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Two-Dimensional Vector Boson Oscillator

Semra GÜRTAŞ DOĞAN[1]

Abstract

We introduce two-dimensional vector boson oscillator by using the generalized vector boson equation that derived as an excited of Zitterbewegung. We write the relativistic vector boson equation for a relativistic spin-1 particle and introduce the oscillator coupling via non-minimal substitutions. The corresponding equation gives a set of equations. By solving this equation set we obtain the components of the corresponding symmetric spinor and accordingly obtain the energy eigen-values for two-dimensional vector boson oscillator. This energy spectrum shows that the oscillator frequency couples with the spin of vector field in question and accordingly we discuss the results.

Keywords: Vector Boson Oscillator, Relativistic Quantum Mechanics, Quantum Oscillator

1. INTRODUCTION

It is well known that obtaining exact solutions of relativistic wave equations such as the Klein-Gordon equation (KGE), the Dirac equation, the Duffin-Kemmer-Petiau (DKP) equation and the well-established many-body equations are regarded as very important in the modern physics. Obtaining exact solutions for relativistic quantum oscillators such as Dirac oscillator (DO) [1], Klein-Gordon oscillator (KGO) [2], Duffin-Kemmer-Petiau oscillator (DKPO) [3,4] and relativistic spin-1 oscillator [5,6] can be considered between among them. The DO has been introduced in terms of a new type of interaction in the Dirac equation and corresponding form of this equation is linear in coordinate and momentum [1]. This form of the Dirac equation has provided the usual harmonic oscillator solution including strong spin-orbit coupling term in the non-relativistic limit. The DO name, which was coined by Moshinsky and Szczepaniak [1], originates from this fact. After the DO oscillator was introduced, the electromagnetic potential associated with the DO interaction has been obtained and the results have shown that this type of interaction describes to the interaction of an anomalous magnetic moment with a linearly growing electric field [7]. Due to this linearly growing characteristic, the DO interaction has been considered to describe the dynamics of quarks [7, 8] and this interaction has been also considered in the context of the two-body systems consisting of fermion-antifermion pairs [9,10]. Furthermore, the DO system has been associated with the quantum optics models such as Jaynes-Cummings [11] (or anti Jaynes-Cummings [12]) model due to the mentioned spin-orbit coupling. In addition to these models, behavior of the DO system has been also investigated by considering the presence of a constant transverse magnetic field [13]. Experimentally, the first microwave realization of the one-dimensional DO has been performed [14] and it has been shown that the results agree well with the theoretical predictions for DO system [14]. Accordingly, the DO system has begun to

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widely studied by several research groups in many areas of the modern physics since this system has an important potential for several physical applications [7, 10, 15, 16, 17,18].

It is well known that the KGE is used to determine the relativistic dynamics of spinless relativistic particles. The KGO interaction, which was considered by inspiring from the DO, has been introduced as the interaction of a spinless relativistic particle (massive) with a complex linear vector potential [2]. This system has attracted a serius attention and has been investigated in many areas of the modern physics. For example, the KGO system has been studied in the presence of external magnetic field in cosmic string spacetime [19], it has been investigated in the presence of Coulomb type potential [20], this relativistic oscillator has been studied by considering the presence of position-dependent mass term [21] and it has been also used to determine the effects of geometric backgrounds having different physical properties on the corresponding physical systems. The studies based on the dynamics of KGO in the Gürses spacetime [22], in the cosmic string background in the context of Kaluza-Klein theory [23], and in the Som-Raychaudhuri spacetime [24] can be considered among such studies.

On the other hand, the DKP equation is used to describe the relativistic dynamics of both spin-0 and spin-1 particles [25, 26, 27]. Similar to the DO interaction, the DKPO system has been introduced via linear non-minimal substitution. This form of the equation has been used to determine the dynamics of spin-0 and spin-1 particles with a linearly varying non-minimal interaction. Several applications have been performed for this form of the DKP equation. For example, this system has been studied under the influence of an external magnetic field [28,29,30], it has been considered in a non-commutative phase space [31,32]. The DKPO has been studied in the context of the minimal length scenarios [33] and it has been investigated by considering the presence of minimal uncertainty in momentum [34]. In three-dimensions, spin-1 sector of the DKP equation has been derived as excited state of Zitterbewegung by considering the associated spinor is a symmetric spinor of rank-two [35]. This spinor, which does not include the spin-zero sector, is constructed by a direct product of two Dirac spinors [36-38]. The vector boson equation (VBE) corresponds to the spin-1 sector of the DKP equation (see also [39]) in 2+1 dimensions [35, 36, 40, 41] and it has been studied in the several contexts of modern physics [5,38-44]. Also, the relativistic oscillator systems are the most useful tools of the mathematical physics, since they are exactly solvable problems, in general. This fact provides to analyse the effects of spacetime topologies on the dynamics of the corresponding physical systems [6,45].

In this research, our aim is to obtain exact results for the two-dimensional vector boson oscillator (VBO), without considering any external effect, in order to discuss the fundamental properties of the system. To achieve this we have used the generalized VBE. We have written this equation in three dimensions in terms of cartesian coordinates and then we have introduced the two-dimensional VBO through non-minimal substitutions. We have solved the corresponding equation by exploiting the angular symmetry of the polar coordinates and have obtained exact results for this relativistic oscillator.

This paper is organized as follows: in section 2, we have written the VBE in three dimensional flat geometric background and have introduced the oscillator coupling, in section 3, by exploiting the angular symmetry of the polar background, we have obtained a set of coupled equations and accordingly we have acquired an exact energy spectrum for this investigated system. Then, in section 4, we have discussed the fundamental properties of this system, such as spin coupling dependence of the spectra and have discussed the behavior of energy levels with respect to the strength of non-minimal coupling.

2. THE VECTOR BOSON EQUATION

In $2+1$-dimensional flat spacetime background, the VBE can be written in terms of cartesian coordinates as the following [5,35], without considering the existence of any external electromagnetic field,

$$\left\{\frac{1}{2}(\gamma^{\mu}\otimes I_2+I_2\otimes\gamma^{\mu})\eth_{\mu}+i\frac{m_b c}{\hbar}I_4\right\}\varphi(\mathbf{x})=0, \quad \mathbf{(1)}$$

$(\mu = 0,1,2.)$.

In Eq. (1), γ^{μ}, I_2, I_4, m_b, c, $\hbar$, φ and x are the Dirac matrices, 2×2 dimensional unit matrix, 4×4 dimensional unit matrix, rest mass of the vector boson, speed of the light in vacuum, the reduced Planck constant, symmetric spinor and spacetime position vector, respectively. Also, in Eq. (1) the symbol $\otimes$ indicates the direct product. In terms of the cartesian coordinates the 2+1-dimensional spacetime background is given by the following metric (with negative signature) [46],

$$ds^2 = c^2dt^2 - dx^2 - dy^2. \qquad (2)$$

According to signature of Eq. (2) the Dirac matrices can be chosen as the following [47],

$$\gamma^t = \frac{1}{c}\sigma^z, \gamma^x = i\sigma^x, \ \ \gamma^y = i\sigma^y, \qquad (3)$$

where, $\sigma^x, \sigma^y, \sigma^z$ are the well-known Pauli spin matrices. Now, we can introduce the two dimensional vector oscillator through the following non-minimal substitutions,

$$\eth_x \longrightarrow \partial_x + \frac{m_b\omega}{\hbar}(\sigma^z \otimes \sigma^z)x,$$
$$\eth_y \longrightarrow \partial_y + \frac{m_b\omega}{\hbar}(\sigma^z \otimes \sigma^z)y\ , \qquad (4)$$

in which ω is the oscillator frequency [5,9]. By assuming the interaction is time-independent we can define the spinor, $\varphi(\mathbf{x})$, as $\varphi(\mathbf{x}) = e^{-i\frac{E}{\hbar}}\Xi(\vec{\boldsymbol{x}})$, in which E is the total energy of the investigated spin-1 field. Under this assumption, by substituting Eq. (3) and Eq. (4) into the Eq. (1) one can obtain the following matrix equation,

$$\left\{2\begin{pmatrix} \mathcal{E}-\mathcal{M} & -\widehat{\Pi}^* & -\widehat{\Pi}^* & 0 \\ -\widehat{\Pi} & -\mathcal{M} & 0 & -\widehat{\Pi}^* \\ -\widehat{\Pi} & 0 & -\mathcal{M} & -\widehat{\Pi}^* \\ 0 & -\widehat{\Pi} & -\widehat{\Pi} & -\mathcal{E}-\mathcal{M} \end{pmatrix} + \kappa\begin{pmatrix} 0 & \mathcal{Z}^* & \mathcal{Z}^* & 0 \\ -\mathcal{Z} & 0 & 0 & -\mathcal{Z}^* \\ -\mathcal{Z} & 0 & 0 & -\mathcal{Z}^* \\ 0 & \mathcal{Z} & \mathcal{Z} & 0 \end{pmatrix}\right\}\Xi(\vec{\boldsymbol{x}}) = 0, \qquad (5)$$

where,

$$\mathcal{E} = \frac{\mathrm{E}}{\hbar c}, \quad \kappa = \frac{m_b\omega}{\hbar}, \quad \mathcal{M} = \frac{m_b c}{\hbar},$$
$$\widehat{\Pi} = \frac{1}{2}\left(\partial_x + i\partial_y\right), \qquad \widehat{\Pi}^* = \frac{1}{2}\left(\partial_x - i\partial_y\right),$$
$$\mathcal{Z} = x + iy, \quad \mathcal{Z}^* = x - iy.$$

To exploit the angular symmetry, we transform the background into the polar coordinates through the following relations [31,46],

$$\partial_x \pm i\partial_y = e^{\pm i\phi}\left(\pm\frac{i}{r}\partial_\phi + \partial_r\right),$$
$$x \pm iy = r\, e^{\pm i\phi}, \qquad (6)$$

and in terms of the new coordinates spatial part of the spinor, $\Xi(\vec{\boldsymbol{x}})$, can be defined as follows,

$$\Xi(\vec{\boldsymbol{x}}) = \begin{pmatrix} \psi_1(r)e^{i(s-1)\phi} \\ \psi_2(r)e^{is\phi} \\ \psi_3(r)e^{is\phi} \\ \psi_4(r)e^{i(s+1)\phi} \end{pmatrix} \qquad (7)$$

which includes all possible spin eigenstates ($s = \pm 1,0$) of the considered vector field.

3. ENERGY SPECTRUM OF TWO-DIMENSIONAL VBO

In this section, we obtain an exact energy spectrum of two-dimensional VBO. Here we should notice that $\psi_2(r)e^{is\phi} = \psi_3(r)e^{is\phi}$ (see Eq. (5) and Eq. (7)). By substituting Eq. (6) and Eq. (7) into the Eq. (5) and then by defining a dimensionless independent variable, $\eta = \kappa r^2$, one can obtain the following equations,

$$\mathcal{E}\psi_+(\eta) - \mathcal{M}\psi_-(\eta) - \frac{s}{\sqrt{\frac{\eta}{\kappa}}}\psi_0(\eta) = 0,$$

$$\mathcal{M}\psi_0(\eta) - \frac{s}{\sqrt{\frac{\eta}{\kappa}}}\psi_-(\eta) + \left(\frac{1}{\sqrt{\frac{\eta}{\kappa}}} + 2\kappa\sqrt{\frac{\eta}{\kappa}}\frac{\mathrm{d}}{d_\eta} + \kappa\sqrt{\frac{\eta}{\kappa}}\right)\psi_+(\eta) = 0,$$

$$\mathcal{E}\psi_-(\eta) - \mathcal{M}\psi_+(\eta) + \kappa\left(\sqrt{\frac{\eta}{\kappa}} - 2\sqrt{\frac{\eta}{\kappa}}\frac{\mathrm{d}}{d_\eta}\right)\psi_0(\eta) = 0, \qquad (8)$$

where, $\psi_{\mp}(\eta) = \psi_1(\eta) \mp \psi_4(\eta)$ and $\psi_0(\eta) = 2\psi_2(\eta)$. By solving these equations for $\psi_0(\eta)$ we obtain a 2^{nd}order wave equation which can be reduced into a well-known form by considering an ansatz function, reads as $\psi_0(\eta) = \eta^{-\frac{1}{2}}\psi(\eta)$,

$$\frac{d^2}{d_\eta^2}\psi(\eta) + \left[\frac{\delta}{\eta} - \frac{1}{4} + \frac{\frac{1}{4}-\xi^2}{\eta^2}\right]\psi(\eta) = 0, \qquad (9)$$

$$\delta = \frac{\mathcal{E}^2-\mathcal{M}^2}{4\kappa} + \frac{\mathcal{E}s}{2\mathcal{M}} - \frac{1}{2}, \quad \xi = \frac{s}{2}.$$

Eq. (9) is in the form of the well-known Whittaker differential equation [40]. Solution function of this equation is obtained as $\psi(\eta)=\mathcal{N}\mathcal{W}_{\delta,\xi}(\eta)$, in which $\mathcal{N}$ is an arbitrary constant and $\mathcal{W}_{\delta,\xi}(\eta)$ is the Whittaker function which can be re-expressed in terms of the generalized Laguerre polynomials (see [48,49]). By using the solution function, $\psi(\eta)=\mathcal{N}\mathcal{W}_{\delta,\xi}(\eta)$, the defined forms of the components in Eq. (8) are obtained

$$\begin{pmatrix}\psi_{+}(\eta)\\ \psi_{0}(\eta)\\ \psi_{-}(\eta)\end{pmatrix}=$$

$$\begin{pmatrix}\frac{\mathcal{N}\sqrt{\kappa}}{\eta(\mathcal{E}^2-\mathcal{M}^2)}(\mathcal{M}(1+2\delta)-s\mathcal{E})\mathcal{W}_{\delta,\xi}(\eta)+2\mathcal{M}\mathcal{W}_{\delta+1,\xi}(\eta)\\ \mathcal{N}\eta^{-\frac{1}{2}}\mathcal{W}_{\delta,\xi}(\eta)\\ \frac{\mathcal{N}\sqrt{\kappa}}{\eta(\mathcal{E}^2-\mathcal{M}^2)}(\mathcal{E}(1+2\delta)-s\mathcal{M})\mathcal{W}_{\delta,\xi}(\eta)+2\mathcal{E}\mathcal{W}_{\delta+1,\xi}(\eta)\end{pmatrix}.$$

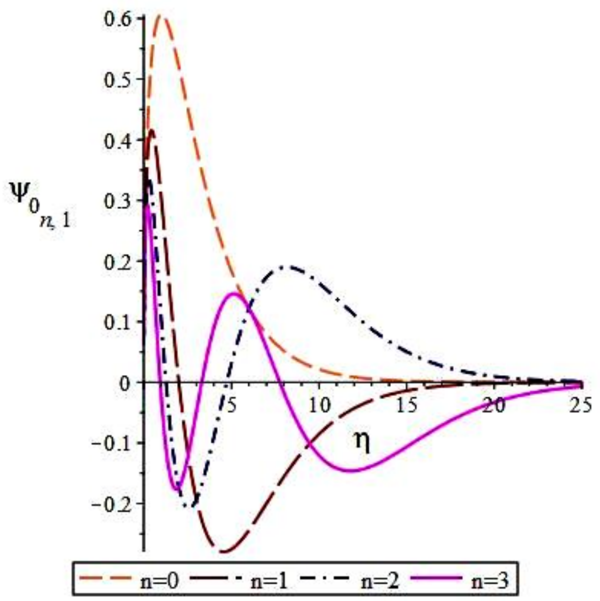

Figure 1 Behaviour of the function $\psi_0(\eta)$ with respect to the varying values of the space coordinate η for a few quantum states. Here we get $m=c=\hbar=\omega=1$.

To be polynomial condition of the solution function, $\psi(\eta)$, is $\frac{1}{2}+\xi-\delta=-n$. This relation gives the following spectrum of energy,

$$E_{n,s}=\mp\omega\hbar s\pm m_bc^2\sqrt{1+\frac{4\omega\hbar}{m_bc^2}\left(n+1+\frac{s}{2}\right)+(\frac{\omega\hbar s}{m_bc^2})^2}. \quad (10)$$

The Eq. (10) gives the exact results of one-dimensional Kemmer oscillator [50] (see also [12]) when $s=0$. For this case ($s=0$), it is very interesting that the result in Eq. (10) agrees well with the recently published results for a fermion-antifermion pair holding via DO coupling [9]. Eq. (10) shows that the oscillator frequency couples with spin of the vector boson. Here, we have seen also that the total energy becomes equal to the rest mass energy of the vector boson when $\omega=0$.

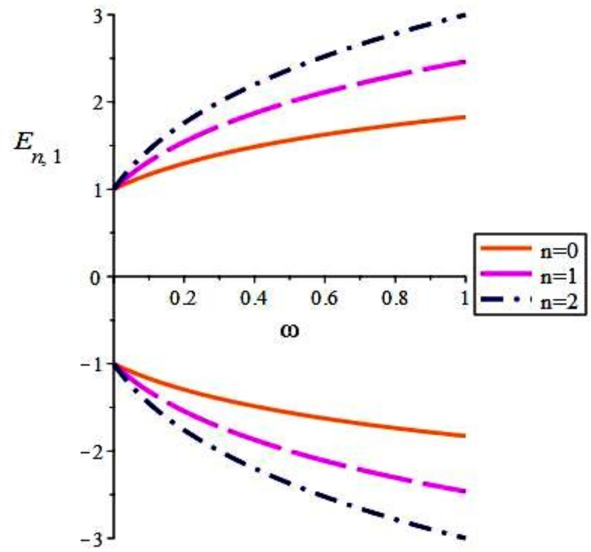

Figure 2 Dependence of the energy levels on the oscillator frequency for $m=c=\hbar=1$

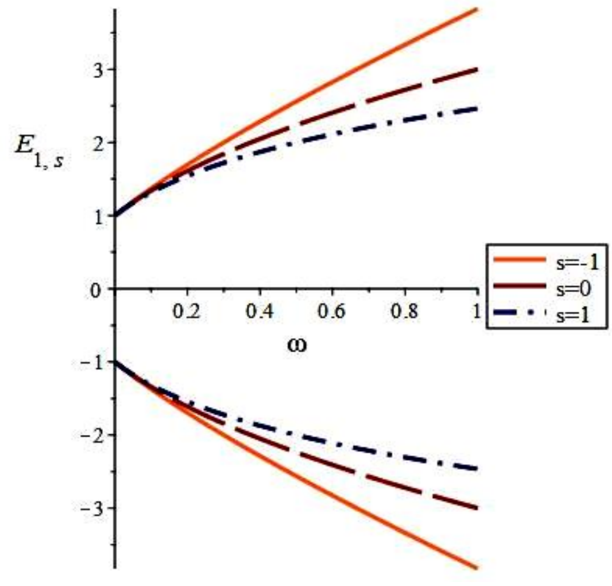

Figure 3 Dependence of the energy levels associated with possible spin eigen-states on the oscillator frequency for $m=c=\hbar=1$

4. RESULTS AND DISCUSSIONS

In this study, we have investigated the relativistic dynamics of two-dimensional VBO by using the VBE. In order to arrive at non-perturvative results for the system in question, we have solved the corresponding form of VBE by defining the oscillator coupling through non-minimal substitutions. The corresponding equation has provided a matrix equation and by solving this equation we have obtained an exact energy spectrum for two-dimensional VBO. This energy expression is given in Eq. (10) and this result agrees well with previously announced results in the literature when the spin coupling vanishes. This fact can be seen by comparing the result in Eq. (10) and the published results in [9,50]. The result in Eq. (10) explicitly shows that the oscillator frequency couples with the spin of vector boson particle. This nature of the considered system has provided to discuss whether the corresponding energy levels can be degenerated or not. The behaviour of the energy levels can be seen in the Figure 2 and Figure 3. We can observe in Figure 3 that there is no any degeneracy in the energy levels associated with the possible spin eigen-states of the VBO. According to the result in Eq. (10) we can see that this spin-1 oscillator does not stop oscillating since there is no physical reason that imposes to stop the oscillation even for the ground state of the system, except for $\omega = 0$. Of course, such a physical reason may occur in the presence of external electromagnetic fields. We have also observed that total energy of the VBO closes to rest mass energy ($E \approx \pm m_b c^2$) of the considered vector field when $\omega \approx 0$. By using the solution function of wave equation in Eq. (9) and the obtained non-perturbative energy expression in Eq. (10) we have shown the behaviour of wave functions, associated with the ground state and a few excited states of the considered system, with respect to the varying values of the defined space coordinate η in Figure 1. In this figure, we can see that amplitude of the wave function(s) closes to zero in the limits of both $\eta \to 0$ and $\eta \to \infty$.Therefore, we can also infer that behaviour of the probability density function, for example $|\psi_0(\eta)|^2$ associated with the solution function, shows similar behaviour. Here, we should also notice that strength of the considered interaction closes to zero when the radial coordinate η (note that $r \propto \sqrt{\eta}$) closes to zero.

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The Declaration of Research and Publication Ethics

The author declares that she complies with the scientific, ethical and quotation rules of SAUJS in all processes of the present paper and that she does not make any falsification on the data collected. Also, the author declares that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that the present study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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