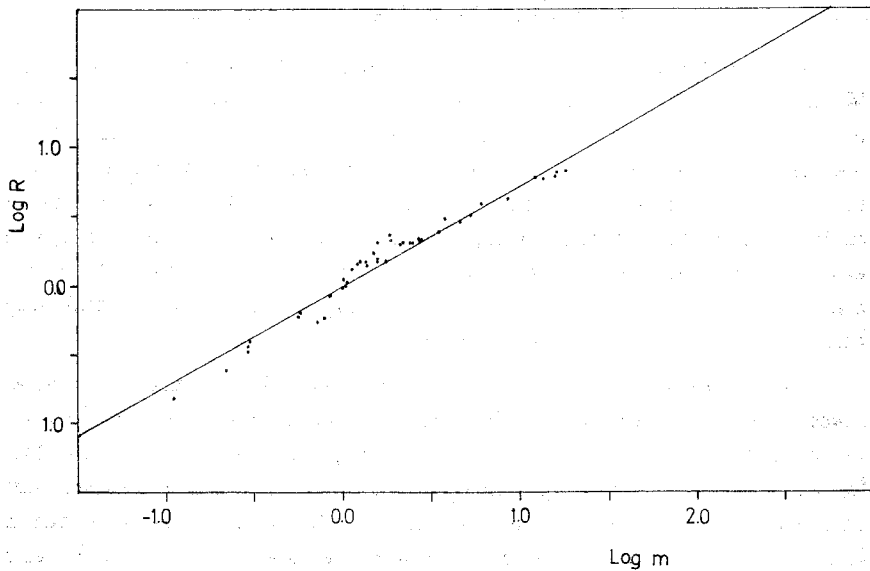


FIGURE 1. The empirical MR relation in logarithmic scale. The masses and radii are from Harmanec (1977). A linear least square fit to all data is shown by solid line. A knee around 1.66 M_{\odot} in the distribution of data is clearly seen.

$$\log R = 0.003 + 0.724 \log M \quad (1)$$

where R and M are in solar units. The linear Pearson correlation coefficient of this correlation was found 0.98. In the present correlation the same knee which was realized by many previous investigators was also

seen. Such change in the slope of MR correlation have been interpreted (see eg. Kopal, 1959) as indication of the transition from proton-proton cycle to the carbon nitrogen cycle as the dominant source of nuclear energy in these stars. The precise location of this knee in the present correlation was estimated around 1.66 ± 0.08 solar mass. This critical location should be important in constraining on the theoretical stellar models.

Two separate linear least square fitting for the two groups of masses divided by 1.66 solar mass revealed the following logarithmic MR correlations:

$$\log R = \begin{cases} 0.0225 + 0.9349 \log M; & \text{if } M < 1.66 \\ 0.1374 + 0.5419 \log M; & \text{if } M > 1.66 \end{cases} \quad (2)$$

THEORETICAL MASS-RADIUS RELATION

In order to obtain theoretical MR relation we have used the zero age MS models from three different sources. The zero age MS models of $M = 0.1, 0.15, 0.3, 0.45, 0.6$ and 0.75 solar mass stars were taken from Vandenberg et al. (1983). These models were calculated for the Helium content $y = 0.25$, and a metal abundance $z=0.02$. The convection parameter α (the ratio of the mixing length to the pressure scale height) was assumed unity in the calculations and noted that the location of the low mass MS stars in the H-R diagram is strongly dependent on metallicity but not so much on the values of y or x .

Twenty-seven zero age MS models between $M=0.7$ and $M=5$ solar mass were taken from Vandenberg and Bridges (1984). These models were calculated for the chemical composition $y=0.26$ and $z=0.0169$ (solar value), and convection parameter $\alpha=1.5$. In addition, seven zero age upper MS models between $M=5$ and $M=120$ solar mass were taken from Stothers (1972). These models were calculated for $x=0.739$ and $z=0.021$.

The radii R (in terms of solar radius) of the zero age MS models taken from three different sources cited above, have been derived by using the given luminosity L (in solar units) and effective temperature T in the well known formula

$$\log L = 2 \log R + 4 \log (T / T_{\odot}) \quad (3)$$

where for the effective temperature T_{\odot} and the bolometric magnitude M_{bol} of the Sun we used the values of 5784 °K and 4.72 magnitude,

respectively. The resulting values of $\log R$ have been plotted against the $\log M$ values in Figure 2. A linear least square fit to all the points leads to

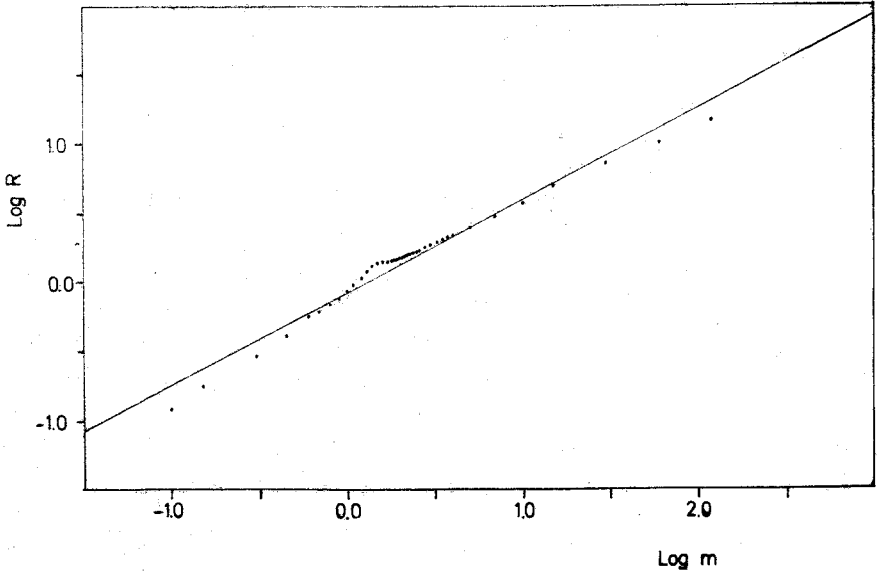


FIGURE 2. Theoretical MR relation for the zero age MS models in logarithmic scale. The models are from three sources (see text). A knee is also seen but around $1.47 M_{\odot}$. A linear least square fit to all points is shown by solid line.

$$\log R = -0.0726 + 0.6684 \log M \quad (4)$$

with the correlation coefficient $r = 0.99$. A knee in the logarithmic form of the correlation is also seen in the theoretical data but in around 1.47 solar mass. As in the empirical MR relation, two separate linear least square fitting for the two groups of masses divided by 1.47 solar mass leads to the following correlation:

$$\log R = \begin{cases} -0.0531 + 0.8824 \log M & \text{if } M < 1.47 \\ 0.0088 + 0.5615 \log M & \text{if } M > 1.47 \end{cases} \quad (5)$$

COMPARISION and DISCUSSION

In order to compare the empirical and theoretical MR relations, the coefficients a and b of the logarithmic linear correlation of the form $\log R = a + b \log M$ are summarized in Table 1.

Table 1. Coefficients of the logarithmic linear MR correlation

	a	b	range of mass
Empirical	0.003	0.724	$0.11 \leq M \leq 37.8$
Theoretical	-0.073	0.668	$0.1 \leq M \leq 120$
Empirical	0.023	0.935	< 1.66
Theoretical	-0.053	0.882	< 1.47
Empirical	0.137	0.542	> 1.66
Theoretical	0.009	0.562	> 1.47

Table 1 shows that the coefficients of theoretical MR correlations, except in the higher mass range, are all smaller than those of empirical correlations. Thus, the zero age MS models reproduce lower envelope of the observed points in the $\log R$ - $\log M$ plane. This known results is not surprising because the stars, for which the empirical correlation have been deduced, are not in zero age but expected to be slightly evolved to varying degrees with constant mass in the direction of the increasing radius. Such evolutionary dispersion in the empirical correlation is expected to be more pronounced statistically for the more massive stars. Since there are not many points in the higher mass end of our empirical data such dispersion is not seen in our empirical $\log R$ - $\log M$ diagram. Moreover, while the theoretical models extends up to $M=120$ solar mass there is no empirical data beyond $M=37.8$ solar mass which corresponds to the spectral type 06 V. Harmance noted the rather large uncertainties in the empirical data for stars of spectral types earlier than B2. Thus, the empirical MR relation given by Equation 2 for the higher mass range should be in large error. For the visual comparison both the empirical and theoretical MR relations are shown in Figure 3. The other important feature seen in Table 1 and Figure 3 is the location of knee in the correlation which is quiet different in theory and observation. If the discrepancy is not purely due to the observational errors but is real, then it can be used in constraining on the theoretical stellar models.

Our results were also compared to previous linear relations between mass and radius of MS stars. Different published values of the constant a and b are given in Table 2.

The coefficients a and b given by different authors agree well with each other within the limits of observational errors of the underlying data. Values of the coefficients are, in fact, expected to be improved while the more accurate data accumulates in time. The present logarithmic MR relation given by Equation 2 can be rewritten as

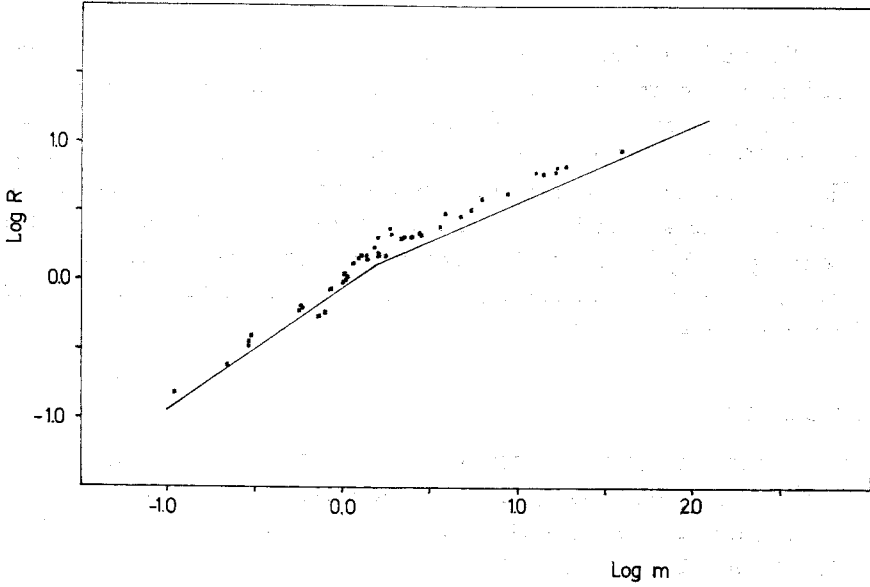


Figure 3. Theoretical MR relation for the zero age MS stars superimposed on the empirical data.

Table 2. Coefficients of the empirical MR relations.

a	b	range of mass	reference
0.00	0.980	< 2	Kopal (1978)
0.10	0.637	> 2	"
-0.020	0.917	< 1.3	Lacy (1979)
0.011	0.640	> 1.3	"
0.053	0.977	< 1.8	Gimenez—Zamorano (1985)
0.153	0.556	> 1.8	"
0.023	0.935	< 1.66	This paper
0.137	0.542	> 1.66	"

$$R = \begin{cases} 1.054M^{0.935} & ; \quad \text{if } M < 1.66 \\ 1.371M^{0.542} & ; \quad \text{if } M > 1.66 \end{cases} \quad (6)$$

which corresponds to a relation between the ratio k of radii and the mass ratio q for the binaries with components not much evolved off the MS. The corresponding new relation can be written as

$$\log k = \begin{cases} 0.935 \log q; & \text{if } M_1 \text{ and } M_2 < 1.66 \\ 0.542 \log q; & \text{if } M_1 \text{ and } M_2 > 1.66 \\ 0.724 \log q; & \text{if } M_1 > 1.66 \text{ and } M_2 < 1.66 \end{cases} \quad (7)$$

CONCLUSION

We conclude that (i) the location of knee in the empirical MR relation is important in constraining the theoretical stellar models. Meantime it looks the theoretical value ($M=1.47$) is slightly lower than the empirical one ($M=1.66$); (ii) any difference between the empirical and theoretical MR relations can be interpreted in terms of the difference between the internal chemical composition and age of the observed stars; (iii) the relation given by Equation (7) between the ratio of radii and the mass ratio can be useful in the light curve analysis of the well-detached binary systems.

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