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Improved Estimators For The Population Mean Under Non-Response

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Highlights

- This paper focuses on the estimation of population mean in the sampling theory.
- A new estimator is proposed using the exponential function in the study.
- A highly precise and more efficient estimation accuracy was obtained under the non-response case.

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Abstract

We propose a novel family of estimators for the population mean under non-response and obtain the MSE equation of the suggested estimator for each situation in theory. These theoretical conditions are applied to three popular data sets in literature and we see that the suggested estimators are more efficient than the traditional estimators, such as ratio, regression estimators, in Case 1; whereas, in Case 2, the suggested estimators are also more efficient than the Unal-Kadilar exponential estimators that are more efficient than the traditional estimators for the same data sets.

1. INTRODUCTION

The ratio, regression, product and exponential type estimators, using the information of the auxiliary variable, have been presented by many authors, such as Cochran [1,2], Bahl and Tuteja [3], Yadav and Kadilar [4], Singh and Pal [5], respectively, as:

$$t_{Ratio} = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{1}$$

$$t_{reg} = \bar{y} + b(\bar{X} - \bar{x}) \tag{2}$$

$$t_{ST} = \bar{y}exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{3}$$

$$t_{YK} = k\ \bar{y}\exp\left(\frac{(a_1\bar{X}+a_2)-(a_1\bar{x}+a_2)}{(a_1\bar{X}+a_2)+(a_1\bar{x}+a_2)}\right), \tag{4}$$

$$t_{SP} = \bar{y}\left(\frac{(a_1\bar{X}+a_2)-(a_1\bar{x}+a_2)}{(a_1\bar{X}+a_2)+(a_1\bar{x}+a_2)}\right)\exp\left(\frac{a_1(\bar{X}-\bar{x})}{a_1(\bar{X}+\bar{x})+2a_2}\right) \tag{5}$$

where $\bar{x}$ and $\bar{y}$ are the sample means of the auxiliary *(x)* and the study *(y)* variables, respectively, $\bar{X}$ represents the population mean of *x*, regression coefficient is symbolized as *b* and (a_1, a_2) is either a real number or a function of known characteristics, such as the population coefficient of variation, standard deviation, skewness, kurtosis.

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Hansen and Hurwitz [6] propose the sub-sampling method as a solution to the non-response problem. Let $S = (S_1, S_2, \dots, S_N)$ consist of *N* units. From *N*, sample size n is drawn by the SRSWOR method. The population size *N* is composed of N_1 and N_2. Here, N_1 is the responding unit while N_2 is the non-responding unit in the population. Similarly, the sample size $n = (n_1 + n_2)$ is divided into 2 parts as responding unit (n_1) and non-responding unit (n_2). The $r = \frac{n_2}{j}\left(j > 1\right)$ units, a sub-sample size, are drawn from n_2, j is the inverse sampling rate. Using these notations, Hansen and Hurwitz [6] proposed the following estimator as

$$t_{HH} = w_1\bar{y}_1 + w_2\bar{y}_{2(r)} \tag{6}$$

where $w_1 = \frac{n_1}{n}$ and $w_2 = \frac{n_2}{n}$, $\bar{y}_1$ and $\bar{y}_{2(r)}$ represent the sample means of the study variable in n_1 units and *r* units, respectively. The variance of t_{HH} is

$$V(t_{HH}) = \bar{Y}^2\left(\lambda C_y^2 + \frac{W_2(j-1)}{n}C_{y(2)}^2\right) \tag{7}$$

where $\bar{Y}$ is the population mean of *y*, $\lambda = \frac{1-f}{n}$, $W_2 = \frac{N_2}{N}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$ and $C_{y(2)}^2 = \frac{S_{y(2)}^2}{\bar{Y}^2}$. Here, $f = \frac{n}{N}$, S_y^2 and $S_{y(2)}^2$ are the population variances of *y* when there is no non-responding and when there are N_2 non-responding units, respectively.

2. MATERIAL METHOD

When non-response is valid only on the study variable and $\bar{X}$ is known (this situation will be called as Case 1), Rao [7] adapts the ratio and regression estimators to Case 1, respectively, as:

$$t_R^* = \frac{\bar{y}^*}{\bar{x}}\bar{X} \tag{8}$$

$$t_{reg}^* = \bar{y}^* + b^*(\bar{X} - \bar{x}) \tag{9}$$

where $\bar{y}^*$ represents the sample mean of *y* under non-response and $b^* = \frac{S_{yx}^*}{S_x^{*2}}$. Here, S_x^{*2} is the population variance under non-response and S_{yx}^* is the population covariance between *x* and *y* under the non-response case.

MSE Equations of (8) and (9) are, respectively,

$$MSE(t_R^*) = \bar{Y}^2\left(\lambda\left(C_y^2 + C_x^2 + 2C_{yx}\right) + \frac{W_2(j-1)}{n}C_{y(2)}^2\right) \tag{10}$$

$$MSE(t_{reg}^*) = \bar{Y}^2\left(\lambda C_y^2\left(1 - 2\rho_{xy}^2\right) + \frac{W_2(j-1)}{n}C_{y(2)}^2\right) \tag{11}$$

where $C_x^2 = \frac{S_x^2}{\bar{X}^2}$, $C_{yx} = \rho_{yx}C_yC_x$. Here, ρ_{yx} is the correlation of the population between the *y* and *x*.

Singh *et al*. [8] adapt the exponential type estimators introduced by Bahl and Tuteja [3] to Case 1, as follows:

$$t_{ST}^* = \bar{y}^* exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{12}$$

and its MSE is given by

$$MSE(t_{ST}^{*}) = \bar{Y}^2 \left(\lambda \left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right) + \frac{W_2(j-1)}{n} C_{y(2)}^2\right). \quad (13)$$

Motivated by Yadav and Kadilar [4] and Singh and Pal [5], Unal and Kadilar [9] propose the novel estimator for Case 1 as follows:

$$t_{UKi}^{*} = k\, \bar{y}^{*} \left(\frac{a_{1i}\bar{X} + a_{2i}}{a_{1i}\bar{x} + a_{2i}}\right)^{\alpha} \exp\left(\frac{a_{1i}(\bar{X} - \bar{x})}{a_{1i}(\bar{X} - \bar{x}) + 2a_{2i}}\right), i = 1,2,\ldots,10 \quad (14)$$

where k is a suitable number for minimizing the MSE of the estimators in (14) and α is a constant taking the values of (-1, 0, 1) to create the family of estimators. The estimator in (14) whose MSE equation is as follows:

$$MSE_{min}(t_{UKi}^{*}) = \bar{Y}^2 \left(1 - \frac{A_1^2}{2A_2}\right), i = 1,2,\ldots,10 \quad (15)$$

where

$$A_1 = \lambda \left(C_x^2 \theta_i^2 \left(\alpha^2 + \frac{3}{4}\right) - C_{yx}\theta_i(1 + 2\alpha)\right) + 2$$

$$A_2 = \left(\lambda\left(2C_y^2 + 2\theta_i^2 C_x^2 + 4\alpha^2\theta_i^2 C_x^2 + 2\alpha\theta_i^2 C_x^2 - 4\theta_i C_{yx} + 8\alpha\theta_i C_{yx}\right) + \frac{W_2(j-1)}{n} C_{y(2)}^2\right).$$

Here

$$\theta_i = \frac{a_i\bar{X}}{a_i\bar{X} + b_i}, i = 1,2,\ldots,10.$$

When non-response is valid on *y* and *x* and $\bar{X}$ is known (this is referred to Case 2), Cochran [2] modifies the traditional ratio estimator in (1) as follows:

$$t_R^{**} = \frac{\bar{y}^*}{\bar{x}^*}\bar{X} \quad (16)$$

where $\bar{x}^*$ represents the sample mean of *x* under non-response.

MSE of (16) is

$$MSE(t_R^{**}) = \bar{Y}^2 \left(\lambda\left(C_y^2 + C_x^2 - 2C_{yx}\right) + \frac{W_2(j-1)}{n}\left(C_{y(2)}^2 + C_{x(2)}^2 - 2C_{yx(2)}\right)\right) \quad (17)$$

where $C_{x(2)}^2 = \frac{S_{x(2)}^2}{\bar{X}^2}$ and $C_{yx(2)} = \rho_{yx(2)C_{y(2)}C_{x(2)}}$. Note that $\rho_{yx(2)}$ is the coefficient of population correlation between *y* and *x* for the non-response group.

Cochran [2] adapts the regression estimator in (2) to Case 2 as

$$t_{reg}^{**} = \bar{y}^* + b^*(\bar{X} - \bar{x}^*) \quad (18)$$

and its MSE equation is given by

$$MSE(t_{reg}^{**}) = \bar{Y}^2 \left(\lambda C_y^2\left(1 - \rho_{xy}^2\right) + \frac{W_2(j-1)}{n}\left(C_{y(2)}^2 + \rho_{xy}^2 \frac{C_y^2}{C_x^2} C_{x(2)}^2 - 2\rho_{xy}\frac{C_y}{C_x} C_{y(2)}^2\right)\right). \quad (19)$$

Singh *et al*. [8] adapt the exponential type estimator in (3) to Case 2 as

$$t_{ST}^{**} = \bar{y}^* exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right) \tag{20}$$

whose MSE is

$$MSE(t_{ST}^{**}) = \bar{Y}^2\left(\lambda\left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right) + \frac{W_2(j-1)}{n}C_{y(2)}^2 + \frac{C_{x(2)}^2}{4} - C_{yx(2)}\right). \tag{21}$$

Unal and Kadilar [9] also propose a family of estimators for Case 2 by adapting (4) and (5) to Case 2 as

$$t_{UKi}^{**} = k\,\bar{y}^*\left(\frac{a_{1i}\bar{X}+a_{2i}}{a_{1i}\bar{x}^*+a_{2i}}\right)^{\alpha} \exp\left(\frac{a_{1i}(\bar{X}-\bar{x}^*)}{a_{1i}(\bar{X}-\bar{x}^*)+2a_{2i}}\right), i = 1,2,\dots,10 \tag{22}$$

and its MSE is

$$MSE_{min}(t_{UKi}^{**}) = \bar{Y}^2\left(1 - \frac{A_3^2}{2A_4}\right), i = 1,2,\dots,10 \tag{23}$$

where

$$A_3 = \theta_i^2\left(\left(\alpha^2 + \frac{3}{4}\right)\left(\lambda C_x^2 + \frac{W_2(j-1)}{n}C_{x(2)}^2\right) - \theta_i(1+2\alpha)\left(\lambda C_{yx} + \frac{W_2(j-1)}{n}C_{yx(2)}\right)\right) + 2$$

$$A_2 = \left(2\theta_i^2(2\alpha^2 + \alpha\alpha + 1)\left(\lambda C_x^2 + + \frac{W_2(j-1)}{n}C_{x(2)}^2\right) - \theta_i(4+8\alpha)\left(\lambda C_{yx} + \frac{W_2(j-1)}{n}C_{y(2)}^2\right)\right.$$
$$\left. + 2\left(\lambda C_y^2 + \frac{W_2(j-1)}{n}C_{y(2)}^2\right) + 2\right).$$

Further, Singh and Kumar [10], Kumar [11], Pal and Singh [12], Khare and Sinha [13] also consider different problems under non-response. Besides, Kumar and Sharma [14] and Sharma and Kumar [15] consider the problem of estimation for the population mean using the transformed auxiliary variable under non-response. In addition, Unal and Kadilar [9] consider the problem of improving the family of estimators for the population mean by using the exponential function in the presence of non-response.

3. THE SUGGESTED CLASSES OF ESTIMATORS

Motivated by Irfan *et al*. [16], we suggest novel ratio-type estimators having the exponential function for the population mean under the Case 1 as follows:

$$\bar{y}_{pro1i} = t_1\bar{y}^*\left(\frac{\bar{X}'}{\bar{x}'}\right) + t_2(\bar{X}' - \bar{x}')\exp\left(\frac{\bar{X}'-\bar{x}'}{\bar{X}'+\bar{x}'}\right), i = 1, 2, \dots, 6 \tag{24}$$

where $\bar{X}' = a_1\bar{X} + a_2$, $\bar{x}' = a_1\bar{x} + a_2$ and $\bar{y}^* = \frac{n_1\bar{y}_1+n_2\bar{y}_{n2}}{n}$.

Using the following notations,

$$\bar{y} = \bar{Y}(1+\varepsilon_0), \bar{x} = \bar{X}(1+\varepsilon_0), E(\varepsilon_0) = E(\varepsilon_1) = 0, E(\varepsilon_0)^2 = \lambda S_x^2,$$
$$E(\varepsilon_1)^2 = \lambda S_y^2 + (j-1)\frac{N_2}{N}\frac{S_{y2}^2}{n}, E(\varepsilon_0\varepsilon_1) = \lambda\rho_{xy}C_xC_y,$$

we obtain the MSE of (24) as follows:

$$MSE(\bar{y}_{pro1i}) = \bar{Y}^2[1 + t_1A + t_2^2R^2\lambda C_x^2 - 2t_1t_2RB - 2t_1C - 2R\emptyset_i\lambda C_x^2] + G, i=1,2,\dots,6, \tag{25}$$

where $\emptyset_1 = 1$, $\emptyset_2 = \frac{\bar{X}S_X}{\bar{X}+\beta_{2(x)}}$, $\emptyset_3 = \frac{\bar{X}\rho_{yx}}{\bar{X}\rho_{yx}+\beta_{2(x)}}$, $\emptyset_4 = \frac{\bar{X}C_X}{\bar{X}C_X+\rho_{yx}}$, $\emptyset_5 = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)}+\rho_{yx}}$, $\emptyset_6 = \frac{\bar{X}}{\bar{X}+\rho_{yx}}$,
$R = \frac{\bar{Y}}{\bar{X}}$, $A = 1 + \lambda C_y^2 + 3\emptyset_i \lambda C_x^2 - 4\emptyset_i \lambda \rho_{xy} C_x C_y$, $B = \lambda \rho_{xy} C_x C_y - \frac{3}{2}\emptyset_i \lambda C_x^2$,
$C = 1 + {\emptyset_i}^2 \lambda C_x^2 - \emptyset_i \lambda \rho_{xy} C_x C_y$.

We obtain the optimal equations of t_1 and t_2 from (25), respectively, as follows:

$$t_{1(\text{opt.})} = \frac{\lambda C_x^2 (2C + B\emptyset_i)}{2(A\lambda C_x^2 - B^2)}, \tag{26}$$
$$t_{2(\text{opt.})} = \frac{2BC + A\emptyset_i \lambda C_x^2}{2R(A\lambda C_x^2 - B^2)}. \tag{27}$$

Using (26) and (27) in (25), we get

$$MSE_{min}(\bar{y}_{pro1i}) = \frac{\bar{Y}^2 \lambda C_x^2}{4D^2}\left[\frac{4D^2}{\lambda C_x^2} - E^2(D - B^2) + F^2 - 2BEF - 4CED - 2\emptyset_i FD\right] + G \tag{28}$$

where $D = A\lambda C_x^2 - B^2$, $E = 2C - B\emptyset_i$, $F = 2BC + A\emptyset_i \lambda C_x^2$, $G = t_1^2 (j-1) \frac{N_2}{N} \frac{S_{y2}^2}{n}$.

For Case 2, we also propose the similar class of estimators in (24) as follows:

$$\bar{y}_{pro2i} = t_3 \bar{y}^* \left(\frac{\bar{X}'}{\bar{x}^{*\prime}}\right) + t_4(\bar{X}' - \bar{x}^{*\prime}) \exp\left(\frac{\bar{X}' - \bar{x}^{*\prime}}{\bar{X}' + \bar{x}^{*\prime}}\right), \; i = 1, 2, \ldots, 6 \tag{29}$$

where $\bar{X}' = a_1 \bar{X} + a_2$ and $\bar{x}^{*\prime} = a_1 \bar{x}^* + a_2$.

MSE of (29) is

$$MSE(\bar{y}_{pro2i}) = \bar{Y}^2[(1 + t_3 A + t_4^2 R^2 \lambda C_x^2 - 2t_3 t_4 RB - 2t_3 C - 2R\emptyset_i \lambda C_x^2) + H(1 + t_3 A' + t_4^2 R^2 \lambda C_{x2}^2 - 2t_3 t_4 RB' - 2t_3 C' - 2R\emptyset_i \lambda C_{x2}^2)], \; i = 1, 2, \ldots, 6, \tag{30}$$

where

$H = \frac{N_2(j-1)}{Nn}$, $A' = 1 + \lambda C_{y2}^2 + 3\emptyset_i \lambda C_{x2}^2 - 4\emptyset_i \lambda \rho_{xy} C_{x2} C_{y2}$,
$B' = \lambda \rho_{xy} C_{x2} C_{y2} - \frac{3}{2}\emptyset_i \lambda C_{x2}^2$, $C' = 1 + {\emptyset_i}^2 \lambda C_{x2}^2 - \emptyset_i \lambda \rho_{xy} C_{x2} C_{y2}$.

We obtain the optimal equations of t_3 and t_4 from (30), respectively, as follows:

$$t_{3(\text{opt.})} = \frac{MI - NJ}{KN - LI}, \tag{31}$$
$$t_{4(\text{opt.})} = \frac{K(MI - NJ) - J(KN - LI)}{I(KN - LI)}, \tag{32}$$

where $N = 2R\lambda(C_x^2 + HC_{x2}^2)$, $I = R(B + HB')$, $J = (C + HC')$, $K = (A + HA')$, $L = (B + HB')$, $M = \lambda\emptyset_i(C_x^2 + HC_{x2}^2)$.

Using (31) and (32) in (30), we get

$$MSE_{min}(\bar{y}_{pro2i}) = \frac{\bar{Y}^2 \lambda C_x^2}{4D^2}\left[\frac{4D^2}{\lambda C_x^2} - E^2(D - B^2) + F^2 - 2BEF - 4CED - 2\emptyset_i FD\right] + H\left[\frac{\bar{Y}^2 \lambda C_{x2}^2}{4{D'}^2}\left[\frac{4{D'}^2}{\lambda C_{x2}^2} - E'^2(D' - B'^2) + F'^2 - 2B'E'F' - 4C'E'D' - 2\emptyset_i F'D'\right]\right] \tag{33}$$

where $D' = A'\lambda C_{x2}^2 - {B'}^2$, $E' = 2C' - B'\emptyset_i$, $F' = 2B'C' + A'\emptyset_i \lambda C_{x2}^2$.

4. NUMERICAL FINDINGS

To demonstrate the efficiency of the proposed estimators, we employed Khare and Sinha [17] data set, which was also utilized in Unal and Kadilar [9] for the Population 1. Table 1 displays the descriptive statistics of the population.

Population 1. [Source: Unal and Kadilar [9]]
The study variable in this population is the number of agricultural laborers and the auxiliary variable is the village's area.

Table 1. *Parameters of Population 1*

$N = 96$	$\bar{X} = 144.87$	$\rho_{yx} = 0.77$	$C_{yx} = 0.8232$
$n = 40$	$\bar{Y} = 137.92$	$\rho_{yx(2)} = 0.72$	$C_{yx(2)} = 1.4077$
$W_2 = 0.25$	$C_y = 1.32$	$C_{y(2)} = 2.08$	$\beta_2(x) = 1.19997$
$\lambda = 0.01458$	$C_x = 0.81$	$C_{x(2)} = 0.94$	$f = 0.4167$

Table 2. *MSE values of suggested and other estimators under Case 1 for Population 1*

Estimators	***j*=3**	***j*=4**	***j*=5**	***j*=6**
t_{HH}	1512.053	2026.406	2540.759	3055.112
t_R^*	1237.294	1751.647	2266.000	2780.353
t_{BT}^*	1329.172	1843.525	2357.878	2872.231
t_{reg}^*	1225.476	1739.829	2254.182	2768.535
t_{UK1}^*	1179.842	1629.580	2057.202	2464.302
t_{UK2}^*	1179.682	1629.426	2057.055	2464.160
t_{UK3}^*	1179.995	1629.727	2057.344	2464.438
t_{UK4}^*	1180.027	1629.758	2057.373	2464.466
t_{UK5}^*	1180.104	1629.832	2057.445	2464.535
t_{UK6}^*	1179.461	1629.214	2056.850	2463.963
t_{UK7}^*	1179.881	1629.618	2057.239	2464.337
t_{UK8}^*	1179.800	1629.54	2057.164	2464.265
t_{UK9}^*	1180.132	1629.858	2057.470	2464.559
t_{UK10}^*	1179.401	1629.156	2056.795	2463.909
$\bar{y}_{pro11}$	**1205.314**	**1710.373**	**2215.433**	**2720.492**
$\bar{y}_{pro12}$	**1303.308**	**1802.645**	**2301.983**	**2801.321**
$\bar{y}_{pro13}$	**1213.734**	**1718.356**	**2222.979**	**2727.602**
$\bar{y}_{pro14}$	**1208.467**	**1713.363**	**2218.259**	**2723.155**
$\bar{y}_{pro15}$	**1206.601**	**1711.594**	**2216.587**	**2721.579**

$\bar{y}_{pro16}$	**1207.874**	**1712.801**	**2217.728**	**2722.654**

Table 3. *MSE values of suggested and other estimators under Case 2 for Population 1*

Estimators	***j*=3**	***j*=4**	***j*=5**	***j*=6**
t_{HH}	1512.053	2026.406	2540.7587	3055.112
t_R^{**}	777.9412	1062.618	1347.294	1631.970
t_{BT}^{**}	1046.972	1420.224	1793.4767	2166.729
t_{reg}^{**}	711.6378	969.7121	1227.7864	1485.861
$t_{UK\,1}^{**}$	702.2648	948.6857	1194.2139	1438.839
$t_{UK\,2}^{**}$	702.1777	948.6378	1194.2025	1438.862
$t_{UK\,3}^{**}$	702.3507	948.7350	1194.2291	1438.823
$t_{UK\,4}^{**}$	702.3692	948.7458	1194.2328	1438.82
$t_{UK\,5}^{**}$	702.4136	948.7723	1194.2425	1438.814
$t_{UK\,6}^{**}$	702.0609	948.5773	1194.1946	1438.903
$t_{UK\,7}^{**}$	702.2868	948.6982	1194.2174	1438.835
$t_{UK\,8}^{**}$	702.2419	948.6729	1194.2105	1438.845
$t_{UK\,9}^{**}$	702.4293	948.7818	1194.2461	1438.812
$t_{UK\,10}^{**}$	702.0302	948.5621	1194.194	1438.916
$\bar{y}_{pro21}$	**177.2127**	**296.0998**	**414.9868**	**533.8738**
$\bar{y}_{pro22}$	**318.9788**	**437.8658**	**556.7528**	**675.6399**
$\bar{y}_{pro23}$	**185.9033**	**304.7904**	**423.6774**	**542.5645**
$\bar{y}_{pro24}$	**180.4613**	**299.3484**	**418.2354**	**537.1224**
$\bar{y}_{pro25}$	**178.5382**	**297.4252**	**416.3123**	**535.1993**
$\bar{y}_{pro26}$	**179.8502**	**298.7372**	**417.6243**	**536.5113**

Population 2. [Source: Khare and Sinha [17]]
The study variable in this population is the number of literate persons in the village and the auxiliary variable is the number of workers in the village (Table 4).

Table 4. *Parameters of Population 2*

$N = 109$	$\bar{X} = 165.26$	$\rho_{yx} = 0.81$	$C_{yx} = 0.3023$
$n = 30$	$\bar{Y} = 145.3$	$\rho_{yx(2)} = 0.78$	$C_{yx(2)} = 1.4077$
$W_2 = 0.25$	$C_y = 0.76$	$C_{y(2)} = 2.68$	$\beta_2(x) = 1.1998$
$\lambda = 0.024$	$C_x = 0.68$	$C_{x(2)} = 0.57$	$f = 0.4167$

Table 5. *MSE values of suggested and other estimators under Case 1 for Population 2*

Estimators	*j*=3	*j*=4	*j*=5	*j*=6
t_{HH}	568.43545	658.2191	748.0027	837.7867
t_R^*	293.67645	383.4601	473.2437	563.0277
t_{BT}^*	385.55445	475.3381	565.1217	654.9057
t_{reg}^*	281.85845	371.6421	461.4257	551.2097
$t_{UK\,1}^*$	236.22445	261.3931	264.4457	246.9767
$t_{UK\,2}^*$	236.06445	261.2391	264.2987	246.8347
$t_{UK\,3}^*$	236.37745	261.5401	264.5877	247.1127
$t_{UK\,4}^*$	236.40945	261.5711	264.6167	247.1407
$t_{UK\,5}^*$	236.48645	261.6451	264.6887	247.2097
$t_{UK\,6}^*$	235.84345	261.0271	264.0937	246.6377
$t_{UK\,7}^*$	236.26345	261.4311	264.4827	247.0117
$t_{UK\,8}^*$	236.18245	261.3531	264.4077	246.9397
$t_{UK\,9}^*$	236.51445	261.6711	264.7137	247.2337
$t_{UK\,10}^*$	235.78345	260.9691	264.0387	246.5837
$\bar{y}_{pro11}$	**261.6964**	**342.186**	**422.677**	**503.167**
$\bar{y}_{pro12}$	**359.6904**	**434.458**	**509.227**	**583.996**
$\bar{y}_{pro13}$	**270.1164**	**350.169**	**430.223**	**510.277**
$\bar{y}_{pro14}$	**264.8494**	**345.176**	**425.503**	**505.830**
$\bar{y}_{pro15}$	**262.9834**	**343.407**	**423.831**	**504.254**
$\bar{y}_{pro16}$	**264.2564**	**344.614**	**424.972**	**505.325**

Table 6. *MSE values of suggested and other estimators under Case 2 for Population 2*

Estimators	*j*=3	*j*=4	*j*=5	*j*=6
t_{HH}	1444.5543	2015.95432	2587.354	3158.754
t_R^{**}	710.44248	1052.16632	1393.889	1735.612
t_{BT}^{**}	979.47328	1409.77232	1840.072	2270.371
t_{reg}^{**}	644.13908	959.26042	1274.382	1589.503
$t_{UK\,1}^{**}$	634.76608	938.23402	1240.809	1542.481
$t_{UK\,2}^{**}$	634.67898	938.18612	1240.798	1542.504
$t_{UK\,3}^{**}$	634.85198	938.28332	1240.824	1542.465
$t_{UK\,4}^{**}$	634.87048	938.29412	1240.828	1542.462
$t_{UK\,5}^{**}$	634.91488	938.32062	1240.838	1542.456
$t_{UK\,6}^{**}$	634.56218	938.12562	1240.79	1542.545

$t^{**}_{UK\,7}$	634.78808	938.24652	1240.813	1542.477
$t^{**}_{UK\,8}$	634.74318	938.22122	1240.806	1542.487
$t^{**}_{UK\,9}$	634.93058	938.33012	1240.841	1542.454
$t^{**}_{UK\,10}$	634.53148	938.11042	1240.789	1542.558
$\bar{y}_{pro21}$	**109.714**	**285.6481**	**461.582**	**637.516**
$\bar{y}_{pro22}$	**251.4801**	**427.4141**	**603.348**	**779.282**
$\bar{y}_{pro23}$	**118.4046**	**294.3387**	**470.273**	**646.207**
$\bar{y}_{pro24}$	**112.9626**	**288.8967**	**464.831**	**640.765**
$\bar{y}_{pro25}$	**111.0395**	**286.9735**	**462.908**	**638.842**
$\bar{y}_{pro26}$	**112.3515**	**288.2855**	**464.220**	**640.154**

Population 3. [Source: Khare and Srivastava [18]]
The study variable in this population is the cultivated area (in acres) and the auxiliary variable is the population of the village (Table 7).

Table 7. *Parameters of Population 3*

$N = 70$	$\bar{X} = 1755.53$	$\rho_{yx} = 0.778$	$C_{yx} = 0.3896$
$n = 35$	$\bar{Y} = 981.29$	$\rho_{yx(2)} = 0.445$	$C_{yx(2)} = 0.10437$
$W_2 = 0.2$	$C_y = 0.6254$	$C_{y(2)} = 0.4087$	$\beta_2(x) = 1.1998$
$\lambda = 0.0143$	$C_x = 0.8009$	$C_{x(2)} = 0.5739$	$f = 0.50$

Table 8. *MSE values of suggested and other estimators under Case 1 for Population 3*

Estimators	***j*=3**	***j*=4**	***j*=5**	***j*=6**
t_{HH}	4694.546	5843.574	6992.602	8141.63
t^{*}_{R}	4419.787	5568.815	6717.843	7866.871
t^{*}_{BT}	4511.665	5660.693	6809.721	7958.749
t^{*}_{reg}	4407.969	5556.997	6706.025	7855.053
$t^{*}_{UK\,1}$	4362.335	5446.748	6509.045	7550.82
$t^{*}_{UK\,2}$	4362.175	5446.594	6508.898	7550.678
$t^{*}_{UK\,3}$	4362.488	5446.895	6509.187	7550.956
$t^{*}_{UK\,4}$	4362.52	5446.926	6509.216	7550.984
$t^{*}_{UK\,5}$	4362.597	5447	6509.288	7551.053
$t^{*}_{UK\,6}$	4361.954	5446.382	6508.693	7550.481
$t^{*}_{UK\,7}$	4362.374	5446.786	6509.082	7550.855
$t^{*}_{UK\,8}$	4362.293	5446.708	6509.007	7550.783

$t^{*}_{UK\ 9}$	4362.625	5447.026	6509.313	7551.077
$t^{*}_{UK\ 10}$	4361.894	5446.324	6508.638	7550.427
$\bar{y}_{pro11}$	**4387.807**	**5527.541**	**6667.276**	**7807.010**
$\bar{y}_{pro12}$	**4485.801**	**5619.813**	**6753.826**	**7887.839**
$\bar{y}_{pro13}$	**4396.227**	**5535.524**	**6674.822**	**7814.120**
$\bar{y}_{pro14}$	**4390.96**	**5530.531**	**6670.102**	**7809.673**
$\bar{y}_{pro15}$	**4389.094**	**5528.762**	**6668.430**	**7808.097**
$\bar{y}_{pro16}$	**4390.367**	**5529.969**	**6669.571**	**7809.168**

Table 9. *MSE values of suggested and other estimators under Case 2 for Population 3*

Estimators	***j*=3**	***j*=4**	***j*=5**	***j*=6**
t_{HH}	5036.613	12309.65	19583.18	26856.72
t^{**}_{R}	4302.501	11345.86	18389.72	25433.58
t^{**}_{BT}	4571.532	11703.47	18835.9	25968.34
t^{**}_{reg}	4236.198	11252.95	18270.21	25287.47
$t^{**}_{UK\ 1}$	4226.825	11231.93	18236.64	25240.45
$t^{**}_{UK\ 2}$	4226.738	11231.88	18236.63	25240.47
$t^{**}_{UK\ 3}$	4226.911	11231.98	18236.66	25240.43
$t^{**}_{UK\ 4}$	4226.929	11231.99	18236.66	25240.43
$t^{**}_{UK\ 5}$	4226.973	11232.01	18236.67	25240.42
$t^{**}_{UK\ 6}$	4226.621	11231.82	18236.62	25240.51
$t^{**}_{UK\ 7}$	4226.847	11231.94	18236.64	25240.45
$t^{**}_{UK\ 8}$	4226.802	11231.91	18236.64	25240.46
$t^{**}_{UK\ 9}$	4226.989	11232.02	18236.67	25240.42
$t^{**}_{UK\ 10}$	4226.59	11231.8	18236.62	25240.53
$\bar{y}_{pro21}$	**3701.773**	**10579.34**	**17457.41**	**24335.48**
$\bar{y}_{pro22}$	**3843539**	**10721.11**	**17599.18**	**24477.25**
$\bar{y}_{pro23}$	**3710.463**	**10588.03**	**17466.10**	**24344.17**
$\bar{y}_{pro24}$	**3705.021**	**10582.59**	**17460.66**	**24338.73**
$\bar{y}_{pro25}$	**3703.098**	**10580.67**	**17458.74**	**24336.81**
$\bar{y}_{pro26}$	**3704.410**	**10581.98**	**17460.05**	**25338.12**

5. DISCUSSION

For Case 1, Table 2 presents the MSE values of the suggested estimators and estimators discussed in Sections 1 and 2 for various *j* values under Case 1 and we see that suggested estimators are more efficient than traditional ones, but not as efficient as estimators of Unal and Kadilar [9]. However, this result changes in Table 3 as Table 3 shows the MSE values of the suggested estimators and the mentioned estimators for various values of *j* under Case 2 and we see that the suggested estimators are more efficient than Hansen and Hurwitz [6] estimator, Cochran [2] ratio and regression estimators, Singh and Pal [5] exponential type estimator, and also Unal and Kadilar [9] family of estimators. When we examine Table 3 in detail, the suggested estimator, $\bar{y}_{pro21}$, is the most efficient estimator for all values of j. We also note that for both Cases in Tables 2 and 3, the MSE values of all estimators get bigger while *j* is increasing.

Similarly, Table 5 presents the MSE values of the suggested estimators and estimators discussed in Sections 1 and 2 for various *j* values under Case 1 and we see that suggested estimators are more efficient than traditional ones, but not as efficient as estimators of Unal and Kadilar [9]. However, this result again changes in Table 6 as Table 6 presents the MSE values of the suggested estimators and the mentioned estimators for various values of *j* under Case 2 and we see that suggested estimators are more efficient than Hansen and Hurwitz [6] estimator, Cochran [2] ratio and regression estimators, Singh and Pal [5] exponential type estimator, and also Unal and Kadilar [9] family of estimators. When we examine Table 6 in detail, the suggested estimator, $\bar{y}_{pro21}$, is again the most efficient estimator for all values of j. We also note that for both Cases in Tables 5 and 6, the MSE values of all estimators get bigger while *j* is increasing.

Same results of Tables 2-3 and Tables 5-6 are also valid for Tables 8-9. It means that for all populations, the most efficient estimator is the suggested estimator, $\bar{y}_{pro21}$ for Case 2 and for Case 1 the suggested estimators are more efficient than the traditional estimators.

6. CONCLUSION

We propose a novel family of estimators for the population mean under the non-response scheme having the exponential function for two scenarios. For both Case 1 and Case 2, the minimum MSE equation of the suggested estimator is obtained. For Case 1, we see that the suggested estimators are more efficient than classical estimators and for Case 2, the suggested estimators are also more efficient than the family of estimators suggested by Unal and Kadilar [9], besides the traditional estimators, by using the popular data sets in literature. Hence, we can conclude that suggested family of estimators is the best in literature under Case 2 in application. In the forthcoming studies, we hope to study the suggested estimators under Case 1 and Case 2 for both the stratified random sampling and for the ranked set sampling, as well.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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REFERENCES

[1] Cochran, W. G., "The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce", The Journal of Agricultural Science, 30(2): 262-275, (1940).

[2] Cochran, W. G., "Sampling Techniques", Third Edition, John Wiley and Sons, New-York, 448, (1977).

[3] Bahl, S., Tuteja, R. K., "Ratio and product type exponential estimators", Journal of Information and Optimization Sciences, 12(1): 159-164, (1991).

[4] Yadav, S.K., Kadilar, C., "Efficient family of exponential estimators for the population mean", Hacettepe Journal of Mathematics and Statistics, 42(6): 671-677, (2013).

[5] Singh, H.P., Pal, S.K., "A new chain ratio-ratio-type exponential estimator using auxiliary information in sample surveys", International Journal of Mathematics and its Applications, 3(4B): 37-46, (2015).

[6] Hansen, M.H., Hurwitz, W.N., "The problem of non-response in sample surveys", Journal of the American Statistical Association, 41(236): 517-529, (1946).

[7] Rao, P. S. R. S., "Ratio estimation with sub sampling the non-respondents", Survey Methodology, 12: 217–230, (1986).

[8] Singh, R., Kumar, M., Chaudhary, M. K., Smarandache, F., "Estimation of mean in presence of non-response using exponential estimator", arXiv preprint arXiv:0906.2462, (2009).

[9] Unal, C., Kadilar, C., "Improved family of estimators using exponential function for the population mean in the presence of non-response", Communications in Statistics-Theory and Methods, 50(1): 237-248, (2021).

[10] Singh, H.P., Kumar, S., "A regression approach to the estimation of finite population mean in presence of non-response", Australian and New Zealand Journal of Statistics, 50(4): 395-408, (2008).

[11] Kumar, S., "Improved exponential estimator for estimating the population mean in the presence of non response", Communications for Statistical Applications and Methods, 20(5): 357-366, (2013).

[12] Pal, S. K., Singh, H. P., "Estimation of mean with two-parameter ratio-product-ratio estimator in double sampling using auxiliary information under non-response", Journal of Modern Applied Statistical Methods, 17(2): 2–32, (2018).

[13] Khare, B. B., Sinha, R. R., "Estimation of product of two population means by multiauxiliary characters under double sampling the non-respondents", Statistics in Transition New Series, 20(3): 81–95, (2019).

[14] Kumar, S., Sharma, V., "Improved chain ratio-product type estimators under double sampling scheme", Journal of Statistics Applications and Probability Letters, 7(2): 87–96, (2020).

[15] Sharma, V., Kumar, S., "Estimation of population mean using transformed auxiliary variable and non response", Revista De Investigacion Operacional, 41(3): 438-444, (2020).

[16] Irfan, M., Javed, M., Lin, Z., "Enhanced estimation of population mean in the presence of auxiliary information", Journal of King Saud University, 31: 1373-1378, (2019).

[17] Khare, B. B., Sinha, R. R., "On class of estimators for population mean using multi-auxiliary characters in the presence of non-response", Statistics in Transition, 10(1): 3-14, (2009).

[18] Khare, B. B., Srivastava, S., "Estimation of population mean using auxiliary character in presence of non-response", National Academic Science Letters India, 16(3): 111-114, (1993).