Turk. J. Math. Comput. Sci.
13(2)(2021) 287–293
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DOI : 10.47000/tjmcs.925182

Hermite-Hadamard Type Inequalities for s-Convex Functions in the Fourth Sense

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Received: 21-04-2021 • Accepted: 18-10-2021

Abstract. In this study, firstly, Hermite-Hadamard type inequalities are examined for functions whose first derivative is s-convex functions in the fourth sense. In addition, Hermite-Hadamard type inequalities are examined for functions whose second derivative is s-convex functions in the fourth sense. Finally, some application examples including special tools and digamma functions are presented.

2010 AMS Classification: 26D07, 26D10, 26D15, 26A51

Keywords: Convex functions, s-convexity, Hermite-Hadamard type inequality, specials means, digamma function.

1. Introduction

The definition of new function classes expands the fields of mathematics and creates new fields of study for researchers. Sometimes this can be achieved by introducing a definition that does not exist in the literature or by generalizing existing functions. One of the most generalized function classes in the literature is convex functions. B- convex functions, B^{-1}-convex functions, s- convex functions etc. can be given as examples [2, 3, 5, 9, 11, 15, 21, 23]. Convex functions as per the definition, are associated with inequalities, and so one of their main characteristics is the satisfaction of some inequalities such as Hermite-Hadamard, Jensen, Ostrowski, Fejer. The expression of these inequalities for generalized classes of convex functions also takes place in the literature [4, 6, 9, 12, 13, 17, 18, 22]. In this study, upper bounds for Hermite-Hadamard inequalities are obtained for s-convex functions in the fourth sense.

Let us recall some basic information we will need in this paper.

Let U be a convex set on vector space X and $\psi : U \to \mathbb{R}$ be a function. ψ is called a convex function on U, if the inequality

$$\psi(\alpha x + \beta y) \leq \alpha\psi(x) + \beta\psi(y). \tag{1.1}$$

holds for all $x, y \in \mathbb{R}^n$ and all $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$.

The convexity concept has been generalized various ways. One of them is achieved by changing the powers of α and β into a positive number less than 1, namely s, in left or right or both side of (1.1) or by replacing the theroem condition $\alpha + \beta = 1$ with $\alpha^s + \beta^s = 1$. In this way, s-convex functions are introduced. The s-convex function in the first sense was defined by Orlicz in 1961 [16], then the idea of s-convex in the second sense was introduced by Hudzik in 1994 [11]. Recently, in the same manner, p-convex functions, s-convex functions in the third and fourth sense have been defined [10, 14, 19, 20].

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Let $s \in (0,1]$ be a fixed number. A function $\psi : \mathbb{R}^n \to \mathbb{R}$ is said to be s-convex function in the fourth sense if

$$\psi(\alpha x + (1-\alpha)y) \leq \alpha^{\frac{1}{s}}\psi(x) + (1-\alpha)^{\frac{1}{s}}\psi(y)$$

where $\alpha \in [0,1]$.

The classes of s-convex functions in the fourth sense are denoted by K_s^4.

If $\psi : \mathbb{R}_+ \to \mathbb{R}$ is s-convex function in the fourth sense, then $\psi(x) \leq 0$ for all $x \in I$. For more information on this class, see [10].

It can be easily seen that for $s = 1$, s-convexity is reduced to the ordinary convexity of functions defined on $\mathbb{R}^n$.

If $\psi : I \to \mathbb{R}$ is a convex function, then the following inequality holds,

$$\psi\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b \psi(x)dx \leq \frac{\psi(a)+\psi(b)}{2},$$

where $a, b \in I$ with $a < b$. This inequality is known as Hermite-Hadamard inequality.

In this paper, some bounds for each side of Hermite-Hadamard inequality are obtained. Some application examples involving special means and digamma functions are presented.

Throughout the paper it will be considered as $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}_+ = [0, \infty)$, $I \subset \mathbb{R}$ and I^o interior of I.

2. Main Results

In this section, for s-convex functions in the fourth sense some inequalities associated with the right side and the left side of the Hermite-Hadamard inequality are derived using certain integral identities equations.

Lemma 2.1. *Let $\psi : I \to \mathbb{R}$ be a s-convex function in the fourth sense, then the inequality*

$$\psi\left(\frac{a+b}{2}\right) \leq \frac{s}{1+s}\frac{\psi(a)+\psi(b)}{2^{\frac{1}{s}-1}}$$

is valid for $0 < s \leq 1$.

Proof. Since ψ is s-convex function in the fourth sense on I, then we get

$$\begin{aligned}
\psi\left(\frac{a+b}{2}\right) &= \psi\left(\frac{\alpha a + (1-\alpha)b}{2} + \frac{(1-\alpha)a + \alpha b}{2}\right) \\
&\leq \left(\frac{1}{2}\right)^{\frac{1}{s}}\left[\psi(\alpha a + (1-\alpha)b) + \psi((1-\alpha)a + \alpha b)\right] \\
&\leq \left(\frac{1}{2}\right)^{\frac{1}{s}}\left[\alpha^{\frac{1}{s}}\psi(a) + (1-\alpha)^{\frac{1}{s}}\psi(b) + (1-\alpha)^{\frac{1}{s}}\psi(a) + \alpha^{\frac{1}{s}}\psi(b)\right] \\
&= \left(\frac{1}{2}\right)^{\frac{1}{s}}(\psi(a)+\psi(b))\left(\alpha^{\frac{1}{s}} + (1-\alpha)^{\frac{1}{s}}\right). \qquad (2.1)
\end{aligned}$$

Integrating (2.1) with respect to α on $[0,1]$, we get

$$\begin{aligned}
\int_0^1 \psi\left(\frac{a+b}{2}\right)d\alpha &\leq \int_0^1 \left(\frac{1}{2}\right)^{\frac{1}{s}}(\psi(a)+\psi(b))\left(\alpha^{\frac{1}{s}} + (1-\alpha)^{\frac{1}{s}}\right)d\alpha \\
&= \frac{\psi(a)+\psi(b)}{2^{\frac{1}{s}}}\int_0^1 \left(\alpha^{\frac{1}{s}} + (1-\alpha)^{\frac{1}{s}}\right)d\alpha \\
&= \frac{2s}{1+s}\frac{\psi(a)+\psi(b)}{2^{\frac{1}{s}}}.
\end{aligned}$$

□

Next two theorems give new upper bound of the left hand Hermite-Hadamard inequality for s-convex functions in the fourth sense.

Theorem 2.2. *Let $\psi : I \to \mathbb{R}$ be a differentiable function on I, $a,b \in I^o$ with $a < b$. If ψ' is s-convex function in the fourth sense on I and $\psi' \in L[a,b]$, then the following inequality holds,*

$$\psi\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b \psi(x)dx \leq \frac{(b-a)^2}{4}\frac{s}{(s+1)(2s+1)}\left(s\left(\psi'(a)+\psi'(b)\right) + 2(s+1)\psi'(\frac{a+b}{2})\right)$$

for $s \in (0,1]$.

Proof. According to [8], if ψ is differentiable function on I, and $\psi' \in L[a,b]$, then the following equality holds,

$$\psi\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b \psi(x)dx = \frac{(b-a)^2}{4}\int_0^1 (1-\alpha)\left(\psi'(\alpha a + (1-\alpha)\frac{a+b}{2}) + \psi'(\alpha b + (1-\alpha)\frac{a+b}{2})\right)d\alpha$$

and s-convexity in the fourth sense of ψ', we can write the following inequality,

$$\begin{aligned}\psi\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b \psi(x)dx &\leq \frac{(b-a)^2}{4}\left\{\int_0^1 (1-\alpha)\left[\alpha^{\frac{1}{s}}\psi'(a) + (1-\alpha)^{\frac{1}{s}}\psi'(\frac{a+b}{2})\right.\right.\\ &\quad \left.\left. +\alpha^{\frac{1}{s}}\psi'(b) + (1-\alpha)^{\frac{1}{s}}\psi'(\frac{a+b}{2})\right]d\alpha\right\}\\ &= \frac{(b-a)^2}{4}\int_0^1 (1-\alpha)(\alpha^{\frac{1}{s}}\left(\psi'(a)+\psi'(b)\right) + 2(1-\alpha)^{\frac{1}{s}}\psi'(\frac{a+b}{2}))d\alpha\\ &= \frac{(b-a)^2}{4}\frac{s}{(s+1)(2s+1)}\left(s\left(\psi'(a)+\psi'(b)\right) + 2(s+1)\psi'(\frac{a+b}{2})\right).\end{aligned}$$

□

Corollary 2.3. *In Theorem 2.2, if we choose $s = 1$, the following inequality is obtained,*

$$\psi\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b \psi(x)dx \leq \frac{(b-a)^2}{24}\left(\psi'(a) + 4\psi'(\frac{a+b}{2}) + \psi'(b)\right).$$

The right hand side of the inequality in Theorem 2.2 is stated in terms of boundary points a, b and midpoint $\frac{a+b}{2}$. By using Lemma 2.1, we can have similar result stated in terms of only boundary points a and b.

Corollary 2.4. *In Theorem 2.2, if we consider Lemma 2.1, then we have the following inequality,*

$$\psi\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b \psi(x)dx \leq \frac{(b-a)^2}{4}\frac{s^2\left(2^{\frac{1}{s}}+4\right)}{(s+1)(2s+1)2^{\frac{1}{s}}}\left(\psi'(a)+\psi'(b)\right).$$

In this inequality, if we take $s = 1$, we get

$$\psi\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b \psi(x)dx \leq \frac{(b-a)^2\left(\psi'(a)+\psi'(b)\right)}{8}. \tag{2.2}$$

Theorem 2.5. *Let $\psi : I \to \mathbb{R}$ be a twice differentiable function on I, $a,b \in I^o$ with $a < b$. If $\psi'' \in L[a,b]$ and s-convex function in the fourth sense on I, then the following inequality holds,*

$$\frac{1}{b-a}\int_a^b \psi(x)dx - \psi\left(\frac{a+b}{2}\right) \leq \frac{(b-a)^2}{16}\frac{s(1+s)(1+2s) + s(2^{\frac{1}{s}+4} - 14s^2 - 7s - 1)}{(1+s)(1+2s)(1+3s)2^{\frac{1}{s}}}\left[\psi''(a)+\psi''(b)\right].$$

Proof. According to [5], if ψ is a twice differentiable function on I, and $\psi'' \in L[a,b]$, then the following equality holds,

$$\frac{1}{b-a}\int_a^b \psi(x)dx - \psi\left(\frac{a+b}{2}\right) = \frac{(b-a)^2}{16}\left[\int_0^1 \alpha^2\psi''(\frac{\alpha}{2}a + \frac{2-\alpha}{2}b)d\alpha + \int_0^1 \alpha^2\psi''(\frac{2-\alpha}{2}a + \frac{\alpha}{2}b)d\alpha\right]$$

and s-convexity in the fourth sense of ψ'', we can write the following inequality,

$$\begin{aligned}\frac{1}{b-a}\int_a^b \psi(x)dx - \psi\left(\frac{a+b}{2}\right) &\leq \frac{(b-a)^2}{16}\left\{\int_0^1 \alpha^2\left[\left(\frac{\alpha}{2}\right)^{\frac{1}{s}}\psi''(a) + \left(\frac{2-\alpha}{2}\right)^{\frac{1}{s}}\psi''(b)\right]d\alpha\right. \\ &\quad \left. + \int_0^1 \alpha^2\left[\left(\frac{2-\alpha}{2}\right)^{\frac{1}{s}}\psi''(a) + \left(\frac{\alpha}{2}\right)^{\frac{1}{s}}\psi''(b)\right]d\alpha\right\} \\ &= \frac{(b-a)^2}{16}\left\{\frac{s}{2^{\frac{1}{s}}(1+3s)}\psi''(a) + \frac{s(2^{\frac{1}{s}+4}s^2 - 14s^2 - 7s - 1)}{(1+s)(1+2s)(1+3s)2^{\frac{1}{s}}}\psi''(b)\right. \\ &\quad \left. + \frac{s(2^{\frac{1}{s}+4} - 14s^2 - 7s - 1)}{(1+s)(1+2s)(1+3s)2^{\frac{1}{s}}}\psi''(a) + \frac{s}{2^{\frac{1}{s}}(1+3s)}\psi''(b)\right\} \\ &= \frac{(b-a)^2}{16}\frac{s(1+s)(1+2s) + s(2^{\frac{1}{s}+4} - 14s^2 - 7s - 1)}{(1+s)(1+2s)(1+3s)2^{\frac{1}{s}}}\left[\psi''(a) + \psi''(b)\right].\end{aligned}$$

□

Next theorem gives new upper bound of the right hand Hermite-Hadamard inequality for functions whose second derivative is s-convex functions in the fourth sense.

Theorem 2.6. *Let $\psi : I \to \mathbb{R}$ be twice differentiable function on I, $a, b \in I^o$ with $a < b$. If $\psi'' \in L[a,b]$ and s-convex function in the fourth sense on I, then the following inequality holds,*

$$\frac{\psi(a)+\psi(b)}{2} - \frac{1}{b-a}\int_a^b \psi(x)dx \leq \frac{(b-a)^2}{2}\frac{s^2}{(2s+1)(3s+1)}\left(\psi''(b) + \psi''(a)\right)$$

for $s \in (0,1]$.

Proof. According to [7] , if ψ is a twice differentiable function on I, and $\psi'' \in L[a,b]$, then the following equality holds,

$$\frac{\psi(a)+\psi(b)}{2} - \frac{1}{b-a}\int_a^b \psi(x)dx = \frac{(b-a)^2}{2}\int_0^1 \alpha(1-\alpha)\psi''(\alpha a + (1-\alpha)b)d\alpha$$

and s-convexity in the fourth sense of ψ'', we get,

$$\begin{aligned}\frac{\psi(a)+\psi(b)}{2} - \frac{1}{b-a}\int_a^b \psi(x)dx &\leq \frac{(b-a)^2}{2}\int_0^1 \alpha(1-\alpha)(\alpha^{\frac{1}{s}}\psi''(a) + (1-\alpha)^{\frac{1}{s}}\psi''(b))d\alpha \\ &= \frac{(b-a)^2}{2}\frac{s^2}{(2s+1)(3s+1)}\left(\psi''(b) + \psi''(a)\right).\end{aligned}$$

□

Corollary 2.7. *In Theorem 2.6, if we choose $s = 1$, the following inequality is obtained;*

$$\frac{\psi(a)+\psi(b)}{2} - \frac{1}{b-a}\int_a^b \psi(x)dx \leq \frac{(b-a)^2\left(\psi''(b) + \psi''(a)\right)}{24}.$$

3. Applications

We consider the applications of our Theorems to the special means. Let us recall the following means for positive real numbers a, b.

Let a, b, p be positive number with $a \neq b$ and $p \neq 1$,

$$\begin{aligned} A(a,b) &= \frac{a+b}{2}, \\ M_p(a,b) &= \left(\frac{a^p+b^p}{2}\right)^{\frac{1}{p}}, \\ L_p(a,b) &= \begin{cases} a & , \text{ if } a=b \\ \left(\frac{a^p-b^p}{p(a-b)}\right)^{1/(p-1)}, & \text{else} \end{cases} \end{aligned}$$

are called Arithmetic mean, Power mean and Stolarsky mean (Generalized logarithmic mean) respectively.

Proposition 3.1. *Let $a, b \in \mathbb{R}_+$ with $a < b$ and s be fixed number in $(0, 1]$. The following inequality holds,*

$$\left[L_{\frac{s}{1+3s}}(a,b)\right]^{\frac{1}{s}+2} + \left[M_{\frac{1+2s}{s}}(a,b)\right]^{\frac{1}{s}+2} \geq \frac{(1+s)(b-a)^2}{(3s+1)}\left[M_{\frac{1}{s}}(a,b)\right]^{\frac{1}{s}}.$$

Proof. The assertion follows from Theorem 2.6 applied to the s-convex function in the fourth sense $\psi''(x) = -x^{\frac{1}{s}}$, $x \in [a, b]$

$$-\frac{s^2\left(a^{\frac{1}{s}+2}+b^{\frac{1}{s}+2}\right)}{2(1+s)(1+2s)} - \frac{1}{b-a}\int_a^b \frac{s^2 x^{\frac{1}{s}+2}}{(1+s)(1+2s)}dx \leq -\frac{(b-a)^2}{2}\frac{s^2}{(1+2s)(1+3s)}\left(b^{\frac{1}{s}}+a^{\frac{1}{s}}\right).$$

After some simple calculations, we get

$$\frac{b^{\frac{1}{s}+3}-a^{\frac{1}{s}+3}}{(b-a)(\frac{1}{s}+3)} + \frac{a^{\frac{1}{s}+2}+b^{\frac{1}{s}+2}}{2} \geq \frac{(1+s)(b-a)^2}{(1+3s)}\frac{b^{\frac{1}{s}}+a^{\frac{1}{s}}}{2}.$$

□

Proposition 3.2. *Let $a, b \in \mathbb{R}_+$ with $a < b$ and s be fixed number in $(0, 1]$. The following inequality holds,*

$$[A(a,b)]^{\frac{1}{s}+1} - \left[L_{\frac{s}{1+2s}}(a,b)\right]^{\frac{1}{s}+1} \geq \frac{(b-a)^2}{2(2s+1)}\left[2(s+1)[A(a,b)]^{\frac{1}{s}} + s\left[M_{\frac{1}{s}}(a,b)\right]^{\frac{1}{s}}\right].$$

Proof. The assertion follows from Theorem 2.2 applied to the function $\psi'(x) = -x^{\frac{1}{s}}$, $x \in [a, b]$, the following inequality is valid,

$$\frac{-\left(\frac{a+b}{2}\right)^{\frac{1}{s}+1}}{\frac{1}{s}+1} - \frac{1}{b-a}\int_a^b \frac{-x^{\frac{1}{s}+1}}{\frac{1}{s}+1}dx \leq \frac{(b-a)^2}{4}\frac{s}{(s+1)(2s+1)}\left[s(-a^{\frac{1}{s}}-b^{\frac{1}{s}}) - 2(s+1)\left(\frac{a+b}{2}\right)^{\frac{1}{s}}\right].$$

After some simple calculations, we get

$$\left(\frac{a+b}{2}\right)^{\frac{1}{s}+1} - \frac{a^{\frac{1}{s}+2}-b^{\frac{1}{s}+2}}{(b-a)\left(\frac{1}{s}+2\right)} \geq \frac{(b-a)^2}{2(2s+1)}\left[2(s+1)\left(\frac{a+b}{2}\right)^{\frac{1}{s}} + \frac{s(a^{\frac{1}{s}}+b^{\frac{1}{s}})}{2}\right].$$

□

Proposition 3.3. *Let $a, b \in \mathbb{R}_+$ with $a < b$ and s be fixed number in $(0, 1]$. The following inequality holds,*

$$[A(a,b)]^{\frac{1}{s}+1} - \left[L_{\frac{s}{1+2s}}(a,b)\right]^{\frac{1}{s}+1} \leq \frac{(b-a)^2(1+s)}{4s}\left[M_{\frac{1}{s}}(a,b)\right]^{\frac{1}{s}}.$$

Proof. Applying inequality (2.2) for convex function $\psi'(x) = x^{\frac{1}{s}}$, $x \in [a, b]$, the following inequality is obtained,

$$\frac{s}{1+s}\left(\frac{a+b}{2}\right)^{\frac{1}{s}+1} - \frac{1}{b-a}\int_a^b \frac{x^{\frac{1}{s}+1}}{\frac{1}{s}+1}dx \leq \frac{(b-a)^2}{8}\left(b^{\frac{1}{s}}+a^{\frac{1}{s}}\right).$$

If we do the necessary calculations, we get the following inequality,

$$\left(\frac{a+b}{2}\right)^{\frac{1}{s}+1}-\frac{b^{\frac{1}{s}+2}-a^{\frac{1}{s}+2}}{(b-a)(\frac{1}{s}+2)}\leq\frac{(1+s)(b-a)^2}{4s}\frac{b^{\frac{1}{s}}+a^{\frac{1}{s}}}{2}.$$

□

Proposition 3.4. *Let $a,b\in\mathbb{R}_+$ with $a<b$. The inequality holds,*

$$\left[M_{\frac{1}{s}+2}(a,b)\right]^{\frac{1}{s}+2}-\left[L_{\frac{s}{3s+1}}(a,b)\right]^{\frac{1}{s}+2}\geq\frac{(s+1)(b-a)^2}{s^2(3s+1)}\left[M_{\frac{1}{s}}(a,b)\right]^{\frac{1}{s}}$$

for all $s\in(0,1]$.

Proof. The assertion follows from Theorem 2.6 applied to the function $\psi''(x)=-x^{\frac{1}{s}}$, $x\in[a,b]$,

$$-\frac{s^2}{(s+1)(2s+1)}\frac{a^{\frac{1}{s}+2}+b^{\frac{1}{s}+2}}{2}+\frac{s}{(s+1)}\frac{1}{(b-a)}\int_0^1\frac{x^{\frac{1}{s}+2}}{\frac{1}{s}+2}dx\leq-\frac{(b-a)^2}{(2s+1)(3s+1)}\frac{a^{\frac{1}{s}}+b^{\frac{1}{s}}}{2}.$$

After some simple calculations, we get

$$\frac{a^{\frac{1}{s}+2}+b^{\frac{1}{s}+2}}{2}-\frac{b^{\frac{1}{s}+3}-a^{\frac{1}{s}+3}}{\left(\frac{1}{s}+3\right)(b-a)}\geq\frac{(s+1)(b-a)^2}{s^2(3s+1)}\frac{a^{\frac{1}{s}}+b^{\frac{1}{s}}}{2}. \tag{3.1}$$

□

Using the theorems given in main results, we can obtain some inequalities involving digamma function. We present only one of them as an example in the following proposition.

Proposition 3.5. *Let $x>5$. Then,*

$$\Psi(x)\leq\frac{x(2x-5)(1+x)}{6(x-4)(x-2)(x-1)}-\gamma,$$

where $\Psi(x)$ is digamma function, i.e.

$$\Psi(x)=\frac{\Gamma'(x)}{\Gamma(x)}\ \textit{for}\ x>0$$

and γ is Euler-Mascheroni constant i.e. $\gamma\approx0.5772156649\ldots$.

Proof. Let us write $t=\frac{a}{b}$ and simplify the expression in (3.1). Then we have

$$\frac{1+t^{\frac{1}{s}+2}}{2}-\frac{s}{(1+3s)}\frac{1-t^{\frac{1}{s}+3}}{(1-t)}\geq\frac{(s+1)(1-t)^2}{s^2(3s+1)}\frac{1+t^{\frac{1}{s}}}{2}.$$

Let us integrate the expression with respect to t on $[0,1]$,

$$\int_0^1\frac{1+t^{\frac{1}{s}+2}}{2}dt-\frac{s}{3s+1}\int_0^1\frac{1-t^{\frac{1}{s}+3}}{1-t}dt\geq\frac{s+1}{2s^2(3s+1)}\int_0^1(1-t)^2\left(1+t^{\frac{1}{s}}\right)dt,$$

hence, the following inequality is obtained

$$\int_0^1\frac{1-t^{\frac{1}{s}+3}}{1-t}dt\leq\frac{(3s+2)(4s+1)(5s+1)}{6(2s+1)(3s+1)}.$$

Using the integral representation of digamma function

$$\Psi(r)=\int_0^1\frac{1-t^{r-1}}{1-t}dt-\gamma,$$

where $r>0$. We have

$$\Psi(4+\frac{1}{s})+\gamma\leq\frac{(3s+2)(4s+1)(5s+1)}{6(2s+1)(3s+1)}.$$

The substitution $x = 4 + \frac{1}{s}$ above yields to

$$\begin{aligned} \Psi(x) + \gamma &\leq \frac{(\frac{3}{x-4}+2)(\frac{4}{x-4}+1)(\frac{5}{x-4}+1)}{6(\frac{2}{x-4}+1)(\frac{3}{x-4}+1)} \\ \Psi(x) &\leq \frac{x(2x-5)(1+x)}{6(x-4)(x-2)(x-1)} - \gamma \end{aligned}$$

for $x > 5$. For more information about the Digamma function, see [1]. □

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

Author Contribution Statement

I have read and agreed to the published version of the manuscript.

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