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SOME ARCHIMEDEAN COPULAS ON PRODUCER PRICE INDEX AND CONSUMER PRICE INDEX: A CASE OF TURKEY

BAZI ARŞIMEDYEN KAPULALAR: ÜFE VE TÜFE İÇIN TÜRKIYE UYGULAMASI

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Abstract

In this paper, copula approach was applied to determine the dependence structure the two indices (PPI and CPI). Ali – Mikhail – Haq, Clayton, Frank and Gumbel – Hougaard from Archimedean family were used. As a result it was found that the Gumbel – Hougaard's family with parameter $\theta = 2.907$ *was the best fitted family which models the dependence structure between the two indices.*

Key Words: Copula, Archimedean copula, dependency structure, goodness of fit chi-square method, Kendall's Tau

Jel Classification: *C02, C10, C40.*

Öz

Bu çalışmada, iki indeks(ÜFE ve TÜFE) arasındaki bağımlılık yapısını belirlemek için copula yaklaşımı uygulanmıştır. Arşimedyen copula ailesinden Ali – Mikhail – Haq, Clayton, Frank ve Gumbel – Hougaard kullanılmıştır. Sonuç olarak, iki indeks arasındaki bağımlılık yapısını en uygun $\theta = 2.907$ *parametresi ile Gumbel – Hougaard ailesinin modellediği bulunmuştur.*

Anahtar Kelimeler: Kapula, Arşimedyen copula, bağımlılık yapısı, Ki-kare uyum iyiliği testi, Kendall's Tau

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Jel Kodları: *C02, C10, C40.*

1. INTRODUCTION

The term copula is that means link or relationship between pieces and was first employed in a mathematical or statistical sense by Abe Sklar in 1959. Sklar has shown that to establish a correlation between multivariate distribution functions and their one-dimensional distribution functions(Nelsen,

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2006: 2). The study of copulas and their applications in statistics is a rather modern phenomenon. Until quite recently, it was difficult to even locate the term "copula" in the statistical literature. Although well known in the statistical literature for a long time, applications of the copula theory in statistical modeling are more a recent popular. Today, there is a fast growing literature on copulas. Nelsen(2006) provide an introduction to the copula theory. Mathematical treatments of copulas are the book by Joe(1997). Matteis(2001) tried to find the most suitable copula for the data set. Cherubini et al.(2004) present a discussion of copula techniques for financial applications. Archimedean copulas are often used because these copulas are easy to handle and have simple. Many other authors worked on Archimedean copulas like Genest and Rivest(1993), Nelsen(2005), Savu and Trede(2007).

The function of copula doesn't require the assumption of normality in relation to a joint distribution between variables and establish the marginal of great number for multi-dimensional joint distribution. The main purpose of copula function is obtained by determining the dependency structure of multivariate distributions, which is the most appropriate observed data. In this paper we aim at reviewing the theory needed to understand copula based modeling and apply it to a given data set. Therefore we used produce price index and consumer price index from 1982 to 2011 in Turkey.

İnflation showing the periodic change in the market prices of selected goods and services are calculated by the average price indices. Therefore, the consumer price index and producer price index inflation rate used in the rate of change. These two indices are expected to be positively related to each other to be able to substitute. A lot of statistical methods are used to determine the relationship. To illustrate, regression analysis that explains with a mathematical formula connection of between dependent and independent variable or variables is one of applied statistical methods. In practice, regression analysis assumes that the relationships between variables are linear and distributions of error terms have normal distributions. But, in practice, these assumptions would not to be able to provide and are deviations. Therefore, copula function was used to determine the dependency structure of the two indices.

This paper is structured as follows. Section 2 briefly reviews the family of Archimedean copulas and copula theory. In Section 3 the results of analysis involves. Finally, section 4 concludes.

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2. MATERIAL and METHODS

This section presents some basic definitions and properties for the class of Archimedean copulas that we use in this paper. Copulas are the functions that join or couple multivariate distribution to their one-dimensional margins possessing uniform distribution on the interval $[0,1]$.

Sklar's Theorem:

Sklar's theorem explains the role that copulas play in the relationship between multivariate distribution functions and their unvariate margins.

Let H be a joint distribution function with margins F and G. Then there exists a copula C such that $H(x,y) = C(F(x), G(y))$. Conversely, for any distribution function F and G and any copula C, the H function defined above is a two-dimensional distribution function with marginals F and G. Further more if F and G are continuous, C is unique (Nelsen, 2006: 17).

Archimedean Copulas:

Recently, Archimedean copulas are used in various fields as a tool for modeling the dependence structure. These copulas are used for several reasons in applications:

1. The easy with which they can be constructed
2. The great variety of different dependence structure
3. The many parametric families of copulas belonging to this class
4. The algebraic properties possessed by the members of this class(Matteis, 2001: 25)

Archimedean approach allows to reduction of a multivariate copula to a simple univariate generator function. For simplicity, consider a bivariate copula from $[0,1]^2$ to $[0,1]$. Let φ be continuous strictly decreasing function from $[0,1]$ to $[0,\infty]$ such that $\varphi(1) = 0$, and let φ^{-1} be the pseudo-inverse of φ defined. Let C be the function from $[0,1]^2$ to $[0,1]$ given by

$$C(u,v) = \varphi^{-1}(\varphi(u) + \varphi(v)) \quad (1)$$

The function φ is called a generator of C and determines Archimedean copula(Nelsen, 2005: 2). Some algebraic properties of Archimedean copulas are shown below.

Theorem: Let C be an Archimedean copula with generator φ. Then:

1. C is symmetric $C(u,v) = C(v,u)$ for all $u, v \in [0,1]$
2. C is associative $C(C(u,v), w) = C(u, C(v,w))$ for all $u, v, w \in [0,1]$
3. If $C > 0$ is any constant then $c\varphi$ is also a generator of C(Matteis, 2001: 26)

There are many copula functions. Joe (1997) and Nelsen (2006) can be looked for extensive information about the functions of copula. The most widely used families of Archimedean copulas are Clayton, Frank, Gumbel – Hougaard and Ali – Mikail – Haq. Some Archimedean copulas which used in the article as below

Ali – Mikail – Haq Copula:

The Ali – Mikail – Haq copula is a symmetric Archimedean copula given by:

$$C_\theta(u,v) = uv[1-\theta(1-u)(1-v)]^{-1} \tag{2}$$

θ parameter that gets a value in the interval $[-1,1]$ allow to positive and negative dependence (Kumar,2010: 660).

Clayton Copula:

The Clayton copula is an asymmetric Archimedean copula, exhibiting greater dependence in the negative tail than in the positive. This copula is given by:

$$C_\theta(u,v) = \left[u^{-\theta} + v^{-\theta} - 1\right]^{-1/\theta} \tag{3}$$

θ gets a value in the interval $[-1\infty), -\{0\}$. The dependence between the observations increases as the value of θ increases, with $\theta \to 0$ implying independence and $\theta \to \infty$ implying perfect dependence (Trivedi and Zimmer, 2005: 18).

Frank Copula:

The Frank copula is a symmetric Archimedean copula given by:

$$C_\theta(u,v) = -\frac{1}{\theta} ln\left[1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right] \tag{4}$$

The Frank copula has support for θ in the interval $(-\infty,\infty)$. The Frank copula is popular for several reasons unlike some other copulas, it permits negative dependence between the marginals. In theory, the Frank copula can be used to model outcomes with strong positive or negative dependence (Trivedi and Zimmer, 2005: 19).

Gumbel – Hougaard Copula:

The Gumbel – Hougaard copula is an asymmetric Archimedean copula, exhibiting greater dependence in the positive tail than in the negative. This copula is given by:

$$C_\theta(u,v) = exp\left\{-\left[(-lnu)^\theta + (-lnv)^\theta\right]^{1/\theta}\right\} \tag{5}$$

The dependence parameter is restricted to the interval $[1,\infty)$ (Joe, 1997: 202-206). These copula families and their generator are shown in Table 1.

Table 1. Some Archimedean Copula Functions and Generators

Family	Copula ($C_\theta(u,v)$)	Generator ($\varphi(\theta)$)	Range of Parameter
Ali-Mikail-Haq	$uv[1-\theta(1-u)(1-v)]^{-1}$	$ln\frac{1-\theta(1-t)}{t}$	$[-1,1]$
Clayton	$\left[u^{-\theta}+v^{-\theta}-1\right]^{-1/\theta}$	$\frac{1}{\theta}(t^{-\theta}-1)$	$[-1,\infty)-\{0\}$
Frank	$-\frac{1}{\theta}ln\left[1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right]$	$-ln\frac{e^{-\theta t}-1}{e^{-\theta}-1}$	$(-\infty,\infty)-\{0\}$
Gumbel-Hougaard	$exp\left\{-\left[(-lnu)^{\theta}+(-lnv)^{\theta}\right]^{1/\theta}\right\}$	$(-lnt)^{\theta}$	$[1,\infty)$

Source: (Frees and Favre, 1998: 10)

Kendall's Tau:

Kendall's tau measure of a pair(X,Y), distributed according to H, can be defined as the difference between the probabilities of concordance and discordance for two independent pairs (X_1,Y_1) and (X_2,Y_2) each with distribution H; that is

$$\tau_{X,Y}=Pr\{(X_1-X_2)(Y_1-Y_2)>0\}-Pr\{(X_1-X_2)(Y_1-Y_2)<0\} \quad (6)$$

These probabilities can be evaluated by integrating over the distribution of (X_2,Y_2). Let X and Y be continuous random variables whose copula is C. Then the population version of Kendall's tau for X and Y in terms of copulas is give by

$$\tau_{X,Y}=\tau_C=4\iint C(u,v)dC(u,v)-1 \quad (7)$$

Let X and Y be random variables with an Archimedean copula C generated by φ in Ω. The population version τ_C of Kendall's tau for X and Y is given by (Nelsen, 2006: 158-162):

$$\tau_C=1+4\int_0^1\frac{\varphi(t)}{\varphi'(t)}dt \quad (8)$$

There exist five methods of estimating copula models. The one step method or exact maximum likelihood (EML) method estimates all parameters of the model at the same time. The second method is the two step estimator or the method of inference functions for margins

(IFM), which first estimates the parameters of the marginals and with these parameters given estimates the copula function. The semiparametric estimation method leave the marginal densities unspecified and uses the empirical probability integral transform in order to obtain the uniform marginals needed to estimate the copula parameters. The last two methods are nonparametric ways of estimating the copula (Manner, 2007: 16). When there are no information about marginal distributions and parameters, the non-parametric estimation methods are used. Therefore, in this study goodness of fit chi-square method was used to obtain copula function. Goodness of fit test procedure for parametric families of Archimedean copulas has three main advantages:

1) It is straightforward, being based on the classical χ^2 statistic, although its asymptotic distribution is not χ^2
2) One can either fully specify an Archimedean copula or just a parametric family Archimedean copulas and most importantly
3) It can handle high-dimensional distributions by exploiting the properties of Archimedean copulas (Savu and Trede, 2007: 109).

Based on differences the observed frequencies and expected frequencies calculate the chi-square statistic in the goodness of fit test. The degrees of freedom associated with goodness of fit statistic were thus calculated using $df = ((I-1)(J-1)) - p - (q-1)$, I is the number of rows, J is the number of columns, p is the the number of parameters estimated and q is the number of cells pooled together. If the table value corresponding to the degree of freedom is greater than the calculated chi-square, the copula family in the null hypothesis is the most suitable one (Genest and Rivest, 1993: 1034-1043).
In this study, it is tried to examine the dependency structure between the Producer Price Index (PPI) and the Consumer Price Index (CPI) of Turkey in the base 1982 by the copula method using the monthly change of rate values for the related indices, including the period 1982:02-2011:02. In nonparametric method, Kendall Tau (τ) is used because of having algebraic formula of copula families and achieving more simple solution.

3. EMPIRICAL RESULTS

This section reports the results of dependency structure between CPI and PPI using copula method. The data set consists of CPI and PPI (base year 1982) using the monthly percentage change of rate values for the related indices in the 1982:01-2011:02 period. Dependency relationships between the data sets were analyzed and Kendall Tau (τ) value was estimated to be 0.656. The most appropriate copula family and their parameters to the data set are estimated using this correlation coefficient. Table 2 presents a $4x4$ cross-classification of the two variables under study. The cell boundaries for two variables were taken as order statistics of rank $\left[349x\frac{j}{4}\right]$ for $j = 1{,}23{,}4$.

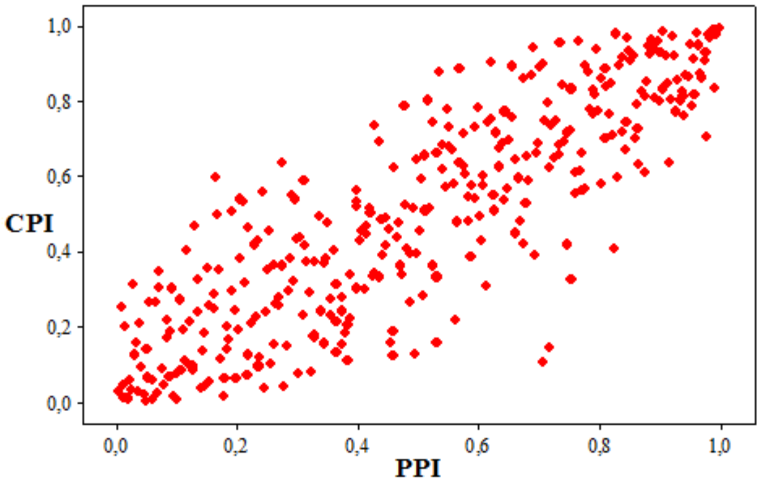

Figure 1. Scatter Plot for Data Set

Figure 1 shows that correlation of between CPI and PPI are positive on the first bisector. That is, it is said that to be positive direction and robust correlation between two indices.

Table 2. The Expected Frequencies for Monthly Percentage Change of CPI and PPI

Variables	$Y_{(87)}$	$Y_{(174)}$	$Y_{(261)}$	$Y_{(349)}$
$X_{(87)}$	58	23	6	0
$X_{(174)}$	25	46	15	1
$X_{(261)}$	4	16	47	20
$X_{(349)}$	0	2	19	67

H_0: The copula family is suitable for data set

H_1: The copula family is not suitable for data set

Table 3. Predicted Frequencies for Some Archimedean Copulas

Variables	$Y_{(87)}$	$Y_{(174)}$	$Y_{(261)}$	$Y_{(349)}$	χ^2	θ
	Ali- Mikhail-Haq predicted frequencies					
$X_{(87)}$	41	23	13	10		
$X_{(174)}$	23	23	23	18		
$X_{(261)}$	13	23	25	27	157.2399	0.836
$X_{(349)}$	10	18	27	32		
	Clayton predicted frequencies					
$X_{(87)}$	73	13	1	0		
$X_{(174)}$	25	46	15	1		

$\mathbf{X}_{(261)}$	1	20	40	27	73.3576	0.814
$\mathbf{X}_{(349)}$	0	6	27	54		
	Frank predicted frequencies					
$\mathbf{X}_{(87)}$	64	20	3	0		
$\mathbf{X}_{(174)}$	20	45	20	2		
$\mathbf{X}_{(261)}$	3	23	43	22	9.385	9.613
$\mathbf{X}_{(349)}$	0	2	22	63		
	Gumbel Hougaard predicted frequencies					
$\mathbf{X}_{(87)}$	60	22	5	0		
$\mathbf{X}_{(174)}$	22	41	23	1		
$\mathbf{X}_{(261)}$	3	23	43	22	6.956	2.907
$\mathbf{X}_{(349)}$	0	3	17	67		

The parameters of copula families that obtained using the Kendall Tau were estimated. Kendall Tau was estimated to be 0.696. The chi-square value between the expected frequencies and the predicted frequencies of some Archimedean copulas were found in Table 3. For each of copulas, the chi-square results were compared with table value. When the chi-square value is compared with the table value, the null hypothesis was rejected at level of significance $\alpha = 0.05$. It was found that the dependency structure of CPI and PPI is suitable with Frank and Gumbel – Hougaard copula families.

4. CONCLUSION

The aim of this paper is to make sample applications by using some Archimedean copulas. To illustrate this, the producer price index (PPI) and the consumer price index (CPI) are used in this study. In paper study, the dependence structure between CPI and PPI values were investigated and copula family had been estimated according to the data set. We found that Frank and Gumbel – Hougaard copula are appropriate for dependency structure between two indices. The Gumbel copula is also an asymmetric copula, but it is exhibiting greater dependence in the positive tail than in the negative (Aas, 2004). The relationship between the Australian Dollar and the Canadian Dollar may be a mixture model of the Gaussian and Gumbel copulas, as may the relationship between the British Pound and the Canadian Dollar (Arnold, 2006). When the calculated values of chi-square analyzed, Gumbel – Hougaard's family with $\theta = 2.907$ paremeter were more appropriate due to having the smallest value of chi-square. Gumbel-Hougaard’s family unlike the other families is modeled only positive dependency structure. Therefore, the dependency structure between PPI and CPI will always be a positive structure. That is, two indices have non-linear comovoement and positive correlation.

The most important feature of copula method is multi-dimensional modeling the dependency structure between variables which have unknown distribution or the distribution is whether normal or not. Therefore the copula analysis is more useful, when the assumptions of other statistical techniques don’t valid.

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