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On Absolute Equivalence of T-Matrices For (C, r) - Bounded Sequences

by

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On Absolute Equivalence of T-Matrices For (C, r) - Bounded Sequences

M. B. ZAMAN

"On absolute equivalence of T-matrices for (Gr)- bounded sequencės"

In this note, the author proved the following main results besides bmmas:-

Theorem 1. The T-matrices A and B are absolutely equivalent for all (C, 1)- bounded sequences iff

$$\text{(i)} \quad \lim_{n \to \infty} \sum_{k=1}^{\infty} k \mid \Delta c_{n,k} \mid = 0,$$

where $C = A - B$

Corollary. A T- matrix A is absolutely translative for all (C-1)- bounded esequences iff

$$\lim_{n \to \infty} \sum_{k=1}^{\infty} k \mid \Delta^2 a_{n,k} \mid = 0.$$

Theorem 2 Let r is a positive integer; then the T-matrices A and B are absolutely equivalent for all (C, r) - bounded sequences iff

$$\text{(i)} \quad \lim_{R \to \infty} R^r c_{n,k} = 0 \text{ for every n, and}$$

$$\text{(ii)} \quad \lim_{n \to \infty} \sum_{k=1}^{\infty} k^j \mid \Delta^j c_{n,k} \mid = 0 \quad (j = 1, 2, 3, \ldots.., r),$$

where $C = A - B$.

Corollary. If r is a positive integer, then a T-matrix A is absolutely translative for all (C, r) - bounded sequences iff

$$\text{(i)} \quad \lim_{K \to \infty} K^r \Delta a_{n,k} = 0 \text{ for every n, and}$$

$$\text{(ii)} \quad \lim_{n \to \infty} \sum_{k=1}^{\infty} k^j \mid \Delta^{j+1} a_{n,k} \mid = 0 \; (j = 1, 2, 3, \ldots\ldots\ldots., r).$$

N. B. The author requests the referee to compare the following if there is any omission in the original M S: - (3.3) on page 5 of M S must be read as (i) of Theorem 1 of this Summary.

(4.1) and (4.2) on page 12 of M S must be read as (i) and (ii) of Theorem 2 of this Summary.

(4.12) and (4.13) on page 15 of M S must be read as (i) and (ii) of corallary of Theorem 2 of this Summary.

1. INTRODUCTION

Cooke ([1], 105, (5. 4, I)) has proved the following theorem:

Theorem A. *A necessary and sufficient condition that* T- *matrices* A *and* B *are absolutely equivalent for all bounded sequences is that*

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} | a_{n,k} - b_{n,k} | = 0 \tag{1.1}$$

Jha ([4], 120) obtained the necessary and sufficient conditions in order that any two infinite matrices are absolutely equivalent for (C,r) – bounded sequence where r being a positive integer.

In this paper, the necessary and sufficient condition for the absolute equivalence of any two T-matrices for all (C, 1) bounded sequences is obtained, and the result is extended for all (C, r) – bounded sequences (r being positive integer).

Finally, necessary and sufficient conditions for the absolute translativity of any T-matrix for the above classes of sequences are found.

2. NOTATION AND DEFINITIONS

Write $\Delta\ d_k = d_k - d_{k+1}$, then

$$\Delta^r d_k = \sum_{p=o}^{r} (-1)^p \binom{r}{p} d_{p+k}, \tag{2.1}$$

and

$$(2.2) \quad \Delta^r (u_k\ w_k) = \sum_{v=o}^{r} \binom{r}{v} \Delta^r u_k\, \Delta^{r-v} w_{k+v}$$

We use the standard notations

$$\overset{r}{\underset{v}{A}} = (r+1)\ (r+2) \ldots (r+v)\ /\ v\ ,\ \overset{r}{\underset{o}{A}} = 1,$$

$$\overset{r}{\underset{k}{S}} = \sum_{v=o}^{k} \overset{r-1}{\underset{k-v}{A}}\ s_v .$$

Write $s_k = \overset{o}{S_k}$ and $\overset{r}{S_k} = \overset{r-1}{S_o} + \overset{r-1}{S_1} + \ldots \overset{r-1}{S_k}$ $(r = 1, 2, \ldots .)$

Definition 1. If $\overset{r}{s_n} = \overset{r}{S_n} / \binom{n+r}{r}$ is bounded for all n,

we say that $\{s_k\}$ is (C,r) – bounded . If $\overset{r}{s_n} \rightarrow s$

as $n \rightarrow \infty$, $\{s_n\}$ is summable – (C,r) to s.

Difinition 2: The matrices $A = (a_{n,k})$ and $B = (b_{n,k})$ are said to be ***absolutely equivalent*** for a given class of sequences $\{s_k\}$ whenever

$$\lim_{n\rightarrow\infty} \left[\sum_{k=1}^{\infty} a_{n,k}\ s_k - \sum_{k=1}^{\infty} b_{n\,k}\ s_k \right] = 0 \text{ (Cooke [1],p.97)}$$

Definition 3. A matrix $A = (a_{n,k})$ is said to be ***absolutely translative*** for a given class of sequences $\{s_k\}$ whenever

$$\lim_{n\rightarrow\infty} \left[\sum_{k=1}^{\infty} a_{n,k}\ s_k - \sum_{k=1}^{\infty} a_{n,k+1}\ s_k \right] = 0 \text{ (Cooke [1], p. 114)}$$

3. OBSOLUTE EQUIVALENCE FOR (C,1) BOUNDED SEGUENCES

In this section we proceed to obtain a necessary and sufficient condition in order that ant two T-matrices are absolutely equivalent for all (C, 1) – bounded sequences. We need the following lemmas to prove our result.

Lemma 1. *If* $\sum_{k=1}^{\infty} k \mid \Delta a_{n,k} \mid \leq M$ *for every* n *and*

$\lim_{k\to\infty} a_{n,k} = o$ *for every* n, *then* $\lim_{k\to\infty} k\, a_{n,k} = O$ *for every* n.

For proof, put $r = 1, \sigma = o$ in Bosanquet's Lemma 7 [2], also Cooke [1], p. 216, p. 218.

Lemma 2. *The necessary and sufficient condition that a matrix A transforms all null sequences into null sequences are that*

(3.1) $\lim_{n\to\infty} a_{n,k} = o$ *for every fixed* k,

and

(3.2) $\sum_{k=1}^{\infty} | a_{n,k} | \leq M$ *for every* n, *where* M *is independent of n.*

For proof see Cooke [1], p. 64. (4. 1, II) and the remarks in italics concerning the case $z = o$; Hardy 3 p. 49.

Lemma 3. *If the matrix* $C = (c_{n,k})$ *is efficient for every* (C,r)-*bounded sequences* (r *being positive integer*), *then it is necessary that* $\lim_{k\to\infty} k^r c_{n,k} = o$ *for every n* (Jha [4], p. 120.).

Lemma 4. If

(i) $\sum_{k=1}^{\infty} | a_{n,k} | \leq M$ *for every* n,

(ii) $\lim_{n\to\infty} a_{n,k} = o$ *for every fixed* k,

(iii) $\lim_{n\to\infty} \sum_{k=1}^{\infty} a_{n,k}\, s_k = o$ *for every bounded* *sequence* $\{s_k\}$, *then*

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} | a_{n,k} | = o$$

This immediately follows from the necessity part of Theorem A. 1. Also see Jha [4] p. 115.

Lemma 5. *Let* $A = (a_{n,k})$ *and* $B = (a_{n,k+1})$ *be infinite matrices. If* A *and* B *are absolutely equivalent for a givenclass of sequences, then* A *is absolutely translative for that class of sequences.*

The proof obviously follows from Definitions 2 and 3.

Theorem 1. *The necessary and sufficient condition that the* T– *matrices* A *and* B *are absolutely equivalent for all* (C, 1) – *bounded sequences is that*

$$(3.3) \qquad \lim_{n\to\infty} \sum_{k=1}^{\infty} k \, | \Delta c_{n,k} | = o$$

where $C = A - B$.

Proof. Put

$$(3.4) \qquad Z_k = (s_1 + s_2 + s_3 + \dots + s_k)/k, \text{ then}$$

$$(3.5) \qquad s_k = k\, Z_k - (k-1)\, Z_{k-1}, \ (k \geq 1).$$

It is obvious that if $\{s_k\}$ is any (C, 1) – bounded sequences, then $\{Z_k\}$, defined by (3. 4), is bounded. Conversely, if $\{Z_k\}$ is any bounded sequence $\{s_k\}$, defined by (3.5), is (C, 1) – bounded. Also

$$(3.6) \qquad \sum_{k=1}^{p} c_{n,k}\, s_k = \sum_{k=1}^{p} c_{n,k} \{k\, Z_k - (k-1)\, Z_{k-1}\}$$

$$= \sum_{k=1}^{p} k\, (c_{n,k} - c_{n,k+1})\, Z_k + P\, c_{n,p+1}\, Z_p.$$

The condition (3.3) implies that

$$(3.7) \qquad \sum_{k=1}^{\infty} k \, | \Delta c_{n,k} | \leq M \text{ for every n.}$$

Since A and B are T – matrices,

$$\lim_{k\to\infty} c_{n,k} = \operatorname{Lim}_{k\to\infty} (a_{n,k} - b_{n,k}) = o \text{ for every } n$$

From (3.7), (3.8) and lemma 1, we have

$$(3.9) \quad \lim_{k\to\infty} k\, c_{n,k} = o \text{ for every } n.$$

Now (3.8) and (3.9) together imply that

$$(3.10) \quad \lim_{p\to\infty} p\, c_{n,p+1} Z_p = o \text{ for every bounded } \{Z_p\}.$$

The condition (3.3) also implies that

$$(3.11) \quad \lim_{n\to\infty} \sum_{k=1}^{\infty} k\, \Delta c_{n,k} Z_k = o \text{ for every bounded}$$

sequence $\{s_k\}$.

Letting $p\to\infty$ in (3.6) and using (3.10) and (3.11) we get

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} c_{n,k} s_k = o \text{ for every } (C,1) \text{ - bounded}$$

sequence $\{s_k\}$. Thus the condition is sufficient.

Conversely, if the T- matrices A and B are absolutely equivalent for all (C, 1) - bounded sequence,

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} c_{n,k} s_k = o \text{ for every } (C,1)\text{– bounded}$$

sequence $\{s_k\} \Longrightarrow$

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} c_{n,k} \{k Z_k - (k-1) Z_{k-1}\} = o \text{ for every}$$

bounded sequence $\{Z_k\} \Longrightarrow$

$$(3.12) \quad \lim_{n\to\infty} [\sum_{k=1}^{\infty} k\, \Delta c_{n,k} Z_k + \lim_{p\to\infty} p\, c_{n,p+1} Z_p] = o$$

for every bounded $\{Z_k\}$.

Since the T- matrices A and B are absolutely equivalent for all (C, 1) – bounded sequences, the matrix C = A – B suems every (C, 1) – bounded sequence to O.

Thus Lemma 3 $\Longrightarrow$ (3. 9). Therefore (3.9) and (3.8) togethet $\Longrightarrow$ (3.10). Now it follows from (3.10) and (3.12) that

$$(3.13) \qquad \lim_{n\to\infty} \sum_{k=1}^{\infty} f_{n,k} Z_k = o \text{ for every bounded } \{Z_k\}$$

where $f_{n,k} = k \,\Delta\, c_{n,k}$.

Now we observe that all null sequences are bounded sequences; hence if $F = (f_{n,k})$ transforms all bounded sequences into null sequences, it must be so far all null sequences. Therefore, from Lemma 2

$$(3.14) \qquad \lim_{n\to\infty} f_{n,k} = o \text{ for every fixed } k,$$

and

$$(3.15) \qquad \sum_{k=1}^{\infty} | f_{n,k} | \leq M \text{ for every } n.$$

Thus it follows from (3. 14), (3. 15), (3. 13) and lemma 4 that

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} | f_{n,k} | = o$$

i.e.

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} k \,| \Delta c_{n,k} | = o,$$

hence the condition is necessary.

Example : If a T – matrix A is defined by

$$a_{n,k} = \frac{1}{n} \; (1 \leq k \leq n), \; = o \; (k > n).$$

and another T- matrix B is defined by

$$b_{n,k} = \frac{1}{n+1} \ (1 \leq k \leq n), \ = o \ k > n.$$

Let $C = A - B$, then

$$\sum_{k=1}^{\infty} k \mid \Delta c_{n,k} \mid = \sum_{k=1}^{\infty} k \mid (a_{n,k} - b_{n,k}) + (a_{n,k+1} - b_{n,k+1}) \mid$$

$$= \sum_{k=1}^{\infty} k \mid (a_{n,k} - a_{n,k+1}) - (b_{n,k} - b_{n,k+1}) \mid$$

$$= n \left| \frac{1}{n} - \frac{1}{n+1} \right| = n \frac{1}{n(n+1)} = \frac{1}{n+1}$$

$$\therefore \lim_{n \to \infty} \sum_{k=1}^{\infty} k \mid \Delta c_{n,k} \mid = o .$$

Thus there are T – matrices A and B which satisfy the condition (3.3).

Corollary : The necessary and sufficient condition that a T-matrix A is absolutely translative for all (C, 1) – bounded sequences is that

$$\lim_{n \to \infty} \sum_{k=1}^{\infty} k \mid \Delta^2 a_{n,k} \mid = o.$$

The result follows at once from Theorem 1 if we put $a_{n,k}$ for $c_{n,k}$ and use Lemma 5.

4. ABSOLUTE EQUIVALENCE FOR (C,r) BOUNDED SEQUENCES

In this section we obtain the necessary and sufficient conditions in order that the T-matrix is absolutely equivalent for all (C, r) - bounded sequences where r is a positive integer. We need the following lemmas.

Lemma 6. *If* r *is a positive integer,* $\{s_n\}$ *is* (c, r) – *bounded if and only if,* $\{Z_n\}$ *is* (C, r-1) – *bounded where* $Z_n = s_1 + s_2 + \dots + s_n) / n$.

This is essentially lemma I of Bosanquet [2].

Lemma 7. *If* $\{s_n\}$ *is* (C, r) – *bounded where* r *is a positive integer, then* $s_n = O(n^r)$ (Jha [4] p. 117).

Lemma 8. *If* $f_{n,k} = k(a_{n,k} - a_{n,k+1})$, *then*

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} k^j \,|\, \Delta^j a_{n,k} \,| = o \quad (j = r-1, r) \text{ implies that}$$

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} k^{r-1} \,|\, \Delta^{r-1} f_{n,k} \,| = o \text{ where } r \text{ is a positive integer}$$

Proof. We have

$$\Delta^{r-1} f_{n,k} = \Delta^{r-1} (k \,\Delta\, a_{n,k})$$

$$= \sum_{v=o}^{r-1} \binom{r-1}{v} \Delta^v k \,\Delta^{r-v}\, an_{n,k+v}$$

$$= k\,\Delta^r a_{n,k} - (r-1)\,\Delta^{r-1} a_{n,k+1}\,.$$

Therefore

$$k^{r-1} \Delta^{r-1} f_{n,k} = k^r \Delta^r a_{n,k} - (r-1)\, k^{r-1} \Delta^{r-1} a_{n,k+1},$$

and hence

$$\sum_{k=1}^{\infty} k^r \,|\,\Delta^{r-1} f_{n,k}\,| \leq \sum_{k=1}^{\infty} k^r \,|\Delta^r a_{n,k}\,| + (r-1) \sum_{k=1}^{\infty} k^{r-1} |\Delta^{r-1} a_{n,k+1}|$$

$$\leq \sum_{k=1}^{\infty} k^r \,|\,\Delta^r a_{n,k}\,| + (r-1) \sum_{k=1}^{\infty} k^{r-1} \,|\,\Delta^{r-1} a_{n,k}$$

$$\to o \text{ as } n \to \infty.$$

Finally, we get

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} k^r \,|\,\Delta^{r-1} f_{n,k}\,| = o.$$

Lemma 9. *If* $f_{n,k} = k\,(a_{n,k} - a_{n,k+1})$,

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} k^{r-1} |\Delta^{r-1} a_{n,k}| = o \textit{ and } \lim_{n\to\infty} \sum_{k=1}^{\infty} k^{r} |\Delta^{r-1} f_{n,k}| = o$$

together imply that $\lim_{n\to\infty} \sum_{k=1}^{\infty} k^r |\Delta^r a_{n,k}| = o$, *where* r *is a positive integer.*

Proof. Proceed as in Lemma 8, we get

$$k^{r-1} \Delta^{r-1} f_{n,k} = k^r \Delta^r a_{n,k} - (r-1) k^{r-1} \Delta^{r-1} a_{n,k+1}$$

or, $k^r \Delta^r a_{n,k} = k^{r-1} \Delta^{r-1} f_{n,k} + (r-1) k^{r-1} \Delta^{r-1} a_{n,k+1}$

Hence

$$\sum_{k=1}^{\infty} k^r |\Delta^r a_{n,k}| \leq \sum_{k=1}^{\infty} k^{r-1} |\Delta^{r-1} f_{n,k}| + (r-1) \sum_{k=1}^{\infty} k^{r-1} |\Delta^{r-1} a_{n,k+1}|$$

$$< \sum_{k=1}^{\infty} k^{r-1} |\Delta^{r-1} f_{n,k}| + (r-1) \sum_{k=1}^{\infty} k^{r-1} |\Delta^{r-1} a_{n,k}|$$

$$\to o \text{ as } n \to \infty.$$

Consequently

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} k^r |\Delta^r a_{n,k}| = o.$$

Theorem 2. *If* r *is a positive integer, the neessary and sufficient conditions that the T-matrices A and* B *are absolutely equivalent for all* (C, r) – *bounded sequences are that*

(4.1) $\lim_{k\to\infty} k^r c_{n,k} = o$ *for every* n, *and*

(4.2) $\lim_{n\to\infty} \sum_{k=1}^{\infty} k^j |\Delta^j c_{n,k}| = o \ (j = 1,2,3, \ldots r)$

where $C = A - B$.

Proof. Let us first prove the sufficient part. We shall use induction on r. Put r = 1 in (4.1) and (4.2), and proceed as in Theorem 1, we at once obtain our theorem for $r=1$. Suppose that the theorem is true for $r = t - 1$; then we shall prove that the theorem is true for $r = t$.

Put

(4.3) $\quad Z_k = (s_1 + s_2 + s_2 + \dots + s_k) / k$, so that

(4.4) $\quad s_k = k Z_k - (k-1) Z_k \ (k > 1)$.

It follows from Lemma 6 that s_k is (C, t) – bounded if, and only if, $\{Z_k\}$ is (C, t–1) – bounded and then by Lemma 7, $Z_k = O(k^{t-1})$. Also we have

$$\sum_{k=1}^{p} c_{n,k} s_k = \sum_{k=1}^{p} c_{n,k} \{k Z_k - (k-1) Z_{k-1}\}$$

$$= \sum_{k=1}^{p} k(c_{n,k} - c_{n,k+1}) Z_k + p\, c_{n,p+1} Z_p$$

Therefore

(4.5) $$\sum_{k=1}^{p} c_{n,k} s_k = \sum_{k=1}^{p} f_{n,k} z_k + p\, c_{n,p+1} z_p,$$

where $f_{n,k} = k (c_{n,k} - c_{n,k+1})$.

The condition (4.1) implies that

(4.6) $$\lim_{p\to\infty} p\, c_{n,p+1} z_p = o, \text{ when } z_k = O(k^{t-1}),$$

since $\lim_{k\to\infty} c_{n,k} = o$ for every n, and

(4.7) $$\lim_{k\to\infty} k f_{n,k} = o \text{ for every n.}$$

By lemma 8, with $r = t$, (4.2) implies that

(4.8) $$\lim_{n\to\infty} \sum_{k=1}^{\infty} k^{t-1} \mid \Delta^{t-1} f_{n,k} \mid = o.$$

Letting $p \to \infty$ in (4.5) and using (4.6) w e get

(4.9) $$\lim_{n\to\infty} \sum_{k=1}^{\infty} c_{n,k} s_k = \lim_{n\to\infty} \sum_{k=1}^{\infty} f_{n,k} z_k .$$

We have already supposed that the theorem is true for $r = t\text{-}1$ and the matrix $F = (f_{n,k})$ satisfies the condition (4.7) and (4.8); then the right-hand side of (4.9) is O i. e.

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} f_{n,k}\, z_k = o.$$

Therefore $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{\infty} c_{n,k}\, s_k = o$ for every (C, t) - bounded sequence. Thus the conditions are sufficient.

If A and B are absolutely equivalent for all (C, r) - bounded sequence, the the matrix C sums all (C, r) - bounded sequences to O. Hence the necessity of the condition (4.1) follows from Lemma 3.

We observe, from Lemma 6, that $\{s_k\}$ is (c, t) – bounded whenever (z_k) is (c, t-1)- bounded.

Transition from (4.5) and (4.9) is justified in this case also because (4.5) is an identity, $z_k = O\,(k^{t-1})$ and (4.1) holds. Therefore

$$\lim_{n\to\infty} \sum_{k=1}^{\infty} c_{n,k}\, s_k = o \text{ for every (C, t) - bounded } \{s_k\}$$

implies, by (4.9). that

$$\text{(4.10)} \quad \lim_{n\to\infty} \sum_{k=1}^{\infty} f_{n,k}\, z_k = o \text{ for every (c, t-1) - bounded } \{z_k\}$$

By the supposition that the theorem is true for $r = t\text{-}1$, then it follows from (4. 10) that

$$\text{(4.11)} \qquad \lim_{n\to\infty} \sum_{k=1}^{\infty} k^{t-1} \mid \Delta^{t-1} f_{n,k} \mid = o$$

is a necessary condition.

Again every (C, t-1) - bounded sequence is (C, t) - bounded, then the necessity of (4.2) for $j = 1, 2, 3 \ldots t\text{-}1$ assumed. Threfore, from Lemma 9, the necessity of the condition (4. 2) follows for $r = t$.

Remark: The condition (4.1) does not follow from (4.2). Of course, the condition (4.2) implies that

$$\sum_{k=1}^{\infty} k^j \mid \Delta^j c_{n,k} \mid \leq M \text{ for every } n \ (j = 1, 2, \ldots, r)$$

but $\sum_{k=1}^{\infty} k^r \mid \Delta^r c_{n,k} \mid \leq M$ for every n, which together with

$c_{n,k} \to o$ as $k \to \infty$ implies that

$$c_{n,k} = o\,(k^{-\sigma-1}),\ \sigma = 0, 1, 2, \ldots, r-1.$$

Cooke [1], 218, also see Bosanquet [2], Lemma 7.

Corollary: *If* r *is a positive integer, the necessary and sufficient conditions that a* T - *matrix A is absolutely translative for all* (C,r) - *bounded sequences are that*

(4.12) $$\lim_{k\to\infty} k^r a_{n,k} = o \text{ for every } n,$$

and

(4.13) $$\lim_{n\to\infty} \sum_{k=1}^{\infty} k^j \mid \Delta^{j+1} a_{n,k} \mid = o \ (j = 1,2,3,\ldots,r)$$

Lemma 5 and Theorem 2 give the corollary.

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