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SOME RESULTS ON THE FUZZY SHEAF OF THE FUNDAMENTAL GROUPS OVER FUZZY TOPOLOGICAL SPACES

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ABSTRACT. Let X be a fuzzy path connected topological space and (H, ψ) be the fuzzy sheaf of fundamental groups over X. Constructing the group of fuzzy sections, it is shown that there is a covariant functor from the category of fuzzy path connected topological spaces and fuzzy continuous mappings to the category of groups of fuzzy sections and homomorphisms. Furthermore, defining the direct sum of the fuzzy sheaves, it is proved that the mapping $P_i = (p_i, p_i^*) : (X_1 \times X_2, H_1 \times H_2) \to (X_i, H_i)$ is a homomorphism for $i = 1, 2$.

1. INTRODUCTION

The concept of a fuzzy set was discovered by Zadeh[5] and one of its earliest branches, the theory of fuzzy topology, was developed by Chang [1] and others. Recently, Zheng [6] introduced the concept of fuzzy path. Using this concept, Salleh and Md Tap [3] constructed the fundamental group of a fuzzy topological space.

Let X be a set and I the unit interval $[0,1]$. A fuzzy set X is characterized by a membership function μ_A which associates with each point $x \in X$ its"grade of membership"$\mu_A(x) \in I$.

Definition 1.1. A fuzzy point in X is *a* fuzzy set with membership function μ_{a_λ} defined by

$$\mu_{a_\lambda}(x) = \begin{cases} \lambda, \ x = a \\ 0, \text{ otherwise} \end{cases}$$

for all $x \in X$. Where $0 < \lambda \leq 1$.

We denoted by k_λ the fuzzy set in X with the constant membership function $\mu_{k_\lambda}(x) = \lambda$ for all $x \in X$.

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Definition 1.2. A fuzzy topology on a set X is a family τ of fuzzy sets in X which satisfies the following conditions:

(i) $k_0, k_1 \in \tau$
(ii) If $A, B \in \tau$, then $A \cap B \in \tau$
(iii) If $A_j \in \tau$ for all $j \in J$, then $\bigcup_{j \in J} A_j \in \tau$.

The pair (X, τ) is called a fuzzy topological space. Every member of τ be called an open fuzzy set. The complement of an open fuzzy set is called a closed fuzzy set.

Definition 1.3. Let (X, τ_1), (Y, τ_2) be two fuzzy topological spaces.A mapping f of (X, τ_1) into (Y, τ_2) is fuzzy continuous iff for each open fuzzy set V in τ_2 the inverse image $f^{-1}(V)$ is in τ_1. On the other hand, f is fuzzy open iff for each open fuzzy set U in τ_1, the image $f(U)$ is in τ_2.

Definition 1.4. A bijective mapping f of fuzzy topological space (X, τ_1) into a fuzzy topological space (Y, τ_2) is called a fuzzy topological mapping if it is fuzzy continuous and fuzzy open.

Definition 1.5. Let τ be a fuzzy topology on a set X. A subfamily $\mathcal{B}$ of τ is called a base for τ if each member of τ can be expressed as the union of members of $\mathcal{B}$ [4].

Definition 1.6. Let (X, τ) be a fuzzy topological space. If $\alpha : (I, \widetilde{\varepsilon}_I) \to (X, \tau)$ is a fuzzy continuous function and a fuzzy set A is connected in $(I, \widetilde{\varepsilon}_I)$ with $A(0) > 0$ and $A(1) > 0$, then the fuzzy set $\alpha(A)$ in (X, τ) is called a fuzzy path in (X, τ).

The fuzzy point $(\alpha(0))_{A(0)} = \alpha(0_{A(0)})$ and $(\alpha(1))_{A(1)} = \alpha(1_{A(1)})$ are called the initial point and the terminal point of the fuzzy path $\alpha(A)$, respectively [2].

Definition 1.7. Let F be a fuzzy set in a fuzzy topological space (X, τ). If for any two fuzzy points a_λ and b_μ in F, there is a fuzzy path from a_λ to b_μ contained in F, then F is said to be fuzzy path connected in (X, τ).

If $F = X$ in the above definition, we call (X, τ) a fuzzy path connected space [3].

Let X be a fuzzy path connected topological space and H_{a_λ} be the fundamental group of X based for any $a_\lambda \in X$, that is $H_{a_\lambda} = \pi_1(X, a_\lambda)$ [4]. Let $X = (X, x_p)$ be a pointed fuzzy topological space for an arbitrary fixed fuzzy point $x_p \in X$. Let H denotes the disjoint union of all fundamental groups obtained for each $a_\lambda \in X$, by H, i.e., $H = \bigvee_{a_\lambda \in X} H_{a_\lambda}$. H is a set over X and the mapping $\psi : H \to X$ defined by

$$\psi(\sigma_{a_\lambda}) = \psi([\alpha(A)]_{a_\lambda}) = a_\lambda$$

for any $\sigma_{a_\lambda} = [\alpha(A)]_{a_\lambda} \in H_{a_\lambda} \subset H$ is onto.

Now, let $W \subset X$ be an open fuzzy set. Define a mapping $s : W \to H$ such that

$$s(a_\lambda) = [\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda}$$

for each $a_\lambda \in W$, where $[\alpha(A)]_{x_p} \in H_{x_p}$ is any element and $[\gamma(G)]$ is an arbitrary fixed fuzzy homotopy class which defines an isomorphism between H_{a_λ} and H_{x_p}. Then the change of s depends on only the change of $\sigma_{x_p} = [\alpha(A)]_{x_p}$. Furthermore, $\psi \circ s = 1_W$. Let us denote the totality of the mappings s defined on W by $\Gamma(W, H)$.

If B is a fuzzy base for X, then $B^* = \{s(W) : W \in B,\ s \in \Gamma(W, H)\}$ is a fuzzy base for H. The mappings ψ and s are fuzzy continuous in this topology. Moreover ψ is a locally fuzzy topological mapping. Then (H, ψ) is a fuzzy sheaf over X. (H, ψ) or only H is called "The fuzzy sheaf of the fundamental groups" over X.

The fundamental group $\pi_1(X, a_\lambda) = \psi^{-1}(a_\lambda)$ is called the stalk of the fuzzy sheaf (H, ψ) over X and denoted by H_{a_λ} for every $a_\lambda \in X$. For any open fuzzy set $W \subset X$, an element s of $\Gamma(W, H)$ is called a fuzzy section of the fuzzy sheaf H over W.

2. SOME RESULTS ON THE FUZZY SHEAF OF THE FUNDAMENTAL GROUPS OVER FUZZY TOPOLOGICAL SPACES

In this section,constructing the group of fuzzy sections, it is shown that there is a covariant functor from the category of fuzzy path connected topological spaces and fuzzy continuous mappings to the category of groups of fuzzy sections and homomorphisms.

We begin by giving the following theorem.

Theorem 2.1. *The set $\Gamma(W, H)$ is a group with the pointwise operation of multiplication.*

Proof. If $s_1, s_2 \in \Gamma(W, H)$ are obtained by the elements $[\alpha_1(A_1)], [\alpha_2(A_2)] \in \pi_1(X, x_p)$, respectively, then $s_1.s_2$ is obtained by the element $[\alpha_1(A_1).\alpha_2(A_2)] \in \pi_1(X, x_p)$. Then,

$$\begin{aligned} s_1(a_\lambda).s_2(a_\lambda) &= [\gamma^{-1}(H) * \alpha_1(A_1) * \gamma(G)]_{a_\lambda}.[\gamma^{-1}(H) * \alpha_2(A_2) * \gamma(G)]_{a_\lambda} \\ &= [\gamma^{-1}(H) * (\alpha_1(A_1) * \alpha_2(A_2)) * \gamma(G)]_{a_\lambda}. \end{aligned}$$

It follows from this definition that the operation of multiplication is well-defined and closed. Clearly, the operation of multiplication is associative.

The mapping $I : W \longrightarrow H$ is the identity element which is obtained by the identity element of $\pi_1(X, x_p)$. The mapping I is defined by $I(a_\lambda) = [\gamma^{-1}(H) * \delta(E) * \gamma(G)]_{a_\lambda}$ for every $a_\lambda \in W$ and $\delta(E)$ is a fuzzy zero path at $x_p \in W$. Then, $I(a_\lambda) = [\gamma^{-1}(H) * \gamma(G)]_{a_\lambda} = [1(Z)]_{a_\lambda}$. Therefore,

$$\begin{aligned} (I.s)(a_\lambda) &= I(a_\lambda).s(a_\lambda) \\ &= [1(Z)]_{a_\lambda}[\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda} \\ &= [1(Z) * (\gamma^{-1}(H) * \alpha(A) * \gamma(G))]_{a_\lambda} \\ &= [\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda} \\ &= s(a_\lambda). \end{aligned}$$

Similarly,

$$(s.I)(a_\lambda) = s(a_\lambda).$$

Furthermore, if $s \in \Gamma(W,H)$ is obtained by the element $[\alpha(A)] \in \pi_1(X, x_p)$, then the mapping $s^{-1} \in \Gamma(W,H)$, which is obtained by the element $[\alpha(A)]^{-1}$, is the inverse element for $s \in \Gamma(W,H)$.The mapping $s^{-1} : W \to H$ is defined by $s^{-1}(a_\lambda) = [\gamma^{-1}(H) * \beta(B) * \gamma(G)]_{a_\lambda}$ for every $a_\lambda \in W$ such that $\beta(B) = (\alpha(A))^{-1}$. Therefore,

$$\begin{aligned}
(s.s^{-1})(a_\lambda) &= s(a_\lambda).s^{-1}(a_\lambda) \\
&= [\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda}.[\gamma^{-1}(H) * \beta(B) * \gamma(G)]_{a_\lambda} \\
&= [(\gamma^{-1}(H) * \alpha(A) * \gamma(G)) * (\gamma^{-1}(H) * \beta(B) * \gamma(G))]_{a_\lambda} \\
&= [\gamma^{-1}(H) * (\alpha(A) * \beta(B)) * \gamma(G)]_{a_\lambda} \\
&= [\gamma^{-1}(H) * \gamma(G)]_{a_\lambda} \\
&= [1(Z)]_{a_\lambda} \\
&= I(a_\lambda).
\end{aligned}$$

Similarly,

$$(s^{-1}.s)(a_\lambda) = I(a_\lambda).$$

□

Thus, the set of fuzzy sections $\Gamma(W,H)$ is a group with the pointwise operation of multiplication.

Definition 2.2. Let $f^* : H_1 \to H_2$ be a mapping. If f^* is a fuzzy continuous, a homomorphism on each stalk of fuzzy sheaf H_1 and maps every stalk of fuzzy sheaf H_1 into a stalk of fuzzy sheaf H_2, then it is called a fuzzy sheaf homomorphism.

Let $f : (X_1, x_p) \to (X_2, f(x_p) = y_q)$ be a fuzzy continuous mapping. We know that the mapping $f^* : H_1 \to H_2$ is a fuzzy sheaf homomorphism. Also, each element $\sigma_{x_p} = [\alpha(A)]_{x_p} \in (H_1)_{x_p}$ defines a unique fuzzy section s^1 over X_1 such that

$$s^1(a_\lambda) = [\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda}$$

for any $a_\lambda \in X_1$. However,

$$f^*([\alpha(A)]_{x_p}) = [(f \circ \alpha)(A)]_{f(x_p)} \in (H_2)_{f(x_p)}$$

and $[(f \circ \alpha)(A)]_{f(x_p)}$ define a section s^2 over X_2 such that

$$s^2(b_\mu) = [\delta^{-1}(F) * (f \circ \alpha)(A) * \delta(E)]_{b_\mu}$$

for any $b_\mu \in X_2$. Then the correspondence

$$[\alpha(A)]_{x_p} \longleftrightarrow [(f \circ \alpha)(A)]_{y_q}$$

between $(H_1)_{x_p}$ and $(H_2)_{y_q}$ gives the correspondence $s_1 \longleftrightarrow s_2$ between $\Gamma(X_1, H_1)$ and $\Gamma(X_2, H_2)$. If we denote this correspondence as

$$\begin{aligned} f_*(s^1(a_\lambda)) &= f^*([\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda}) \\ &= [\delta^{-1}(F) * (f \circ \alpha)(A) * \delta(E)]_{b_\mu} \\ &= s^2(b_\mu), \end{aligned}$$

then the mapping $f_* : \Gamma(X_1, H_1) \to \Gamma(X_2, H_2)$ is a homomorphism. In fact, for any two sections $s_1^1, s_2^1 \in \Gamma(X_1, H_1)$ and any point $b_\mu \in X_2$,

$$\begin{aligned} (f_*(s_1^1))(b_\mu) &= f^*([\gamma^{-1}(H) * \alpha_1(A_1) * \gamma(G)]_{a_\lambda}) \\ &= [\delta^{-1}(F) * (f \circ \alpha_1)(A_1) * \delta(E)]_{b_\mu} \end{aligned}$$

and

$$\begin{aligned} (f_*(s_2^1))(b_\mu) &= f^*([\gamma^{-1}(H) * \alpha_2(A_2) * \gamma(G)]_{a_\lambda}) \\ &= [\delta^{-1}(F) * (f \circ \alpha_2)(A_2) * \delta(E)]_{b_\mu}. \end{aligned}$$

Therefore,

$$\begin{aligned} (f_*(s_1^1).f_*(s_2^1))(b_\mu) &= [\delta^{-1}(F) * ((f \circ \alpha_1)(A_1) * (f \circ \alpha_2)(A_2)) * \delta(E)]_{b_\mu} \\ &= [\delta^{-1}(F) * (f \circ \alpha_1(A_1)\alpha_2(A_2)) * \delta(E)]_{b_\mu} \\ &= (f_*(s_1^1.s_2^1))(b_\mu). \end{aligned}$$

Hence we have the following theorem.

Theorem 2.3. *Let $f : X_1 \to X_2$ be a fuzzy continuous mapping. Then there exists a homomorphism $f_* : \Gamma(X_1, H_1) \to \Gamma(X_2, H_2)$.*

Theorem 2.4. *Let $f_1 : X_1 \to X_2$, $f_2 : X_2 \to X_3$ be fuzzy continuous mappings. Then, there exists a homomorphism*

$$f_* = (f_2 \circ f_1)_* : \Gamma(X_1, H_1) \to \Gamma(X_3, H_3)$$

such that $(f_2 \circ f_1)_* = f_{2*} \circ f_{1*}$.

Proof. Since the mappings f_1, f_2 are fuzzy continuous, the mapping $f = f_2 \circ f_1$ is fuzzy continuous. Therefore, there exists a homomorphism

$$f_* : \Gamma(X_1, H_1) \to \Gamma(X_3, H_3)$$

by Theorem 2.2. We show that $f_* = (f_2 \circ f_1)_* = f_{2*} \circ f_{1*}$.

In fact,

$$\begin{aligned} f_*(s^1(a_\lambda)) &= (f_2 \circ f_1)_*(s^1(a_\lambda)) \\ &= (f_2 \circ f_1)_*([\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda}) \\ &= [\gamma^{-1}(H) * (f_2 \circ f_1 \circ \alpha(A)) * \gamma(G)]_{a_\lambda} \\ &= f_{2*}[\gamma^{-1}(H) * (f_1 \circ \alpha(A)) * \gamma(G)]_{a_\lambda} \\ &= (f_{2*} \circ f_{1*})[\gamma^{-1}(H) * \alpha(A) * \gamma(G)]_{a_\lambda} \\ &= (f_{2*} \circ f_{1*})(s^1(a_\lambda)) \end{aligned}$$

for any $s^1 \in \Gamma(X_1, H_1)$ and $a_\lambda \in X$. □

Let C be the category of fuzzy path connected topological spaces and fuzzy continuous mappings and D be the category of fuzzy section groups and homomorphisms. Let us define a mapping $F : C \to D$ with $F(X) = \Gamma(X, H)$ and $F(f) = f_*$ for any element $X \in C$ and morphism $f : X_1 \to X_2$. Then F is a covariant functor. In fact, the followings are satisfied.

1. If $f = 1_X$, then $F(1_X) = (1_X)_*$ and $(1_X)_*(s) = s$ for any $s \in \Gamma(X, H)$. Thus $F(1_X) = 1_{F(X)}$.
2. Let $f_1 : X_1 \to X_2, f_2 : X_2 \to X_3$ any morphisms. Then $f_2f_1 = f_2 \circ f_1 : X_1 \to X_3$ is a morphism and
$$F(f_2f_1) = (f_2f_1)_* : \Gamma(X_1, H_1) \to \Gamma(X_3, H_3).$$
Moreover,
$$(f_2f_1)_*(s^1) = f_{2*}(f_{1*}(s^1)) = (f_{2*}f_{1*})(s^1).$$
Hence,
$$F(f_2f_1) = F(f_2).F(f_1).$$

We then have the following theorem.

Theorem 2.5. *There is a covariant functor from the category of fuzzy path connected topological spaces and fuzzy continuous mappings to the category of groups of fuzzy sections and homomorphisms.*

Now, let $f : X_1 \to X_2$ be a fuzzy topological mapping. Then there is the mapping $f^{-1} : X_2 \to X_1$ such that $ff^{-1} = 1_{X_2}, f^{-1}f = 1_{X_1}$. From Theorem 2.3,

$$\begin{aligned}(ff^{-1})_* &= f_*(f^{-1})_* = 1_{F(X_2)} \\ (f^{-1}f)_* &= (f^{-1})_*f_* = 1_{F(X_1)}.\end{aligned}$$

Hence $(f^{-1})_* = (f_*)^{-1}$. Therefore, f_* is an isomorphism.

So, we can give the following corollary.

Corollary 1. *Let $f : X_1 \to X_2$ be a fuzzy topological mapping. Then, the corresponding groups $\Gamma(X_1, H_1)$ and $\Gamma(X_2, H_2)$ are isomorphic.*

Now, let H_1 and H_2 be fuzzy sheaves of fundamental groups over X_1, X_2, respectively. Then,

$$H_1 = \bigvee_{a_{\lambda_1} \in X_1} (H_1)_{a_{\lambda_1}}, \quad H_2 = \bigvee_{a_{\lambda_2} \in X_2} (H_2)_{a_{\lambda_2}}$$

and so

$$H_1 \times H_2 = \bigvee_{(a_{\lambda_1}, a_{\lambda_2}) \in X_1 \times X_2} (H_1)_{a_{\lambda_1}} \times (H_2)_{a_{\lambda_2}}.$$

$H_1 \times H_2$ is a fuzzy set over the fuzzy topological space $X_1 \times X_2$. Moreover, $H_1 \times H_2$ is also a fuzzy topological space, since H_1, H_2 are fuzzy topological spaces.

Let us now define a mapping

$$\Phi : H_1 \times H_2 \rightarrow X_1 \times X_2$$

as follows:

If $((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}) \in H_1 \times H_2$, then

$$\Phi((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}) = (\psi_1((\sigma_1)_{a_{\lambda_1}}), \psi_2((\sigma_2)_{a_{\lambda_2}})) = (a_{\lambda_1}, a_{\lambda_2}) \in X_1 \times X_2.$$

We assert that $\Phi = (\psi_1, \psi_2)$ is a locally fuzzy topological mapping. In fact, since the mappings $\psi_1 : H_1 \rightarrow X_1$, $\psi_2 : H_2 \rightarrow X_2$ are locally fuzzy topological mappings, there exist open fuzzy neighborhoods $U_1((\sigma_1)_{a_{\lambda_1}}) \subset H_1$, $W_1(a_{\lambda_1}) \subset X_1$, $U_2((\sigma_2)_{a_{\lambda_2}}) \subset H_2$, $W_2(a_{\lambda_2}) \subset X_2$ such that

$$\psi_1|_{U_1} : U_1 \rightarrow W_1, \ \psi_2|_{U_2} : U_2 \rightarrow W_2$$

are fuzzy topological mappings. Therefore

$$U((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}) = U_1((\sigma_1)_{a_{\lambda_1}}) \times U_2((\sigma_2)_{a_{\lambda_2}})$$

and

$$W = W_1(a_{\lambda_1}) \times W_2(a_{\lambda_2})$$

are open fuzzy neighborhoods of the fuzzy points $((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}})$ and $(a_{\lambda_1}, a_{\lambda_2})$, respectively. Finally, it is clearly seen that $\Phi|_U : U \rightarrow W$ is a fuzzy topological mapping. Thus, $(H_1 \times H_2, \Phi)$ is a fuzzy sheaf over $X_1 \times X_2$. Moreover, $\Gamma(W, H_1 \times H_2)$ is a group, for any open fuzzy set $W \subset X_1 \times X_2$.

Definition 2.6. Let H_1 and H_2 be fuzzy sheaves of fundamental groups over X_1 and X_2, respectively. Then the fuzzy sheaf $H_1 \times H_2$ is called the direct sum of the fuzzy sheaves H_1 and H_2.

Let $H_1 \times H_2$ be denoted as the pair $(X_1 \times X_2, H_1 \times H_2)$.

Definition 2.7. Let the pairs (X_1, H_1) and (X_2, H_2) be given. It is said that there is a homomorphism between these pairs and it is written

$$F = (f, f^*) : (X_1, H_1) \rightarrow (X_2, H_2),$$

if there exists a pair $F = (f, f^*)$ such that

(1) $f : X_1 \rightarrow X_2$ is a fuzzy continuous mapping,
(2) $f^* : H_1 \rightarrow H_2$ is a fuzzy continuous mapping,
(3) f^* preserves the stalks with respect to f,
(4) For every $a_{\lambda_1} \in X_1$, $f^*|_{(H_1)_{a_{\lambda_1}}} : (H_1)_{a_{\lambda_1}} \rightarrow (H_2)_{f(a_{\lambda_1})}$ is a homomorphism.

Now, we can give the following theorem.

Theorem 2.8. *Let the pairs (X_1, H_1) and (X_2, H_2) be given. Then the mapping*

$$P_i : (p_i, p_i^*) : (X_1 \times X_2, H_1 \times H_2) \rightarrow (X_i, H_i)$$

is a homomorphism for $i = 1, 2$.

Proof. Let us first show that p_i^* is a stalk preserving mapping with respect to p_i. In fact,

$$\begin{aligned}(p_i \circ \Phi)((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}) &= p_i(\Phi((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}})) \\ &= p_i(a_{\lambda_1}, a_{\lambda_2}) \\ &= a_{\lambda_i}\end{aligned}$$

and

$$\begin{aligned}(\psi_i \circ p_i^*)((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}) &= \psi_i(p_i^*((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}})) \\ &= \psi_i((\sigma_i)_{a_{\lambda_i}}) \\ &= a_{\lambda_i}\end{aligned}$$

and so $p_i \circ \Phi = \psi_i \circ p_i^*$, for each $((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}) \in H_1 \times H_2$.

p_i^* is a homomorphism on each stalk. Indeed,

$$\begin{aligned}p_i^*(((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}).((\mu_1)_{b_{\lambda_1}}, (\mu_2)_{b_{\lambda_2}}))) &= p_i^*((\sigma_1)_{a_{\lambda_1}}.(\mu_1)_{b_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}.(\mu_2)_{b_{\lambda_2}}) \\ &= (\sigma_i)_{a_{\lambda_i}}.(\mu_i)_{b_{\lambda_i}}\end{aligned}$$

for $i = 1, 2$. Moreover,

$$(\sigma_i)_{a_{\lambda_i}}.(\mu_i)_{b_{\lambda_i}} = p_i^*((\sigma_1)_{a_{\lambda_1}}, (\sigma_2)_{a_{\lambda_2}}).p_i^*((\mu_1)_{b_{\lambda_1}}, (\mu_2)_{b_{\lambda_2}})$$

for $i = 1, 2$.

The mappings $p_i^* : H_1 \times H_2 \to H_i$, $p_i : X_1 \times X_2 \to X_i$ are fuzzy continuous, since they are fuzzy projections.

Thus P_i is a homomorphism between the pairs $(X_1 \times X_2, H_1 \times H_2)$ and (X_i, H_i), $i = 1, 2$.

ÖZET. X bir fuzzy eğrisel irtibatlı topolojik uzay ve (H, ψ), X üzerinde esas grupların fuzzy demeti olsun. Fuzzy kesitlerin grubu oluşturularak, fuzzy eğrisel irtibatlı topolojik uzaylar ve fuzzy sürekli fonksiyonlar kategorisinden fuzzy kesitlerin grubu ve homomorfizmler kategorisine bir kovaryant funktorun var olduğu gösterilmiştir. Ayrıca, fuzzy demetlerin direkt toplamı tarif edilerek, $i = 1, 2$ için $P_i = (p_i, p_i^*) : (X_1 \times X_2, H_1 \times H_2) \to (X_i, H_i)$ dönüşümünün bir homomorfizm olduğu ispatlanmıştır. □

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