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RULED SURFACE WITH CONSTANT SLOPE ACCORDING TO OSCULATING PLANE OF BASE CURVE IN GALILEAN 3−SPACE

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Abstract. The object of this paper is to investigate the properties of the ruled surface which direction vector has a constant slope with osculating plane of the base curve in Galiean 3−space. We obtain some properties of this kind of ruled surface by calculating the geometric invariants. Also, we give an application on the example and their graphs are visualized by using the Mathematica program.

1. Introduction

Inertial reference frame is defined as a coordinate system moving at a constant velocity. In 1632, Galileo first described the principle "the laws of motion are the same in all inertial frames" using the example of a ship travelling at constant velocity. According to this principle, any observer below the deck would not be able to tell whether the ship was moving or stationary. The Galilean transformation between two inertial frames (x, y, z) and (x', y', z') is defined as

$$\begin{aligned} x' &= a + x, \\ y' &= b + cx + (\cos\varphi)\, y + (\sin\varphi)\, z, \\ z' &= d + ex - (\sin\varphi)\, y + (\cos\varphi)\, z, \end{aligned}$$

where a, b, c, d, e, and φ are some constants. In Galilean space, since two inertial frames are related by a Galilean transformation, all physical laws are the same in all inertial reference frames.

In differential geometry, various surfaces have been extensively studied by the authors in the special spaces: extrinsically and intrinsically [3, 5, 6, 7, 8, 11, 12]. Ruled surface is one of these surfaces and is defined as a surface formed by moving the generating vector along a base curve [14]. Many authors studied on the characaterizations of the ruled surfaces [1, 4, 9, 10, 13].

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In this paper, we investigate the ruled surface whose generator vector has a constant slope according to osculating plane of the base curve in Galilean 3−space and we obtained some important results of this ruled surface. Also, we give some properties of this kind of ruled surface using its invariant curvatures. Finally, we present an example of such a ruled surface in Galilean 3−space.

2. Preliminaries

The standard metric of Galilean 3−space $\mathbb{G}_3$ is defined as

$$\langle x,y\rangle=\begin{cases} x_1y_1, & x_1\neq 0 \text{ or } y_1\neq 0\\ x_2y_2+x_3y_3, & x_1=0=y_1,\end{cases} \tag{2.1}$$

where x_i and y_j $(i,j=1,2,3)$ are shown the coefficients of the vectors x and y, respectively. The cross product in Galilean 3−space is defined by

$$x\times y=\begin{cases} \begin{vmatrix} 0 & e_2 & e_3\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{vmatrix}, & x_1\neq 0 \text{ or } y_1\neq 0,\\ \\ \begin{vmatrix} e_1 & e_2 & e_3\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{vmatrix}, & x_1=0=y_1,\end{cases} \tag{2.2}$$

[12].

Let $\alpha: I\subset\mathbb{R}\to\mathbb{G}_3$ be a unit speed curve with the parametrization $\alpha(s)=(s,y(s),z(s))$. The Frenet frame is defined $\{T(s)=\alpha'(s),N(s),B(s)\}$ for the curve $\alpha(s)$ in Galilean 3−space. The Frenet equations are given by

$$\begin{aligned} T'(s) &= \kappa(s)N(s),\\ N'(s) &= \tau(s)B(s),\\ B'(s) &= -\tau(s)N(s),\end{aligned} \tag{2.3}$$

with the curvature $\kappa(s)=\|\alpha''(s)\|$ and the torsion $\tau(s)=\frac{1}{\kappa^2(s)}\det(\alpha',\alpha'',\alpha''')$ [2].

Let $X(u,v)=(x(u,v),y(u,v),z(u,v))$ be a parametric surface in Galilean 3−space. The interior geometry of the parametric surface $X(u,v)$ at the point $X(u_0,v_0)$ is obtained by the first fundamental form. The first fundamental form of the surface is

$$I=(g_1du+g_2dv)^2+\varepsilon(h_{uu}du^2+2h_{uv}dudv+h_{vv}dv^2) \tag{2.4}$$

where $g_1:=x_u=\frac{\partial x}{\partial u}$, $g_2:=x_v$, $h_{uv}:=y_uy_v+z_uz_v$, $h_{uu}:=y_u^2+z_u^2$, $h_{vv}:=y_v^2+z_v^2$, and

$$\varepsilon=\begin{cases} 0, & \text{if the direction } du:dv \text{ is non-isotropic},\\ 1, & \text{if the direction } du:dv \text{ is isotropic}.\end{cases}$$

The Gauss map of the surface $X(u,v)$ is defined as

$$U=\frac{1}{W}(0,-x_uz_v+x_vz_u,x_uy_v-x_vy_u) \tag{2.5}$$

where $W = \sqrt{(x_u z_v - x_v z_u)^2 + (x_v y_u - x_u y_v)^2}$. The second fundamental form is given by

$$II = L_{11}(du)^2 + 2L_{12}dudv + L_{22}(dv)^2$$

where

$$L_{ij} = \frac{1}{g_1}\left\langle g_1\left(0, y_{,ij}, z_{,ij}\right) + g_{i,j}\left(0, y_u, z_u\right), U\right\rangle \text{ for } g_1 \neq 0 \tag{2.6}$$

or

$$L_{ij} = \tfrac{1}{g_2}\left\langle g_2\left(0, y_{,ij}, z_{,ij}\right) + g_{i,j}\left(0, y_v, z_v\right), U\right\rangle \text{ for } g_2 \neq 0$$

where $y_{,ij} = \frac{\partial y}{\partial u_i u_j}$, $j = 1, 2$ and $u_1 := u$, $u_2 := v$. The invariant curvatures K and H of the surface are calculated as:

$$K := \frac{L_{11}L_{22} - L_{12}^2}{W^2} \text{ and } H := \frac{g_2^2 L_{11} - 2g_1 g_2 L_{12} + g_1^2 L_{22}}{2W^2} \tag{2.7}$$

where K, H are called as Gaussian curvature and mean curvature of the surface, respectively. A surface in Galilean 3−space is called as flat (resp. minimal) surface if its Gaussian (resp. mean) curvature is zero [3, 11]. The principal curvatures k_1 and k_2 of the surface X are given as

$$k_1 = 2H \text{ and } k_2 = \frac{L_{11}L_{22} - L_{12}^2}{g_2^2 L_{11} - 2g_1 g_2 L_{12} + g_1^2 L_{22}}. \tag{2.8}$$

3. Constant Slope Ruled Surface in Galilean 3−space

In this section, we will analyze the properties of the ruled surfaces whose director vector make a constant slope with the osculating plane of the base curve α. Then, we will obtain some properties of this kind of surfaces.

In Galilean 3−space, we construct the ruled surface with constant slope according to the osculating plane of the base curve as follows:

$$X(s, \lambda) = \alpha(s) + \lambda D(s) \tag{3.1}$$

where $\alpha(s) = (s, y(s), z(s))$ is the director curve and $D(s) = \cos(\theta(s))\, T(s) + \sin(\theta(s))\, N(s) + \omega B(s)$ is the generator vector of the ruled surface $X(s, \lambda)$. The coefficients of the principal fundamental form are given by

$$\begin{gathered} g_1 = 1 - \lambda\theta'(s)\sin(\theta(s)), \ g_2 = \cos(\theta(s)), \\ g_{1,1} = \tfrac{d}{ds}\left(1 - \lambda\theta'(s)\sin(\theta(s))\right), g_{1,2} = -\theta'(s)\sin(\theta(s)), \text{ and } g_{2,2} = 0. \end{gathered} \tag{3.2}$$

The normal vector U of the surface $X(s, \lambda)$ is calculated as

$$\begin{aligned} U = &\frac{1}{\kappa W}\{0, -A(z''\sin\theta + \omega y'') + \lambda\cos\theta\left(Bz'' + \tau\sin\theta y''\right), \\ &A(y''\sin\theta - \omega z'') - \lambda\cos\theta\left(By'' - \tau\sin\theta z''\right)\} \end{aligned} \tag{3.3}$$

where $W = \left(A^2(\sin^2\theta + \omega^2) + \lambda^2\cos^2\theta(B^2 + \tau^2\sin^2\theta)\right)^{1/2}$, $A = 1 - \lambda\theta'\sin\theta$, and $B = \kappa\cos\theta + \theta'\cos\theta - \omega\tau$. The coefficients of the second fundamental form are calculated as follows:

$$L_{11}=\frac{1}{\kappa W}\left\{\begin{array}{c}[-A(z''\sin\theta+\omega y'')+\lambda\cos\theta\,(Bz''+\tau\sin\theta y'')]\\ \left[2A_sy'+Ay''+\left(\frac{\lambda A_s}{\kappa A}-\frac{\lambda\kappa'}{\kappa^2}\right)(By''-\tau\sin\theta z'')+\frac{\lambda}{\kappa}(By''-\tau\sin\theta z'')_s\right]\\ +[A(y''\sin\theta-\omega z'')-\lambda\cos\theta\,(By''-\tau\sin\theta z'')]\\ \left[2A_sz'+Az''+\left(\frac{\lambda A_s}{\kappa A}-\frac{\lambda\kappa'}{\kappa^2}\right)(Bz''+\tau\sin\theta y'')+\frac{\lambda}{\kappa}(Bz''+\tau\sin\theta y'')_s\right]\end{array}\right\},$$

$$L_{12}=\frac{1}{\kappa WA}\left\{\begin{array}{c}[-A(z''\sin\theta+\omega y'')+\lambda\cos\theta\,(Bz''+\tau\sin\theta y'')]\\ \left[-2Ay'\theta'\sin\theta+\frac{1}{\kappa}(A-\lambda\theta'\sin\theta)\,(By''-\tau\sin\theta z'')\right]\\ +[A(y''\sin\theta-\omega z'')-\lambda\cos\theta\,(By''-\tau\sin\theta z'')]\\ \left[-2Az'\theta'\sin\theta+\frac{1}{\kappa}(A-\lambda\theta'\sin\theta)\,(Bz''+\tau\sin\theta y'')\right],\end{array}\right\},$$

and $L_{22}=0$, where the derivatives are taken according to the parameter s.

Now, we will examine some properties of the ruled surfaces with constant slope relative to special curves in Galilean space.

Case 1. If the base curve $\alpha(s)$ of the ruled surface is a **plane** curve with the parametric equation $\alpha(s)=(s,y(s),0)$, then the ruled surface with constant slope is given by

$$X(s,\lambda)=\alpha(s)+\lambda D(s)$$

where $D(s)=\cos\theta(s)T(s)+\sin\theta(s)N(s)+\omega\overrightarrow{e}_3$ is the generator vector, ω is an arbitrary constant, and $\overrightarrow{e}_3=(0,0,1)$. The Gauss map of the ruled surface is calculated as follows:

$$U(s,\lambda)=\frac{1}{W}(0,-\omega(1-\lambda\theta'\sin\theta),(1-\lambda\theta'\sin\theta)\sin\theta-\lambda\cos^2\theta(\theta'+\kappa))$$

where $W=\left(\omega^2(1-\lambda\theta'\sin\theta)^2+(\sin\theta-\lambda\theta'-\lambda\kappa\cos^2\theta)^2\right)^{1/2}$.

Theorem 3.1. *The ruled surface $X(s,\lambda)$ with the base curve $\alpha(s)=(s,y(s),0)$ is developable if and only if the following equation is satisfied*

$$(1-\lambda\theta'\sin\theta)(\cos\theta(\theta'+\kappa)-2\theta'\sin\theta y')-\lambda\theta'\sin\theta\cos\theta(\theta'+\kappa)=0.$$

Proof. From Eq. (2.6), the coefficients of the first and second fundamental forms of $X(s,\lambda)$ are calculated as

$$g_1=1-\lambda\theta'\sin\theta\qquad \textit{and}\qquad g_2=\cos\theta,$$

$$\begin{aligned}(3.4)\quad L_{11}&=-\frac{\omega}{W}(1-\lambda\theta'\sin\theta)\left(\begin{array}{c}(1-\lambda\theta'\sin\theta)\kappa+2y'\frac{d(1-\lambda\theta'\sin\theta)}{ds}\\ -\lambda\theta'\sin\theta(\theta'+\kappa)+\lambda\cos\theta(\theta''+\kappa')\\ +\frac{\lambda\cos\theta(\theta'+\kappa)\frac{d(1-\lambda\theta'\sin\theta)}{ds}}{1-\lambda\theta'\sin\theta}\end{array}\right),\\ L_{12}&=-\frac{\omega}{W}(1-\lambda\theta'\sin\theta)\left(\begin{array}{c}-2\theta'\sin\theta y'+\cos\theta(\theta'+\kappa)\\ -\frac{\lambda\theta'\cos\theta\sin\theta(\theta'+\kappa)}{1-\lambda\theta'\sin\theta}\end{array}\right),\\ L_{22}&=0.\end{aligned}$$

The Gauss curvature of the surface $X(s,\lambda)$ is

$$K=-\frac{\omega^2}{W^4}(1-\lambda\theta'\sin\theta)^2\left(\begin{array}{c}-2\theta'\sin\theta y'+\cos\theta(\theta'+\kappa)\\ -\frac{\lambda\theta'\cos\theta\sin\theta(\theta'+\kappa)}{1-\lambda\theta'\sin\theta}\end{array}\right)^2.$$

The ruled surface $X(s,\lambda)$ with the base curve $\alpha(s)=(s,y(s),0)$ is to be developable, its Gauss curvature must be zero. So, we obtain the desired result. □

Theorem 3.2. *The ruled surface $X(s,\lambda)$ with the base curve $\alpha(s)=(s,y(s),0)$ is minimal if and only if*

$$\cos\theta\begin{bmatrix} -(1-\lambda\theta'\sin\theta)\kappa \\ +2y'\frac{d(1-\lambda\theta'\sin\theta)}{ds} \\ +\lambda\theta'\sin\theta(\theta'+\kappa) \\ +\lambda\cos\theta(\theta''+\kappa') \\ +\frac{\lambda\cos\theta(\theta'+\kappa)\frac{d(1-\lambda\theta'\sin\theta)}{ds}}{1-\lambda\theta'\sin\theta}\end{bmatrix} - 2\theta'(1-\lambda\theta'\sin\theta)\left(-2\sin\theta y'+\cos\theta\right)=0.$$

Proof. If we substitute the components of Eq. (3.4) into the Eq. (2.7), we obtain the mean curvature of the ruled surface $X(s,\lambda)$. For the surface to be minimal, its mean curvature is equal to zero. So, we get the desired differential equation for $1-\lambda\theta'\sin\theta\neq 0$. □

Corollary 3.3. *If the function θ is a constant, then the ruled surface $X(s,\lambda)$ has the Gauss curvature and the mean curvature as follows:*

$$K=-\frac{\omega^2}{W^4}(\kappa^2(s)\cos^2\theta) \qquad \textit{and} \qquad H=-\frac{\omega\cos^2\theta}{2W^3}(-\kappa(s)+\lambda\kappa'(s)\cos\theta).$$

Corollary 3.4. *If the function θ is a constant and the surface $X(s,\lambda)$ is developable, then the base curve is the straight line in Galilean space.*

Corollary 3.5. *If the function θ is a constant and the surface $X(s,\lambda)$ is minimal, then the base curve α has the curvature $\kappa(s)=e^{s/\lambda\cos\theta}+c$.*

There exists a common perpendicular to two constructive rulings in the ruled surface, then the foot of the common perpendicular on the main rulings is called a central point. The locus of the central point is called the striction curve.

Theorem 3.6. *The following conditions are satified for the striction curve of the ruled surface $X(s,\lambda)$:*
(i) If the function θ is an arbitrary constant, then the striction curve is
$\beta(s)=\left(s-\frac{1}{\kappa(s)},y(s)-\frac{1}{\kappa(s)\cos\theta}(\cos\theta y'(s)+\sin\theta),-\frac{\omega}{\kappa(s)\cos\theta}\right).$
(ii) If the function θ is not a constant, then the striction curve is
$\beta(s)=\left(s-\frac{1}{(\theta'(s))^2}\cot^2\theta(s),y(s)-\frac{\cos\theta(s)}{(\theta'(s))^2\sin^2\theta(s)}(\cos\theta(s)y'(s)+\sin\theta(s)),-\frac{\omega\cos\theta}{(\theta'(s))^2\sin^2\theta(s)}\right).$

Proof. The striction curve of the ruled surface is two types depending on whether D' is isotropic, or non-isotropic vector in Galilean space. The striction curve of the ruled surface is calculated the following formula

$$\beta(s)=\alpha(s)-\frac{\langle T(s),D(s)\rangle}{\langle D'(s),D'(s)\rangle}D(s).$$

If the derivative of the generator vector with respect to s is an isotropic vector, then the striction curve is calculated in (i) and if $D'(s)$ is a non isotropic vector, then the striction curve is calculated as in (ii). □

Case 2. If the base curve $\alpha(s)$ of the ruled surface is a **plane** curve with the parametric expression $\alpha(s)=(s,0,z(s))$, then the ruled surface with constant slope is given by

$$X(s,\lambda)=\alpha(s)+\lambda D(s)$$

where $D(s) = \cos\phi(s)T(s) + \sin\phi(s)N(s) - \sigma\overrightarrow{e}_2$ is the generator vector, σ is an arbitrary constant, and $\overrightarrow{e}_2 = (0,1,0)$. The Gauss map of the ruled surface is calculated as follows:

$$U(s,\lambda) = \frac{1}{W}(0, -(1-\lambda\phi'\sin\phi)\sin\phi - \lambda\cos^2\phi(\phi'+\kappa), -\sigma(1-\lambda\phi'\sin\phi))$$

where $W = \left(\sigma^2(1-\lambda\phi'\sin\phi)^2 + ((1-\lambda\phi'\sin\phi)\sin\phi + \lambda\cos^2\phi(\phi'+\kappa))^2\right)^{1/2}$.

Theorem 3.7. *The ruled surface $X(s,\lambda)$ with the base curve $\alpha(s) = (s,0,z(s))$ is developable if and only if the differential equation is satisfied*

$$(1-\lambda\phi'\sin\phi)(\cos\phi(\phi'+\kappa) - 2\phi'(\sin\phi)z') - \lambda\phi'\sin\phi\cos\phi(\phi'+\kappa) = 0.$$

Proof. From Eq.(2.6), the coefficients of the first and second fundamental forms of $X(s,\lambda)$ are calculated as

$$g_1 = 1-\lambda\phi'\sin\phi \qquad \textit{and} \qquad g_2 = \cos\phi,$$

$$\begin{aligned}
(3.5)\; L_{11} &= -\frac{\sigma}{W}(1-\lambda\phi'\sin\phi)\begin{pmatrix} (1-\lambda\phi'\sin\phi)\kappa + 2z'\frac{d(1-\lambda\phi'\sin\phi)}{ds} \\ -\lambda\phi'\sin\phi(\phi'+\kappa) + \lambda\cos\phi(\phi''+\kappa') \\ +\frac{\lambda\cos\phi(\phi'+\kappa)\frac{d(1-\lambda\phi'\sin\phi)}{ds}}{1-\lambda\phi'\sin\phi} \end{pmatrix}, \\
L_{12} &= -\frac{\sigma}{W}(1-\lambda\phi'\sin\phi)\begin{pmatrix} -2\phi'\sin\phi z' + \cos\phi(\phi'+\kappa) \\ -\frac{\lambda\phi'\cos\phi\sin\phi(\phi'+\kappa)}{1-\lambda\phi'\sin\phi} \end{pmatrix}, \\
L_{22} &= 0.
\end{aligned}$$

The Gauss curvature of the surface $X(s,\lambda)$ is

$$K = -\frac{\sigma^2}{W^4}(1-\lambda\phi'\sin\phi)^2\begin{pmatrix} -2\phi'\sin\phi z' + \cos\phi(\phi'+\kappa) \\ -\frac{\lambda\phi'\cos\phi\sin\phi(\phi'+\kappa)}{1-\lambda\phi'\sin\phi} \end{pmatrix}^2.$$

The ruled surface $X(s,\lambda)$ with the base curve $\alpha(s) = (s,0,z(s))$ is to be developable, its Gauss curvature must be zero. So, we obtain the desired result. $\square$

Theorem 3.8. *The ruled surface $X(s,\lambda)$ with the base curve $\alpha(s) = (s,0,z(s))$ is minimal if and only if the following differential eqaution is fulfilled*

$$\cos\phi\begin{bmatrix} -(1-\lambda\phi'\sin\phi)\kappa \\ +2z'\frac{d(1-\lambda\phi'\sin\phi)}{ds} \\ +\lambda\phi'\sin\phi(\phi'+\kappa) \\ +\lambda\cos\phi(\phi''+\kappa') \\ +\frac{\lambda\cos\phi(\phi'+\kappa)\frac{d(1-\lambda\phi'\sin\phi)}{ds}}{1-\lambda\phi'\sin\phi} \end{bmatrix} - 2\phi'(1-\lambda\phi'\sin\phi)(-2\sin\phi z' + \cos\phi) = 0.$$

Proof. If we substitute the components of Eq. (3.5) into the Eq. (2.7), we obtain the mean curvature of the ruled surface $X(s,\lambda)$. For the surface to be minimal, its mean curvature is zero. From here, we get the desired differential equation for $1-\lambda\phi'\sin\phi \neq 0$. $\square$

Corollary 3.9. *If the function ϕ is a constant, then the ruled surface $X(s,\lambda)$ has the Gauss curvature and the mean curvature as follows:*

$$K = -\frac{\sigma^2}{W^4}(\kappa^2(s)\cos^2\phi) \qquad \textit{and} \qquad H = -\frac{\sigma\cos^2\phi}{2W^3}(-\kappa(s) + \lambda\kappa'(s)\cos\phi).$$

Corollary 3.10. *If the function ϕ is a constant and the surface $X(s,\lambda)$ is developable, then the base curve is the straight line in Galilean space.*

Corollary 3.11. *If the function ϕ is a constant and the surface $X(s,\lambda)$ is minimal, then the base curve α has the curvature $\kappa(s) = e^{s/\lambda\cos\phi} + c_1$.*

Theorem 3.12. *The following conditions are satified for the striction curve of the ruled surface $X(s,\lambda)$:*

(i) If the function ϕ is an arbitrary constant, then the striction curve is $\beta(s) = \left(s - \frac{1}{\kappa(s)}, \frac{\sigma}{\kappa(s)\cos\phi}, z(s) - \frac{1}{\kappa(s)\cos\phi}(\cos\phi z'(s) + \sin\phi)\right)$.

(ii) If the function ϕ is not a constant, then the striction curve is $\beta(s) = \left(s - \frac{1}{(\phi'(s))^2}\cot^2\phi(s), \frac{\sigma\cos\phi}{(\phi'(s))^2\sin^2\phi(s)}, z(s) + \frac{\cos\phi(s)}{(\phi'(s))^2\sin^2\phi(s)}(\cos\phi(s)z'(s) + \sin\phi(s))\right)$.

Proof. The striction curve of ruled surface is two types depending on whether the vector D' is isotropic or non-isotropic in Galilean space. The striction curve of the ruled surface is calculated from the formula

$$\beta(s) = \alpha(s) - \frac{\langle T(s), D(s)\rangle}{\langle D'(s), D'(s)\rangle} D(s).$$

If the derivative of the generator vector with respect to s is isotropic vector, then the striction curve is calculated in (i) and if $D'(s)$ is non isotropic vector, then the striction curve is calculated as in (ii). $\square$

Example

In this section, we give the ruled surface whose base curve is $\alpha(s)$ and generator vector $D(s)$. Let $\alpha = \alpha(s)$ be an admissible unit speed curve with the parametrization

$$\alpha(s) = \left(s, \frac{1}{4}\left[(3-4s)\cos(2\sqrt{s}) + 6\sqrt{s}\sin(2\sqrt{s})\right], \frac{1}{4}\left[(3-4s)\sin(2\sqrt{s}) - 6\sqrt{s}\cos(2\sqrt{s})\right]\right)$$

with the Frenet frame apparatus

$$\begin{aligned} T(s) &= \left(1, \frac{1}{2}\cos(2\sqrt{s}) + \sqrt{s}\sin(2\sqrt{s}), \frac{1}{2}\sin(2\sqrt{s}) - \sqrt{s}\cos(2\sqrt{s})\right), \\ N(s) &= \left(0, \cos(2\sqrt{s}), \sin(2\sqrt{s})\right), \\ B(s) &= \left(0, -\sin(2\sqrt{s}), \cos(2\sqrt{s})\right), \end{aligned}$$

$\kappa(s) = 1$ and $\tau(s) = 1/\sqrt{s}$. The ruled surface $X(s,\lambda)$ with constant slope according to osculating plane of the curve $\alpha(s)$ is given as follows

$$X(s,\lambda) = \alpha(s) + \lambda D(s),$$

where $D(s) = \cos s^3 T(s) + \sin s^3 N(s) + \omega B(s)$ and ω is an arbitrary constant in Figure 1.

4. Conclusion

This study is important in terms of finding invariants of the ruled surface with constant slope in Galilean 3−space. The striction curve of this surface is calculated. Also, the conditions for the surface to be minimal and developable are obtained. It is also examined the special cases and the results are obtained.

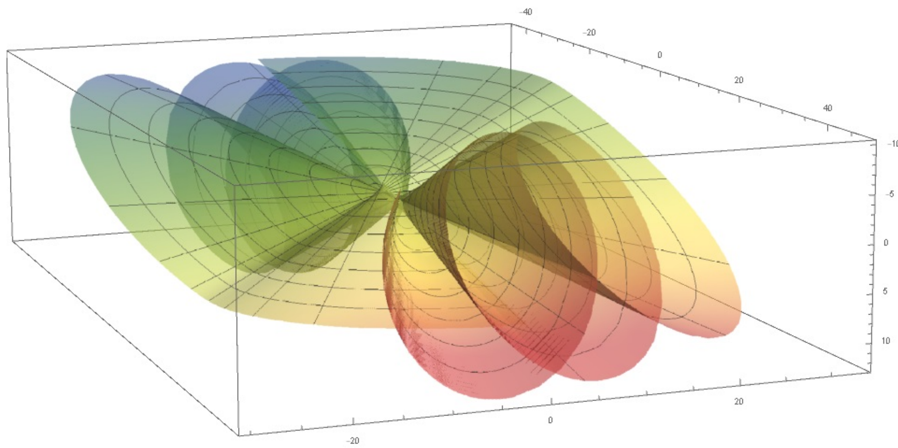

FIGURE 1. The ruled surface with constant slope $\omega = 3$.

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