The apparent variations of the solar diameter

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(Submitted by the director of the Department of Astronomy)

Özet: Güneş çapının rasadlanan değişime otokorelasyon metodu tatbik edildi. Neticeler esas itibariyle M. Cimino tarafından elde edilen neticelerle mutabıktır.

Değişim peryodu 23 sene civarındadır, fakat bu hakikî manâda bir peryot değildir.

Leke sayılarının değişimi ile çap değişimi arasındaki faz değişimi teyid edilmektedir.

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Abstract: The autocorrelation method is applied to the observed variation of the solar diameter.

The results substantially agree with those obtained by M. Cimino. The period of the variation is around 23 years, but this is not a period in a strict sense. The phase shift between the variations in diameter and the variation in sunspot numbers is confirmed.

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1. In the past various authors, such as Secchi[1], Wolf[2] and Meyermann[3], have discussed the observed variations of the apparent solar diameter. The results are not quite consistant inter se.

Opinions are divided whether the observed variations are merely due to the effect of probable error or that a periodic variation occurs.

Recently the latter view has especially been advocated by M. Cimino[4] who had at his disposal the extensive material collected at the Observatory of Campidoglio.

In the present paper the powerful method of autocorrelation analysis, as developed by Kendall[5], has been applied to this problem. This method requires a series of observations of suitable length and homogenity. These requirements are met by

the series of mean yearly solar diameters as published by the Observatory of Campidoglio and which have also been used by M. Cimino [4]

2. We first computed the values $\Delta R_i = R_i - \overline{R}$ where R_i is the observed solar diameter in a given year and $\overline{R}$ the mean diameter in the interval of 63 years. The observed distribution of the values ΔR appear in table 1.

TABLE 1.

Observed frequencies N of the different values ΔR (unit o'' o')

ΔR	N	ΔR	N
< -90	1	$> +90$	0
-90 to -70	2	$+70$ to $+90$	0
-70 to -50	6	$+50$ to $+70$	3
-50 to -30	7	$+30$ to $+50$	14
-30 to -10	7	$+10$ to $+30$	12
-10 to $+10$	11		

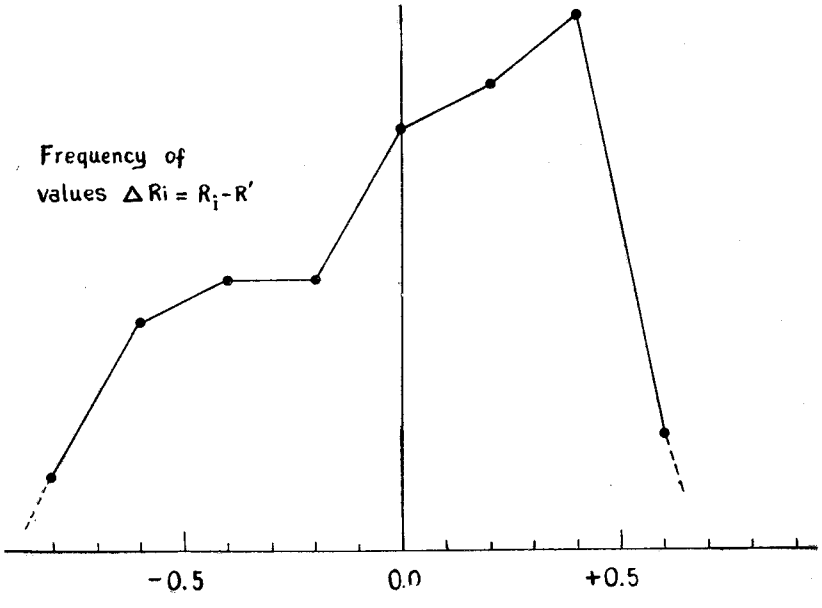

Fig. 1.

It is seen from fig. 1, in which the values of table 1 have graphically been represented, that the distribution curve of the values ΔR is strongly a symmetrical and cannot be represented by a Gaussian error curve.

3. Next from the values ΔR we have computed the numerical values of the autocorrelation coefficients

$$r_k = \frac{\Sigma u_i \, u_{i+k}}{\Sigma u_i^2 - k u^{-2}}$$

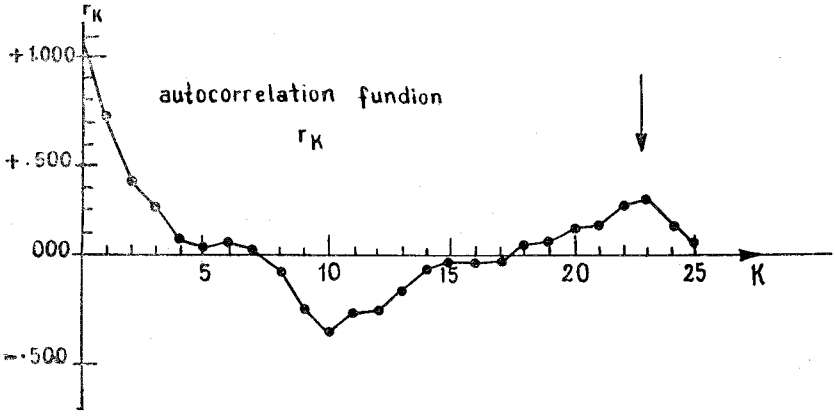

Fig. 2.

With a view to the limited number of term ΔR_i which are available, only the values r_k from $K = 1$ up to $K = 25$ were computed. By definition $r_0 = 1$. The numerical results of our computations appear in table 2, while the shape of the curve r_k is indicated in figure 2.

TABLE 2.

Values of the autocorrelation coefficients

k	r_k	k	r_k	k	r_k
0	+ 1.000	9	— 0.225	18	+ 0.038
1	+ 0.675	10	— 0.305	19	+ 0.073
2	+ 0.368	11	— 0.265	20	+ 0.104
3	+ 0.227	12	— 0.255	21	+ 0.145
4	+ 0.093	13	— 0.142	22	+ 0.231
5	+ 0.066	14	— 0.066	23	+ 0.252
6	+ 0.078	15	— 0.035	24	+ 0.165
7	+ 0.023	16	— 0.006	25	+ 0.096
8	— 0.051	17	— 0.006		

In general our results confirm those previously obtained by M. Cimino[4]. In the variations a "period„ of around 23 years is apparent. However, the curve in figure 2 is a damped harmonic, with a fairly strong damping and from this we conclude that the variations in the solar diameter are not periodic in a very strict sense. Because of this strong damping of the autocorrelation function we have refrained from determining an amplitude. Such an amplitude would have a very restricted meaning only.

4. We next consider the degree of correlation between the variations of the apparent diameter and the variations of the sunspot numbers. The "period" in the diameter is $\pm$ 23 years or about a double sunspot cycle.

If therefore there is a connection between the two phenomena, the sign in the polarity of the sunspots cannot be neglected. Therefore, starting with an arbitrary sunspot cycle, the observed numbers of sunspots in the first cycle were taken positive, those in the second cycle negative, the third cycle positive, etc. Next the correlation coefficients

$$r_{k'} = \frac{[\Delta R_i \cdot \Delta N_i]}{\sqrt{\left(\sum_{0}^{N-K} \Delta R_i^2\right)\left(\sum_{K}^{N} \Delta N_{i+k}\right)}}$$

were computed. Here $\Delta N_i = N_i - \bar{N}$ where N_i is the observed number of sunspots in a given year. For obvious reasons $\bar{N} \to 0$ The resulting values $r_{k'}$ appear in table 3.

TABLE 3.

Values of $r_{k'}$ indicating function correlation between the variations of solar diameter and the variations of sunspot numbers.

k	$r_{k'}$	k	$r_{k'}$	k	$r_{k'}$	k	$r_{k'}$
0	+ 0.101	9	— 0.249	18	+ 0.337	27	— 0.673
1	— 0.046	10	— 0.197	19	+ 0.389	28	— 0.592
2	— 0.287	11	— 0.076	20	+ 0.038	29	— 0.460
3	— 0.455	12	+ 0.106	21	+ 0.151	30	— 0.286
4	— 0.459	13	+ 0.197	22	+ 0.002		
5	— 0.400	14	+ 0.291	23	— 0.048		
6	— 0.279	15	+ 0.332	24	— 0.203		
7	— 0.193	16	+ 0.352	25	— 0.312		
8	— 0.223	17	+ 0.323	26	— 0.479		

It is evident that in this case it is not necessary that r_0' be $r_0' = 1.000$.

The first minimum in the correlation function r_k' occurs at $K = 4$. To determine the period we had to extend our values beyond the limit $K = 25$ and now the second minimum occurs at K = 27. In the functional correlation between ΔR_i and ΔN_i therefore there also occurs a period of 23 years.

This is also very evident from the curve r_k' in figure 3 which is based on the numbers in table 3.

With the curve 3 there is more guarantee that the 23 years is the correct value of the period than with the curve 2. With the curve 2 we were forced to use the theoretical maximum at $K = 0$, but as Kendall [5] has pointed out, for determining a period it is better to use the subsequent maxima only. With the curve 3 we are able to use two directly observed minima. While the epochs of the minima seem to be pretty well fixed, we cannot claim any high accuracy for the numerical values of r_k' as given in the final column of our table 3.

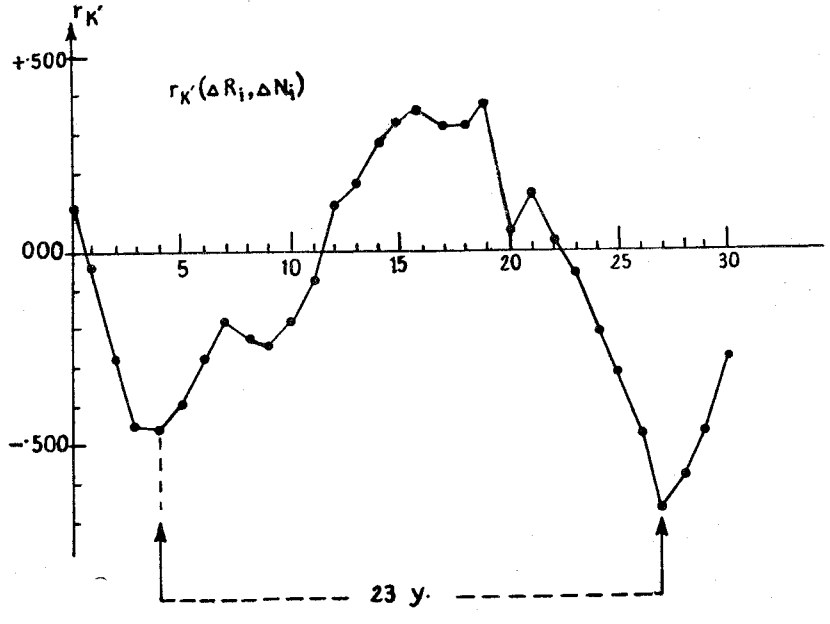

Fig. 3.

The whole series of observation compromises an interval of 63 years. Consequently there is a great danger that for the hig-

her values of K the numerical results have been influenced by sampling errors.

Both in fig. 3 and in table 3 it is indicated that there may be a faseshift between the variations of the solar diameter and the variations in sunspot numbers. This phaseshift of about 4 years is very apparent in the period at which the two minima appear. Also the maximum around $K = 19$ seems to be displaced. On the other hand no such shift appears when the periods of up cross and down cross are considered.

For the present we must therefore leave the question undecided, whether the apparent phaseshift indicated by the minima in the r'_k curve is real or not.

5. Our results can be summarized as follows.

a) The observed variations of the solar diameter have a mean period of 23, but the variations are not very strictly periodic.

b) If the sign of the polarity is taken into account, there is a close correlation between the variations of the numbers of sunspots and the variations of the solar diameter. This gives a better proof of the reality of the 23 years period.

c) There may be a phase shift between the curve indicating the sunspot cycles and the one indicating the variations of the solar diameter, but as to this the evidence is not conclusive.

d) Our results therefore largely support the conclusions which M. C*i*mino has derived. [4]

e) We have not been able to apply this method to the material collected by Meyermann, because his observations cover too short an interval of time. [5]

f) When we conclude that there occurs a cyclical variation of the solar diameter, we do not consider this variation as indicating a pulsation in the solar atmosphere. With a view to the observed 23 years period, it seems possible that certain variations in the constitution or transparency of the atmosphere are synchrone with variations in the orientation of the magnetic fields around sunspots. Anyhow, both phenomena seem to have a common origin.

References:

[1] Secchi, A; C. R **73,** 97. 1871
[2] Wolf, R; Hdb. d. Astr. **4,** 437, 1892
[3] Meyermann, B; A. N. **279,** 45, 1950
[4] Cimino, M; Atti acad. hindei, **1,** 624, 1946
[5] Kendall, M. G; The Advanced theory of statistics, **2,** 396 ft 1946

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