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AN APPLICATION OF THE SUM OF LINEAR OPERATORS IN INFINITE MATRIX THEORY

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ABSTRACT

We present in this work a new application of the sum of linear operators in non differential case. We apply Labbas-Terreni's [6] approach in infinite-matrix theory and then we give some concrete applications: in the theory of continued fractions, in numerical schemes and in the development of some analytic function on a given basis.

1. INTRODUCTION

In the complex Banach space E, the following equation

$$A.X + B.X - \lambda.X = Y \ , \lambda > 0 \tag{1}$$

where Y is given in E and A, B are two closed linear operators with domains D(A), D(B), has been considered by Da Prato and Grisvard [1] and by many authors, under differents hypotheses and then many applications in partial differential equation of elliptic, parabolic or hyperbolic type, are given. For example, see R. Labbas and B. Terreni [6] or M. Furhman [3]. Our first aim, in this work, is to present a new application of the sum theory in non differential case. Equation (1) is regarded as an infinite linear system in space $l^\infty = s_1$, that is A + B is considered as the sum of two particular infinite matrices defined respectively on $D(A) = s_r$ $(0 < r < 1)$ and $D(B) = s_1$ (see section 2) [In infinite matrix theory it means that A belongs to the matrix class $(s_{1/\alpha}, s_1)$, see I.J. Maddox [9]]. In our case, we prove that (-A) and (-B) are generators of analytic semigroups not strongly continuous at $t = 0$ since the density of their domains is not true. The classical perturbation theory, as in Kato [4] or in Pazy [10] [for example, the relative boundedness with respect A or B] is not verified here. So we use the results of Labbas-Terreni [6] in our non commutative situation. The choice of these two infinite matrices is motivated by the resolution of a class of non symmetric linear infinite tridiagonal systems.

The plan of this paper is as follows. In section 2 we recall some results about some regular sets of infinite matrices taken from R. Labbas and B. de Malafosse [5]. Section 3 contains the hypotheses and the main results of the sum-strategy as in Labbas-Terreni [6]. In section 4 we give an example of two infinite matrices A and B regarded as two unbonded linear operators on $l^\infty = s_1$; then we study their spectral properties and we showh that the Labbas-Terreni [6] sum-strategy can be applied. In section 5 we explicite Lions-Peetre's [8] interpolation spaces between D(A) and $E = s_1$ for obtaining maximal regularity results for the solution of (1). In the last paragraph we give simple examples which can be regarded by the approach presented here: in the analytic theory of continued fractions, in the study of numerical schemes by finite differences for some differential equations and in the development of some analytic function on a particular basis.

2. THE SPACES S_r AND s_r

Consider a positive real number r and a Banach space G and denote by S_r the set of infinite matrices $M = (a_{nm})_{n,m=1,...}$ of linear bounded operators from G to itself, and s_r the set of infinite column matrices $X = {}^t(x_1,x_2,x_3,...) = (x_n)$ of elements x_n in G which satisfy respectively

$$\begin{cases} \sup_{n\geq 1}\left(\sum_{m\geq 1} \|a_{nm}\|_{L(G)} r^{m-n}\right) < \infty \\ \sup_{n\geq 1}\left(\dfrac{\|x_n\|_G}{r^n}\right) < \infty . \end{cases} \tag{2}$$

These spaces are naturally normed by

$$\begin{cases} \|M\|_{S_r} = \sup_{n\geq 1}\left(\sum_{m\geq 1} \|a_{nm}\|_{L(G)} r^{m-n}\right) , \\ \|X\|_{S_r} = \sup_{n\geq 1}\left(\dfrac{\|x_n\|_G}{r^n}\right) . \end{cases} \tag{3}$$

More generally, we can consider another sequence $c = (c_n)$, $c_n > 0$, instead of (r^n) in definition of S_r and s_r; we obtain the spaces S_c and s_c characterized by

$$\begin{cases} \sup_{n\geq 1} \left(\sum_{m\geq 1} \|a_{nm}\|_{L(G)} \frac{c_m}{c_n} \right) < \infty , \\ \sup_{n\geq 1} \left(\frac{\|x_n\|_G}{c_n} \right) < \infty . \end{cases} \tag{4}$$

with the corresponding norms. L(G) denotes the Banach space of all bounded linear operators from G into itself. In R. Labbas and B. de Malafosse [5] we have studied some interesting properties of these spaces from which we recall, in particular, the following result:

Proposition 1. For any positive real number r the sets s_r and s_c are Banach spaces and the sets S_r and S_c are unit Banach algebras.

3. SUM OF LINEAR OPERATORS

We simply quote here some results taken from Da Prato-Grisvard [1] and Labbas-Terreni [6]. Consider a complex Banach space E and two closed linear operators A and B defined on their domains $D(A) \subset E$ and $D(B) \subset E$. Their sum is defined by setting:

$$SX = AX + BX \quad , X \in D(S) = D(A) \cap D(B) \tag{5}$$

Now we shall make the following assumptions on A and B

$$(H.1) \begin{cases} \exists C_A, C_B > 0 \ , \ \varepsilon_A, \varepsilon_B \in \,]0,\pi[\text{ such that} \\ \text{i) } \rho(A) \supset \sum_{\varepsilon_A} = \{z \in C \ / \ |\mathrm{Arg}(z)| < \pi - \varepsilon_A\} \\ \|(A - zI)^{-1}\|_{L(E)} \leq C_A \ / \ |z| \ , \ \forall z \in \sum_{\varepsilon_A} - \{0\} , \\ \text{ii) } \rho(B) \supset \sum_{\varepsilon_B} = \{z \in C \ / \ |\mathrm{Arg}(z)| < \pi - \varepsilon_B\} \\ \|(B - zI)^{-1}\|_{L(E)} \leq C_B \ / \ |z| \ , \ \forall z \in \sum_{\varepsilon_B} - \{0\} , \\ \text{iii) } \varepsilon_A + \varepsilon_B < \pi . \end{cases}$$

If we consider the commutative case:

$$\begin{aligned} &(A - \xi I)^{-1} (B - \eta I)^{-1} - (B - \eta I)^{-1} (A - \xi I)^{-1} \\ = &\left[(A - \xi I)^{-1} ; (B - \eta I)^{-1}\right] = 0 \ ; \ \forall \xi \in \rho(A) \ , \ \forall \eta \in \rho(B) \end{aligned} \tag{6}$$

and if we assume that D(A) and D(B) are densely defined in E, it is well known (See [1]) that the following bounded linear operator (for all $\lambda > 0$)

$$L_\lambda = -\frac{1}{2i\pi}\int_\Gamma (B + zI)^{-1}(A - \lambda I - zI)^{-1}\,dz \tag{7}$$

coincides with $(\bar{S} - \lambda)^{-1}$ where $\bar{S} = \overline{A + B}$. Γ is a simple infinite sectorial curve lying in $\rho(A - \lambda I) \cap \rho(-B)$. In our applications the commutativity and the density are not true, so we shall assume that:

$$\text{(H.2)}\begin{cases} \exists C > 0\ ,\ h \in N\ ,\ (\tau_i)_{i=1,\dots h}\ ,\ (\rho_i)_{i=1,\dots h} \text{ such that} \\ \qquad \forall i \geq 1\ ;\ 0 \leq 1 - \tau_i < \rho_i \leq 2 \text{ and} \\ \|\mu A(A - \lambda I)^{-1}[A^{-1}\ ;\ (B + \mu I)^{-1}]\|_{L(E)} \leq C\sum_{i=1}^{h}|\lambda|^{-\tau_i}.|\mu|^{-\rho_i} \\ \qquad \text{for } |\lambda|\ ,\ |\mu| \to \infty\ ;\ \lambda \in \rho(A)\ ,\ \mu \in \rho(-B). \end{cases} \tag{8}$$

Let us put:

$$\delta = \min_{1\leq i\leq h}(\tau_i + \rho_i - 1) > 0. \tag{9}$$

For this commutator see Labbas-Terreni [6], [7]. Now, for any $\sigma \in]0,1[$, let us introduce the real interpolation spaces $D_A(\sigma,\infty)$ between D(A) and E [or $D_B(\sigma,\infty)$ between D(B) and E] characterized in Grisvard [2] by:

$$D_A(\sigma,\infty) = \left\{X \in E \ /\ \sup_{z\in\Gamma}\|z^\sigma A(A - zI)^{-1}X\|_E < \infty\right\}, \tag{10}$$

and which equals, in our case

$$\left\{X \in E \ /\ \sup_{t\geq 0}\|t^\sigma A(A + tI)^{-1}X\|_E < \infty\right\}.$$

It is a Banach space with the natural following norm:

$$\|X\|_{D_A(\sigma,\infty)} = \|X\|_E + \sup_{t\geq 0}\|t^\sigma A(A + tI)^{-1}X\|_E\ ,$$

which is equivalent to $\sup_{t\geq 0}\|t^\sigma A(A + tI)^{-1}X\|_E$ when A is invertible (as in our case). Now we recall the main result proved in [6]:

Theorem 2. Under assumptions (H.1) and (H.2) theer exists λ^* such that $\forall\lambda \geq \lambda^*$ and $\forall Y \in D_A(\sigma,\infty)$ equation $AX + BX - \lambda X = Y$ has a unique solution $X \in D(A) \cap D(B)$ which satisfies

i) $(A - \lambda I)X \in D_A(\theta,\infty)\quad \forall\theta \leq \mathrm{Min}(\sigma,\delta)$,

ii) $BX \in D_A(\theta,\infty) \qquad \forall\theta \leq \mathrm{Min}(\sigma,\delta)$,

iii) $(A - \lambda I)X \in D_B(\theta,\infty) \quad \forall\theta \leq \mathrm{Min}(\sigma,\delta)$.

Remark 1. Clearly one has a similar result when replacing A by B. Notice the unexpected regularity result in the statement iii).

4. DEFINITION OF OPERATORS A AND B

In this section we consider the Banach space $E = s_1 = l^\infty$ and the two linear operators characterized by the two following infinite matrices:

$$A = \begin{pmatrix} a & b_1 & & & & \\ & a^2 & b_2 & & O & \\ & & \cdot & \cdot & & \\ & & & a^n & b_n & \\ & O & & & \cdot & \cdot \\ & & & & & \cdot \end{pmatrix}, \tag{11}$$

such that

$$\begin{cases} \text{i) } a > 1 \text{ and} \\ \text{ii) } \exists M_A > 0 \ / \ \forall n \geq 1 \ |b_n| \leq M_A \end{cases} \tag{12}$$

and

$$B = \begin{pmatrix} \beta_1 & & & & \\ \gamma_2 & \beta_2 & & O & \\ & \cdot & \cdot & & \\ & & \gamma_n & \beta_n & \\ & O & & \cdot & \cdot \\ & & & & \cdot \quad \cdot \end{pmatrix}, \tag{13}$$

under the following assumptions:

$$\begin{cases} \text{i) } \beta_{2n} = 1 \ ; \ \beta_{2n+1} = (2n+1)! \\ \text{ii) } \gamma_{2n} = O(1) \ (n \to \infty) \\ \text{iii) } \gamma_{2n+1} = o(1) \ (n \to \infty) \ . \end{cases} \tag{14}$$

Remark 2. We have choosed the first diagonal of A in a particular form only to simplify calculus. All the following results in this work are true if we replace this diagonal by any non decreasing strictly positive sequence (α_n) which tends to infinity. The same remark is true for B; we can replace i) in (14) by

$$(\beta_{2n}) \to L \neq 0 \quad \text{and} \quad \left(\frac{\beta_{2n+1}}{\alpha_{2n+1}}\right) \to +\infty .$$

Then the spectral properties of A and B are given by:

Proposition 3. In the Banach space s_1 the two linear operators A and B are closed and verify

i) $D(A) = s_{1/\alpha}$,

ii) $D(B) = s_{1/\beta} = \left\{X \in s_1 \,/\, |x_{2n}| = O(1)\,,\, |x_{2n+1}| = O\left(\frac{1}{(2n+1)!}\right)\ (n \to \infty)\right\}$,

iii) $\overline{D(B)} \neq s_1$ and $\overline{D(A)} \neq s_1$,

iv) There exist positive numbers ε_A, ε_B, M (with $\varepsilon_A + \varepsilon_B < \pi$) such that

$$\|(A - \lambda I)^{-1}\|_{L(s_1)} \leq \frac{M}{|\lambda|}\ ,\ \forall\lambda \neq 0 \text{ and } |\text{Arg}(\lambda)| \geq \varepsilon_A$$

$$\|(B + \mu I)^{-1}\|_{L(s_1)} \leq \frac{M}{|\mu|}\ ,\ \forall\mu \neq 0 \text{ and } |\text{Arg}(\mu)| \leq \pi - \varepsilon_B .$$

Statement i) Let (X_p) be a sequence that converges to X in s_1 and such that $X_p \in s_{1/\alpha}$ for every p, and (AX_p) converges to Y in s_1. Let $X_p = (x_{np})$, then we obviously have

$$\begin{cases} \forall n \geq 1 & x_{np} \to x_n \ (p \to \infty) \\ \forall n \geq 1 & a^n x_{np} + b_n x_{n+1,p} \to y_n \ (p \to \infty)\ , \end{cases}$$

which implies that

$$\forall n \geq 1 \qquad a^n x_n + b_n x_{n+1} = y_n.$$

However this is not enough to guarantee that $X = (x_n)$ belongs to $s_{1/\alpha}$. We deduce this from (12) since $\exists K > 0$ such that

$$a^n |x_n| \leq |a^n x_n + b_n x_{n+1} - b_n x_{n+1}| \leq |y_n| + |b_n||x_{n+1}| \leq K.$$

So the operators A is closed. The proof is similar for B. It is not difficut to see, from (11), (12), (13) and (14) that $D(A) = s_{1/\alpha}$ and $D(B) = s_{1/\beta}$. For the non density of D(A), it is sufficient to see that the vector $(1, 1, ..., ...) \in s_1$ cannot be approximated by a sequence $X_n = (x_{n,p})_p$ in $s_{1/\alpha}$ otherwise this would imply that $x_{n,m}$ tends to 1, when m tends to infinity for every n; this is not possible since all sequence in $s_{1/\alpha}$ tend to zero. An analogous reasoning is true for B. Now it is easy to see that the sequence $X = (0, 1, 0, 1, ...)$ belongs to $s_{1/\beta} = D(B)$ but not to D(A), and the sequence $X = (1/a, 0, 1/a^3, 0, 1/a^5, ...)$ belongs to D(A) and not to D(B). This last remark implies that the embedding relative boundedness with respect to A or B cannot be applied here.

For fixed $\varepsilon_0 \in]0, \pi/2[$, let us define the infinite sectorial set:

$$\Pi_{\varepsilon_0} = \{\lambda \in C \;/\; |\mathrm{Arg}(\lambda)| \le \varepsilon_0\} \tag{15}$$

and for all complex $\lambda \notin \Pi_{\varepsilon_0}$ let

$$D_\lambda = \left(\frac{1}{a^n - \lambda}\,\delta_{nm}\right)_{n,m} \qquad \delta_{nm} = 1 \text{ if } n = m \text{ and } 0 \text{ elsewhere},$$

$$D_\lambda.(A - \lambda I) = \begin{pmatrix} 1 & q_1 & & & \\ & 1 & q_2 & & O \\ & & \ddots & \ddots & \\ & & & 1 & q_n \\ & O & & & \ddots & \ddots \\ & & & & & \ddots \end{pmatrix} \quad \text{with } q_n = \frac{b_n}{a^n - \lambda}.$$

It is easy to see that

$$\left|\frac{b_n}{a^n - \lambda}\right| \le \frac{M_A}{a^n.\sin\varepsilon_0} \qquad \forall\lambda \notin \Pi_{\varepsilon_0},$$

since

$$|a^n - \lambda| = d(\lambda, a^n) \ge a^n.\sin\varepsilon_0 \qquad \forall\lambda \notin \Pi_{\varepsilon_0},$$

from which we deduce that there exists a larger integer $n_0 = n(\varepsilon_0)$ such that

$$n \ge n_0 \Rightarrow \left|\frac{b_n}{a^n - \lambda}\right| \le \frac{1}{2}.$$

Now, we consider the following infinite matrix

$$Q_\lambda = \begin{pmatrix} & T_\lambda & \vdots & & O \\ \cdot & \cdots & 1 & & \\ & & & 1 & \\ & O & & & \ddots \end{pmatrix},$$

where T_λ is the finite matriq of order n_0 defined by

$$T_\lambda = \begin{pmatrix} 1 & q_1 & & O \\ & \cdot & \cdot & \\ & & 1 & q_{n_0-1} \\ O & & & \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -q_1 & q_1q_2 & -q_1q_2q_3 & \cdot & \cdot \\ & 1 & -q_2 & +q_2q_3 & \cdot & \cdot \\ & & 1 & -q_3 & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \\ & O & & & 1 & -q_{n_0-1} \\ & & & & & 1 \end{pmatrix}.$$

Then

$$R_\lambda = D_\lambda.(A - \lambda I).Q_\lambda = \begin{pmatrix} 1 & & & & \\ & \cdot & & O & \\ & & 1 & q_{n_0} & \\ & & & 1 & q_{n_0+1} \\ & O & & & \cdot & \cdot \\ & & & & & \cdot \end{pmatrix}.$$

This product is defined on any space s_r, $r > 0$. Hence

$$\|I - R_\lambda\|_{s_{1/\alpha}} = \sup\left\{\frac{1}{a}\left|\frac{b_n}{a^n - \lambda}\right| / n \geq n_0\right\} \leq \frac{1}{2a} < 1.$$

So that, for any $\lambda \notin \Pi_{\varepsilon_0}$, R_λ is invertible in the Banach algebra $S_{1/\alpha}$ and

$$(A - \lambda I)^{-1} = Q_\lambda R_\lambda^{-1} D_\lambda. \tag{16}$$

Now we can verify that for $Y \in s_1$, we have $D_\lambda Y \in s_{1/\alpha}$, $R_\lambda^{-1}(D_\lambda Y) \in s_{1/\alpha}$, $Q_\lambda R_\lambda^{-1}(D_\lambda Y) \in s_{1/\alpha}$ and

$$\|(A - \lambda I)^{-1} Y\|_{s_1} = \|Q_\lambda R_\lambda^{-1} D_\lambda Y\|_{s_1} \leq \|Q_\lambda\|_{s_1} \|R_\lambda^{-1}\|_{s_1} \|D_\lambda\|_{s_1} \|Y\|_{s_1},$$

but

$$\|D_\lambda\|_{s_1} = \sup_{n\geq 1}\left(\left|\frac{1}{a_n - \lambda}\right|\right) \leq \begin{cases} \dfrac{1}{|\lambda| \sin\theta} & \text{for } \lambda = |\lambda| e^{i\theta} \notin \mathbb{R}^- \\ \dfrac{1}{|\lambda|} & \text{for } \lambda \in \mathbb{R}^-, \end{cases}$$

since $|a^n - \lambda| = d(\lambda, a^n) \geq d(\lambda, p(\lambda)) = |\lambda| \sin\theta$ for $\lambda = |\lambda| e^{i\theta} \notin \mathbb{R}^-$ where $p(\lambda)$ is the orthogonal projection of λ on the real axis.

And

$$\|I - R_\lambda\|_{s_1} = \sup_n\left(\left|\frac{b_n}{a_n - \lambda}\right|\right) \leq \frac{1}{2},$$

which implies that

$$\|R_\lambda^{-1}\|_{s_1} \le 2.$$

Moreover we can see that there exists some constant K depending only on n_0 such that

$$\|Q_\lambda\|_{S_1} = \sup\{(1 + |q_n| + |q_n q_{n+1}| + \dots + |q_n q_{n+1} \dots q_{n_0-1}|);\ n \le n_0 - 1\} \le K$$

since all the terms q_n, for $n \in \{1, 2, \dots, n_0\}$, are bounded.

So the estimate of $(A - \lambda I)^{-1}$ in statement ii) holds.

For fixed $\varepsilon_1 \in]0, \pi/2[$, let $\Sigma_{\varepsilon_1} = \{\mu \in C \ / \ |Arg(\mu)| \le \pi - \varepsilon_1\}$. From (14) we deduce that there eists $n(\varepsilon_1) = n_1$ such that for any $\mu \in \Sigma_{\varepsilon_1}$ we have

$$2k + 1 \ge n_1 \Rightarrow |\gamma_{2k+1}| \le \frac{1}{2} \,.\, \sin \varepsilon_1 ,$$

$$2k \ge n_1 \Rightarrow \left|\frac{\gamma_{2k}}{(1+\mu)(2k-1)!}\right| \le \frac{1}{2} .$$

Now consider the following matrices:

$$D'_\mu = \left(\frac{1}{\beta_n + \mu}\,\delta_{nm}\right)_{n,m} ,$$

$$T'_\mu = \begin{pmatrix} 1 & & & O \\ q'_2 & 1 & \cdot & \\ & \cdot & \cdot & \\ O & & q'_{n_1} & 1 \end{pmatrix}^{-1}, \quad Q'_\mu = \begin{pmatrix} & T'_\mu & \vdots & & O & \\ \cdot & \cdot & 1 & & & \\ & & & 1 & & \\ & O & & & \cdot & \\ & & & & & \cdot \end{pmatrix},$$

where $q'_n = \dfrac{\lambda_n}{\beta_n + \mu}$, $(n \ge 2)$.

$$R'_\mu = Q'_\mu . D'_\mu .(B + \mu I) = \begin{pmatrix} 1 & & & & & \\ & \cdot & & & O & \\ & & 1 & & & \\ & & q'_{n_1+1} & 1 & & \\ & O & & q'_{n_1+2} & 1 & \cdot \\ & & & & \cdot & \cdot \end{pmatrix}.$$

Using (4) one has

$$\|I - R'_\mu\|_{s_{1/\beta}} = \sup\left\{\left|\frac{\gamma_n}{\beta_n + \mu}\frac{\beta_n}{\beta_{n-1}}\right| \ ; \ n \geq n_1\right\},$$
$$= \text{Max}\ (\tau_1\ ,\ \tau_2)$$

where

$$\tau_1 = \sup\left\{\left|\frac{\gamma_{2k}}{1+\mu}\right|\frac{1}{(2k-1)!}\ ;\ k \geq n_1/2\right\},$$
$$\tau_2 = \sup\left\{\left|\frac{\gamma_{2k+1}}{(2k+1)! + \mu}\right|(2k+1)!\ ;\ k \geq (n_1 - 1)/2\right\}.$$

Now it is clear that

$$\tau_2 \leq 1/2,$$
$$\tau_1 \leq 1/2,$$

which impies that $\|I - R'_\mu\|_{s_{1/\beta}} \leq 1/2$. Thus $\exists C > 0$ such that for any $Y \in s_1$

$$\|(B + \mu I)^{-1}\ Y\|_{s_1} = \|(R'_\mu)^{-1} Q'_\mu D'_\mu Y\|_{s_1} = \frac{2C}{|\mu|\ \sin\ \varepsilon_1}\ \|Y\|_{s_1}\ .$$

The proposition 3 is proved.

The operators (-A) and (-B) verify the hypothesis (H.1) with $\varepsilon_{-A} = \varepsilon_0$ and $\varepsilon_{-B} = \varepsilon_1$. It is clear that σ(-A) and σ(B) do not intersect, so $\rho(A) \cup \rho(-B) = C$.

We have proved that (-A) and (-B) are generators of analytic semigroup $e^{(-A)t}$ and $e^{(-B)t}$ not strongly continuous at $t = 0$.

Now we shall prove that (H.2) is satisfied under the following additional assumption on A:

$$\sup_{n\geq 1}\left(\frac{|b_{n-1}|\ n!}{a^n}\right) < \infty\ . \tag{17}$$

Remark 3. It follows from (14) that the sequence (γ_n) is bounded by some constant C_B, so we can always assume that there exists $\varepsilon_1 \in\ ]0, \pi/2[$ such that

$$\sup_{n\geq 1}\left(|\gamma_{2n}|\ ,\ \frac{|\gamma_{2n-1}|}{(2n-1)!}\right) = \frac{1}{2}\ \sin\ \varepsilon_1\ ,$$

otherwise it would be sufficient to consider $\dfrac{\sin\ \varepsilon_1}{2C_B}\ B$ instead of B.

Proposition 4. Under (12), (14) and (17) there exists a constant $K(\varepsilon_A,\varepsilon_B) > 0$ such that

$$\|\mu A(A-\lambda I)^{-1}\ ;\ [A^{-1}\ ;\ (B+\mu I)^{-1}]\|_{L(E)} \le K(\varepsilon_A,\varepsilon_B) \sum_{i=1}^{2} \frac{1}{|\lambda|^{\tau_i}|\mu|^{\rho_i}},$$

$\forall\lambda \notin \Pi_{\varepsilon_A}$, $\forall\mu \notin \Pi_{\varepsilon_B}$; with $(\tau_1,\rho_1) = (1,1)$, $(\tau_2,\rho_2) = (0,2)$. (The condition $0 \le 1 - \tau_i < \rho_i \le 2$ is satisfied).

Proof. Put

$$C = \mu A(A-\lambda I)^{-1}\ [A^{-1};\ (B+\mu I)^{-1}] = \mu A(A-\lambda I)^{-1}C_0$$

and consider the vector-space $S_1^{\beta} = \{(B_n.a_{nm})/(a_{nm}) \in S_1\}$ which is an unit algebra containing S_1, then we can write in this algebra the following equality (for all $\mu \notin \Pi_{\varepsilon_1}$)

$$B + \mu I = D_\mu + B_0 = D_\mu.(I + D_\mu^{-1}B_0),$$

where

$$D_\mu = ((\beta_n + \mu)\ \delta_{nm})_{n,m}\ ,\ B_0 = \begin{pmatrix} 0 & & & O \\ \gamma_2 & 0 & & \\ & \gamma_3 & 0 & \\ O & & . & . \end{pmatrix},$$

since $(I + D_\mu^{-1}B_0) \in S_1$. It follows from (14), and (17) that

$$\|D_\mu^{-1}B_0\|_{S_1} = \sup_{n\ge2}\left(\frac{|\gamma_n|}{|\beta_n+\mu|}\right) \le \frac{1}{2},$$

which gives the classical Neumann's series (in the Banach algebra S_1):

$$(B+\mu I)^{-1} = \sum_{n=0}^{\infty}(-1)^n(D_\mu^{-1}B_0)^n D_\mu^{-1},$$

from which we have:

$$\begin{aligned} C_0 &= [A^{-1};\ (B+\mu I)^{-1}] = A^{-1}(B+\mu I)^{-1} - (B+\mu I)^{-1}A^{-1} \qquad (18)\\ &= D_\mu^{-1}A^{-1} - A^{-1}D_\mu^{-1} - [D_\mu^{-1}B_0D_\mu^{-1}]A^{-1} + A^{-1}[D_\mu^{-1}B_0D_\mu^{-1}]\\ &+ \sum_{n=2}^{\infty}(-1)^n[(D_\mu^{-1}B_0)^{n-2}(D_\mu^{-1}B_0)^2D_\mu^{-1}A^{-1} - A^{-1}(D_\mu^{-1}B_0)^{n-2}(D_\mu^{-1}B_0)^2D_\mu^{-1}]\\ &= C_1 + R, \end{aligned}$$

where C_1 is the sum of the first four elements of C_0:

$$\begin{aligned} C_1 &= D_\mu^{-1}(I - B_0 D_\mu^{-1})A^{-1} - A^{-1}D_\mu^{-1}(I - B_0 D_\mu^{-1}) \\ &= K_\mu A^{-1} - A^{-1}K_\mu \\ &= (c_{nm})_{n,m=1,\dots} \end{aligned}$$

with

$$K_\mu = D_\mu^{-1}(I - B_0 D_\mu^{-1}) = \begin{pmatrix} \zeta_1 & & & O \\ \eta_2 & \zeta_2 & & \\ & \eta_3 & \cdot & \\ O & & \cdot & \cdot \end{pmatrix},$$

and

$$\begin{cases} \zeta_n = \dfrac{1}{\beta_n + \mu} \\ \eta_n = -\dfrac{\gamma_n}{(\beta_{n-1} + \mu)(\beta_n + \mu)} \end{cases}.$$

Now, using the explicite following formula for the upper triangular infinite matrix

$$A^{-1} = (\alpha_{nm})$$

with

$$\begin{aligned} \alpha_{nm} &= (-1)^{m-n}(b_n b_{n+1} \dots b_{m-1}).a^{-\frac{(m-n)(m+n)}{2}}, \quad \forall m > n \geq 1 \\ \alpha_{nm} &= \frac{1}{a^n}, \end{aligned}$$

it follows that

$$\begin{cases} c_{n,n-1} = (\alpha_{n-1,n-1} - \alpha_{nn})\eta_n & n > 1 \\ c_{nm} = \alpha_{nm}(\zeta_n - \zeta_m) + \eta_n \alpha_{n-1,m} - \alpha_{n,m+1}\eta_{m+1} & m \geq n \end{cases}$$

which yields

$$c_{n,n-1} = \frac{\gamma_n}{(\beta_{n-1} + \mu)(\beta_n + \mu)} \frac{(a-1)}{a^{n-1}} \quad \forall n > 1,$$

$$\begin{aligned} c_{nm} &= \alpha_{nm}\left[\frac{\beta_m - \beta_n}{(\beta_n + \mu)(\beta_m + \mu)}\right] \\ &+ \alpha_{nm}\left[\frac{b_m}{a^{m+1}} \frac{\gamma_{m+1}}{(\beta_m + \mu)(\beta_{m+1} + \mu)} - \frac{b_{m-1}}{a^{n-1}} \frac{\gamma_n}{(\beta_{n-1} + \mu)(\beta_n + \mu)}\right] \end{aligned}$$

$$= \alpha_{nm}.k_{nm} + \alpha_{nm}k'_{nm} \qquad \forall m \geq n.$$

We easily verify that

$$|k'_{nm}| = \left|\frac{b_m}{a^{m+1}} \frac{\gamma_{m+1}}{(\beta_m + \mu)(\beta_{m+1} + \mu)} - \frac{b_{m-1}}{a^{n-1}} \frac{\gamma_n}{(\beta_{n-1} + \mu)(\beta_n + \mu)}\right| \tag{19}$$
$$\leq \frac{M_A.M_B}{|\mu|^2.\sin^2\varepsilon_1}.$$

So the commutator C_1 in (18) may be written as:

$$C_1 = C_{11} + C_{12} + C_{13},$$

with respectively

$$C_{11} = \begin{pmatrix} 0 & & & O \\ c_{21} & 0 & & \\ & c_{32} & . & \\ O & & . & . \end{pmatrix}; \; C_{12} = (\alpha_{nm}.k_{nm}) \; ; \; C_{13} = (\alpha_{nm}.k'_{nm}) \; \forall m \geq n.$$

Using a direct calculation and the fact that in infinite matrix theory, the property $({}^tM)^{-1} = {}^t(M^{-1})$ holds for any invertible matrix M, the matrix $A(A - \lambda I)^{-1}$ can be written as a sum of a diagonal matrix and an upper triangular matrix

$$A(A - \lambda I)^{-1} = D_1 + T_1,$$

where

$$D_1 = \left(\frac{a^n}{a^n - \lambda}\,\delta_{nm}\right), \; T_1 = (t_{nm}),$$

and

$$t_{nm} = \lambda \frac{(-1)^{m-n}(b_n b_{n+1} \ldots b_{m-1})}{(a^n - \lambda)\,(a^{n+1} - \lambda)\,\ldots\,(a^{m-1} - \lambda)\,(a^m - \lambda)} \quad \forall m \geq n + 1.$$

Now we have in algebra S_1:

$$\|(D_1 + T_1)(C_{11} + C_{12} + C_{13})\|_{S_1} \leq \|D_1C_{11}\| + \|D_1C_{12}\| + \|D_1C_{13}\| + \|T_1C_{11}\| + \|T_1C_{12}\| + \|T_1C_{13}\|$$

It is easily seen, from hypotheses (12) and (17) that there exists a constant K_0 such that

$$\forall m \geq n \Rightarrow |b_n b_{n+1} \dots b_m| \leq K_0, \tag{20}$$

then for any $\lambda \notin \Pi_{\varepsilon_A}$ and any $\mu \notin \Pi_{\varepsilon_B}$, we have

$$\|D_1 C_{11}\|_{S_1} = \sup_{n\geq 2} \left| \frac{a^n}{a^n - \lambda} c_{n,n-1} \right| = \sup_{n\geq 2} \left| \frac{a^n}{a^n - \lambda} \frac{\gamma_n}{(\beta_{n-1} + \mu)(\beta_n + \mu)} \frac{(a-1)}{a^{n-1}} \right|$$
$$\leq \frac{K_1(\varepsilon_0, \varepsilon_1)}{|\lambda|\ |\mu|^2},$$

$$\|D_1 C_{12}\|_{S_1} = \sup_{n\geq 2} \sum_{m=n+1}^{\infty} \left| \frac{(b_n b_{n+1} \dots b_{m-1})}{a^n a^{n+1} \dots a^{m-1}} \frac{1}{a^m} \frac{a^n}{a^n - \lambda} \left[\frac{\beta_m - \beta_n}{(\beta_n + \mu)(\beta_m + \mu)} \right] \right|$$
$$\leq \frac{K_0.K_2(\varepsilon_0, \varepsilon_1)}{|\lambda|\ |\mu|^2} \sup_{n\geq 2} \sum_{m=n+1}^{\infty} \frac{|b_{m-1}.m!|}{a^{n+1} \dots a^{m-1} a^m},$$

and from (17), there exists some constant K' such that

$$\|D_1 C_{12}\|_{S_1} \leq K' \frac{K_0.K_2(\varepsilon_0, \varepsilon_1)}{|\lambda|\ |\mu|^2} \sup_{n\geq 2} \sum_{m=n+1}^{\infty} \frac{1}{a^{n+1} \dots a^{m-1}}$$
$$\leq K' \frac{K_0.K_2(\varepsilon_0, \varepsilon_1)}{|\lambda|\ |\mu|^2} \sup_{n\geq 2} \sum_{m=n+1}^{\infty} \frac{1}{a^{n+m}}$$
$$\leq K' \frac{K_0.K_2(\varepsilon_0, \varepsilon_1)}{|\lambda|\ |\mu|^2} \frac{a}{a-1}.$$

On the other hand, it follows from (19)

$$\|D_1 C_{13}\|_{S_1} = \sup_{n\geq 2} \sum_{m=n+1}^{\infty} \left| \alpha_{nm}.k'_{nm} \frac{a^n}{(a^n - \lambda)} \right|$$
$$\leq \frac{K_0 M_A M_B}{|\mu|^2 .\sin^2 \varepsilon_1} \sup_{n\geq 2} \left[\left| \frac{b_n}{(a^n - \lambda)} \right| + \sum_{m=n+2}^{\infty} \left| \frac{b_n b_{n+1} \dots b_{m-1}}{a^{n+1} \dots a^{m-1}} \frac{1}{(a^n - \lambda)} \right| \right]$$
$$\leq \frac{K_3(\varepsilon_0, \varepsilon_1)}{|\lambda|\ |\mu|^2} \left(\sup_{n\geq 2} \sum_{m=n+2}^{\infty} \left| \frac{1}{a^{n+1} \dots a^{m-1}} \right| + K'' \right)$$
$$\leq \frac{K_3(\varepsilon_0, \varepsilon_1)}{|\lambda|\ |\mu|^2} \left(\frac{a}{a-1} + K'' \right).$$

Finally

$$
\begin{aligned}
\|T_1C_{11}\|_{S_1} &\le \|T_1\|_{S_1}\ \|C_{11}\|_{S_1} \\
&= \sup_{n\ge2} |c_{n,n-1}| \sup_{n\ge1} \sum_{m=n+1}^{\infty} \left| \lambda \frac{(b_n b_{n+1} \dots b_{m-1})}{(a^n-\lambda)(a^{n+1}-\lambda)\dots(a^{m-1}-\lambda)} \frac{1}{(a^m-\lambda)} \right| \\
&\le \sup_{n\ge2} |c_{n,n-1}| \sup_{n\ge1} \left(\left| \frac{\lambda b_n}{(a^n-\lambda)(a^{n+1}-\lambda)} \right| \right) \\
&+ \sup_{n\ge2} |c_{n,n-1}| \sup_{n\ge1} \sum_{m=n+2}^{\infty} |\lambda| \frac{|b_n b_{n+1}|\ |b_{n+2} \dots b_{m-1}|}{|(a^n-\lambda)(a^{n+1}-\lambda)|\ |(a^{n+2}-\lambda)\dots(a^m-\lambda)|} \\
&\le \frac{|\lambda|\ M_A\ M_B}{|\lambda|^2\ |\mu|^2\ \sin^2\varepsilon_1\ \sin^2\varepsilon_0} + \frac{|\lambda|\ M_B\ K_0}{|\lambda|^2\ |\mu|^2\ \sin^2\varepsilon_1} \sup_{n\ge1} \sum_{m=n+2}^{\infty} \frac{1}{\left(a^{\frac{m+n+2}{2}} \sin\varepsilon_0\right)^{m-n-1}} \\
&\le \frac{K_4\ (\varepsilon_A,\varepsilon_B)}{|\lambda|\ |\mu|^2}\ .
\end{aligned}
$$

To bound the next term we shall need the following formula

$$
\|T_1C_{12}\|_{S_1} \le \sup_{n\ge1} \left(\sum_{m=n+2}^{\infty} \left(\sum_{j=n+1}^{m-1} \left| \frac{\lambda b_n \dots b_{j-1}}{(a^n-\lambda)\dots(a^j-\lambda)} \frac{b_j \dots b_{m-1}}{a^j \dots a^m} \frac{(\beta_j-\beta_m)}{(\beta_m+\mu)(\beta_m+\mu)} \right| \right) \right) ;
$$

we use also the fact that $(\beta_j - \beta_m) \le m!$ and (17). Therefore it follows

$$
\begin{aligned}
\|T_1C_{11}\|_{S_1} &\le \frac{K_0 C}{|\lambda|\ |\mu|^2\ \sin^2\varepsilon_B} \sup_{n\ge1} \sum_{m=n+1}^{\infty} \sum_{j=n+1}^{m-1} \frac{1}{\left(a^{\frac{m+n+2}{2}} \sin\varepsilon_0\right)^{j-n-1}} \\
&\le \frac{K_5\ (\varepsilon_0,\varepsilon_1)}{|\lambda|\ |\mu|^2}\ .
\end{aligned}
$$

The last term is easily bounded as

$$
\|T_1C_{13}\|_{S_1} \le \|T_1C_{13}\|_{S_1}\ \|C_{13}\|_{S_1} \le \frac{K_6\ (\varepsilon_0,\varepsilon_1)}{|\lambda|\ |\mu|^2}\ .
$$

Now if we group all these estimates, we deduce

$$
\begin{aligned}
&\left\|\mu A(A-\lambda I)^{-1}\ [A^{-1};(B+\mu I)^{-1}]\right\| \\
&= \left\|\mu A(A-\lambda I)^{-1}\ C_0\right\| \\
&\le \left\|\mu A(A-\lambda I)^{-1}\ (C_1+R)\right\| \\
&\le \left\|\mu A(A-\lambda I)^{-1}\ C_1\right\| + \left\|\mu A(A-\lambda I)^{-1}\ R\right\| \\
&\le \left\|\mu(D_1+T_1)\ (C_{11}+C_{12}+C_{13})\right\| + \left\|\mu A(A-\lambda I)^{-1}\ R\right\| \\
&\le K(\varepsilon_0,\varepsilon_1) \left[\frac{1}{|\lambda|\ |\mu|} + \frac{1}{|\mu|^2} \right] .
\end{aligned}
$$

We have used the fact that $\|A(A-\lambda I)^{-1} R\| = O(\|R\|) = O\left(\frac{1}{|\mu|^3}\right)$ when $|\lambda|$, $|\mu|$ tend to infinity, which can be deduced from (18). The proposition 4 is proved and the numer δ defined in (9) is equal to 1.

5. REAL INTERPOLATION SPACES

We shall apply theorem 2 given in section 3, so we must explicite the interpolation Banach space $D_A(\theta,\infty)$ between $D_A = s_{1/\alpha}$ and $E = s_1$. We shall use property (10).

Proposition 5. For any $\theta \in \,]0,1[$ the space $D_A(\theta,\infty)$ coincides with s_{1/α^θ}.

Proof. We shall prove only that $s_{1/\alpha^\theta} \subset D_A(\theta,\infty)$; for the other embedding, the same techniques can be used. Given $X = (x_n) \in s_{1/\alpha^\theta}$, then

$$\forall m \geq 1 \qquad |x_m| \leq \|X\|_{s_{(1/a)^\theta}} \cdot \left(\frac{1}{a^\theta}\right)^m . \tag{21}$$

Since A is invertible, we must only estimate the semi-norm $\sup_{t>0}\|t^\theta A(A+tI)^{-1}X\|_E$. Let t_0 be such that

$$\forall t \geq t_0 \Rightarrow \frac{M_A}{t} \leq \frac{1}{2} \tag{22}$$

then it sufficies to estimate $\sup_{t\geq t_0}\|t^\theta A(A+tI)^{-1}X\|_E$. Let us write that $A(A+tI)^{-1} = I - t(A + tI)^{-1} = (r^t_{nm})_{n,m=1,\dots}$ where

$$r^t_{nm} = \begin{cases} 0 & \text{if } m < n \\ \dfrac{a^n}{a^n + t} & \text{if } m = n \\ t\,\dfrac{(-1)^{m-n-1} b_n b_{n+1} \dots b_{m-1}}{(a^n + t)(a^{n+1} + t)\dots(a^m + t)} & \text{if } m > n \end{cases}$$

Then from $t^\theta A(A + tI)^{-1}X = Y = (y_n)_{n\geq 1}$, we get

$$y_n = t^\theta \sum_{m\geq n}^{\infty} r^t_{nm} x_m .$$

(The convergence is justified by (16) for example). Recalling (21) and (22) it yields

$$y_n \leq t^\theta \sum_{m\geq n}^{\infty} |r_{nm}^t x_m|$$

$$\leq \frac{t^\theta a^n}{a^n + t} |x_n| + \sum_{m\geq n+1}^{\infty} \frac{t.|b_n b_{n+1} \dots b_{m-1}|.t^\theta |x_m|}{(a^n + t)(a^{n+1} + t)\dots(a^m + t)}$$

$$\leq \frac{t^\theta a^n}{a^n + t} \frac{\|X\|_{s_{(1/a)^\theta}}}{a^{n\theta}} + \sum_{m\geq n+1}^{\infty} \frac{t.|b_n b_{n+1} \dots b_{m-1}|.t^\theta \|X\|_{s_{(1/a)^\theta}}}{(a^n + t)(a^{n+1} + t)\dots(a^m + t)a^{m\theta}}$$

then, from

$$\sup_{t\geq 0} \frac{t^\theta a^n}{a^n + t} = (1 - \theta)^{1-\theta} \theta^\theta a^{n\theta}$$

we have

$$|y_n| \leq K_1 \|X\|_{s_{(1/a)^\theta}} + K_2 \sum_{m\geq n+1}^{\infty} \frac{(M_A)^{m-n}}{t^{m-n-1} a^m} \|X\|_{s_{(1/a)^\theta}}$$

$$\leq K_1 \|X\|_{s_{(1/a)^\theta}} + (M_A)^{-1} K_2 \sum_{j=0}^{\infty} \frac{1}{(2a)^j} \|X\|_{s_{(1/a)^\theta}}.$$

Applying theorem 2, one has

Theorem 6. Let A and B be the two infinite matrices defined by (1), (13) such that (12), (14) and (17) hold. Then there exists λ^* such that $\forall\lambda \geq \lambda^*$ and $\forall Y \in s_{1/\alpha^\theta}$ the linear infinite system $[-A - B - \lambda I].X = Y$ has a unique solution $X \in s_{1/\alpha} \cap s_{1/\beta}$ such that:

i) $(A + \lambda I)X \in s_{1/\alpha^\theta}$,

ii) $BX \in s_{1/\alpha^\theta}$.

In our case it is easily seen that $s_{1/\alpha} \cap s_{1/\beta}$ coincides with s_c, where the sequence (c_n) is defined by

$$c_n = \begin{cases} \dfrac{1}{a^{2k}} & \text{if } n = 2k \\[2ex] \dfrac{1}{(2k + 1)!} & \text{otherwise.} \end{cases} \tag{23}$$

6. EXAMPLES AND REMARKS

6.1. In continued fractions theory. Let us define the following particular sequences

$$\omega_{n-1} = b_{n-1} = \gamma_n = \frac{c_n a^n}{n!}, \quad n \geq 2$$

$$\kappa_n = \begin{cases} a^{2k} + 1 & \text{if } n = 2k \\ a^{2k+1} + (2k+1)! & \text{if } n = 2k+1, \end{cases}$$

where the sequence (c_n) is assumed to be bounded. Consider now the following infinite system:

$$\begin{cases} (\kappa_1 + \lambda)x_1 - \omega_1 x_2 = y_1 \\ -\omega_1 x_1 + (\kappa_2 + \lambda)x_2 - \omega_2 x_3 = y_2 \\ \dots\dots\dots = \dots \\ -\omega_{m-1}x_{m-1} + (\kappa_m + \lambda)x_m - \omega_m x_{m+1} = y_m \\ \dots\dots\dots = \dots \end{cases}$$

For the particular second member $(1, 0, 0, \dots) \in s_{1/\alpha^\theta}$, this system gives formally the following continued fraction

$$x_1 = \cfrac{1}{\kappa_1 + \lambda - \cfrac{\omega_1^2}{\kappa_2 + \lambda - \cfrac{\omega_2^2}{\kappa_2 + \lambda - \dots}}}$$

and in virtue of theorem 6, we see that this fraction converges, since, by using the right inverse of $A + B + \lambda I$, the first element x_1 of the solution $X = (x_1, x_2, \dots)$ coincides exactly with the first component of

$$(A + B + \lambda I)^{-1}(1, 0, 0, \dots),$$

with the following regularity:

$$\forall\ \theta \in\ ]0, 1[\ \sup_{n \geq 1} \left(a^{n\theta} \left|a^n x_n + b^n x_{n+1}\right|\right) < \infty .$$

6.2. In numerical schemes. Our approach can be applied in studying differential equations that are approximated by finite difference schemes of type

$$(*) \begin{cases} a_{n-1}u_{n-1} + (b_n + \lambda)u + c_{n+1}u_{n+1} = f_n \\ n = 0, \pm 1, \pm 2, \pm 3, \dots \end{cases}$$

where (f_n) is given in space l^∞. The hypotheses concerning sequences (a_n), (b_n), (c_n) mean that (*) is of elliptic type. Theorem 6 gives the resolution of (*) in the adapted space l^∞, see [5].

6.3. Development of some analytic function on a some given basis. For $\theta \in]0,1[$, $\alpha > 1$ and $\rho \in]0,1[$ consider the analytic function

$$f(z) = \frac{a^\theta}{a^\theta - z} = \sum_{n=0}^{\infty} \frac{1}{a^{n\theta}} z^n \quad \text{for } |z| \le \rho$$

then thanks to theorem 6, there exists a unique sequence $X = (x_n)_{n\ge0}$ belonging to the space s_c (defined in (23)) such that

$$f(z) = \frac{a^\theta}{a^\theta - z} \tag{24}$$

$$= \sum_{n=0}^{\infty} \frac{1}{a^{n\theta}} z^n = x_0 + \sum_{n=1}^{\infty} x_n\left[b_{n-1}z^{n-1} + (a^n + \beta_n + \lambda)z^n + \gamma_{n+1}z^{n+1}\right]$$

for $|z| \le \rho$ and a large $\lambda > 0$. Applying the result of regularity, we have also:

$$\sup_{n\ge0}(a^{n\theta}|a^n x_n + b_n x_{n+1}|) < \infty .$$

In fact, we can see that equation (24) is equivalent to (1) with $Y = {}^t(1, 1/\alpha^\theta, 1/\alpha^{2\theta}, ...)$ and $b_0 = 0$; $(b_n)_{n\ge1}$, $(\beta_n)_{n\ge1}$, $(\gamma_n)_{n\ge2}$ are three sequences verifying (12), (14) and (17). This is justified by the absolute convergence of double series $\sum_{m\ge1}\sum_{n\ge1} |a_{nm}(\lambda)|\,|x_m|\,|z|^n$ when $|z| \le \rho$ and $(x_n) \in s_c$ (see(23)), here $(a_{nm}(\lambda))_{nm} = -A - B - \lambda I$ and A, B are defined as in (11), (13).

Remark. When E is a Hilbert space, M. Fuhrman [3] has proved, under (H.1) and (H.2), that for large λ and for any Y in E the unique solution $X \in D(A) \cap D(B)$ of (1), exists and satisfies

$$\|A.X\|_E + \|B.X\|_E \le K(\lambda)\|Y\|_E,$$

this estimate is important for studying stability and convergence of numerical schemes of type (*). This approach, in Hilbert case, gives also the resolution of infinite linear systems when the second member is not regular but only in E.

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