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Investigation of Secondary School Students' Strategies for Solving Routine and Non-Routine Problems

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ABSTRACT

The aim of this study is to determine the situations of secondary school students in solving routine and non-routine problems and the strategies they use while solving these problems. The research is in the survey model, and the sample of the study consists of 430 students studying in the 6th, 7th, and 8th grades of two official secondary schools by homogeneous sampling which is one of the purposeful sampling methods. Routine and non-routine problem test and problem evaluation rubric were used as data collection tools in the study. As a result of the research, it was concluded that students were more successful in routine problems and these students mostly used arithmetic strategies in solving routine and non-routine problems. Moreover, it was revealed that students used the equation strategy in solving routine problems, and the guess and check strategy in solving non-routine problems. As a result of the study, it showed that non-routine problems should be included more in the classroom and textbooks in mathematics lessons. In addition, it was concluded that students never used drawing, make a list, logical reasoning strategies in the process of problem-solving, but never using the strategies of finding a pattern, elimination, and working backwards.

Keywords: Routine problem, non-routine problem, problem-solving strategies, mathematics, secondary school students

Ortaokul Öğrencilerinin Rutin ve Rutin Olmayan Problemleri Çözme Stratejilerinin İncelenmesi

ÖZ

Bu araştırmanın amacı, ortaokul öğrencilerinin rutin ve rutin olmayan problemleri çözme durumlarını ve bu problemleri çözerken kullandıkları stratejileri belirlemektir. Araştırma tarama modelindedir. Araştırmanın örneklemini, amaçlı örnekleme yöntemlerinden homojen örnekleme ile belirlenen 6., 7. ve 8. sınıflarında öğrenim görmekte olan toplam 430 öğrenci oluşturmaktadır. Araştırmada veri toplama aracı olarak rutin ve rutin olmayan problem testi ve problem değerlendirme rubriği kullanılmıştır. Araştırmada öğrencilerin rutin problemlerde daha başarılı oldukları ve öğrencilerin rutin ve rutin olmayan problemlerin çözümünde en çok aritmetiksel strateji kullandıkları sonucuna ulaşılmıştır. Ayrıca rutin problemlerin çözümünde öğrencilerin denklem kurma stratejisini kullanırken rutin olmayan problemlerin çözümünde ise tahmin kontrol stratejisini kullandıkları ortaya çıkmıştır. Çalışmanın sonucunda matematik derslerinde rutin olmayan problemlere sınıf içerisinde ve ders kitaplarında daha fazla yer verilmesi gerekliliği ortaya çıkmıştır. Ayrıca öğrencilerin çizim yapma, sistematik liste yapma, mantıksal akıl yürütme stratejilerini problem çözme sürecinde kullanmakla beraber bağıntı bulma, eleme ve geriye doğru çalışma stratejilerini hiç kullanmadıkları sonucuna ulaşılmıştır.

Anahtar kelimeler: Rutin problem, rutin olmayan problem, problem çözme stratejileri, matematik, ortaokul öğrencileri

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INTRODUCTION

Mathematic requires many activities such as abstraction, classification, and generalization. Classification means understanding the group of various objects according to the similarity and generalization which mean that an object based on knowledge is developed through specific examples. Moreover, math is learnt to use and replace symbols (Sappaile & Djam, 2017). From this aspect, teaching mathematics can also be defined as providing mathematical knowledge and skills that a person will need in the daily life. Teaching problem-solving and providing a way of thinking evaluate events according to the approach of problem-solving (Ministry of National Education [MoNE], 2009). There are many definitions related to the concept of problem in literature. According to Blum and Niss (1991), the problem is that the situations in which a person does not have any methodological, procedural, or algorithmic knowledge to answer. It consists of open questions and mentally challenging individuals. According to Morgan (1995), the problem is a situation of conflict in which the individual faces obstacles in the process of achieving his/her goal. According to Olkun and Toluk Uçar (2003), the problem is a situation that creates curiosity to solve and it does not have a ready solution However, itcan be solved by using one's knowledge and experience. Many researchers categorize the types of problems as routine and non-routine problems in the teaching of mathematics (Altun, 2015; Anderson, 2009; Cai, 2000; Laterell, 2013; Lee & Kim, 2005; Reusser & Stebler, 1997). In this context, routine problems' solutions are known before. (Mayer & Hegarty, 1996), and they are frequently encountered in the daily life. They can be solved by knowing and applying four operational skills correctly (Altun, 2015). On the other hand, non-routine problems are problems that do not have a known- method or formula for solution, require the student to analyze the data carefully, make a creative attempt, and use one or more strategies (Artut & Tarım, 2006; Sita Pramayudi, Sudiarta, & Astawa, 2020).

According to many mathematics educators, the solution of both routine and non-routine problems has a unifying importance in connecting learners' mathematical experiences as a meaningful whole (Posamentier & Krulik, 2008). In this context, Schunk (2012) defines problem solving as an effort to reach a goal in a situation where the individual does not have an automatic solution. In addition, in order for individuals to solve problems, they need to develop, combine and change the information they previously learned to obtain a successful solution (Fülöp, 2015; Posamentier & Krulik, 2008). On the other hand, teaching problem solving in mathematics teaching can be applied in three different ways (Schroeder & Lester, 1989; Van de Walle, Karp & Bay-Williams, 2013). These are teaching for problem solving, teaching about problem solving, and teaching with problem solving. In this context, teaching for problem-solving will be presented. It offers a traditional approach, and the problem is solved after the students are taught. Teaching about problem-solving is teaching the student problem to solve the steps or strategies. Finally, in teaching with problem-solving the student learns the problem through real contexts, situations, and models. According to this approach, skills are formed in the problem-solving process, and learning happens through problem-solving.

It is seen that individuals also try to find out many different strategies in the process of problem-solving (Altun, 2015; Baykul, 2019; Fan & Zhu, 2007). A strategy is a group of mental or physical actions designed to solve a problem. Strategies can be taught or they can appear spontaneously as a result of the rearrangement of plans (Biddlecomb & Carr, 2010). According to Posamentier and Krulik (2008), strategies that start with simple applications and are used in increasingly difficult and complex problems also provide students with the opportunity to use their problem-solving skills in the daily life. In literature, problem-solving strategies generally include arithmetic strategy, finding a pattern strategy, guess and check strategy, drawing strategy and equation strategy (Fan & Zhu, 2007; Fong & Hsui, 1999; Posamentier & Krulik, 2008; Van de Walle et al., 2013; Yazgan & Arslan, 2019). From this point of view, arithmetic strategy can be generally said as a type of strategy in which the student writes a mathematical expression that includes one or more operations by using the numbers given in the problem (Fong & Hsui, 1999). Find a pattern strategy is to create a different sequence by applying the arithmetic or geometric rules involved in the problems. On the other hand, it can be said that the guess and check strategy, also called as "try and see," (Van de Walle et al., 2013) is a type of strategy that we use unconsciously in our daily lives (Posamentier & Krulik, 2008). Drawing strategy, on the other hand, is a type of strategy that can be used not only in the geometry problems but also in all kinds of problems (Polya, 1957), and enables the relationship between datas to be seen (Yazgan & Arslan, 2019). Finally, the strategy of establishing an equation is the equations created by using various symbols such as x, a instead of the unknown in the solution of mathematical problems.

When the relevant literature is examined in the context of problem types and solution strategies of these problems, some studies in the field focus on problem-solving strategies applied in routine problems (Atay, 2017; Avcu, 2012; Demir, 2019; Gür & Hangül, 2015; Yılmaz, 2019) and non-routine problems (Altun, Memnun &

Yazgan; 2007; Altun & Memnun, 2008; Andrade, Fortes and Mabilangan, 2020; Arslan & Yazgan, 2015; Elia, van den Heuvel-Panhuizen & Kolovou, 2009; Gürbüz & Güder, 2016; Gürsan & Yazgan, 2020; Mogari & Chirove, 2017; Saygılı, 2017). However, in the context of current literature, a limited number of studies (Bozkurt & Topal, 2019; Karakoca, 2011) that examines the strategies in the solution processes of both types of problems stand out. In this context, Arslan and Yazgan (2015) concluded in their study that sixth, seventh, and eighth grade students with their high achievement levels, in general, reach solutions to non-routine problems by using appropriate strategies. In addition, there is another important issue obtained from the research that students mostly try to find out the finding pattern and drawing strategies. Gür and Hangül (2015) also found in their study that although students were successful in the strategies of searching for patterns, working backwards, writing equations, and making lists, they were less successful in drawing and using predictive control strategies. Besides, Atay (2017) in his study revealed with the aim of determining the ability of secondary school for seventh grade students to use problem-solving strategies. It is stated that all students applied to the equation strategy; however, he found that students with average and low achievement in mathematics applied for the drawing strategy. On the other hand, Mogari and Chirove (2017) in their study while examining non-routine problem-solving strategies for the high school students, they concluded that students mostly used prediction, control, and modeling strategies. Moreover, another important finding in the study is that students of 11th grade got the highest average score in non-routine problems while students of 10th grade got the lowest score. Similarly, Saygılı (2017) examines the non-routine problem-solving skill levels of high school students as a result of their studies, and the strategies they use concluded that students most frequently use the strategies of making systematic lists, finding pattern, logical thinking, and drawing. Yılmaz (2019) also found that students mostly used prediction-checking, making a list and using equation strategies as a result of his study to determine the strategies used by undergraduate students in classroom education in the process of problem-solving. Last, Andrade et al. (2020) concluded that students mostly use arithmetic strategy, prediction control and drawing strategies in solving non-routine problems. As seen from the studies mentioned-above, when the relevant literature is examined in the context of problem types and the solution strategies of these problems, it is seen that some of the studies focus on problem-solving strategies applied in the routine problems. Others focus on the solution strategies of non-routine problems. In this context, routine problems develop students' four-operation skills and they reinforce their newly learned knowledge. On the other hand, non-routine problems provide the development of thinking skills such as adapting, organizing, classifying, and associating the knowledge gained through routine problems to different situations. It is important to determine the success levels of the students in these types of problem by considering both types together. They should know the strategies they apply and determine the existing situations of the students by considering two types of problem in terms of being understood by both teachers and mathematics educators. However, in the context of the current literature, it is seen that there are a limited number of studies examining the strategies in the solution processes of both types of problems. It is aimed to determine the situations of secondary school students in solving routine and non-routine problems and the strategies they use while solving these problems. For these purposes, the following questions are tried to be solved in the study:

1. What is the level of secondary school students' solving routine and non-routine problems?
2. What strategies do secondary school students use while solving routine problems?
3. What strategies do secondary school students use while solving non-routine problems?

METHOD

In the study which aims to determine the students' situations of solving routine and non-routine problems for the secondary school students and to reveal the strategies they use while solving these problems, the survey model was used. Survey model is the gathering of information about a large sample in relating to a specific subject or situation (Fraenkel, Wallen, Hyun, 2012). In this study the secondary school students' situations about solving the routine and non-routine problems and strategies they use in this process were determined.

Population and Sample

The population of the research consists of 6th, 7th and 8th grade secondary school students affiliated to Adıyaman Directorate of National Education. The sample of the study involves 430 students studying in the 6th, 7th, and 8th grades of two public secondary schools determined by homogeneous sampling. Purposeful sampling enables in the research by selecting well-developed information. Homogeneous sampling is to examine subgroups in a detailed way by taking a homogeneous sample (Patton, 2014). The study group determined by the homogeneous sampling method consists of the successful students of the best two secondary schools in the region from the upper secondary socio-economic level. It is seen that 52.6% (226) of the students that make up the sample of the study are girls, and 47.4% (204) of them are boys. According to the grade level, 38.8% (167) of the students are 6th grade, 30.2% (130) for 7th grade, 30.9% (133) for 8th grade students. Besides, students' mathematics grade is classified according to the criteria within the scope of the national education examination passing regulation (MEB, 2019). According to this information, when the mathematics reports scores of the students were examined, 53.5% (220) of the students in question were good; 18.3% (69) were in good, 8% (30) were in medium, 6.4% (24)were passed and 8.8% were not passing the exam(33).

Data Collection Tools

In the research "Routine and Non-Routine Testing Problem" and "Problem Evaluation Rubric" were used as data collection tools in order to determine students' situations of solving routine and non-routine problems. In this context, the Routine and Non-Routine Problem Test were designed by the researchers to determine the situation of students solving routine and non-routine problems by making use of the current literature (Cai, 2000; Karakoca, 2011) (Appendix 1: Routine and Non-routine Problem Testing). While choosing these problems, students should have the quality to solve problems by producing more than one strategy. In addition, attention has been paid to ensure that it is suitable for both the students and the secondary school mathematics curriculum. Content validity of the prepared problems was presented to the views of two mathematics educators. However, two of the questions were about algebraic expression, and they were not suitable for the level of sixth grade students, since it is considered that the other two problems can be solved with a single strategy. These four questions were excluded from the test. The remaining four routine and four non-routine problems were applied to 64 students outside the study group. In this context, there was no problem in understanding the questions and it was decided that two hours in a lesson were sufficient. Furthermore, the reliable value of the test was calculated as α=.86. If this result is higher than .80, it can be accepted as an indicator that the test is reliable (Özdamar, 2013). On the other hand, the Problem Evaluation Rubric (Appendix 2: Problem Evaluation Rubric) used in the study was scored according to the five-point rubric created by Cai (2000). According to this rubric, each question scores are between 0 and 4. Accordingly, if the student's explanation and the solution process are correct and complete, the student will get 4 points. The student's explanation and solution process are basically correct, but if there are minor mistakes, the student will have 3 points. The solution process of the student is a bit understandable, but if the result is not reached, the student will score with 2 points. If the student has very limited knowledge, the student is scored with 1 point, and if the student solve the problem incorrectly or leave it blank, the student is scored with 0 points.

Data Collection

During the data collection process, the necessary permissions were obtained from the Adıyaman Provincial Directorate of National Education and the ethical approval board of the university, and the school administration was informed about the purpose of the study. In this context, the pilot study was carried out with 64 (6th grade: 19, seventh grade: 21 and eighth grade: 20). Students at the school work with at least one of the researchers. It took approximately nine weeks to collect data. The process of data collection was completed in two hours of class (totally 80 minutes).

Data Analysis

In the process of data analysis, SPSS 22 (IBM Corp, 2017) program was used. The data were also analyzed using descriptive statistics and descriptive analysis methods. In this context, eight questions in the Routine and Non-Routine Problem tests were firstly coded in a separate way by the researchers according to the problem evaluation rubric in order to determine the students' problem-solving status. Then the obtained codes were compared, and a consensus was achieved among the researchers. In the next stage an expert in problem-solving and mathematics education worked as the second coder, and they coded the answers of 30 randomly selected students. The agreement percentage of the data encoded by the researchers and the expert suggested by Miles and Huberman (1994) and it was calculated as .95. On the other hand, the strategies used by students in routine and non-routine problem tests were analyzed by descriptive analysis method. In the descriptive analysis, the data are interpreted by organizing them into the previously determined themes or titles (Yıldırım & Simsek, 2016). In this study, problem-solving strategies were prepared by using the current literature (Cai, 2000; Posamentier & Krulik, 2008; Yazgan & Arslan, 2019). The strategies used by the students were coded separately by the researchers, and then the codes obtained were compared and a consensus was achieved in cases where there was a difference of opinion. In the next stage, the same mathematics educator worked as the second coder and he/she coded the randomly selected data collection tool. The agreement percentage of the data coded by the researchers and the expert were calculated as .91 according to the formula suggested by Miles and Huberman (1994). On the other hand, codes such as S1, S2, S3, S4, S5, S6, S7, S8, and S9 were used to keep the identities of the students participating in the study. In this context, for example, S1 code refers to the first student's answer sheet while S2 code refers to the second student's answer sheet.

Research Ethics

Ethical principles and rules were followed during the planning, data collection, analysis, and reporting of the research. Ethical compliance approval was obtained for this research in accordance with the decision of the Scientific Research and Publication Ethics Committee in the field of Social and Human Sciences of Çukurova University Ethics Committee dated on 02.06.2020 and numbered with 95704281.

FINDINGS

According to the first sub-purpose of the study, the arithmetic mean and standard deviation scores regarding the students' level of solving routine and non-routine problems are given in Table 1.

Table 1. The Arithmetic Mean and Standard Deviation Distributions of Students for Each of the Routine and Non-Routine Problems

Problem Type Problems	Problems	$\bar{X}$	ss
Routine Problems	Washing Machine	2.69	1.48
	Map	2.75	1.47
	Pizza	2.21	1.82
	Camping	2.31	1.97
	Total	2.49	1.23
Non-Routine Problems n	Restaurant	1.85	1.99
	Blok	1.80	1.99
	Bus	1.89	1.98
	Island	2.07	1.74
	Total	1.90	1.40

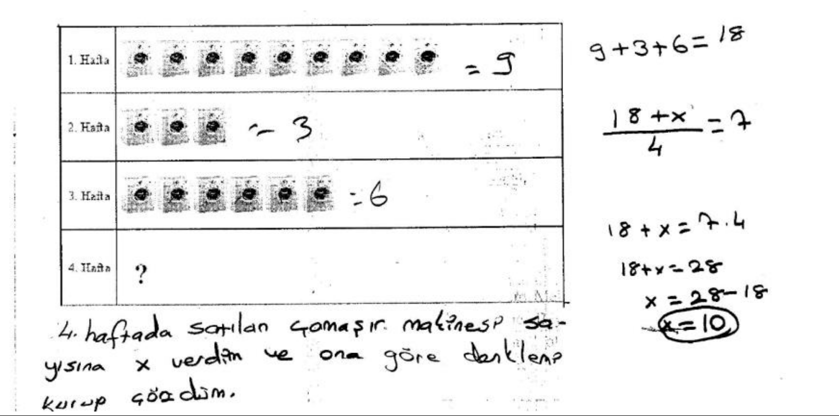
When Table 1 is examined, it is seen that the problem with the highest average in routine problems is the washing machine problem ($\bar{X}$=2.69), and the problem with the lowest average is the pizza problem ($\bar{X}$=2.21). In non-routine problems, it is seen that the highest mean is the island problem ($\bar{X}$=2.07), and the problem with the lowest average is the block problem ($\bar{X}$=1.69). When analyzed in terms of total score, it is seen that the average of routine problems ($\bar{X}$=2.31) is higher than the average of non-routine problems ($\bar{X}$=1.90).

According to the second sub-purpose of the study, the frequency and percentage values of the strategies used in solving routine problems of the students are included in Table 2.

Table 2. Percentage and Frequency Values of the Strategies Used by Students to Solve Routine Problems

Routine problems	Arithmetic strategy		Guess and check strategy		Equation making strategy		Logical Reasoning strategy		Drawing strategy		Total	
	f	%	f	%	f	%	f	%	f	%	f	%
Washing Machine	142	54	63	24	59	22					264	100
Map	157	66			81	34					238	100
Pizza	170	65					46	18	44	17	260	100
Camping	225	100									225	100
Total	694	70	63	6	140	14	46	5	44	5	987	100

When Table 2 is examined, it is seen that students mostly use arithmetic strategy (70%) and least use logical reasoning (5%) and drawing (5%) strategies in solving routine problems. Accordingly, the solution examples of the students are listed below in order. As the first of the routine problems it is seen that in the washing machine problem the students mostly used the arithmetic strategy (54%), then the guess and check strategy (24%) and the least equation strategy (22%). The solution of the student with the code of S197 in this matter is seen in Figure 1.



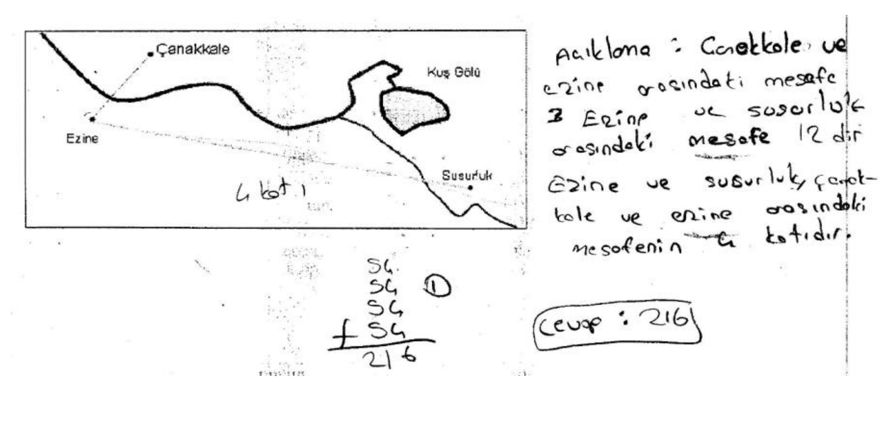
Student solution:

"I gave x to the number of washing machines sold in the 4th week and I created the equation and solved it accordingly."

Figure 1. *The Solution Made by A 7th Grade Student with Code S197 in the Washing Machine Problem*

Figure 1 shows the solution form of the 7th grade student (S197) who solves the washing machine problem in accordance with the strategy of establishing an equation. Accordingly, the student showed the situation with x (unknown) because he did not know 9 machines sold in the first week, 3 machines sold in the second week, 6 machines sold in the third week, and the number of machines sold in the 4th week. Then, he summed up the number of machines sold each week (18+x) and divided the expression by 4 to 7, and solved the equation to reach the correct answer.

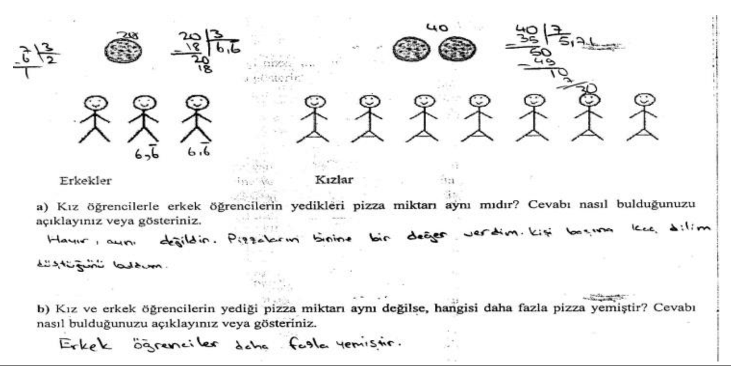
In the map problem, one of the routine problems is seen that the students mostly (66%) used the arithmetic strategy and 34% of them used the equation strategy. In this context, for example, the procedure for the solution of the map question of the 6th grade student with the code of S51 is shown in Figure 2.



Student solution: "Explanation: The distance between Canakkale and Ezine is 3 units, and the distance between Ezine and Susurluk is 12 units. Ezine and Susurluk are 4 times in accordance to the distance between Canakkale and Ezine."

Figure 2. *The Solution Made by A 6th Grade Student with Code S51 in the Map Problem*

When Figure 2 is examined, it is seen that the 6th grade student uses the arithmetic strategy in the map problem. According to it, the student coded S51 shows that the distance between Çanakkale and Ezine is 3 units, and the distance between Ezine and Susurluk is 12 units. In the next stage, the student found that the length between Ezine and Susurluk was 4 times longer than the distance between Çanakkale and Ezine (12 ÷ 3 = 4). Moreover, in order to find out how many kilometers is he has reached the desired result by summing 54 of them in 4 times (54 + 54 + 54 + 54 = 216). On the other hand, it was concluded that students mostly (65%) used arithmetic strategy in pizza problem that is one of the routine problems. Furthermore, 18% of these students used the logical reasoning strategy while 17% used the drawing strategy. In this context, the procedure for solving the map question of the 6th grade student with code S152 is shown in Figure 3.

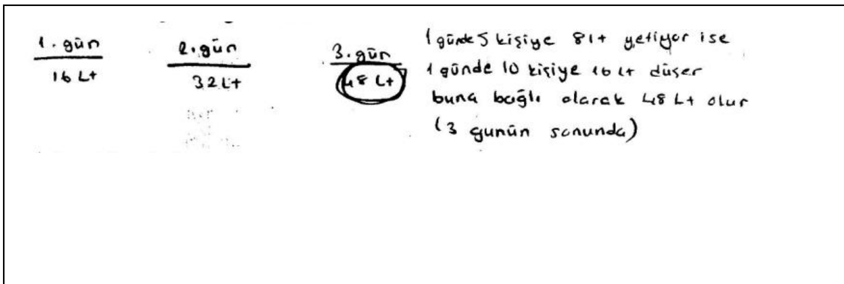


Student solution: "a) No, it is not the same. I gave a value to one of the pizzas. I found out how many slices per person. b) Male students ate more."

Figure 3. *The Solution Made by A 6th Grade Student with Code S152 in the Pizza Problem*

When Figure 3 is examined, it is seen that the 6th grade student with the code of S152 used the arithmetic strategy in solving the pizza problem. Accordingly, S152 coded student pizzas were divided into 20 slices. While 6.6 (20/3 = 6.6) slices of pizza were dropped for each male student, 5.7 slices of pizza were dropped for each female student (40/7 = 5.7). In the next option, he concluded that men ate more pizza as more slices were dropped by the male students.

Among the last of the routine problems, the camping problem is seen that all the students reach their conclusion by solving the arithmetic strategy. In this context, the procedure for solving the map question of the student with the code of S182 is shown in Figure 4.



Student solution: "If 8 liters is enough for 5 people in a day, 16 liters will be for 10 people in a day. Accordingly, it becomes 48 liters at the end of three days"

Figure 4. *The Solution Made by A 7th Grade Student with Code S182 in the Camp Problem*

When Figure 4 is examined, it is seen that the 7th grade student with the code S182 used the arithmetic strategy in the camp problem. Accordingly, the student coding S182 calculated that 16 liters of water would be enough for 10 people using the correct proportion. In the next step, the student coding S182 calculated three times the value he found by using the proportion to find the three-day amount of water (16x3 = 48).

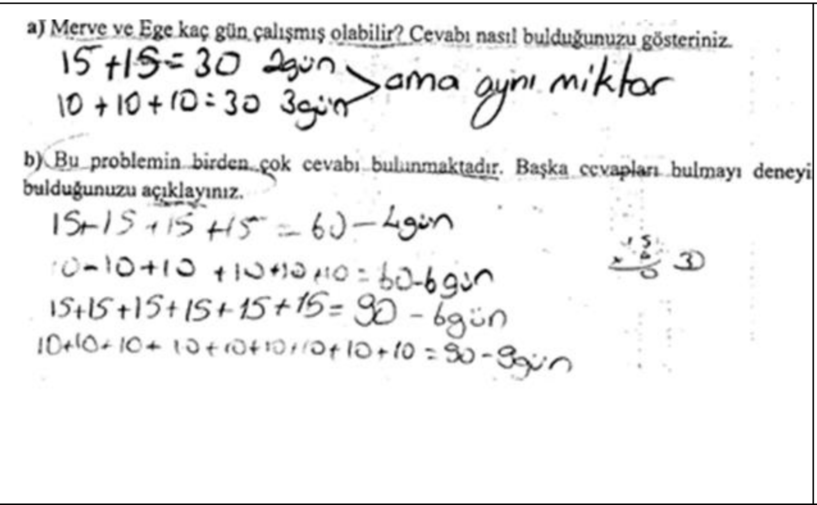
According to the third sub-purpose of the research, the frequency and percentage values of the strategies used in solving routine problems of the students are given in Table 3.

Table 3. Percentage and Frequency Values of the Strategies Used by Students to Solve Non-Routine Problems

Non-Routine Problems	Arithmetic strategy		Guess and check strategy		Systematic listing strategy		Total	
	f	%	f	%	f	%	f	%
Restaurant	140	87	-	-	21	13	161	100
Blok	64	44	81	56	-	-	145	100
Bus	192	100	-	-	-	-	192	100
Island	125	70	53	30	-	-	178	100
Total	521	76	134	20	21	4	676	100

When Table 3 is examined, it is seen that the students mostly used the arithmetic strategy (76%) in the solution of non-routine problems, and the systematic list strategy (4%) was used at least. Accordingly, examples of solutions made by students are listed below.

In the restaurant problem which is in the first place in non-routine problems, it is seen that students mostly use the arithmetic strategy (87%) and at least the systematic list strategy (13%). In this context, the procedure for the solution of the restaurant problem by a 7th grade student with the code of S167 is shown in Figure 5.



Student solution:

"a) 15+15=30 (2 days)

10+10+10=30(3 days, but the same amount)

b) 15+15+15+15=60 (4 days)

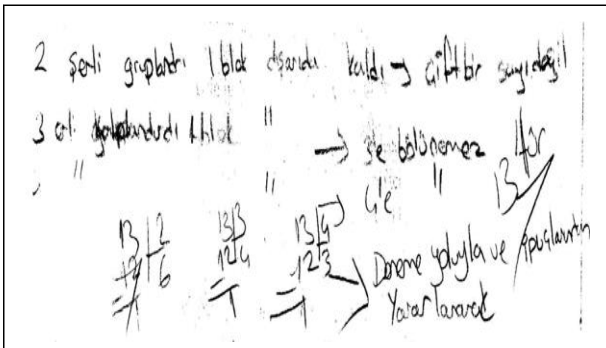
10+10+10+10+10+10=60 (6 days)

15+15+15+15+15+15=90 (6 days)

10+10+10+10+10+10+10+10+10=90 (9 days)"

Figure 5. *The Solution Made by A 7th Grade Student with Code S167 in the Restaurant Problem*

When Figure 5 is examined, it is seen that the 7th grade student used S167 the arithmetic strategy in the solution phase of the restaurant problem. Accordingly, the student reached the conclusion by adding numbers to the solution of the problem. Besides, the student found that Merve, who is in one of the options in the problem, earned 30 TL when she worked for two days, and earned 30 TL when Ege worked for three days. It is seen the problem reached the result by equalizing the money they earned. On the other hand, in the non-routine block problem it is seen that 56% of the students apply the guess and check strategy, and 44% in the arithmetic strategy. In this context, the procedure for the solution of the block problem by the 6th grade student with the code of S86 is shown in Figure 6.



Student solution: "Grouped by 2, 1 block left out – not an even number

Grouped by 3, 1 block left out – not divided by 3

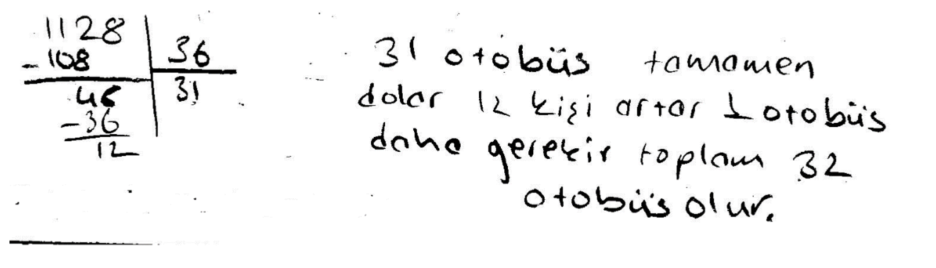
Grouped by 4, 1 block left out – not divided by 4

I found it by trial and using hints"

Figure 6. *The Solution Made by A 6th Grade Student with the Code S86 in the Block Problem*

When Figure 6 is examined, it is seen that the 6th grade student with S86 used the guess and check strategy in the block problem. Accordingly, the student stated that in solving the problem the blocks were grouped into two, three and four, but they increased one block at a time. Moreover, the same student stated that when the blocks are grouped by two, the total number of blocks cannot be double because it increases by 1 block. When it is grouped by three, it cannot be divided into three, and in the groupings of four the total number of blocks cannot be a multiple of two, three and four since one block will increase by not being divided into four. Then, he reached the conclusion by using the clues he found and the number 13 gave the desired answer.

In the non-routine bus problem, it is seen that all the students solved the problem with the arithmetic strategy. In this context, the procedure for solving the bus problem of the 7th grade student with the code of S200 is shown in Figure 7.

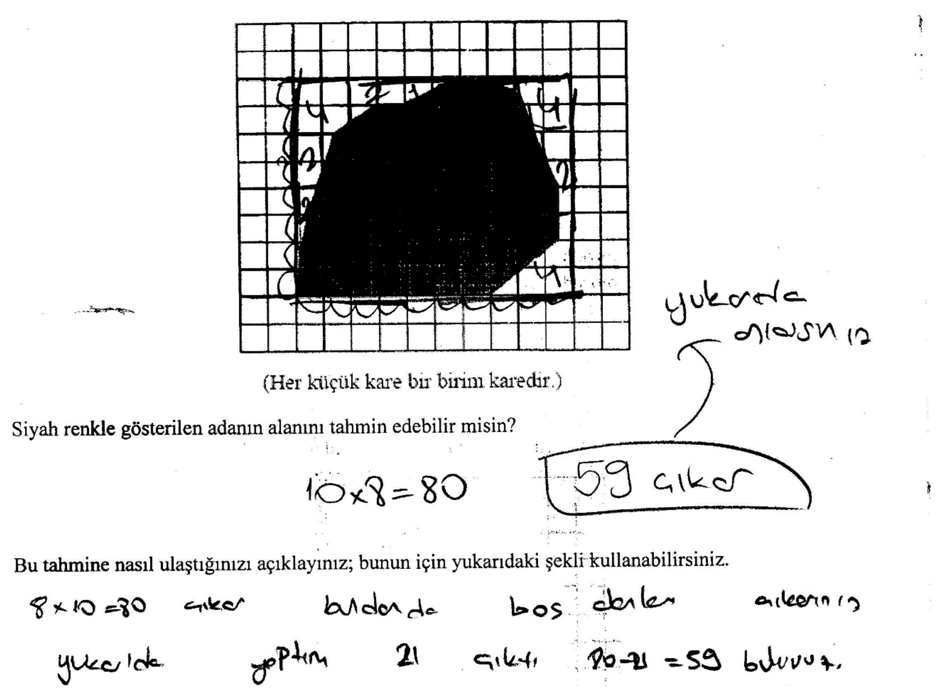


Student solution: “31 buses are fully filled. 12 people left. 1 more bus is needed. There will be 32 buses in total.”

Figure 7. *The Solution Made by A 7th Grade Student with Code S200 in the Bus Problem*

When Figure 7 is examined, it is seen that the 7th grade student uses the arithmetic strategy in the bus problem. Accordingly, in the solution of the problem the student divided 1128 into 36 in order to get places to 1128 students on buses of 36 people and found the result of the division as 31. For 12 students who remain, he added 1 more bus and reached 32 answers.

Finally, in the non-routine island problem 70% of the students used the arithmetic strategy. It is seen that 30% of them apply the estimation and control strategy. In this context, the procedure for the solution of the island problem by the 8th grade student with the code of S354 is shown in Figure 8.



Student solution: “8.10=80. I solved the problem above. The result 80-21=59 is found.”

Figure 8. *The Solution of the 8th Grade Student with Code S354 in the Island Problem*

When Figure 8 is examined, it is clearly seen that the solution of the 8th grade student coded S354 according to the arithmetic strategy in the island problem. Accordingly, the student in question drew a rectangle around the black island and calculated the area of this rectangle (10×8=80). In the next step he calculated the unit squares for the white area inside the rectangle he drew. At last, by subtracting the sum of the white unit squares from the area of the rectangle, he concluded (80-21).

DISCUSSION & CONCLUSION

This study was conducted to determine the situations of secondary school students to solve routine and non-routine problems, and the strategies they use while solving these problems. Accordingly, when students' solving routine and non-routine problems were examined, it was concluded that the students participating in the study were more successful in solving routine problems than non-routine problems. This result obtained from the study is also like the studies in literature (Cai, 2000; Bozkurt & Topal, 2019; Karakoca, 2011; Taskın et al., 2012). In this context, Karakoca (2011) found that sixth grade students' success in routine questions was higher than their success in non-routine questions in a study that examined the solution process of routine and non-routine problems. Similarly, Bozkurt and Topal (2019) concluded that sixth grade students were successful in routine problems but could not perform well in non-routine problems. Finally, Taskın et al. (2012) also found that students scored higher in routine problems than non-routine problems in the achievement test of routine and non-routine problems they applied to high school students. On the other hand, when the findings of the research regarding the strategies used in problem-solving are examined, it is clearly seen that the students participating in the study mostly use arithmetic and guess and check strategies in both routine and non-routine problems. This result obtained from the research points to similar results with the studies conducted in literature (Andrade et al., 2020; Avcu, 2012; Karakoca, 2011; Yılmaz, 2019). In this context, Karakoca (2011) stated in his study that he conducted sixth grade students used the guess and check strategy more than the others. Moreover, Andrade, et al. (2020) found in their studies that arithmetic strategy and prediction control strategies are the most-used strategies by students in solving non-routine problems. Finally, Yılmaz (2019) also revealed that teacher candidates in the class mostly used the strategies of guess and check, systematic listing and equation.

Other important finding obtained from the research is that equation setting and logical reasoning strategies are only applied in the solution process of routine problems. In this context, when the current literature is examined, it is seen that this result is partially similar to the studies conducted in the field (Atay, 2017, Demir, 2019; Gür & Hangül, 2015; Yılmaz, 2019). In this context, for example, Yılmaz (2019), as a result of his study to determine the strategies used by prospective classroom teachers in the routine problem-solving process, revealed that classroom teacher candidates mostly used prediction-checking, make a list and equation using strategies. Again, Gür and Hangül (2015) found in their study that students were successful in equation and make a list strategies, but they were less successful in drawing and using guess and check strategies. Similarly, Atay (2017) also preferred the strategy of making equations most of the secondary school students. They reached the conclusion.

Conclusion and Recommendations

It is thought that this study, which was conducted with the aim of determining the situation of solving routine and non-routine problems of secondary school students and the strategies they use while solving these problems contributes to researchers in terms of mathematics education. In this context, in the study, it is clear that students mostly resort to arithmetic strategies, guess and check strategies, and systematic listing strategies in the context of non-routine problems. In this context, Saygılı (2017), as a result of his study examining the non-routine problem-solving skill levels of high school students and the strategies they use, concluded that students mostly use the strategies of making systematic lists, finding patterns-correlations, and logical thinking. Similarly, Mogari and Chirove (2017) found that high school students mostly used the prediction control strategy in their study, where they examined non-routine problem solving strategies.

At the end of the study which was conducted with the aim of determining the situation of secondary school students in solving routine and non-routine problems and the strategies they use while solving these problems, it was revealed that students were generally more successful in solving routine problems than non-routine problems. For this reason, it is recommended to include non-routine problems more in the classroom and in textbooks in the lessons of mathematics. Moreover, it was concluded that in the process of solving routine and non-routine problems, arithmetic strategy and prediction control strategies were mostly used by the students who participated in the study. However, it was revealed that students used drawing, making a list and logical reasoning strategies, but they never used the strategies of finding pattern, elimination, and working backwards. In the context of this result, it is thought that eliminating the deficiencies of students in learning different strategies and increasing students' awareness will significantly contribute to problem-solving processes. On the other hand, the reasons why students use some strategies little or not at all should be investigated. Subjects in the class of mathematics and teachers' inclusion of all kinds of problem-solving strategies in the classroom can help students to gain a different perspective, improve their problem-solving skills, and look at events from a broader perspective. The fact that this study was conducted only with secondary school students can be expressed as a limitation. In this context, longitudinal studies can be conducted with students at different education levels to examine the developmental levels of the students in the process. Moreover, other limitation of the study is the collection of data only by quantitative research method. In future studies it may be suggested to conclude interviews with teachers by using qualitative research methods or to examine how the problem-solving process is carried out and which strategies are applied by making classroom observations. In addition, it can be suggested to conduct studies on encouraging students to use the least used strategies such as drawing, systematic listing, finding correlations, elimination and drawing in the non-routine problem-solving process by interviewing the teachers.

Statements of Publication Ethics

As authors of the research, we declare that the study has no unethical problem and we observed research and publication ethics. Ethical principles and rules were followed during the planning, data collection, analysis, and reporting of the research. Ethics Committee Approval was obtained from the Scientific Research and Publication Ethics Committee in the field of Social and Human Sciences of Çukurova University (the letter dated on 02.06.2020 and numbered with E. 95704281).

Researchers' Contribution Rate

This work is a product of the first author's master's thesis. The data were collected by the first author and conducted under the supervision of the second author.

Conflict of Interest

There is no conflict of interest for this study.

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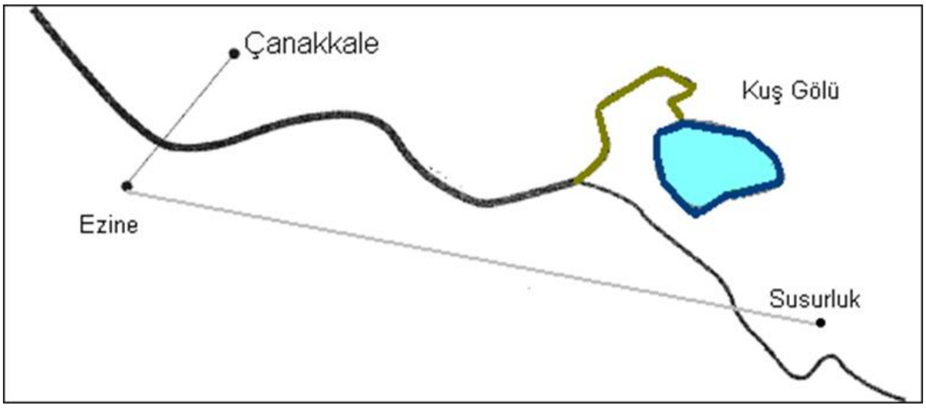
APPENDIX

Appendix 1. Routine and Non-routine Problem Test

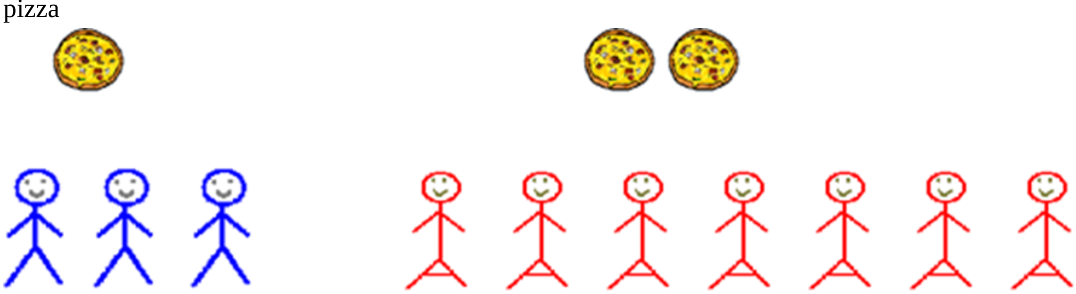
Routine problems

1) Washing Machine Problem: Mr. Mehmet owns a white goods store. The picture-below shows the number of washing machines Mr. Mehmet sold in the first three weeks of January. How many washing machines should Mr. Mehmet sell on the 4th week so that the average of the number of washing machines sold in a month is 7? Show how you found the answer.

2) Map Problem: The actual distance between Çanakkale and Ezine is 54 km. On the map, the distance between Çanakkale and Ezine is 3 cm. Accordingly, if the distance between Ezine and Susurluk is 12 cm on the map, what is the actual distance between Ezine and Susurluk? Show how you found the answer.



3) Pizza Problem: There are 7 female and 3 male students below. 7 girls will share 2 pizzas, 3 boys will share 1 pizza equally.



Male Female

a) Is the amount of pizza eaten by female students and male students the same? Explain or show how you found the answer.

b) If the amount of pizza eaten by male and female students is not the same, which one ate more pizza? Explain or show how you found the answer.

4) Camping Problem: A group of 10 people will go to a 3-day scout camp. However, since there is no water in the place where they will go, they must take water to drink. Because of it, they saw in the scout guide book they read that 8 liters of water is enough for 5 people for a day. In this case, how much water should the group of 10 people who will go to the summer camp take with them? Show how you found the answer.

Non-routine Problems

1) Restaurant problem: Merve and Ege are two friends working together in the same restaurant. While Merve's job is to sell hamburgers, Ege's job is to clean the tables where the customers sit. While Merve earns 15 TL in a day, Ege earns 10 TL. While the total number of days Merve and Ege worked are not equal, the total amount they earn is equal to each other. According to this information,

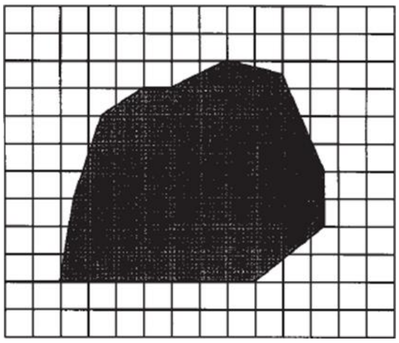
a) How many days could Merve and Ege have worked? Show how you found the answer.

b) This problem has multiple answers. Try to find other answers and explain how you found the answer.

2) Block problem: Ayşenur's father, Mr. Mustafa, asks his daughter what she is doing in today's math class. Ayşenur answers as follows: 'Today we used blocks in math class. When I grouped the blocks in my hand as 2 sets , 1 block was left out. When I grouped them in 3, 1 block was left out. When I grouped 4 of them, 1 block was left out.' Ayşenur's father, Mr. Mustafa, said to his daughter: 'How many blocks did you have?' What do you think was Ayşenur's response to her father? Explain how you found the answer.

3) Bus problem: With the spring, 13 Kasım Primary School decides to take a trip to Istanbul by renting a bus. There are 1128 people in total who will participate into the voyage. If there are 36 seats on each bus, how many buses are needed in total?

4) Island problem: The region shown in black represents an island below.



a) Can you guess the area of the island in black?

b) Explain how you arrived at this estimate. You can use the figure metioned-above for it.

Appendix 2: Problem Evaluation Rubric

The criteria of the problem-solving rubric created by Cai (2000) is given as follows:

4 points: The student's explanation and solution process are correct and their explanations are complete.

3 points: The student's explanation and solution process are basically correct, but there are some minor errors and uncertainties.

2 points: The student did not fully understand the problem and could not reach a conclusion.

1 point: It is understood that the student has limited knowledge in the explanation and solution process.

0 points: The student did not understand the problem at all or left it unanswered.

In the scoring process, each problem was evaluated according to the scoring criteria graded over 4 points. Furthermore,

in the scoring of 2-stage problems, the first stage was evaluated as 2 points, and the second stage was evaluated as 2 points.