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TRANSIT FREQUENCY OPTIMIZATION USING FIREFLY ALGORITHM AND EVALUATION OF THE PARAMETERS

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ABSTRACT: Over the last few decades, rapidly growing cities in terms of population and land use have led to many transportation-based problems such as longer travel times, traffic congestion, traffic crashes, and air and noise pollution. Increasing the modal share of transit systems appears to be one of the most effective methods to solve transportation-based problems. However, transit systems, particularly in countries having limited resources, should be used efficiently to achieve sustainable urban mobility. Even only adjusting frequencies of transit lines, with no infrastructure investment cost requirements, can provide a more efficient transit system. In this paper, a transit frequency setting model based on the Firefly Algorithm (FA), which is a relatively new metaheuristic for the transportation network design problems, is presented to minimize total user cost under a fleet size constraint. The proposed model is performed on a 10-route Mandl's Test Network using different combinations of parameters to demonstrate the effect of parameter values on the solution quality. After that, the best solution of 30 solutions obtained by the calibrated parameter values is compared to the existing frequency set of the 10-route transit network. The results show that the FA can obtain better frequency sets by selecting the proper values for the parameters.

Keywords: Transit network frequency setting problem, Firefly algorithm, Bi-level optimization.

1. INTRODUCTION

Transportation-based problems, which arise from increased population, mobility, and vehicles, have become a challenging issue for decision-makers in the last decades. Transit systems, a backbone for cities in terms of mobility needs, can mitigate transportation-based problems by enhancing their modal share in the total number of trips. However, considering the potential costs of transit systems, transit systems must be utilized more efficiently, especially in countries with limited sources.

Urban Transportation Network Design Problem (UTNDP), which is related to the planning, design, and management of transportation, includes mainly Road Network Design Problem (RNDP) and Public Transportation Network Design Problem (PTNDP) [1]. PTNDP is a set of interdependent problems that must be considered for an efficient transit system. The sub-problems of PTNDP span network design, frequency setting, transit network timetabling, vehicle scheduling problem, and driver scheduling problem [2]. Ideally, all of the strategic,

tactical, and operational problems regarding the transit system must be solved simultaneously. However, such complex problems can only be tackled by dividing them into sub-problems due to their computational difficulty.

Transit Network Frequency Setting Problem (TNFSP) is solely to determine the number of vehicle departures in a given line within a given planning period or to determine the time interval of subsequent vehicles in transit lines. Determining transit service frequency is considered a tactical decision and is related to more efficient use of sources. Moreover, generally, no monetary investment is required. The demand for the transit system may vary depending on the hours of days, days of the week, and the different seasons of the year. Although a preliminary setting is determined in the strategic decision phase, a comprehensive study should be conducted in the tactical decision process [3].

Transportation network users are willing to reach their destinations by minimizing their total travel costs that consist of total trip duration (i.e., access/egress time, wait time, in-vehicle travel time) and monetary costs. Operators also desire to minimize the operating costs such as fleet cost, personnel costs, maintenance, and repair costs of vehicles. A higher level of service is likely to lead to lower user costs and higher operator costs. Conversely, lower operator costs cause a lower level of service for users or higher user costs. Transit planning should be handled in consideration of both user and operator costs considering this trade-off.

Solving TNFSPs using exact solution methods is very difficult because of NP-Hard nature with a combinatorial optimization structure, where obtaining an optimal solution is too time-consuming, especially for those having a vast search space [4]. Another challenge is the non-convexity of the problem [5]. Thus, metaheuristic algorithms, which can obtain a near-optimal solution in reasonable times, are convenient methods in solving such complex problems.

Transportation network design problems diverge slightly from the problems in other disciplines since any changes on the network influence the travel choice of the users. Thus, transportation network design problems are generally formulated as a bi-level problem. The upper level is the problem of decision-maker who designs or manages networks by forecasting the travel behaviors of users in the face of changes in the network. On the other hand, the lower level is the problem of users who plans their travel choices according to the changes in the network. Hence, the bi-level structure enables to design the network in terms of both users and operators.

TNFSP has been tackled many times in the literature employing various objective functions, constraints, transit assignment methods. [6] determines the optimal bus frequencies to minimize the total travel time of passengers subject to the constraint of the fleet size of each operator on the transit network, employing an iterative solution approach which consists of a Genetic Algorithm and a label-marking method. Using a gradient projection method, [7] presents a transit frequency setting algorithm that aims to maximize demand on variable-demand networks with the constraints of the fleet size and frequency bounds. They test the optimization model onto a small test network. [8] proposes a frequency setting model to minimize the weighted sum of the operator and user costs. The optimization algorithm is based on the parallel genetic algorithm, in which a coarse-grained strategy and a local search algorithm based on Tabu search are embedded to increase the performance of the Genetic Algorithm. The model proposed by [9] obtains both optimal frequencies and optimal bus sizes by minimizing the sum of the total user cost and operator cost. The algorithm determines a set of frequencies using the Hooke-Jeeves algorithm while performing the congested transit assignment using ESTRAUS simulation software. The proposed algorithm is performed on the Santander transit system with

a 19-line network and a fleet of six different bus sizes. The results provide a reduction of 6% of the total cost when compared with the current frequency set and bus sizes. [10] develops a transit frequency setting model minimizing the sum of the user's travel time, operator cost, and variance in user travel time onto a real-size network by Genetic Algorithm. [3] compares the exact and approximated solution approaches on TNFSP. The exact and approximated approaches use CPLEX and Tabu Search Algorithm, respectively. [11] presents a solution method facilitating the coordination between bus rapid transit and feeder bus systems with the objective of minimizing the total cost, which consists of the bus operator and users' costs, using Genetic Algorithm. [12] develops a transit frequency setting model to maximize wait time savings under the constraints of the budget, fleet size, vehicle load, and policy headway, implementing the proposed model on the Chicago Transit network for the morning-peak hours. In the work of [13], a multi-objective frequency setting model, which aims to minimize the total travel time of all users and required fleet size for the operators by Tabu Search, is performed over a real medium-sized network in terms of two data sets corresponding to morning-peak and off-peak periods. In a recent study, [14] compares two different frequency determination methods named optimum frequency and demand-based frequency methods. Frequencies are determined for two methods on the routes obtained by Ant Colony Optimization. Finally, [15] presents a frequency setting model based on a novel objective function, which aims to slow the spread of COVID-19 caused by crowding at public transportation stops. The proposed model minimizes the total infection risk cost occurring at stops under a limited fleet size, using the Differential Evolution algorithm.

The recent researches tackle frequency setting in TNDFSP in which determines the frequency of each transit line besides route design. The work of [16] addresses the problem of determining the frequencies and routes of buses in the multi-modal transit network to minimize the sum of the operator cost, user's cost, and penalty cost for unsatisfied demand. In the proposed TNDFSP model, the frequencies are obtained by an iterative frequency setting procedure, while the transit assignment is simulated using EMME software. The model is applied to the Rome transit network. [17] determines the frequencies by a proposed descent direction method in the TNDFS study. The proposed algorithm is applied to a small-scale network and the real-size bus network of Tin Shui Wai, Hong Kong. [18] aims to obtain a set of routes and their associated frequencies simultaneously using Bee Colony Algorithm. The work determines the frequency set with the objective of minimizing the total travel time for all users and minimizing the required fleet size utilizing a Pareto optimality optimization method to optimize both objectives. To minimize the weighted sum of total travel time and the total number of transfers for all users, [19] handles TNDFSP utilizing a Memetic Algorithm in which local search operators are embedded to Genetic Algorithm to obtain better solutions. [20] presents a TNDFS model to determine the optimal set of transit routes and their associated frequency simultaneously. The proposed algorithm, which is based on the Differential Evolution, generates Pareto-optimal solutions for the minimizations of the total travel time and the required fleet size. [21] assigns the frequency values to the transit lines in consideration of minimizing the passenger time and operating cost employing the Multi-Objective Particle Swarm Optimization algorithm. [22] address a TNDFSP in which both the total travel time of network users and the CO₂ emission are minimized. The model employs a Memetic Algorithm considering a heterogeneous fleet.

The studies mentioned above concerning TNFSP, tackle the problem using exact, heuristic, and metaheuristic solution approaches. However, recent studies focus intensively on the use of metaheuristics due to their time-saving advantages. [1] expresses that the Genetic Algorithm (GA) and Simulated Annealing (SA) have been frequently used for TNDFSP studies and emphasize that recently developed metaheuristic like Firefly Algorithm (FA) should be applied

to TNDSP. Similarly, [23] states that most of the transit scheduling studies are based on well-known metaheuristics such as GA, Tabu Search (TS), and that there is no investigation on novel population-based metaheuristics like FA.

FA is one of the well-known metaheuristics, which has been composed of inspiring fireflies' behavior. Its natural feature enables to converge fast and to lead to obtaining optimal solution early since each firefly moves towards the brighter firefly in each searching step [24]. [25] states that since fireflies gather more closely around each optimum (local or global) without jumping around, FA is more successful than other metaheuristics. For these reasons, we employ FA in the proposed transit frequency setting model.

There are two major motivations for this study. The first one is that, to the best of our knowledge, there is no work concerning transit frequency setting problems employing FA. Thus, the performance evaluation of FA in TNFSP is open for research. The second one is that the planners may be interested in exploring different solution methods concerning the transit frequency setting problem.

In this study, we present a bi-level TNFSP model that aims to minimize the total user cost subject to a constraint of a fleet size utilizing FA. The proposed model is tested on 10-route Mandl Test Network with the different combinations of parameter values of FA to evaluate the effect of the algorithm parameter values. After that, the best solution among the 30 solutions generated by the calibrated parameter values is compared with the existing frequency sets of the 10-route network. The structure of this article is as follows: Section 2 presents the mathematical model and passenger assignment method of this study, while Section 3 describes the solution algorithm based on FA. Section 4 gives the results of computational experiments. Finally, Section 5 presents concluding remarks and future researches.

2. PROBLEM DESCRIPTION

This paper proposes a bi-level TNFSP whose upper level determines the optimal frequency values of transit lines. The lower level is the transit assignment process that defines the path choice model of the users. Section 2.1 below explains the mathematical models, that is, the objective function and the constraint, utilized in obtaining the optimal frequency values, while Section 2.2 describes extensively how users are assigned to the transit network.

2.1. Mathematical Model

The proposed model determines a frequency set for transit lines, which minimizes total user cost subject to a fleet size constraint. User cost is a generalized cost that consists of waiting time, in-vehicle time, and transfer penalty. The expected wait time for lines is calculated as half of the line headways, assuming random arrivals of passengers. To better represent the perception of waiting time and making transfers, coefficients are applied to the costs of relevant trip stages. The objective function of the optimization model can be formulated as follows:

$$\min F = \sum_{u \in U} IVT_u + c_{wt} \times WT_u + c_{t1} \times \delta_u^1 + c_{t2} \times \delta_u^2 \quad (1)$$

- U : the set of transit users
- u : index of the transit user
- IVT_u : in-vehicle travel time of user u

- WT_u : total waiting time of user u
 δ_u^1 : binary variable, 1 if user u makes the first transfer during the trip, 0 otherwise
 δ_u^2 : binary variable, 1 if user u makes the second transfer during the trip, 0 otherwise
 c_{wt} : coefficient of total wait duration
 c_{t1} : penalty of the first transfer
 c_{t2} : penalty of the second transfer

In order to obey the fleet size constraint, the number of vehicles exceeding the fleet size is multiplied by an enormous penalty coefficient. Thus, it is guaranteed that the algorithm suppresses the number of required vehicles below the allowed fleet size. To determine the number of vehicles exceeding the fleet size constraint, the required fleet size necessary to operate the transit system is calculated as follows:

$$RF = \sum_{l \in L} \text{round} \left(\frac{t_l}{h_l} \right) \quad (2)$$

- RF : required fleet size to operate transit network
 L : the set of transit lines
 l : index of the transit line
 t_l : roundtrip time of line l
 h_l : headway of line l

2.2. Transit Assignment

The transit assignment process is necessary to calculate the total user travel costs. In this study, we adopt a transit assignment method similar to the recent transit network design studies such as [18], [20], [21], [26]-[28]. The transit assignment method of this study is as follows: Users search for paths of three categories to fulfill their transportation demands. Initially, users search for direct trip paths between their origin and destination nodes. In case that there is no direct trip, users search for a one-transfer path. Further, if users do not have any one-transfer path, they try to find a two-transfer path. Users, who cannot find paths in these three categories are called unserved demand. In case users find multiple paths in a category, it is assumed that they select the path based on the logit model, as presented in Eq. (3). The utility of each path is calculated as presented in Eq. (4).

$$p_{i,j,k} = \frac{e^{-U_{i,j,k} \times R}}{\sum_{k \in P_{i,j}} e^{-U_{i,j,k} \times R}} \quad \forall i \in N, \forall j \in N \quad (3)$$

$$U_{i,j,k} = IVT + c_{wt} \times WT + c_{t1} \times \delta^1 + c_{t2} \times \delta^2 \quad (4)$$

- N : set of nodes
 $P_{i,j}$: set of paths between the origin i and the destination j
 $p_{i,j,k}$: probability to choose the path k between the origin i and the destination j
 $U_{i,j,k}$: cost/disutility of path $p_{i,j,k}$ between the origin i and the destination j
 R : coefficient of path cost (disutility)
 IVT : total in-vehicle time of path $p_{i,j,k}$
 WT : total waiting time of path $p_{i,j,k}$

3. SOLUTION ALGORITHM

In this study, we employ the FA developed by [29] to solve complex problems, which is a relatively new metaheuristic, especially in solving transportation problems. FA is a swarm intelligence-based optimization algorithm inspired by the behavior of fireflies that produce flashes to attract mating partners. In FA, three idealized rules are followed: (1) All fireflies are attracted to each other regardless of their sex, (2) attractiveness is proportional to a fireflies' brightness; less bright fireflies will move toward the brighter fireflies, and if there is no brighter firefly, they will move randomly, (3) the brightness is determined by the landscape of objective function.

In FA, it is assumed that the brightness of a firefly determines its attractiveness. The attractiveness value, β , is relative to each firefly couple. Attractiveness depends on the distance between firefly i and firefly j , r_{ij} , as presented in Eq. (5).

$$\beta = \beta_0 e^{-\gamma r_{ij}^2} \quad (5)$$

Where β_0 the light intensity, typically set to 1 in literature. γ is the coefficient of light absorption, which is essential in determining the speed of convergence. γ ranges between 0.1 and 10.0. r is the Cartesian distance between any two fireflies, which can be calculated by the following equation.

$$r_{ij} = |x_i - x_j| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \quad (6)$$

Where $x_{i,k}$ is the spatial coordinate of firefly i on dimension k , d is the number dimensions (*i.e.*, variables of the decision vector). If the firefly j is brighter, the position of the firefly i after moving toward firefly j is calculated by;

$$x_i = x_i + \beta_0 e^{-\gamma r^2} (x_j - x_i) + \alpha \epsilon_i \quad (7)$$

The second term of Eq. (7) is related to attractiveness. The third term is the randomization term consists of the randomization parameter α and the vector of random numbers ϵ_i drawn from the Gaussian distribution.

The parameter α takes a value between 0 and 1, and ϵ_i ranges in $rand(0,1) - 1/2$ where $rand$ is a random number generation function used with boundary value parameters. The steps of FA associated with the proposed transit frequency setting model are as follows:

Step 0: Initialization. Set stopping criteria and values of parameters α , γ , β_0 , $nPop$, z and generate a frequency set $\mathbf{f}_i = \{f_{i,1}, f_{i,2}, \dots, f_{i,d}\} \forall d \in L, i \in \{1, 2, \dots, nPop\}$ for each firefly in the population with size $nPop$, and calculate the cost c_i of each frequency set \mathbf{f}_i .

Step 1: Loop. Compare each frequency set \mathbf{f}_i with other frequency sets $\mathbf{f}_j, j \in \{1, 2, \dots, nPop\}, i \neq j$

Step 1.1: Update Frequency Set. Update the frequency set \mathbf{f}_i if $c_i < c_j$

Step 1.2: Transit Assignment. Assign users to the network using updated frequency set \mathbf{f}_i to calculate the total user cost and the required fleet size.

Step 1.3: Cost Calculation. Calculate the objective function cost, c_i , as a result of updated frequency set f_i .

Step 2: Termination. If maximum generation number z is reached, stop and output the best solution. Otherwise, return to Step 1.

4. COMPUTATIONAL EXPERIMENTS

The proposed model is performed on Mandl’s Test Network, using the 10-route bus network found as the best solution in the TNDFSP study of [24]. Mandl’s Test Network, which has been used in many studies by researchers, consists of 15 nodes and 21 bi-directional links. The travel time of links and transit demand between each node pair at peak hour is depicted in Fig. 1. The details of the 10-route bus network, which needs a fleet of 76 buses to operate the transit system, are given in Table 1.

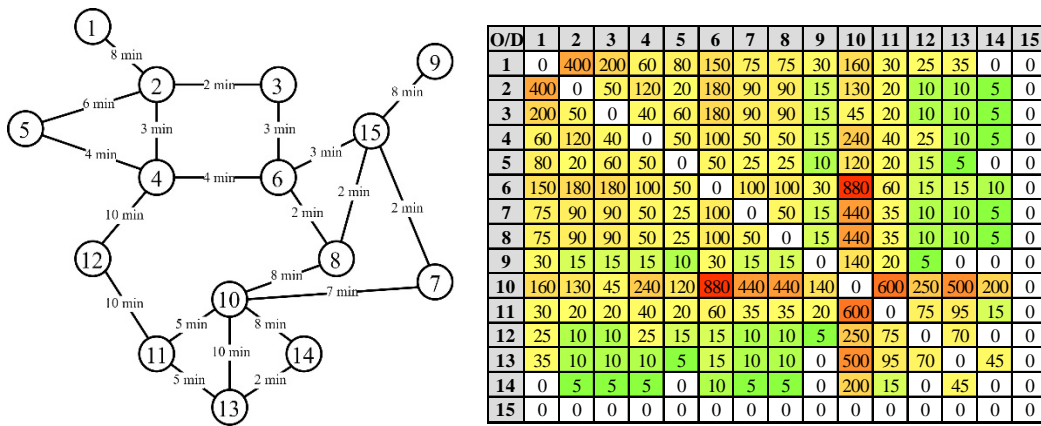


Figure 1. (a) Mandl’s transit network, (b) Demand matrix at peak hour.

Table 1. Details of the 10-route transit network

Route No	Initial Node	Arrival Node	Node Number	Node Order	Fleet Size	Travel Time (min)	Frequency (/h)
1	1	13	8	1-2-3-6-8-10-11-13	12	66	10.91
2	9	12	6	9-7-15-7-10-11-12	9	64	8.44
3	7	5	8	7-15-8-6-3-2-4-5	4	36	6.67
4	2	14	8	2-4-6-8-10-11-13-14	9	58	9.31
5	13	4	8	13-14-10-8-6-3-2-4	8	56	8.57
6	1	12	5	1-2-5-4-12	3	56	3.21
7	11	1	8	11-10-7-15-6-3-2-1	13	60	13.00
8	5	11	6	5-4-6-8-10-11	9	46	11.74
9	13	1	7	13-11-12-4-5-2-1	5	86	3.49
10	9	12	8	9-15-8-6-3-2-4-12	4	60	4.00

Metaheuristics are known to be sensitive to parameter values. In problems having vast search space, improper parameter values can lead to failure in finding the global optimum solution. Hence, to better demonstrate the effect of different parameter values on the performance of FA, the optimization model is executed with the various combinations of parameter values.

It is expected that the increase in $nPop$ and z values will affect the quality of the solutions positively. The light intensity parameter β_0 is usually taken as 1.0, and the effect of the parameters γ and α on the transit frequency setting problem is focused in this study. The parameters γ and α values are chosen from the sets $\gamma = \{0.1, 0.5, 1.0, 5.0, 10.0\}$ and $\alpha = \{0.00, 0.25, 0.50, 0.75, 1.00\}$, respectively.

In this study, we execute three independent runs for 25 parameter combinations, which results in 75 runs of optimizations. Population size $nPop$, and the maximum number of generations z is set to 50 and 20, respectively. Both FA and transit assignment algorithms are coded in MATLAB. The tests are performed on an Intel Core i7 computer with 3.4 GHz CPU and 16 GB of RAM. The average execution time is approximately 10 min per run. The parameters used in the experiments are given in Table 2.

Table 2. Details of the 10-route transit network

Parameter	Value
Number of fireflies ($nPop$)	50
Light intensity (β_0)	1
Maximum generation (z)	20
Coefficient of path disutility (R)	0.1
Bus capacity (C)	50 passengers/bus
The penalty of the first transfer (c_{t1})	30 min/transfer
The penalty of the second transfer (c_{t2})	70 min/transfer
Coefficient of waiting time (c_{wt})	2
The minimum frequency allowed for lines (f_{min})	1/h
The maximum frequency allowed for all lines (f_{max})	60/h
The maximum fleet size allowed	76 buses

Fig. 2 presents the results of 75 solutions belonging to the parameter optimization, showing that FA is not successful enough in obeying the fleet size constraint. Due to the trade-off structure between the user cost and the fleet size, the user cost reduces as the fleet size increases or vice versa. Only 20 solutions are equal or below the constraint of 76 buses, while the remaining solutions are unsuccessful in mitigating the fleet size constraint. All optimization runs of the combinations $\{\alpha=0.75, \gamma=1\}$, $\{\alpha=1, \gamma=0.1\}$, $\{\alpha=1, \gamma=1\}$, and $\{\alpha=1, \gamma=5\}$ can reach acceptable solutions not exceeding the fleet size constraint. However, the best combination of parameter values is $\{\alpha=1, \gamma=1\}$, with a minimum average user cost of 271,503.

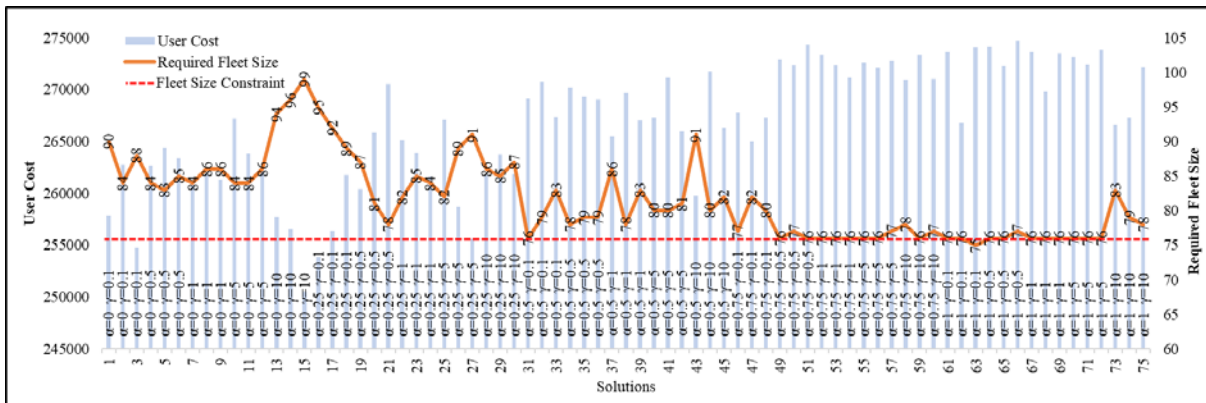


Figure 2. Comparative results of all parameter combinations in terms of required fleet size and user cost.

Fig. 3 illustrates the average fleet size and user cost values with respect to the values of α and γ . The increase in parameter α improves the quality of the solutions by decreasing the vehicle number exceeding the constraint. On the other hand, parameter γ obtains better solutions when defined as 0.5. Considering that the best combination of parameters is $\{\alpha=1, \gamma=1\}$, Figs. 2 and 3 show significantly that parameter values should be treated as a combination, not individually.

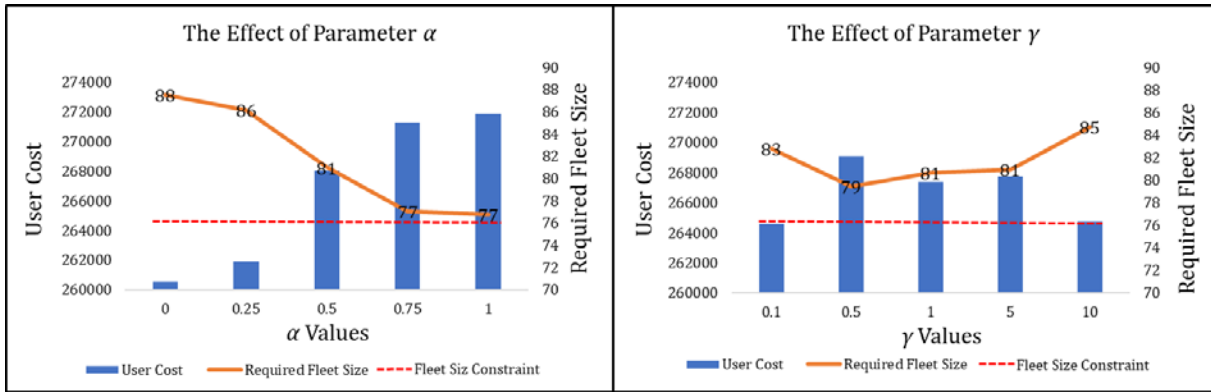


Figure 3. The effects of parameters α and γ on the quality of solutions.

To demonstrate the proposed algorithm's improvement by the calibrated parameter values in terms of stability and robustness, 30 independent optimization runs are executed by the calibrated parameter values $\{\alpha=1, \gamma=1\}$. The findings of 30 runs are depicted in Fig. 4. The results clearly show that all solutions fulfill to obey the fleet constraint the contrary to the solutions in parameter optimization in Fig. 2. The best solution is obtained in 9th run with the user cost of 265,505. All 30 runs have an average user cost of 274,972, a standard deviation of 3,103, and a coefficient of variation of 1%. Details for the best solution obtained among 30 optimization runs are given in Table 3, proving that passenger loads, even in the busiest segments, do not exceed the line capacities.

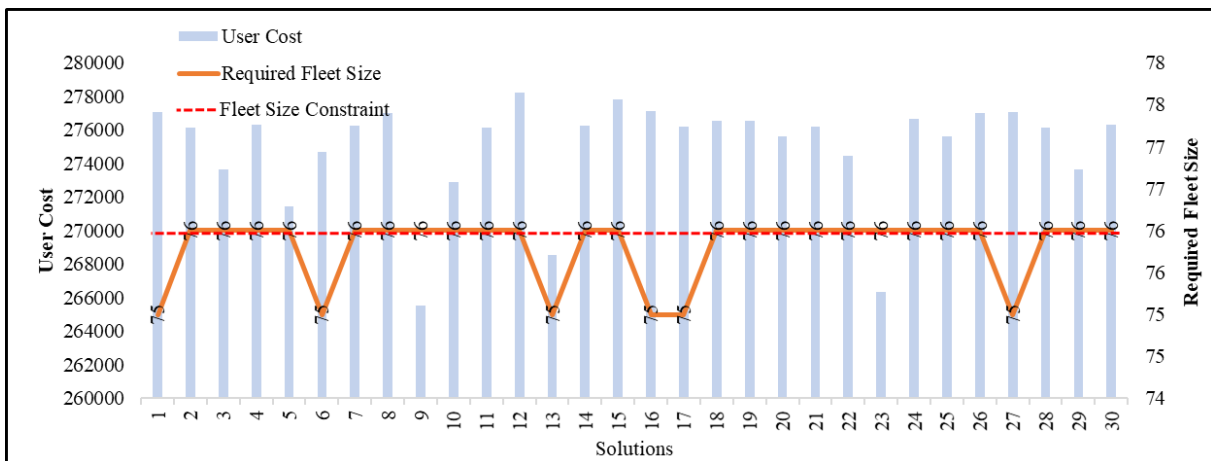


Figure 4. 30 independent runs with the calibrated parameter values.

Table 3. The corresponding details of the best frequency set obtained the calibrated parameter values

Route No	Frequency (/h)	Headway (min)	Required Buses Number	Line Capacity	Peak Segment	Max. Occupancy Rate (%)	Peak Load
1	13.78	4.35	15	689	6-8	96	665
2	8.82	6.80	9	441	10-11	96	425
3	5.73	10.47	3	286	3-2	96	276
4	1.00	60.00	1	50	6-7	34	17
5	12.96	4.63	12	648	10-8	90	581
6	5.08	11.81	5	254	1-2	81	207
7	12.52	4.79	13	626	15-6	99	624
8	16.22	3.70	12	811	8-10	94	764
9	2.36	25.42	3	118	11-12	81	95
10	3.36	17.86	3	168	6-3	81	137

Table 4 presents a comparison between the best frequency set obtained by the proposed model and the frequency set of the existing network. It should be noted that the user cost and fleet size

values of Arbex and Cunha solution are obtained by our transit assignment model and that the assignment methods of both studies are quite similar. The best frequency set generated by the proposed model provides a reduction of 4% in the user cost using the same fleet size. The results show that the frequency optimization model proposed in this study can slightly improve the solution quality. The significant changes occur in the frequency values of lines 1, 4, 5, and 8.

Table 4. Comparison of the best solutions

Solution	User Cost	Fleet Size	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
Existing frequency set	276,433	76	10.91	8.44	6.67	9.31	8.57	3.21	13.00	11.74	3.49	4.00
The best of our proposed model	265,505	76	13.78	8.82	5.73	1.00	12.96	5.08	12.52	16.22	2.36	3.36

Table 5 compares the performance outputs concerning the best frequency set of the proposed model with the performance outputs related to the frequency set of the existing network, in terms of required buses number, peak segments, line capacity, and peak load for each line. The left side of the slash gives the outputs belonging to the best frequency set of the proposed model, while the right side gives the outputs regarding the existing frequency set. The findings show a remarkable difference in the required bus number of line 4 between the solutions. The peak segments of lines are about the same for both solutions except for lines 3, 4, 7, and 9. Also, the significant difference in capacity values of lines 4, 5, and 8, which stems from frequency values, takes attention.

Table 5. Comparison of the best solutions

Lines	Required buses number	Line Capacity	Peak Load	Peak Segment
L1	15/12	689/546	665/574	6-8/6-8
L2	9/9	441/422	425/406	10-11/10-11
L3	3/4	286/334	276/287	3-2/6-3
L4	1/9	50/466	17/464	6-7/8-10
L5	12/8	648/429	581/402	8-10/8-10
L6	5/3	254/161	207/117	1-2/1-2
L7	13/13	626/650	624/607	15-6/10-7
L8	12/9	811/587	764/597	8-10/8-10
L9	3/5	118/175	95/141	11-12/1-2
L10	3/4	168/200	137/150	6-3/6-3

5. CONCLUSION AND FUTURE RESEARCH

In this study, we present an optimization model for TNFSP employing the Firefly Algorithm, which is a relatively new metaheuristic for transportation network design problems. The objective of the optimization model is to minimize the total user cost under a fleet size constraint. The proposed model is applied to Mandl's Network with 10 routes, by different combinations of parameters to show the effect of different parameter values on the quality of the solutions. The best of 30 solutions obtained by the calibrated parameter values is compared with the existing frequency set of the 10-route network.

The results show that the FA is not successful enough to satisfy the fleet size constraint in every optimization run by the randomly chosen parameter values. However, the FA can reach better solutions by selecting proper parameter values without exceeding the fleet size constraint. The best solutions are obtained with the combination of parameter values $\alpha = 1$ and $\gamma = 1$. The best

frequency set found by the proposed model slightly decreases the user cost by about 4% using the same fleet size compared to the existing frequency set of the 10-route bus network.

For future research, we plan to compare the efficiency of FA in the transit frequency setting problem with other well-known metaheuristics such as the Genetic Algorithm, Differential Evolution algorithm, and Particle Swarm Optimization algorithm. Moreover, we will use a real-sized network to test the applicability of the proposed model.

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