Inquiries into the Idea of Space:
Aurel Bejancu, A Biographical Note

Bogdan D. Suceavă

(Dedicated to the memory of **Prof. Dr. Aurel BEJANCU (1946 - 2020)**)

ABSTRACT

John F. Nash Jr.'s Embedding Theorem, published originally in 1956, states that every Riemannian manifold can be isometrically embedded into some Euclidean space. This fundamental result is a very beautiful and extremely important result in differential geometry, and especially in the geometry of submanifolds. One of the researchers with outstanding contributions in the geometry of submanifolds, as well as in other areas of differential geometry, including its connections with physics, with a long creative career spanning from his first research paper in 1971 to his last in 2016, was Aurel Bejancu. In this biographical note we present his life and we remind with great respect his contributions.

Figure 1. Prof. Aurel Bejancu (19 August 1946 - 3 April 2020).

Remembered today as an influential geometer with an inspiring career spanning almost five decades, Aurel Bejancu was born on 19 August 1946 in Pociovaliştea, Romania, and passed away on 3 April 2020. The *International Electronic Journal of Geometry* had the honor of having Prof. A. Bejancu among his editors, and it is with our greatest respect that we present here a few personal pieces of information, to better place in an appropriate historical context his life and his work.

Aurel Bejancu graduated in June 1968 from the Faculty of Mathematics-Mechanics at the University of Timişoara, with a specialization in Geometry. After graduation, his first academic position was with the Iaşi branch of the Romanian Academy, working there as researcher from 1 September 1968 until 15 June 1974; after this date, his position became of researcher with the Mathematics Institute in Iaşi. It was in this period that he worked on and ultimately defended on 26 April 1973 his doctoral dissertation titled *Lie-Banach Groups and Generalizations* at the University *Alexandru Ioan Cuza* Iaşi. His dissertation was written under Prof. Gheorghe Gheorghiev's scientific direction. Prof. Aurel Bejancu joined the faculty at University *Alexandru Ioan Cuza* Iaşi as researcher from 1 April 1975 to 15 September 1978, when he moved to a teaching position as Lecturer with the Technical University *Gheorghe Asachi* Iaşi. His work was notably recognized through the Simion Stoilow Prize of the Romanian Academy for the year 1978, a prize he shared in that year with Gheorghe Micula.

Figure 2. Prof. Aurel Bejancu delivering a presentation in the Romanian-Japanese Colloquium on Finsler Geometry, in Iaşi, at the Myller Seminar, 16 August 1984.

Aurel Bejancu became full Professor at the Technical University *Gheorghe Asachi* Iaşi on 1 October 1990. Besides his academic work, Aurel Bejancu served also in various editorial or administrative positions, including a term as Associate Dean of the Department of Electronics and Telecommunications (1992-1996) at the Technical University *Gh. Asachi* from Iaşi. His transition to the University of Kuwait took place first as Visiting Professor, then as Professor since 1 September 1999, until his retirement in 2016. He served as a member of the Editorial Board of the *Kuwait Journal of Science and Engineering* (2010 – 2012). He joined the Editorial boards of the *Bulletin of Society of Mathematicians Banja Luka* (Bosnia) in 1995, and of the *International Electronic Journal of Geometry* in 2008. In the 1990s, Prof. Bejancu served as a member of the National Council for Academic Scientific Research, a consulting body within the Ministry of Education in Romania, where the top experts in the nation are invited to assess the academic work of their colleagues and the performance of the academic programs in their respective specialities. Prof. Bejancu was a prolific researcher, having published over 160 mathematical journal papers and 5 research monograph books with prestigious publishing houses.

During the 1970s and 1980s, the academic environment in Iaşi was particularly fertile and inspiring for geometry. The geometry school in Iaşi was the result of several decades of very successful academic life, which can be traced back to the foundational efforts of Alexandru Myller (1879-1965), a former doctoral student of David Hilbert. Prof. Bejancu's doctoral advisor, Gheorghe Gheorghiev, was a doctoral student of Alexandru Myller. Élie Cartan visited and presented a conference in Iaşi when he toured Romania in 1931, and Kentaro Yano published in 1938 his Parisian doctoral thesis in the the Iaşi *Annals* [18]. The importance of the journal published in Iaşi becomes evident if we mention that T. J. Willmore published in it in 1965 the paper where the famous Willmore Conjecture was first stated [19]. This important result which sparked much research in the geometry of submanifolds was proved in 2013 by Fernando C. Marquez and André Neves [14].

In the last decades of the 20th century, several influential geometers lived and worked in Iaşi. They were either working at the two Universities in this historical city, or had close academic ties with the doctoral programs in Iaşi. Besides Gheorghe Gheorghiev, we remind here Professors Radu Miron, Mihai Anastasiei, Vasile Cruceanu, Vasile Oproiu, and Dumitru Motreanu, among others. Outstanding results in the area of

Finsler geometry were obtained by Professor Gheorghe Munteanu, whose career continued at the University in Braşov, but whose academic ties with Iaşi were meaningful. These geometers' contributions and their academic interactions include the development of Lagrange generalized metrics, Finsler geometry, the geometry of submanifolds, special classes of connections, among other themes central to the investigations of the geometry school in Iaşi.

This inspiring environment nurtured A. Bejancu's stellar work. Throughout his career, he constantly expressed his deep gratitude towards his mentor and advisor, Prof. Gheorghe Gheorghiev. Writing about Aurel Bejancu's contributions in an academic document from 1998, Prof. Radu Miron (member of the Romanian Academy) presented the range of his academic interests as follows: "Prof. Aurel Bejancu pursues his research in the area of differential geometry and its applications to physics. He obtained important results in the following directions: Lie-Banach groups, CR-submanifolds in Kähler manifolds, gauge theory, and Finlser manifolds." The document points out that "A part of Prof. Aurel Bejancu's results became fundamental results in the respective research areas, with citations in over 200 research papers and in 8 monographs. Due to the interests in his work from several research groups from all over the world, he was invited to visit and present his research in several universities from the U.S.A., Canada, Spain, Poland, and was invited speaker to a variety of academic conferences."

Figure 3. Prof. Aurel Bejancu at the University of Kuwait in the spring of 2016.

A theme that attracted many citations and proved to be based on a very inspiring idea was A. Bejancu's study of CR submanifolds of a Kähler manifold [1, 2]. Prof. Koichi Ogiue discussed this paper published originally in *Proc. AMS* in [15]: "This paper provides an introduction to differential geometry of CR submanifolds of a Kähler manifold, which generalize totally real and complex analytic submanifolds. After studying some basic properties, the author is concerned mainly with totally geodesic, totally umbilical and minimal CR submanifolds." An interesting result due to Bejancu in [2] states that there exist no totally umbilical proper CR submanifolds in an elliptic or hyperbolic complex space form. As Aurel Bejancu's mastery of the geometry of CR submanifolds grew, he wrote in 1986 the monograph [3]. It is quite remarkable, in terms of number of citations and actual impact, that all A. Bejancu's monographs attracted a lot of attention over the years. Writing about Prof. Bejancu's volume [3] in the *Bulletin of the American Mathematical Society*, Prof. Bang-Yen Chen wrote [10]: "The study of complex submanifolds of a Kaehlerian manifold, in particular, of a complex projective space, is one of the most important fields in differential geometry." Furthermore, commenting on the style and the thoughtful selection of the content, Prof. Bang-Yen Chen appreciated the importance of [3]

noting in [10]: "In summary, the author did a very good job of arranging all of the important results on CR-submanifolds in one place. Since the original material is widely scattered in the literature, readers interested in CR-submanifolds will find it enormously helpful to have the results assembled in one place with a unified exposition. This book should be a valuable addition to all research libraries."

To better describe the importance of Prof. Aurel Bejancu's contribution by initiating the study of CR submanifolds through his paper [1], we can appreciate how much this research area flourished over time. One of the expressions of this development is the volume edited by Sorin Dragomir, Mohammad Hasan Shahid, and Falleh R. Al-Solamy [11], where Prof. Aurel Bejancu's initial work is widely cited and his insight is much appreciated.

Aurel Bejancu maintained close research contacts with highly respected geometers from around the world and his work received utmost attention and appreciation. One of the most successful papers written by Aurel Bejancu was the outcome of his collaboration with Masahiro Kon and Kentaro Yano [7]. In his review written for MathSciNet, Shoshichi Kobayashi discussed this contribution [13]: "The authors study CR-submanifolds M of arbitrary dimension of a complex space form $\bar{M}$. In addition to the classical second fundamental form, they use the tensor field $P \in End(TM)$ which sends $X \in TM$ to the tangential component of JX and obtain several local and global results on CR-submanifolds. For example, under the assumption that P is parallel, they prove a structure theorem for CR-submanifolds."

Prof. Bejancu's most cited work is his monograph co-authored with Krishan L. Duggal [12], which represents a systematic study of degenerate submanifolds in semi-Riemannian geometry.

S. S. Singh wrote [17] about the monograph *Finsler geometry and applications* [4] that "The study of spaces with generalized metrics was initiated by P. Finsler in 1918. Global methods appeared in Finsler geometry in the 1960s following the significant work of M. Matsumoto concerning a global theory of Finsler connections." Furthermore: "In Chapter 6 a geometric framework for a multidimensional relativity theory is given. As a pleasant surprise, Finsler geometry offers a mathematical apparatus suitable for the study of the differential geometry of a supermanifold. The last chapter deals with deformations of oriented media via Finsler geometry." The value of Aurel Bejancu's vision is pointed out by S. S. Singh in eulogizing terms [17]: " This is the first time that Finsler geometry has been treated following the same lines as Riemannian geometry, and the book is the first to point out that parameter dependent physics can be successfully treated by using Finsler geometry." This interest in the applications to physics remained central to A. Bejancu's research up until his later years; some important publications appeared in 2015 [5, 6], right before his retirement.
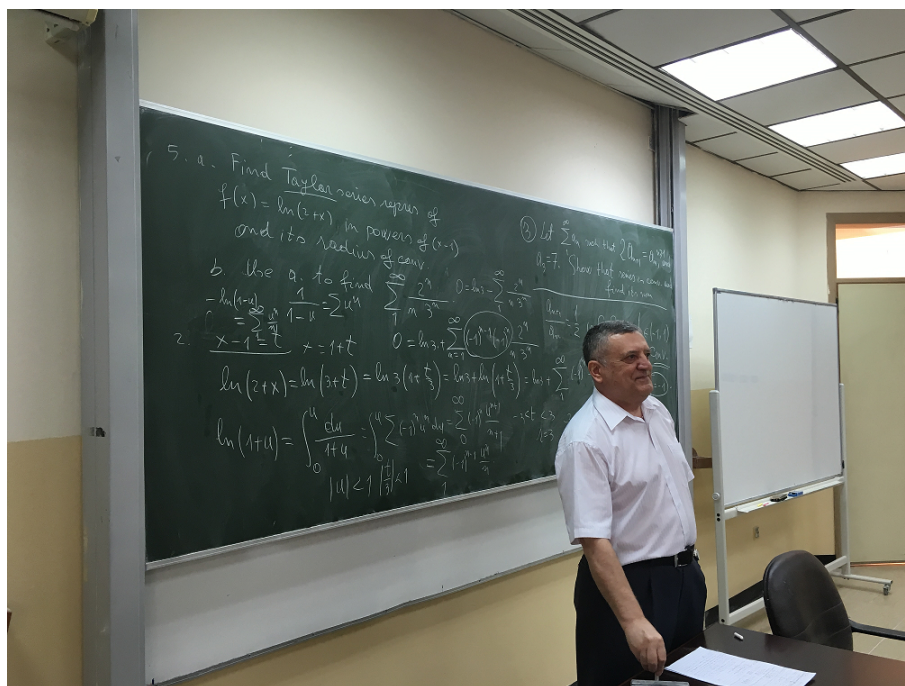

Figure 4. Prof. Aurel Bejancu taught at the University of Kuwait until the end of the academic year 2015-2016. This image is taken before his retirement.

Commenting on the monograph [8], Zhongmin Shen [16] pointed out that "The geometry of Finsler

submanifolds is a very difficult subject, due to the lack of PDE techniques. So far very few global results have been obtained. The present book is the first in the literature that is entirely devoted to the study of the geometry of Finsler submanifolds."

Our present note approached just a few of the themes present in Prof. Aurel Bejancu's work; we did not attempt to convey the depth of his results. Prof. Aurel Bejancu's gift to the mathematical community are his papers and his books, which are still cited and inspire many of us. His ethical model in pursuing a lifetime of research and academic inquiries represents a model for many geometers from all over the world.

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Affiliations

BOGDAN D. SUCEAVĂ
ADDRESS: California State University, Fullerton, Department of Mathematics, 800 N. State College Blvd., Fullerton, CA 92831,U.S.A.
E-MAIL: bsuceava@fullerton.edu
ORCID ID: 0000-0003-3361-3201