Osmangazi Journal of Educational Research Volume 8(1), Spring 2021

RESEARCH Open Access

Suggested Citation: Yenilmez, K., & Karakoca-Gürler, A. (2021). Sixth graders' levels of using mathematical thinking in problem-solving. *Osmangazi Journal of Educational Research, 8*(1), 111-128.

Submitted: 26/04/2021 **Revised:** 16/05/2021 **Accepted:** 16/05/2021

Sixth Graders' Levels of Using Mathematical Thinking in Problem-Solving

***Kürşat Yenilmez, **Ayşe Karakoca Gürler**

Abstract. In this study, using of mathematical thinking of sixth grade students in problem-solving were examined and whether these skills vary with students' gender, pre-school education in terms of the students' achievements in mathematics. The sample of this study constituted 1114 students whom are chosen stratified sampling method sixth grade students of Cankaya, Kecioren and Yenimahalle towns' schools of Ankara. Turkish translation of Mathematical thinking scale of Cai (2000) was used for collecting data. The scale consisted of 12 questions and the initial six questions were routine questions and the rests were not- routine. Frequencies, percentage, arithmetic mean, standard deviation, t test, one-way analysis of variance (ANOVA) were used for analyzing the data. Besides, while analyzing the reasoning and thinking skills in problem-solving process, qualitative study used to survey students' strategies on problems. Based on the findings of the study, students' mathematical thinking states are not changed by gender but pre-school education and mathematics success variables showed significant difference to their mathematical thinking states. In addition, the students' routine questions average higher than the average of not-routine questions. Result of qualitative research indicates the students have problem in reasoning, communication and flexible thinking skills. Moreover the students are observed to rank mostly routine algorithms and strategies that lead to solutions.

Key Words. Thinking, mathematical thinking, problem solving.

* Prof. Dr., Eskisehir Osmangazi University, Faculty of Education, Eskisehir, Turkey

e-mail: kyenilmez@ogu.edu.tr

** Mathematics Teacher, Ministry of National Education, İstanbul, Turkey

e-mail: aysegurler87@gmail.com

One of the important factors making difference in individuals' thinking skills and perspectives on life is teaching mathematics. One of the prominent concepts in this context is mathematical thinking as well, and moreover, learning and developing mathematical thinking ways in teaching mathematics should be set as a goal (NCTM, 2000; Haylock & Cockburn, 2003; Stacey, 2006). Components of mathematical thinking have been defined by various researchers (Liu, 2003; Mason, Burton & Stacey, 1985; Mubark, 2005; Tall, 2002). While Mason, Burton and Stacey (1991) offered four stages of mathematical thinking, i.e. specializing, generalizing, conjecturing, justifying and convincing, Hacısalihoglu et al. (2003), on the other hand, referring to the study of Mason, Burton and Stacey, stated that mathematical thinking process covers the components of elaboration, generalization, prediction and convincing. Tall (2002) suggested that mathematical thinking includes the components such as abstraction, synthesizing, generalizing, modeling, problem-solving, and proof. Analyzing the above mentioned statements, it can be said that mathematical thinking is a higher-degree thinking process that requires the use of skills in different dimensions of the problem by emphasizing them, rather than finding the answer to any problem. Previous studies highlight that discussing a problem from various aspects and setting a strategy is important to ensure the development of this high-level skill in students (Ferri, 2003; NCTM, 2000). As students focus on a problem, they learn to form new strategies and to solve new types of problems by redesigning the strategies they use. In this way, choosing the ways to be applied to achieve success in problem-solving is called strategy (Baykul, 2009). Understanding how students think and make an inference when solving a mathematical problem can shed light on how their learning occurs (Yeşildere & Türnüklü, 2007). Therefore, several different strategies can be used to solve a problem and there is no best or only one strategy for solving problems (Billstein et al., 2004). New elementary mathematics curriculum, too, points out that while evaluating students' problem-solving skills, problems that can be solved using different strategies should be included (MEB, 2009).

When the literature is reviewed, it is seen that there is a great deal of studies on subjects such as mathematical thinking conditions and development of students through problem-solving and strategies used in problem processes (Cai & Kenney, 2000; Cai, 2003; Pape, Bell & Yetkin, 2003; Lee, 2006), examining the effect of different teaching approaches-based learning processes on students' achievement and mathematical thinking processes (Bukova, 2008; Taşdemir, 2008; Bulut, 2009), the effect of different components such as gender and school location on the mathematical thinking process (Duran, 2005; Ma'Moon, 2005), and formation process of mathematical thinking (Yeşildere, 2006; Arslan & Yıldız, 2010).

In the light of the abovementioned points and researches conducted, this study, departing from the assumption that there cannot be a single strategy in the problem-solving process, aims to examine the use of mathematical thinking in the problem-solving process, which is thought to be effective in the process of students' use of mathematics and understanding its importance. In research, answers to the following questions were sought in line with this general purpose.

1. What are the levels of the elementary sixth graders in using mathematical thinking in problem-solving?

2. Is there a significant difference between the mathematical thinking levels of male and female students in problem-solving?

3. Is there a significant difference between the students who received pre-school education and who did not, in terms of mathematical thinking levels in problem-solving?

4. Is there a significant difference between the students who had different mathematics achievements in terms of mathematical thinking levels in problem-solving?

5. What are the strategies practiced by sixth graders when using mathematical thinking in problem-solving?

Method

Research Model

A mixed model was used in this study by discussing qualitative and quantitative research dimensions together properly to the research questions and the focus of the research. Triangulation, which refers to the use of multiple data collection methods, is a frequently used way. In the same study, using appropriate qualitative methods such as interview, observation, examination of records in addition to quantitative tools such as questionnaires, scales, is a triangulation (Patton, 1990). While the determination of students' use of mathematical thinking in the problem-solving process generates the quantitative dimension of the research; specifying the strategies they use in problem-solving, close monitoring, in-depth description and interpretation of events and facts in the situation studied constitute the qualitative aspect of the research.

Sampling

The research population includes sixth graders studying at elementary schools located at Çankaya, Keçiören and Yenimahalle districts of Ankara province, Turkey. The sample covers a total of 1114 students, including 597 males and 517 females, who study at sixth grades of 12 schools located at Çankaya, Keçiören and Yenimahalle districts of Ankara province and who were selected using stratified sampling method.

Data Collection

The Mathematical Thinking Scale, which was applied by Cai (2000) as a data collection tool in 2000, was used as data collection tool in the study, adopting into Turkish. This scale consists of 12 questions, where the first six items are of routine questions and the last six of non-routine questions. In these questions, students are expected to answer the questions and explain their answers.

Data Analysis

In the analysis of the data; independent samples t-test was used to determine whether the scores that express the mathematical thinking skills of elementary second stage students, differ by the gender variable and pre-school education, while one-way analysis of variance (ANOVA) was used to determine whether it differs by the pass mark. Answers given by students to the 12-item mathematical thinking scale of Cai (2000), which was applied after adopted into Turkish, were converted into qualitative and quantitative data and then analyzed.

In quantitative analysis, a grading key with a range of 0-4 were used. Each question was scored on 4 points, while students were evaluated on 48 points. For scoring 2-step questions, stage 1 was given 2 points and stage 2 was given 2 points, and for scoring 3-step questions, on the other hand, the first two stages were given 1 point each and final stage was given 2 points. The quantitative analysis score descriptions made considering the scoring criteria used by Cai (2000) in his study were given in Table 1 below.

Table 1.

Score Descriptions of Quantitative Analysis Questions

Score	**Score Descriptions**
4 points	It was given to answers that were correct in solving and explaining the problem, that express their thoughts with correct mathematical notation and symbols and that clearly express the line of reasoning and indicate that it has a full understanding.
3 points	It was given to answers that were correct except for a few minor errors or uncertainties in the way and explanation of solving the problem that express their thoughts with correct mathematical notation and symbols, and that clearly express the line of reasoning and indicates that it has a full understanding.
2 points	It was given to answers that indicate that the statements for the solution were inadequate in some respects, although the way and description of solving the problem show some understanding of the problem.
1 point	It was given to answers that show that they have limited knowledge about the way and explanation of solving the problem.
0 point	It was given to answers where the problem was solved incorrectly or in which the problem was left unanswered.

Different strategies used by students in problem-solving were categorized and analyzed in line with content analysis, one of the qualitative analysis. Content analysis refers to classifying and summarizing verbal and written data in terms of a specific problem or purpose, measuring certain variables or concepts and categorizing them by scanning in order to infer (Fox, 1969; adopted by Tavşancıl & Aslan, 2001). In terms of the content analysis techniques, on the other hand, frequency analysis was used. In the simplest terms, frequency analysis refers to quantitative (percentage and ratio) frequency of units. This type of analysis allows to understand the intensity and importance of a particular item (Tavşancıl & Aslan, 2001).

The qualitative analysis stages performed in three categories are given in Figure 1 below.

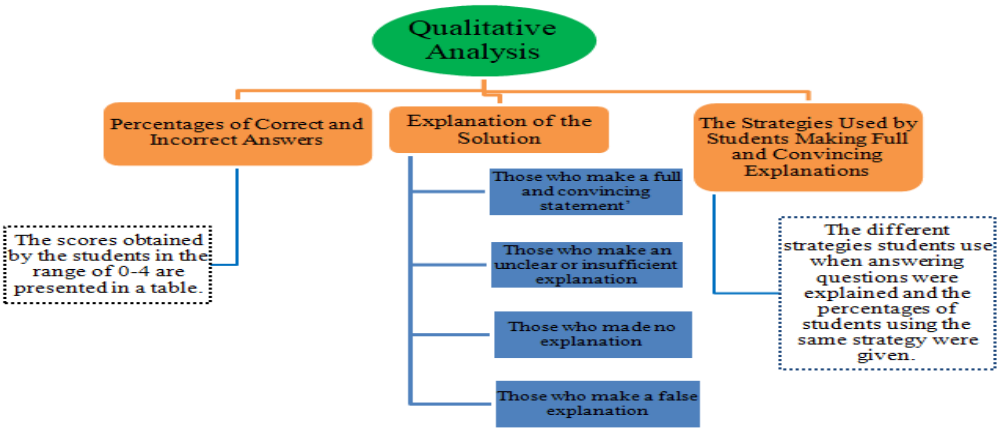

Figure 1. Qualitative Analysis Categories.

Categories given in Figure 1 are as follows: 'Percentages of Correct and Incorrect Answers', 'Explanation of the Solution', and 'The Strategies Used by Students Making Full and Convincing Explanations'. In the category of Percentages of Correct and Incorrect Answers, the scores obtained by the students in the range of 0-4 are presented in a table. In the category of Explanation of the Solution, the answers of the students were examined under four titles. Answers of students whose solution is exactly correct are given under the title 'those who make a full and convincing statement'. Under the title 'those who make an unclear or insufficient explanation', the situations in which the student's explanation is not understood and is insufficient and the answers of the students who answered the question but did not make any explanation are meant. Students who left the question blank and did not answer were given under the title 'those who made no explanation'. The title 'those who make a false explanation' covers the answers of the students who solved and explained the question incorrectly. In the category of 'The Strategies Used by Students Making Full and Convincing Explanations', the different strategies students use when answering questions were explained and the percentages of students using the same strategy were given.

Results

Mathematical thinking levels of sixth graders in the problem-solving process, examination of mathematical thinking levels by some variables, and findings regarding the strategies they used in the problem-solving process were detailed below.

Mathematical Thinking Levels of Sixth Graders in the Problem-Solving Process

Mean and standard deviation of the scores representing the mathematical thinking levels of sixth graders in the problem-solving process is given in Table 2.

Table 2.

Mean and Standard Deviation of the Scores Representing the Mathematical Thinking Levels of Sixth Graders in The Problem-Solving Process

General Situation	N	Mean	Standard Deviation
Total (12 questions)	1114	26,42	11,281
First 6 questions	1114	14,84	6,605
Last 6 questions	1114	11,58	5,967

As seen in Table 2, students' mean score of mathematical thinking in the problem-solving process was found to be 26.42 out of 48 points in 12 questions. This score indicates that students have an average level of success. In addition, the average of the first 6 questions was 14.84, while

11.58 of the last 6 questions. The result of the dependent samples t-test applied to determine whether the scores expressing the mathematical thinking levels in the first 6 and the last 6 questions in the problem-solving process of the students were significant was given in Table 3.

Table 3.

Comparison of the Scores Expressing Mathematical Thinking Levels in the First 6 and Last 6 Questions in the Problem-Solving Process

	Mean	N	Standard Deviation	t	p
First 6 questions	14,84	1114	6,605	19,463	<0,001
Last 6 questions	11,58	1114	5,967		

As seen in Table 3, the difference between students' scores expressing their mathematical thinking levels in the first 6 and the last 6 questions was found to be significant (p<0,001). This finding indicates that students' achievement in the first 6 questions was higher, although their mean scores were close to each other.

Examination of Sixth Graders' Mathematical Thinking Levels in the Problem-Solving Process in Terms of Some Variables

Sixth graders' mathematical thinking levels in the problem-solving process were examined whether differed by students' gender, pre-school education and mathematics achievement. Students' t-test results, which was applied to determine whether their mathematical thinking levels in the problem-solving process differed by gender, were given in Table 4.

Table 4.

The Difference of Students' Mathematical Thinking Levels in the Problem-Solving Process by Gender Variable

	Gender	**N**	**Mean**	**Standard Deviation**	***t***	***p***
Total	Male	597	26,41	11,359	-0,037	0,971
	Female	517	26,44	11,202		
First 6 questions	Male	597	14,98	6,545	0,781	0,435
	Female	517	14,67	6,675		
Last 6 questions	Male	597	11,43	6,100	-0,934	0,350
	Female	517	11,76	5,810		

As seen in Table 4, students' mathematical thinking levels in the problem-solving process were found to show insignificant difference by gender variable (p=0,971 >α=0,05). In addition, it can be said that the average scores obtained by male and female students from the questions were close to each other in all three categories.

Students’ t-test results, which was applied to determine whether their mathematical thinking levels in the problem-solving process differed by pre-school education, were given in Table 5.

Table 5.

The Difference of Students’ Mathematical Thinking Levels in the Problem-Solving Process by Pre-School Education Variable

	Pre-school education	**N**	**Mean**	**Standard Deviation**	***t***	***p***
Total	Yes	566	29,81	10,525	10,671	< 0,001
	No	548	22,93	10,975		
First 6 questions	Yes	566	16,67	5,921	9,782	< 0,001
	No	548	12,95	6,744		
Last 6 questions	Yes	566	13,13	5,857	9,148	< 0,001
	No	548	9,98	5,652		

As seen in Table 5, students’ mathematical thinking levels in the problem-solving process were found to show significant difference by pre-school education variable ($p<0,001$). Besides, it is seen that total mean scores, mean scores of the first six questions and mean scores of the last six questions of students who received pre-school education were found to be higher in all three categories than students who did not.

The variance analysis results of students, which was applied to determine whether their mathematical thinking levels in the problem-solving process differed by mathematics achievement, were given in Table 6. Tukey’s multiple comparison test was used to determine which groups the differences obtained from variance analysis resulted from. Statistically significant difference level was taken as 0.05.

Table 6.

The Difference of Students' Mathematical Thinking Levels in the Problem-Solving Process by Mathematics Achievement Variable

Mathematical Thinking	Source	Sum of Squares	SD	Mean Squares	*t*	*p*	Difference
Total	Inter-group	85554,080	4	21388,520	422,877	< 0,001	G-Z, O-Z, İ-Z, PZ, O-G, İ-G, P-G, İ-O, P-O, P-İ
	Intra-group	56091,626	1109	50,579			
	Total	141645,706	1113				
First 6	Inter-group	24734,344	4	6183,586	287,894	< 0,001	G-Z, O-Z, İ-Z, PZ, O-G, İ-G, P-G, İ-O, P-O, P-İ
	Intra-group	23819,893	1109	21,479			
	Total	48554,238	1113				
Last 6	Inter-group	18458,283	4	4614,571	241,752	< 0,001	G-Z, O-Z, İ-Z, PZ, O-G, İ-G, P-G, İ-O, P-O, P-İ
	Intra-group	21168,619	1109	19,088			
	Total	39626,902	1113				

As seen in Table 6, students' mathematical thinking levels in the problem-solving process were found to show a significant difference by pass mark variable (p<0,001). In all three categories; mathematical thinking levels of students in the problem-solving process were found to show significant different, whose mathematics achievement level was 'fail' compared to those with 'pass, average, good and very good' pass marks; whose mathematics achievement level was 'pass' compared to those with 'average, good and very good' pass marks; whose mathematics achievement level was 'average' compared to those with 'good and very good' pass marks; whose mathematics achievement level was 'good' compared to those with 'very good' pass marks. This difference is thought to be due to the increase in mathematical thinking levels of students as their pass marks increase and due to the fact that they orient to the subjects as they study mathematics and thus their skills such as prediction and interpretation improve

Examination of Using Mathematical Thinking in the Process of Solving Routine Problems

Elementary sixth graders' mean and standard deviation of Questions 1-6 in using mathematical thinking levels in problem-solving process were given in Table 7.

Table 7.

Mean and Standard Deviation of Students' Scores Regarding Questions 1-6

Questions (1-6)	N	Mean	Standard Deviation
Question 1	1114	3,53	1,044
Question 2	1114	2,38	1,681
Question 3	1114	1,45	1,669
Question 4	1114	2,54	1,834
Question 5	1114	2,51	1,497
Question 6	1114	2,43	1,695

Examining students' mean scores regarding the first six questions in Table 7, it is seen that the highest mean score expected from each question out of 24 points was 4. It is seen that the students got the highest mean score out of 4 points in the first question, in which they were expected to use the arithmetic mean. It is observed that the mean scores of Questions 2, 4, 5 and 6, which do not require the use of interpretation and prediction skills, were close to each other. Although one of the ways that enable students solve question 2, as in Question 1, was the arithmetic mean, it is observed that students' mean score in question 1 was 3, 53, while 2,38 in question 2. The reason for this decrease is thought to be the fact that the students were asked the arithmetic mean in the second question from a different aspect rather than directly. Students were found to get the lowest mean score in Question 3. The reason why students got low scores from question 3 was thought to be due to inability to decide which operation they would perform in the problem and their lack of knowledge in 'area and ratio'.

Examination of Using Mathematical Thinking in the Process of Solving Non routine Problems

Elementary sixth graders' mean and standard deviation of questions 7-12 in using mathematical thinking levels in problem-solving process were given in Table 8.

Table 8.

Mean and Standard Deviation of Students' Scores Regarding Questions 7-12

Questions (7-12)	N	Mean	Standard Deviation
Question 7	1114	1,86	1,697
Question 8	1114	1,25	1,693
Question 9	1114	2,43	1,735
Question 10	1114	2,37	1,471
Question 11	1114	2,00	1,383
Question 12	1114	1,68	1,345

Examining students' mean scores regarding the last six questions in Table 8, it is seen that the highest mean score expected from each question out of 24 points was 4. It is seen that the students got the highest mean score out of 4 points in Question 9, and that the mean scores of questions 9 and 10 had similar values. Most of the students were found to perform division operation in Question 9 correctly but did not interpret on the situation by using their prediction skills. This may be related to the interpretation skills of the students or may be due to the lack of their non-math skills. In Question 10, where students were asked to predict the area of the island using the given, it was observed that they thought that their predictions were not necessarily correct and expressed their opinions by their own opinions rather than using the information given in the question, and that they did not give any explanation by writing an approximate answer. Students were found to get the lowest score in Question 8. When examining the answers given to the question, it was observed that many students try to reach the solution by trying the numbers one by one. This is thought to be due to the lack of information on 'LCM' (least common multiple), one of the ways that provides students with a shortcut. Analyzing Table 7 and Table 8, it is observed that the mean scores of the students in Questions 1-6, except for Question 3, were higher than in Questions 7-12. This finding indicates that students were more successful with routine questions compared to non-routine questions.

Strategies Practiced by Sixth Graders in the Problem-Solving Process

This part examines the findings regarding the strategies practiced by students in the problem-solving process, through Question 4.

Findings regarding Question 4: The actual distance between Çanakkale and Ezine is 54 km. The distance between Çanakkale and Ezine on the map is 3 cm. Accordingly, if the distance between Ezine and Susurluk is 12 cm on the map, what is the actual distance between Ezine and Susurluk? Explain how you found the answer.

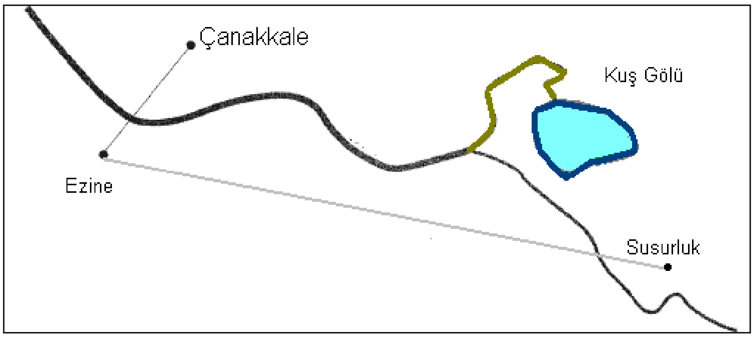

Figure 2. Map for Question 4.

In Question 4, students were expected to answer the desired point in the problem using the distance on the map and the actual distance. As it necessitates students place information about the problem on the given map and think about the situation, this question requires the ability to use visual and quantitative information in problem-solving.

Percentages of Correct and Incorrect Answers

58.4% of the students gave the correct answer to the question, while 41.6% gave the incorrect answer. Table 9 shows the distribution of scores given to students' answers.

Table 9.

Distribution of scores got from Question 4

Question 4	**f**	**%**
0	360	32,3
1	2	0,2
2	74	6,6
3	27	2,4
4	651	58,4
TOTAL	1114	100

Explanation of Solutions

Table 10 shows the findings regarding students'' explanation of their solutions.

Table 10.

Percentages of students who made different levels of explanation in Question 4

Category	**A Sample Answer**	**f**	**%**
Those Who Make a Full and Convincing Explanation	Shown in Strategy 1 and 2	651	58
Those Who Make an Unclear or Insufficient Explanation	$54 \times 12 = 648$	103	9
Those Who Made No Explanation	...	186	17
Those Who Make a False Explanation	$\frac{54 \times 3}{12}$	174	16
Total		1114	100

58% of the students correctly explained their thoughts on the solution of the problem. 9% of the students solved half of the point that will reach them to the correct answer but did not complete it (like 54x12=648).

Strategies Used by Students Making a Full and Convincing Statement

The findings regarding the strategies used by students who made a full and convincing statement were given in Table 11.

Table 11.

The strategies used by the students who made a full and convincing statement in Question 4

	f	**%**
Strategy 1: The students first find the equivalent of 1 cm of the actual distance between Çanakkale and Ezine (18x3=54). Then they find the actual distance by multiplying 12 with the result obtained (18x12=216).	303	46
Strategy 2: The students first find the equivalent of 1 cm of the distance between Çanakkale and Ezine on the map (4 x3=12). Then they find the actual distance by multiplying 54 with the result obtained (54x4=216).	171	26
Strategy 3: The student reaches the result by making a direct proportion to find the actual distance.($\frac{3}{12}=\frac{54}{x}$, x=216)	154	24
Strategy 4: The student finds the equivalent of actual distance of 54 km between Çanakkale and Ezine given in the question in 1 cm ($\frac{54}{3}=18$) and reaches the solution by adding 18 to each 1 cm (1cm=18, 2cm=36, 3cm=54,...12cm=216).	23	4
	651	100

When Table 11 was examined, it was seen that the students used four strategies for the Question 4. Students were found to reach the solution using similar mathematical sentences in Strategy 1 and Strategy 2, while using ratio and proportion in Strategy 3, and 4% of the students were found to express the statement in Strategy 1 as in Strategy 4, by using the addition operation.

Conclusion and Discussion

Students were found to have an average success in mathematical thinking skills in the problem-solving process. Besides, although students' mean scores of routine questions and non-routine questions were close to each other, their success levels in routine questions were found to be higher. However, in routine questions, asking the students the question from a different aspect rather than directly led students to have difficulty answering and their success to decline. In his study conducted with American and Chinese students, Cai (2000) found that there was a significant difference in routine questions in favor of students in China while a significant difference in non-routine questions in favor of American students. This finding led us to the conclusion that students in Turkey show similarities with Chinese students rather than American students in their reasoning and mathematical thinking skills in the problem-solving process. Students' mathematical thinking skills in problem-solving did not differ by the gender variable. However, there were different results regarding this situation in the literature. For example, Duran (2005) concluded that gender is an important factor in determining mathematical thinking skills and male students have higher mathematical thinking skills than female students. Ma'Moon (2005) suggested that female students have a significantly higher average than male students in total test scores and in three of the six dimensions of mathematical thinking. When the mathematical thinking skills of students in problem-solving are examined by the pre-school education variable, there was a significant difference in favor of students who received pre-school education. Duran (2005) concluded that students who receive pre-school education have better mathematical thinking skills compared to those who did not.

When the mathematical thinking skills of students in problem-solving are examined by the mathematics achievement variable, there was a significant difference in favor of students with high mathematics achievement. Taşdemir (2008) stated that students who use mathematical processes at a high level in Science and Technology course problems use problem-solving processes effectively, moreover, he pointed out that students who display the mathematical processes in the problems at medium and low levels reach the results with the intuitionistic solution without using mathematical reasoning and formulation in problem-solving.

It was concluded that students' skills to use visual and quantitative information were better in the first 6 questions consisting of closed-ended questions compared to the last 6 questions consisting of open-ended questions. In addition, students more successful in operational problems that can be solved using the information provided directly, compared to problems requiring reasoning, interpretation and flexibility skills. A similar result was obtained in the study conducted by Yeşildere and Türnüklü (2007). The answers given by the students showed that they experienced problems in applying the information given in the question and in explaining the solution of the problem and it is thought that this was due to the fact that they did not acquire mathematical information conceptually and due to lack of communication skills. This result coincides with the results of similar studies in the literature (Blitzer, 2003; Bukova, 2008; Dreyfus, 1991; Tall, 1997).

It was observed that students preferred to give examples instead of solving some questions by explaining them with mathematical statements. The study of Yeşildere (2006) gives similar results. Besides, as they gave only one answer, students were found to be unable to use their flexible thinking skills sufficiently in problems with multiple solutions, and it was thought that the reason for this may be due to the fact that they reached several values but thought that they finished the problem solution before completing it, since multiple cases were requested in the problem. Similar results to this situation were also stated in the studies of Arslan and Yıldız (2010) and Özer and Arıkan (2002).

It was observed that students who made a full and convincing explanation used the prediction and control strategy more than other strategies. However, it was seen that they thought that their predictions were not necessarily correct in some points, and made predictions based on their own opinions rather than using what was given in the question. Similar to this conclusion, Altun and Arslan (2006) found that students expressed the question using the prediction and control strategy, with case studies, randomly, and by giving values systematically, instead of making a mathematical explanation. When the literature was reviewed, it was seen that Bukova and Alkan (2005) reached different conclusions regarding the use of prediction skill. In their study, they concluded that the prospective teachers did not use their predictive skills, and they performed a lot of unnecessary procedures. When the strategies used by students who made a full and convincing explanation in problem-solving were examined as a percentage; it was identified that the percentages of strategies that require visual skills such as drawing diagrams and creating tables strategies were lower than the strategies that provided a solution with routine algorithms and symbolic statements.

In line with these results, for students to acquire important skills in problem-solving process such as interpreting, predicting, reasoning; it can be stated that it would be beneficial to include problems that do not have a single correct answer but can be solved with different strategies in the education process, or the problems with incomplete or excessive information, or the problems that require the interpretation of tables and graphics in addition to drawing figures or drawings, and real life related problems.

About Authors

First Author: Kürşat Yenilmez is a member of Eskişehir Osmangazi University. He works at the Faculty of Education. He is currently working at the Mathematics and Science Education Department. He completed his doctorate at Eskişehir Osmangazi University and his subject is on Mathematics. He mainly works in the fields of Mathematics Education.

Second Author: Ayşe Karakoca Gürler is a teacher at Ministry of National Education. She works at Hikmet Uluğbay Middle school as a mathematics teacher. She completed her master's degree at Eskişehir Osmangazi University and her PhD at Anadolu University. She mainly works in the fields of mathematics education.

Conflict of Interest

It has been reported by the authors that there is no conflict of interest.

Funding

No funding was received

Ethical Standards

We have carried out the research within the framework of the Helsinki Declaration. The participants are volunteers.

ORCID

Kürşat Yenilmez https://orcid.org/0000-0001-6256-4686

Ayşe Karakoca Gürler https://orcid.org/0000-0002-6218-6950

References

Alkan, H., & Bukova-Güzel, E. (2005). Öğretmen adaylarında matematiksel düşünmenin gelişimi. *Gazi Eğitim Fakültesi Dergisi, 25*(3), 221-236.

Altun, M., & Arslan, Ç. (2006). İlköğretim öğrencilerinin problem çözme stratejilerini öğrenmeleri üzerine bir çalışma. *Uludağ Eğitim Fakültesi Dergisi*, *19*(1), 1-21.

Arslan, S., & Yıldız, C. (2010). 11. sınıf öğrencilerinin matematiksel düşünmenin aşamalarındaki yaşantılarından yansımalar. *Eğitim ve Bilim, 35*(156), 17-31.

Baykul, Y. (2009). İlköğretimde matematik öğretimi 6-8. sınıflar için. Ankara: Pegem A Yayıncılık.

Billstein, R., Libeskind, S., & Johnny, W. (2004). A problem solving approach to matematics for elementary school teachers. New York: Addison Wesley Longman.

Blitzer, R. (2003). Thinking mathematically. New Jersey, Prentice Hall.

Bukova, E. (2008). Yapılandırmacı öğrenme yaklaşımının matematik öğretmen adaylarının matematiksel düşünme süreçlerine olan etkisi. *e-Journal of New World Sciences Academy*, *3*(4), 678-688.

Bulut, M. (2009). İşbirliğine dayalı yapılandırmacı öğrenme ortamlarında kullanılan bilgisayar cebiri sistemlerinin (BCS), matematiksel düşünme, öğrenci başarısına ve tutumuna etkisi. [Unpublished doctoral dissertation]. Gazi Üniversitesi, Ankara.

Cai, J., & Kenney, P.A. (2000). Fostering mathematical thinking through multiple solutions. *Mathematics Teaching in the Middle School*, *5*(8), 534-539.

Cai, J. (2000). Mathematical thinking involved in U.S. and Chinese students' solving of process-constrained and process-open problems. *Mathematical Thinking and Learning*, *2*(4), 309–340.

Cai, J. (2003). Singaporean students mathematical thinking in problem solving and problem posing: an exploratory study. *International Journal of Mathematics Education in Science and Technology*, *34*(5), 719-737.

Dreyfus, T. (1991). Advanced mathematical thinking processes. In Advanced Mathematical Thinking D. Tall (Ed.), pp. 25-41. Dordrecht: Kluwer.

Duran, N. (2005). Matematiksel düşünme becerilerine ilişkin bir araştırma. [Unpublished graduate dissertation]. Hacettepe Üniversitesi, Ankara.

Ferri, R. B. (2003). Mathematical thinking styles-An emprical study. Retrieved from www.erzwiss.unihamburg.de/Personal/Gkaiser/pdf-dok/borrom2.pdf on 11.04.2011.

Hacısalihoğlu, H. H., Mirasyedioğlu, S., & Akpınar, A. (2003). Matematik öğretimi: İlköğretim 1-5. Ankara: Asil Yayın Dağıtım.

Haylock, D., & Cockburn, A. (2003). Understanding mathematics in the lower primary years. London: Paul Champman Publishing.

Lee, K. (2006). Teacher's knowledge of middle school students' mathematical thinking in algebra word problem solving. Dissertation, UMI: AAT 3247827

Liu, P. H. (2003). Do teachers need to incorporate the history of mathematics in their teaching?. *The Mathematics Teacher*, *96*(6), 416-421.

Ma'Moon, M. M. (2005). Mathematical thinking and mathematics achievement of students in the year 11 scientific stream in Jordan. Dissertation.

Mason, J., Burton, L., & Stacey, K. (1991). Thinking mathematically. England, Addison-Wesley Publishers, Wokingham.

MEB (2009). İlköğretim matematik dersi 6-8. sınıflar öğretim programı ve kılavuzu, Ankara: TTK Başkanlığı.

Mubark, M. (2005). *Mathematical thinking and mathematical achievement of students in the year of 11 scientific stream in Jordan*. [Unpublished doctoral dissertation]. New Castle.

National Council of Teachers of Mathematics (2000). Principles and standarts for school mathematics, Reston/VA: National Council of Teachers of Mathematics.

Özer, Ö., & Arıkan, A. (2002, Eylül). *Lise matematik derslerinde öğrencilerin ispat yapma düzeyleri*. V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, Orta Doğu Teknik Üniversitesi, Ankara.

Pape, S. J., Bell, C. V., & Yetkin, I. E. (2003). Developing mathematical thinking and self-regulated learning: a teaching experiment ın a seventh- grade mathematics classroom. *Educational Studies in Mathematics, 53,* 179– 202.

Patton, M. Q. (1990). Qualitative evaluation and research methods (2. edition). London: Sage Pub.

Stacey, K. (2006). What is mathematical thinking and why is it important? *APECTsukuba International Conference,* Tokyo and Sapporo, Japan. Retrieved from http://www.apecneted.org/resources/files/12_3-4_06_1_Stacey.pdf on 01.04.201.

Tall, D. (1997). From school to university: the transition from elementary to advanced mathematical thinking. Australasian Bridging Conference in Mathematics, New Zealand.

Tall, D. (2002). Advanced mathematical thinking. USA: Kluwer Academic Publishers.

Taşdemir, A. (2008). Matematiksel düşünme becerilerinin ilköğretim öğrencilerinin fen ve teknoloji dersindeki akademik başarıları, problem çözme becerileri ve tutumları üzerine etkileri. [Unpublished doctoral dissertation]. Gazi Üniversitesi, Ankara.

Tavşancıl, E., & Aslan, E. A. (2001). Sözel, yazılı ve diğer materyaller için içerik analizi ve uygulama örnekleri, Ankara: Epsilon Yayınları.

Yeşildere, S. (2006). Farklı matematiksel güce sahip ilköğretim 6, 7 ve 8. sınıf öğrencilerinin matematiksel düşünme ve bilgiyi oluşturma süreçlerinin incelenmesi. [Unpublished doctoral dissertation]. Dokuz Eylül Üniversitesi, İzmir.

Yeşildere, S., & Türnüklü, E. (2007). Öğrencilerin matematiksel düşünme ve akıl yürütme süreçlerinin incelenmesi. *Ankara Üniversitesi Eğitim Bilimleri Fakültesi Dergisi*, *40*(1), 181–213.

Yıldırım, A., & Şimşek, H. (2004). Sosyal bilimlerde nitel araştırma yöntemleri. Ankara: Seçkin Yayıncılık.